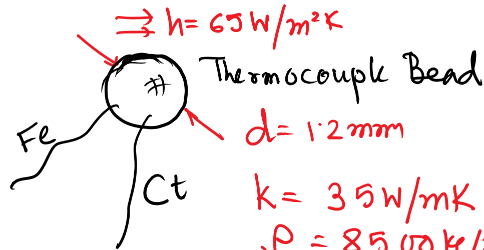
Transient Conduction

Tuesday, March 23, 2021 3:31 PM

The temperature of a gas stream is to be measured by a thermocouple whose junction can be approximated as a 1.2-mm-diameter sphere. The properties of the junction are $k = 35$ W/m · °C, $\rho = 8500$ kg/m³, and $C_p = 320$ J/kg · °C, and the heat transfer coefficient between the junction and the gas is $h = 65$ W/m² · °C. (Determine how long it will take for the thermocouple to read 99 percent of the initial temperature difference.) *Answer: 38.5 s*



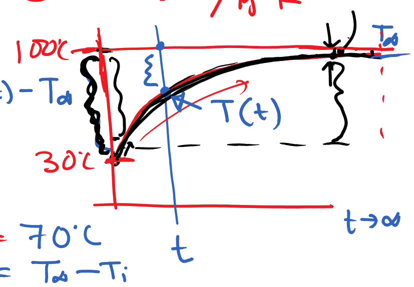
$k = 35$ W/mK

$\rho = 8500$ kg/m³

$C = 320$ J/kg K

CHECK: $Bi = \frac{hL}{k_s} = \frac{65 \times 0.0002}{35} = < 0.0004$

$$\frac{\theta}{\theta_i} = \frac{T - T_\infty}{T_i - T_\infty} = \exp\left[-\left(\frac{hA_{s,c}}{\rho \forall c}\right)t\right] = \exp(-t/\tau)$$



$\theta(t) = T(t) - T_\infty$

$\theta_i = 70°C = T_\infty - T_i$

$\theta(t) = 100 - T(t)$

$$\frac{\theta}{\theta_i} = \frac{T_\infty - T}{T_\infty - T_i} = \frac{0.01\,(T_\infty - T_i)}{(T_\infty - T_i)} = 0.01$$

$\boxed{Bi < 0.1 \Rightarrow \text{Lumped Capacitance model holds good.}}$

$$L_{sph.} = \frac{V}{A_{HT}} = \frac{d}{6} = \frac{0.0012}{6} = 0.0002\text{ m}$$

$$\tau = RC = \frac{1}{hA} \cdot \rho \forall C$$

$$= \frac{8500 \times \frac{4}{3}\pi r^3 \times 320}{65 \times 4\pi r^2}$$

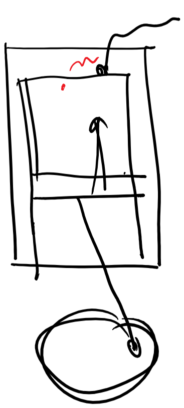
$$= \frac{8500 \times 0.0006 \times 320}{65 \times 3} = \boxed{8.37\text{ s}}$$

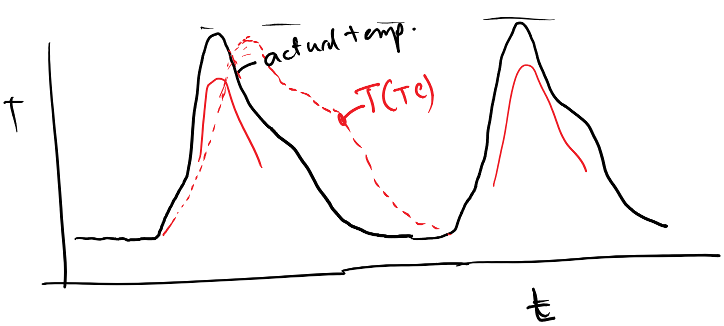
Time Const. = 8.37 s

$$\frac{\theta}{\theta_i} = \exp(-t/8.37) = 0.01$$

$$\Rightarrow \boxed{t = -8.37 \times \ln(0.01) = 38.54\text{ s}}$$

$\boxed{\text{Measurement of transient temp by TC is a challenge}}$





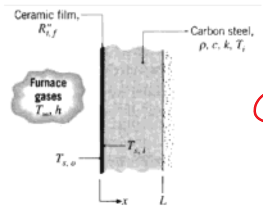
Infra-red Thermography →

Energy Storage

Sample Problem

P 5.15 From Incropera & Dewitt, 5th Edition

A plane wall of a furnace is fabricated from plain carbon steel ($k = 60$ W/m · K, $\rho = 7850$ kg/m^3, $c = 430$ J/kg · K) and is of thickness $L = 10$ mm. To protect it from the corrosive effects of the furnace combustion gases, one surface of the wall is coated with a thin ceramic film that, for a unit surface area, has a thermal resistance of $R''_{t,f} = 0.01$ m^2 · K/W. The opposite surface is well insulated from the surroundings.

