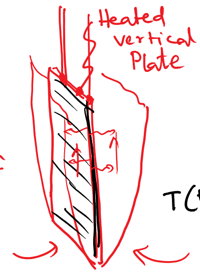
1.22 The free convection heat transfer coefficient on a thin hot vertical plate suspended in still air can be determined from observations of the change in plate temperature with time as it cools. Assuming the plate is isothermal and radiation exchange with its surroundings is negligible, evaluate the convection coefficient at the instant of time when the plate temperature is 225°C and the change in plate temperature with time (dT/dt) is -0.022 K/s. The ambient air temperature is 25°C and the plate measures 0.3×0.3 m with a mass of 3.75 kg and a specific heat of 2770 J/kg · K.

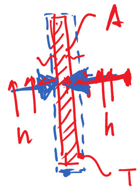


Free Convection / Natural Convection

" Buoyancy "

1st Law of thermodynamics / Energy Conservation Eqn. [Energy Eqn.] 25°C

$$\boxed{\frac{dE}{dt} = \sum \dot{Q} - \sum \cancel{W}}$$



Area = 2A

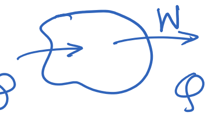
• $T_\infty = 25°C$

$$\dot{Q} = -A_{total} \times h \times (T_s - T_\infty)$$

$$= -2A \times h\,(T_s - T_\infty)$$

$$mc\frac{dT}{dt} = \sum \dot{Q}$$

$$mc\frac{dT}{dt} = -2Ah\,(T_s - T_\infty)$$

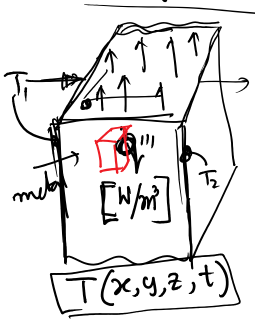


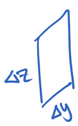
Q +ve ⇒ Heat added to the system

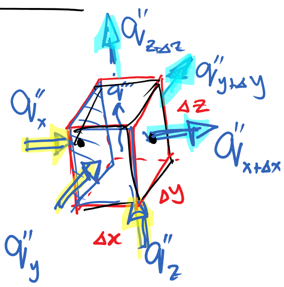
$$3.75 \times 2770 \times (\cancel{+}0.022) = \cancel{+}\,2 \times (0.3 \times 0.3)\,(225 - 25)\,h$$

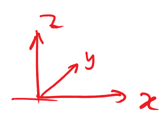
$$\Rightarrow h = \frac{3.75 \times 2770 \times 0.022}{2 \times 0.09 \times 200} \quad N/m^2K$$

Equation for Conduction Heat Transfer / Energy Conservation Eqn.









$\dot{q}'''$ volumetric heat generation
[e.g. Joule heating, Iron Loss in a transformer, Nuclear fuel $\frac{W}{m^3}$]

1st Law for the differential element of metal

x, y, z, t } independent variables

T } dependent variable

$$\boxed{\frac{d}{dt}(E_{cv}) = \sum \dot{E}_{in} - \sum \dot{E}_{out} + \dot{E}_{gen}}$$

$$\frac{\partial}{\partial t}\left[\rho C T\, \Delta x \Delta y \Delta z\right] = (q''_x\, \Delta z \times \Delta y) + (q''_y\, \Delta x\, \Delta z) + (q''_z \times \Delta x \Delta y)$$

$$- (q''_{x+\Delta x}\, \Delta z \Delta y) - q''_{y+\Delta y}\, \Delta x \Delta z - q''_{z+\Delta z}\, \Delta x \Delta y$$

$$+ \left(\Delta x \Delta y \Delta z\right)\dot{q}'''$$

$$= -\left[\left(q''_{x+\Delta x} - q''_{x}\right)\Delta z \Delta y + \left(q''_{y+\Delta y} - q''_{y}\right)\Delta x \Delta z + \left(q''_{z+\Delta z} - q''_{z}\right)\Delta x \Delta y\right] + \dot{q}'''\Delta x \Delta z \Delta y \quad \text{(A)}$$

