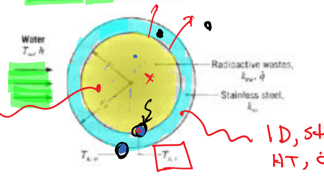
March 11

Thursday, March 11, 2021 9:56 AM

Cylindrical

11. Radioactive wastes (k_{rw} = 20 W/m · K) are stored in a [redacted], stainless steel (k_{ss} = 15 W/m · K) container of inner and outer radii equal to r_i = 0.5 m and r_o = 0.6 m. Heat is generated volumetrically within the wastes at a uniform rate of $\dot{q} = 10^5$ W/m³, and the outer surface of the container is exposed to a water flow for which h = 1000 W/m² · K and T_∞ = 25°C.

→ Try for spherical geometry



1D (steady) HT, $\dot{q} \neq 0$

1D, steady HT, $\dot{q}''' = 0$

(a) Evaluate the steady-state outer surface temperature, $T_{s,o}$.

(b) Evaluate the steady-state inner surface temperature, $T_{s,i}$.

(c) Obtain an expression for the temperature distribution, $T(r)$, in the radioactive wastes. Express your result in terms of r_i, $T_{s,i}$, k_{rw}, and $\dot{q}$. Evaluate the temperature at $r = 0$.

Fuel $\dot{q}''' = 10^5$ W/m³ , $k_r = 20$ W/mK

Container $r_i = 0.5$ m

$r_o = 0.6$ m

$k = 15$ W/mK

Outside fluid

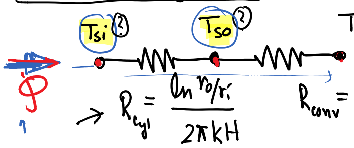
$h = 1000$ W/m²K

$T_\infty = 25$°C

$$R_{sp} = \frac{1}{4\pi k}\left[\frac{1}{r_i} - \frac{1}{r_o}\right]$$

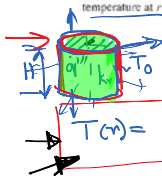
$$R_{conv} = \frac{1}{h\, 4\pi r_o^2}$$

For a cylinder length 1 H



$$\rightarrow R_{cyl} = \frac{\ln r_o/r_i}{2\pi k H} \qquad R_{conv} = \frac{1}{h\, 2\pi r_o H}$$

$$\boxed{\dot{Q} = \pi r_i^2 H \dot{q}'''}$$



$$\Rightarrow T_0 = f(\dot{q}''', k_r, r)$$

$$\boxed{T(r) = T_0 + \frac{\dot{q}'''}{4k}(R^2 - r^2)}$$

T_{si} k_r r_i

Heat Generated inside the cylinder

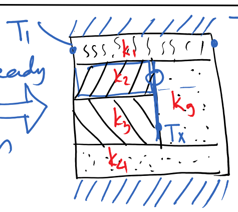
$$\dot{Q} = \dot{q}''' \times \pi r_i^2 H$$

$$\rightarrow T_{so} = T_\infty + \dot{Q} \times R_{conv}$$

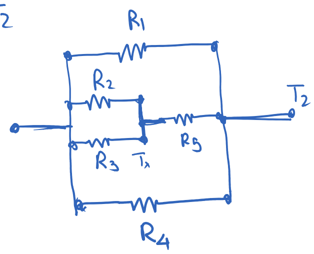
$$= T_\infty + \frac{\cancel{\pi} r_i^2 \cancel{H} \dot{q}'''}{h\, 2\cancel{\pi} r_o \cancel{H}}$$

$$= T_\infty + \frac{\dot{q}''' r_i^2}{2 h r_o} \quad \left[\frac{W}{m^3}\ \frac{m\ m^2 K}{W}\right]$$

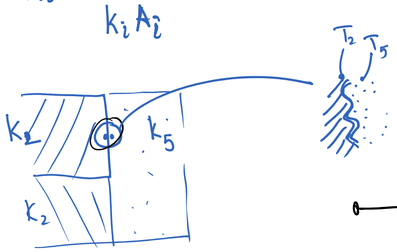
$$\rightarrow T_{si} = T_{so} + \dot{Q} \times R_{cyl.}$$