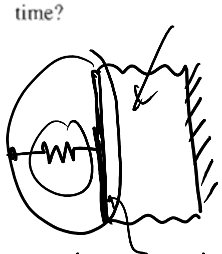


At furnace start-up the wall is at an initial temperature of $T_i = 300$ K, and combustion gases at $T_\infty = 1300$ K enter the furnace, providing a convection coefficient of $h = 25$ W/m^2 K at the ceramic film. Assuming the film to have negligible thermal capacitance, how long will it take for the inner surface of the steel to achieve a temperature of $T_{s,i} = 1200$ K? What is the temperature $T_{s,o}$ of the exposed surface of the ceramic film at this time?

$$\frac{1}{hA} + \frac{R''_{tf}}{A}$$

$$= \frac{1}{A}\left[\frac{1}{25} + 0.01\right]$$

$$= \frac{0.05}{A} \quad K/W$$



$$\frac{\theta}{\theta_i} = e\left(-t/\tau\right)$$

$$R_{eq} = \left(\frac{1}{hA} + \frac{R''_{tf}}{A}\right)$$

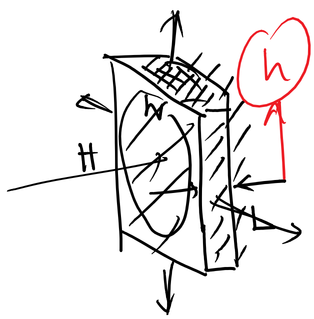
$$\boxed{\begin{aligned}\tau &= RC \\ &= R_{eq} \times (\rho V c)\end{aligned}}$$

$$R_{conv} = \frac{1}{hA} \quad \frac{K}{W}$$

$$R_{cer} = \frac{R''_t}{A} \quad \frac{K}{W}$$

$$Bi = \frac{hL}{k_s} = \frac{25 \times 0.01}{60} \ll 0.1$$

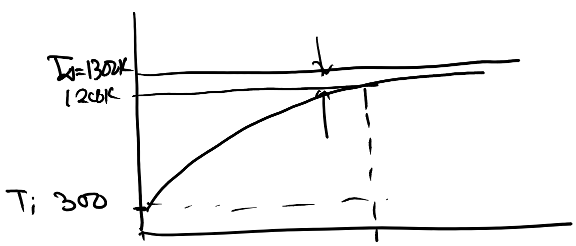
$\Rightarrow$ Lumped capacitance holds good



$$L_c = \frac{V}{HT\ Area} = \frac{HWL}{HW} = L$$

$$\tau = \frac{0.05}{\cancel{A}} \times 7850 \times 430 \times 0.01 \times \cancel{A}$$

$$= \underline{168.775} \quad s$$



$$\frac{\theta}{\theta_i} = e\left(-t/168.775\right)$$

$$\frac{1300-1200}{1300-300} = exp\left(-\frac{t}{168.775}\right)$$

$$\Rightarrow \quad t =$$

Summary 1) Generalized Conduction eqn.

derived it from 1st Law of thermodynamics

For 1-D heat transfer

$$\rho c \frac{\partial T}{\partial t} = \frac{1}{r^n}\frac{\partial}{\partial r}\left(r^n k \frac{\partial T}{\partial r}\right) + \dot{q}'''$$

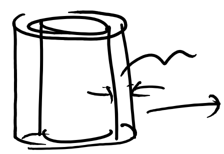
For $k = \text{const}$, $\Rightarrow$ $\frac{\partial T}{\partial t} = \frac{1}{\alpha} \frac{1}{r^n}\frac{\partial}{\partial r}\left(r^n \frac{\partial T}{\partial r}\right) + \frac{\dot{q}'''}{\rho c}$

$\alpha = \frac{K}{\rho C}$

For Steady $\Rightarrow$ $\frac{\partial T}{\partial t} = 0 \Rightarrow$ $0 = \frac{1}{r^n}\frac{d}{dr}\left(r^n \frac{dT}{dr}\right) + \frac{\dot{q}'''}{k}$

$\boxed{\text{Boundary cond}^{n}\text{s}}$ $T(r)$, $T(x)$ ....

Critical thickness of Insulation



→ Thermal Resistance analogy

→ Composite Walls & contact resistance

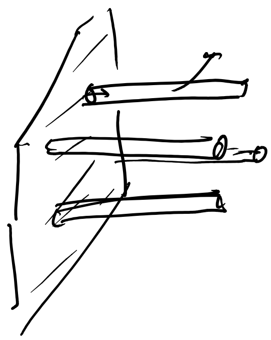
Fins → Infinitely long fin

→ Finite length

→ Adiabatic fin tip

→ Convective BC

$\underline{\underline{\epsilon}}$, η , $\underline{\underline{\epsilon_{\text{finned surface}}}}$, $\eta_{\text{finned surface}}$



Transient conduction →

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→ Lumped Capacitance Method

valid only for $Bi \ll 1$

$$Bi = \frac{h \times L_c}{k_s}$$