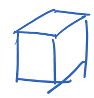
Taylor Series expansion

$$q''_{x+\Delta x} = q''_{x} + \frac{\partial q''_x}{\partial x}\Delta x + \cancel{\frac{\partial^2 q''_x}{\partial x^2}\frac{\Delta x^2}{2!}} + \dots$$

$$\Rightarrow \boxed{q''_{x+\Delta x} - q''_{x} = \frac{\partial}{\partial x}\left(q''_x\right)\Delta x}$$

$$\boxed{q''_{y+\Delta y} - q''_{y} = \frac{\partial}{\partial y}\left(q''_y\right)\Delta y}$$

$$\boxed{q''_{z+\Delta z} - q''_{z} = \frac{\partial}{\partial z}\left(q''_z\right)\Delta z}$$



Substituting in (A)

$$\frac{\partial}{\partial t}\left[\rho\left(\Delta x \Delta y \Delta z\right) \times c \times T\right] = -\left[\frac{\partial q''_x}{\partial x}\Delta x \Delta y \Delta z + \frac{\partial q''_y}{\partial y}\Delta y \Delta x \Delta z + \frac{\partial q''_z}{\partial z}\Delta z \Delta x \Delta y\right] + \dot{q}'''\Delta x \Delta y \Delta z$$

Divide both sides by $\Delta x \Delta y \Delta z \Rightarrow$

$$\boxed{\frac{\partial}{\partial t}\left(\rho c T\right) = -\left(\frac{\partial q''_x}{\partial x} + \frac{\partial q''_y}{\partial y} + \frac{\partial q''_z}{\partial z}\right) + \dot{q}'''}$$

$$q''_x = -k\frac{\partial T}{\partial x}$$

$$q''_y = -k\frac{\partial T}{\partial y}$$

$$q''_z = -k\frac{\partial T}{\partial z}$$

Energy eqn

Rate of Change of Stored energy

Net influx of energy by conduction

volumetric heat generation

Fourier's Law of heat conduction

$$\vec{q''} = q''_x \hat{i} + q''_y \hat{j} + q''_z \hat{k}$$

$$= \underbrace{\left(-k\frac{\partial T}{\partial x}\right)}_{q''_x}\hat{i} + \underbrace{\left(-k\frac{\partial T}{\partial y}\right)}_{q''_y}\hat{j} + \underbrace{\left(-k\frac{\partial T}{\partial z}\right)}_{q''_z}\hat{k}$$

$$\boxed{\frac{\partial}{\partial t}\left(\rho c T\right) = \frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right) + \frac{\partial}{\partial y}\left(k\frac{\partial T}{\partial y}\right) + \frac{\partial}{\partial z}\left(k\frac{\partial T}{\partial z}\right) + \dot{q}'''} \quad \text{(A)}$$

Generalized Conduction Equation

If the heat Transfer is steady (No change with time)

$$\frac{\partial}{\partial t}(\rho C T) = 0$$

$$\boxed{\frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right) + \frac{\partial}{\partial y}\left(k\frac{\partial T}{\partial y}\right) + \frac{\partial}{\partial z}\left(k\frac{\partial T}{\partial z}\right) + \dot{q}''' = 0}$$

ρ = density, C = sp. heat k = thermal conductivity

If these ~~properties~~ properties are constant, then from (A) we can say

$$\Rightarrow \rho C\frac{\partial T}{\partial t} = k\left[\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2}\right] + \dot{q}'''$$

$$\left[\frac{W}{m^3};\ \frac{kg}{m^3}\cdot\frac{J}{kg\,K}\cdot\frac{K}{s};\ \frac{W}{mK}\cdot\frac{K}{m^2}\right]$$

or

$$\boxed{\frac{\partial T}{\partial t} = \frac{k}{\rho C}\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2}\right) + \frac{\dot{q}'''}{\rho C}}$$

Energy ~~density~~ eqn.

Generalized Conduction eqn. for a material with Constant properties

$\frac{k}{\rho C_p}$ = thermal diffusivity m^2/s

Differential equation $\longrightarrow$ partial differential eqn of t, x, y, z

2nd order p.d.e. (in space)

1st order p.d.e (in time)

Linear p.d.e.

Linear differention Eqn.

non- " " "

power of the dependent variable $1 \rightarrow$ Linear Pde

Navier Stokes Equation

Chapter 8, Som & Biswas

Prof. Suman Chakraborty

$$\boxed{\rho C\frac{\partial T}{\partial t} = k\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2}\right) + \dot{q}'''}$$

If $\dot{q}''' = 0$, like there is no Joule heating / volumetric heating

$$\rho C\frac{\partial T}{\partial t} = k\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2}\right)$$

or

$$\frac{1}{\alpha}\frac{\partial T}{\partial t} = \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2}\right)$$

[Diffusion Eqn.]

