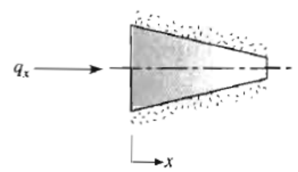
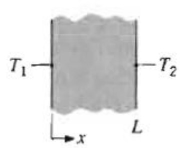
Assignment 2: General Equation of Conduction

1. Assume steady-state, one-dimensional heat conduction through the symmetric shape shown.



Assuming that there is no internal heat generation, derive an expression for the thermal conductivity $k(x)$ for these conditions: $A(x) = (1 - x)$, $T(x) = 300(1 - 2x - x^3)$, and $q = 6000$ W, where A is in square meters, T in kelvins, and x in meters.

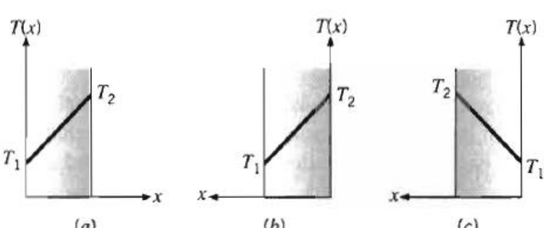
2. Consider steady-state conditions for one-dimensional conduction in a plane wall having a thermal conductivity $k = 50$ W/m · K and a thickness $L = 0.25$ m, with no internal heat generation.



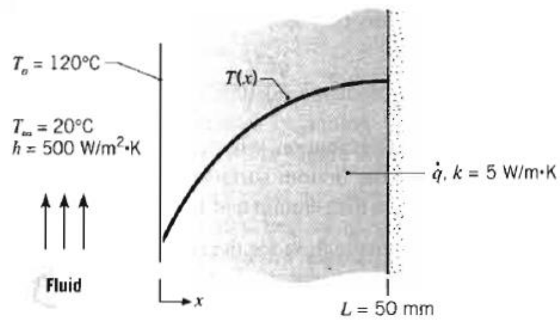
Determine the heat flux and the unknown quantity for each case and sketch the temperature distribution, indicating the direction of the heat flux.

Case	T_1(°C)	T_2(°C)	dT/dx (K/m)
1	50	−20	
2	−30	−10	
3	70		160
4		40	−80
5		30	200

3. Consider a plane wall 100 mm thick and of thermal conductivity 100 W/m · K. Steady-state conditions are known to exist with $T_1 = 400$ K and $T_2 = 600$ K. Determine the heat flux q_x'' and the temperature gradient dT/dx for the coordinate systems shown.



4. One-dimensional, steady-state conduction with uniform internal energy generation occurs in a plane wall with a thickness of 50 mm and a constant thermal conductivity of 5 W/m · K. For these conditions, the temperature distribution has the form, $T(x) = a + bx + cx^2$. The surface at $x = 0$ has a temperature of $T(0) \equiv T_o = 120$°C and experiences convection with a fluid for which $T_\infty = 20$°C and $h = 500$ W/m^2 · K. The surface at $x = L$ is well insulated.



(a) Applying an overall energy balance to the wall, calculate the internal energy generation rate, $\dot{q}$.

(b) Determine the coefficients a, b, and c by applying the boundary conditions to the prescribed temperature distribution. Use the results to calculate and plot the temperature distribution.

(c) Consider conditions for which the convection coefficient is halved, but the internal energy generation rate remains unchanged. Determine the new values of a, b, and c, and use the results to plot the temperature distribution. *Hint:* recognize that $T(0)$ is no longer 120°C.

(d) Under conditions for which the internal energy generation rate is doubled, and the convection coefficient remains unchanged ($h = 500$ W/m^2 · K), determine the new values of a, b, and c and plot the corresponding temperature distribution. Referring to the results of parts (b), (c), and (d) as Cases 1, 2, and 3, respectively, compare the temperature distributions for the three cases and discuss the effects of h and $\dot{q}$ on the distributions.

5. Consider one-dimensional, steady-state heat conduction in a plate with constant thermal conductivity in a region $0 \leq x \leq L$. Energy is generated in the medium at a rate of $g_0 e^{-\beta x}$ W/m^3, while the boundary surfaces at $x = 0$ are kept insulated and at $x = L$ dissipate heat by convection into a medium at temperature T_∞ with a heat transfer coefficient h W/(m^2 · °C). Write the mathematical formulation of this heat conduction problem.

6. A solid cylinder of radius r and height H has a uniform heat generation of g_0 W/m^3. The cylindrical boundaries are maintained in melting ice at 0°C, while the top and the bottom walls of the cylinder are insulated. Is the heat transfer one-, two- or three-

dimensional? Write the corresponding the governing equation of heat conduction and evaluate the temperature profile inside the cylinder under a steady operating condition.

6 Consider one-dimensional, steady-state heat conduction in a hollow cylinder with constant thermal conductivity in the region $a \leq r \leq b$. Heat is generated in the cylinder at a rate of g_0 W/m^3, while heat is dissipated by convection into fluids flowing inside and outside the cylindrical tube. Heat transfer coefficients for the inside and outside fluids are h_a and h_b, respectively, and temperatures of the inside and outside fluids are T_a and T_b, respectively. Write the mathematical formulation of this heat conduction problem.