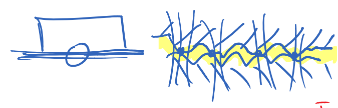


1D, steady HT. with $\dot{q}''' = 0$

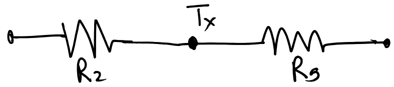


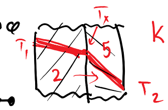
$$R_i = \frac{L_i}{k_i A_i}$$





With zero contact resistance





$$k_2 \frac{dT}{dx}\bigg|_{x-} = k_5 \frac{dT}{dx}\bigg|_{x+}$$

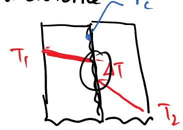
$$T_{x-} = T_x = T_{x+}$$

But if we have finite contact resistance r_c

Thermal contact resistance

$$\boxed{r_c} = \frac{1}{h_c} = \frac{m^2K}{W}$$

Thermal contact conductance [W/m²K]



ΔT = Temp jump at the contact

$$k_2 \frac{dT}{dx}\bigg|_{x-} = k_5 \frac{dT}{dx}\bigg|_{x+}$$

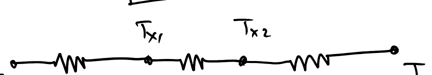
$$T_{x-} \neq T_{x+}$$

$\Rightarrow$

$\dot{Q}, \Delta T$

$\Rightarrow \dot{S}_{gen}$

$$R = \frac{1}{h_c A} = \frac{r_c}{A}$$



R_2 r_c/A R_5

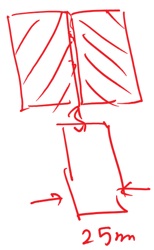
or

K

T_1 R_2 r_c/A or $\frac{1}{h_c A}$ R_5 T_2

$\frac{K}{W}$

Prob Thermal contact conductance at the interface of two 1-cm Al plates is 11000 W/m²K. Determine the thickness of the Al plate whose thermal resistance is equal to the resistance offered by the contact



$$\boxed{r_c = \frac{1}{h_c} = \frac{1}{11000} \frac{m^2 K}{W}}$$

$\Rightarrow$ for 1 m² plate are

$$R_{contact} = \frac{r_c}{A} = \frac{1}{h_c A} = \frac{1}{11000}\left[\frac{m^2 K}{W} \cdot \frac{1}{m^2}\right]$$

For Al. plate

$$R_{cond} = \frac{L}{kA} = \frac{L}{k}$$

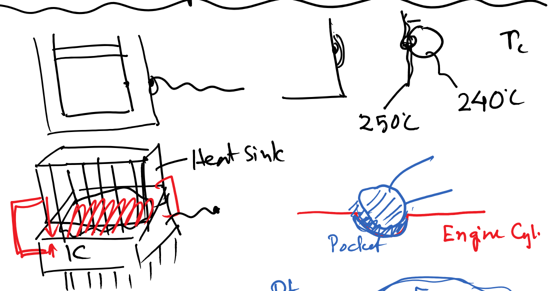
$k_{Al} = 237 \frac{W}{mK}$

$$R_{contact} = R_{conduction}$$

$$\frac{L}{k \cdot 1} = \frac{1}{11000}$$

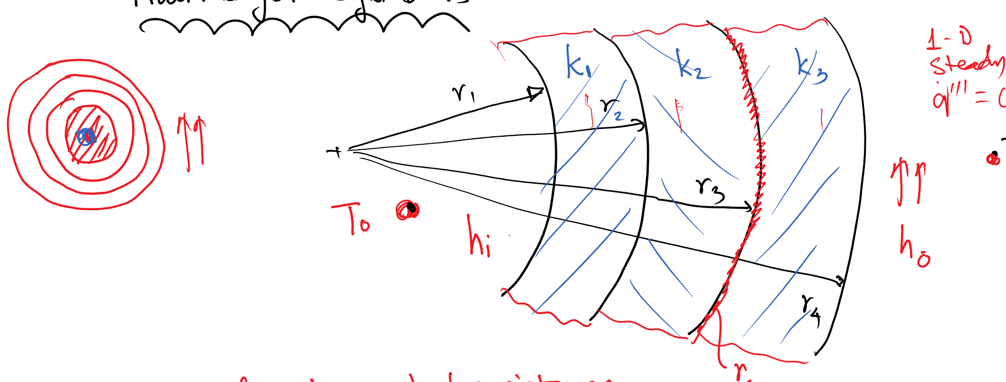
$$\Rightarrow L = \frac{237}{11000} m = 21.5 mm$$