α – thermal Diffusivity

$$\boxed{\vec{\nabla} = \frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}}$$

Fourier's Law $\vec{q} = -k\frac{\partial T}{\partial x}\hat{i} - k\frac{\partial T}{\partial y}\hat{j} - k\frac{\partial T}{\partial z}\hat{k}$

$$\boxed{\vec{q} = -k\vec{\nabla}T}$$

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} = \boxed{\vec{\nabla}\cdot(\vec{\nabla} T)} =$$

$\vec{\nabla}\cdot\vec{A}$

$= \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$

$$\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}+\frac{\partial^2 T}{\partial z^2} = \boxed{\nabla\cdot(\nabla T)} =$$

$$= \frac{\partial}{\partial x}\left(\frac{\partial T}{\partial x}\right)+\frac{\partial}{\partial y}\left(\frac{\partial T}{\partial y}\right)+\frac{\partial}{\partial z}\left(\frac{\partial T}{\partial z}\right)$$

$$= \frac{\partial A_x}{\partial x}+\frac{\partial A_y}{\partial y}+\frac{\partial A_z}{\partial z}$$

$$\boxed{\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}+\frac{\partial^2 T}{\partial z^2} = \nabla^2 T}$$

$$\boxed{\frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right)+\frac{\partial}{\partial y}\left(k\frac{\partial T}{\partial y}\right)+\frac{\partial}{\partial z}\left(k\frac{\partial T}{\partial z}\right) = \left[\vec{\nabla}\cdot\left(k\vec{\nabla}T\right)\right]}$$

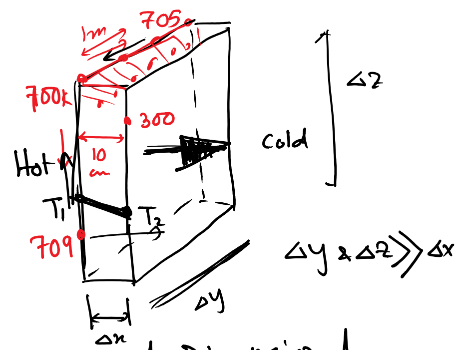
Gen Governing eqn. for conduction

$$\rightarrow \boxed{\frac{\partial}{\partial t}(\rho C T) = \frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right)+\frac{\partial}{\partial y}\left(k\frac{\partial T}{\partial y}\right)+\frac{\partial}{\partial z}\left(k\frac{\partial T}{\partial z}\right)+\dot{q}'''}$$

$T(t, x, y, z)$

$$\frac{\partial T}{\partial x} \gg \frac{\partial T}{\partial y} \;\&\; \frac{\partial T}{\partial z}$$

$$4000\,\frac{k}{m} \qquad 5\,\frac{k}{m} \qquad 9\,\frac{k}{m}$$



1-Dimensional

$$\frac{\partial}{\partial y} \;\&\; \frac{\partial}{\partial z} \approx 0$$

also $k = $ Const.

also $\frac{\partial}{\partial t} = 0$

Also $\dot{q}''' = 0$

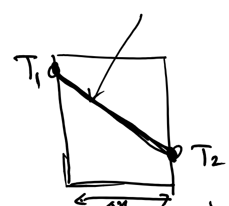
$$\cancel{\frac{\partial}{\partial t}}(\rho C T) = \frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right)+\cancel{\frac{\partial}{\partial y}\left(k\frac{\partial T}{\partial y}\right)}+\cancel{\frac{\partial}{\partial z}\left(k\frac{\partial T}{\partial z}\right)}+\cancel{\dot{q}'''} \to 0$$

$$0 = \frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right) + 0$$

$$T(x) \Rightarrow \frac{\partial}{\partial x} = \frac{d}{dx}$$

$$\Rightarrow \frac{d}{dx}\left(k\frac{dT}{dx}\right) = 0$$

$$\Rightarrow k\frac{dT}{dx} = \text{Const.}$$



$$q'' = k\left(\frac{T_1 - T_2}{\Delta x}\right)$$

If $k = $ Const $\quad \boxed{\frac{dT}{dx} = \text{Const}}$

1-D, steady heat conduction for constant k