Measuring Temperature by Thermocouples



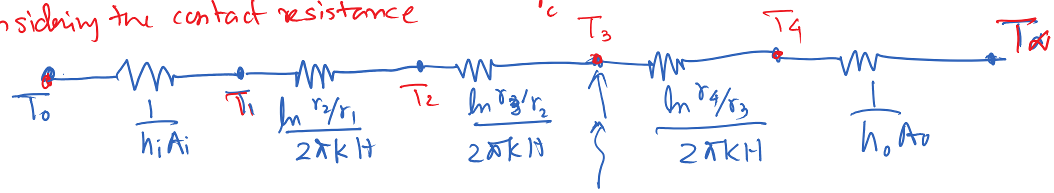
1) r_c is a fn of roughness of the mating surfaces
2) r_c is a fn of the contact pressure
3) Intervening medium (eg. Silver paste, Thermal paste)

Multi layer Cylinders

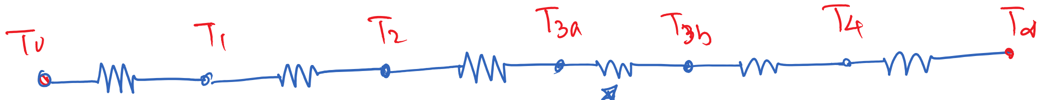


1-D
Steady
$\dot{q}''' = 0$

without considering the contact resistance

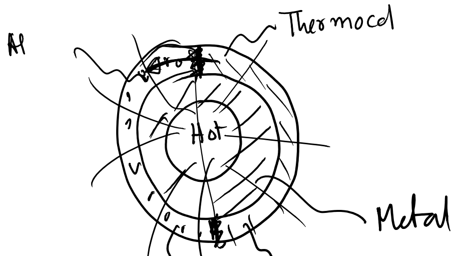


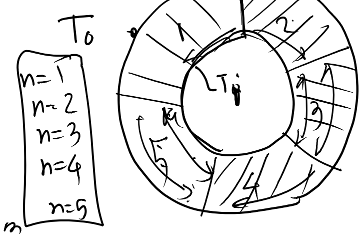
With the contact resistance



$$\boxed{r_c / A_c}$$

$$\boxed{\begin{aligned} A_c &= 2\pi r_3 H \\ r_c &= \frac{1}{h_c} \end{aligned}}$$

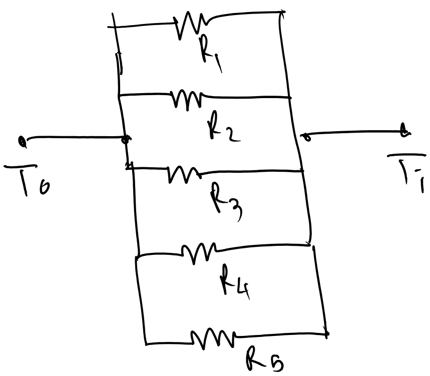


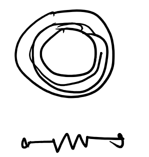


k_s are not too dissimilar so that H.T. is still axisymmetric

Composites along the azimuthal direction

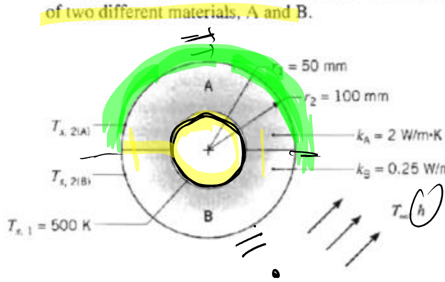
$$R_n = \frac{\ln r_o/r_i}{\left(\frac{2\pi}{n} k H\right)} \equiv$$





$$\left(\frac{\ln r_o/r_i}{2\pi k H}\right)$$

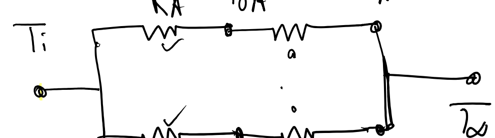
7 Steam flowing through a long, thin-walled pipe maintains the pipe wall at a uniform temperature of 500 K. The pipe is covered with an insulation blanket comprised of two different materials, A and B.



The interface between the two materials may be assumed to have an infinite contact resistance, and the entire outer surface is exposed to air for which T_∞ = 300 K and h = 25 W/m² · K.

(a) Sketch the thermal circuit of the system. Label (using the above symbols) all pertinent nodes and resistances.

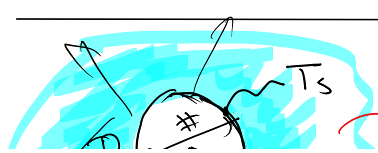
(b) For the prescribed conditions, what is the total heat loss from the pipe? What are the outer surface temperatures $T_{s,2(A)}$ and $T_{s,2(B)}$?



$$R_A = \frac{\ln r_o/r_i}{\left(\frac{2\pi}{2}\right) k_A H}, \quad R_B = \frac{\ln r_o/r_i}{\frac{2\pi}{2} k_B H}$$

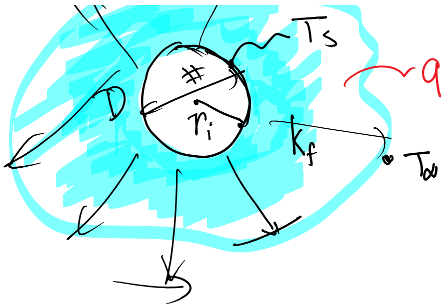
$$R_{conv,A} = \frac{1}{hA} = \frac{1}{h\left(\frac{2\pi}{2}\right) r_o H} = R_{conv,B}$$

||

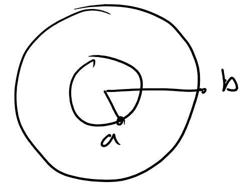


quiscent fluid

Find h =



Find $h =$



$$R_{sp} = \frac{1}{4\pi k r_i}$$

$$R_{sp} = \frac{1}{4\pi k}\left(\frac{1}{a} - \frac{1}{b}\right)$$

$a = r_i$

$b \Longrightarrow \infty$

$$\Rightarrow \quad q = \frac{\Delta T}{R_{sph}}$$

$$h A \cancel{\Delta T} = \frac{\cancel{\Delta T}}{R_{sp}}$$

$$h = \frac{1}{A R_{sp}} = \frac{4\pi k r_i}{4\pi r_i^2} = \frac{k}{r_i}$$

$(h = k/r_i)$, $\boxed{h = \frac{2k_f}{D}}$ where $\boxed{D = 2r_i}$

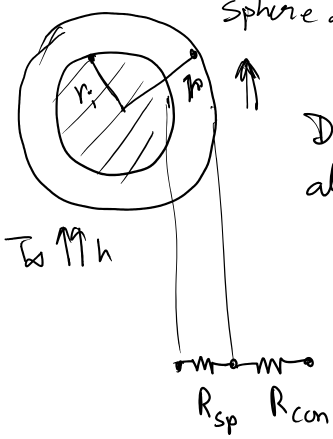
$\boxed{Re = \frac{\rho V D}{\mu} = \frac{VD}{\nu}}$ $\boxed{\text{Reynolds Number}}$

D = hydraulic diameter

$\nu = \mu/\rho$ = kinematic viscosity (momentum diffusivity)

Critical insulation thickness for spheres

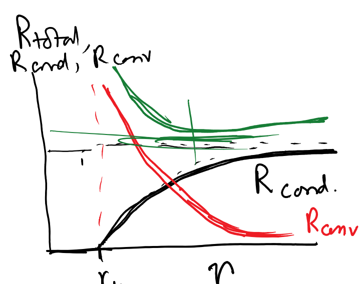
Sphere of radius r_i, covered by an insulation of thickness $\delta = (r - r_i)$ $\boxed{r = \text{variable outer radius}}$



Does a critical insulation thickness appear here also?

$$R_{sp} = \frac{1}{4\pi k}\left[\frac{1}{r_i} - \frac{1}{r}\right]$$

$$R_{con} = \frac{1}{hA} = \frac{1}{4\pi r^2 h}$$



$$R_{total} = \frac{1}{4\pi k}\left[\frac{1}{r_i} - \frac{1}{r}\right] + \frac{1}{4\pi r^2 h} \rightarrow$$

$$\frac{dR_{tot}}{dr} = 0$$

$$\frac{d^2 R_{tot}}{dr^2} > 0$$

Home Work: Find r_{crit} for spherical shell