Heat Exchangers

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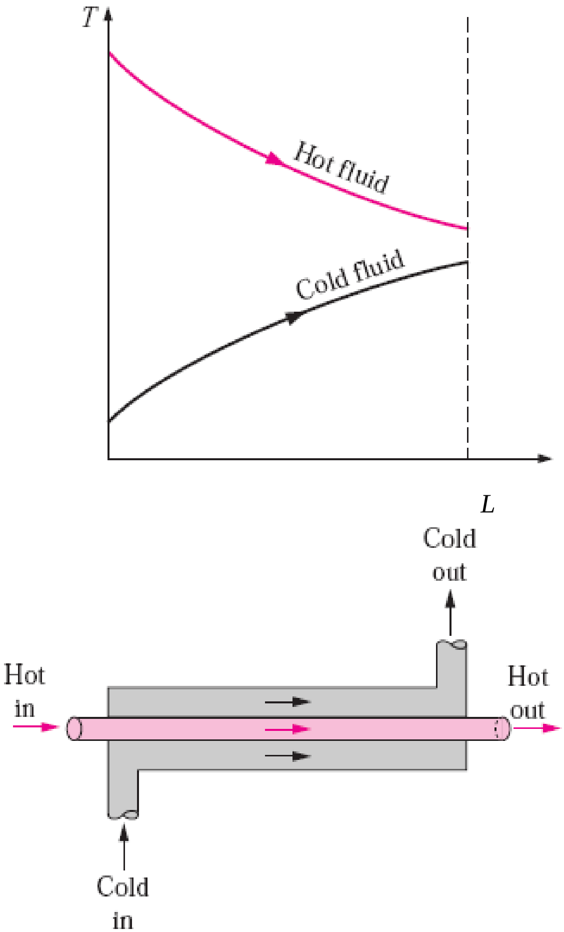
Purpose of heat exchangers

- Purpose: To transfer heat from one fluid to another, achieving heating or cooling of the target fluid
  - Uses the energy content (latent and/or sensible) of one fluid to alter energy content of the other fluid
- Examples:
  - Power plants: Boiler, condenser, cooling tower, regenerative feedwater heaters, oil coolers, etc.
  - Process plants: process heat exchangers
  - Car radiators
  - Electronics cooling systems
  - AC and ventilation systems
  - …

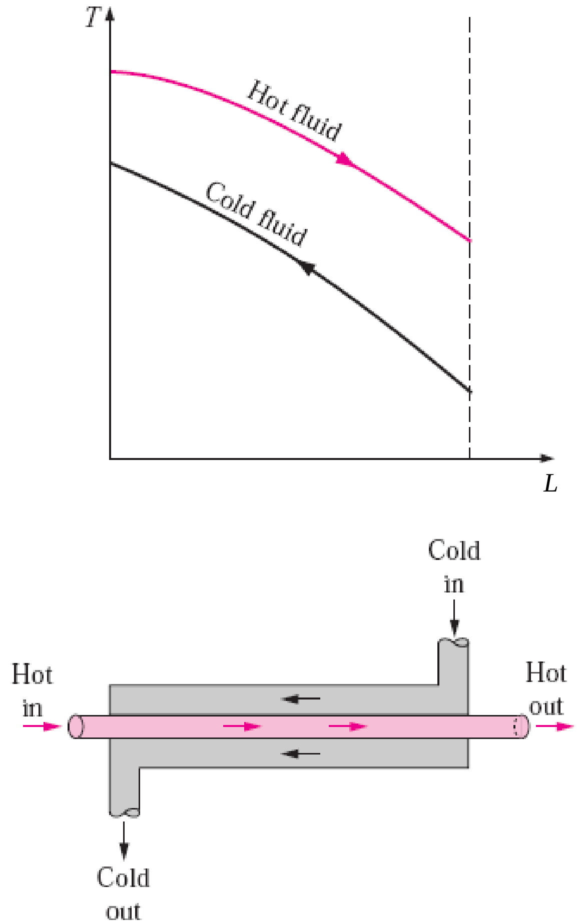
Classification of heat exchangers

- Flow arrangement
  - Parallel-flow
  - Counter-flow
  - Cross-flow
- Geometry of construction
  - Concentric tubes
  - Shell and tube
  - Plate
- Heat transfer mechanism
  - Direct and contact Indirect contact (surface) types
  - Single phase or phase change
  - Recuperative and regenerative

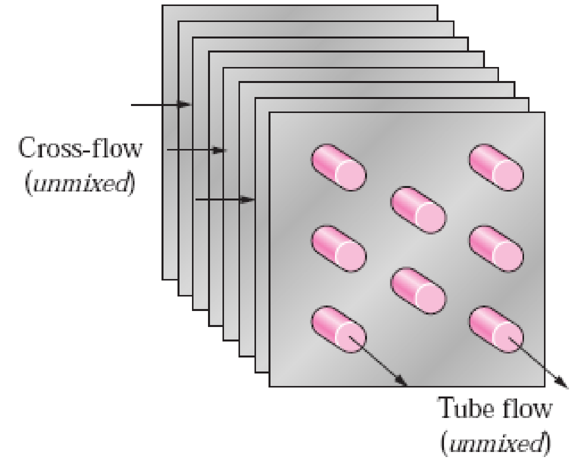
Parallel, counter and cross flows



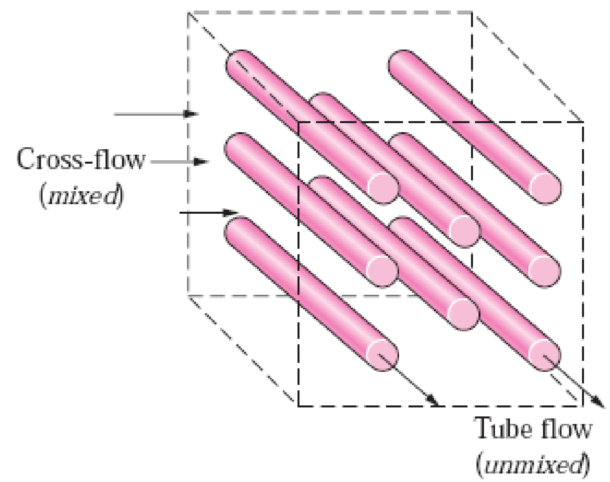
Parallel flow,
concentric tube type



counter flow,
concentric tube type



Cross flow
(both fluids unmixed)

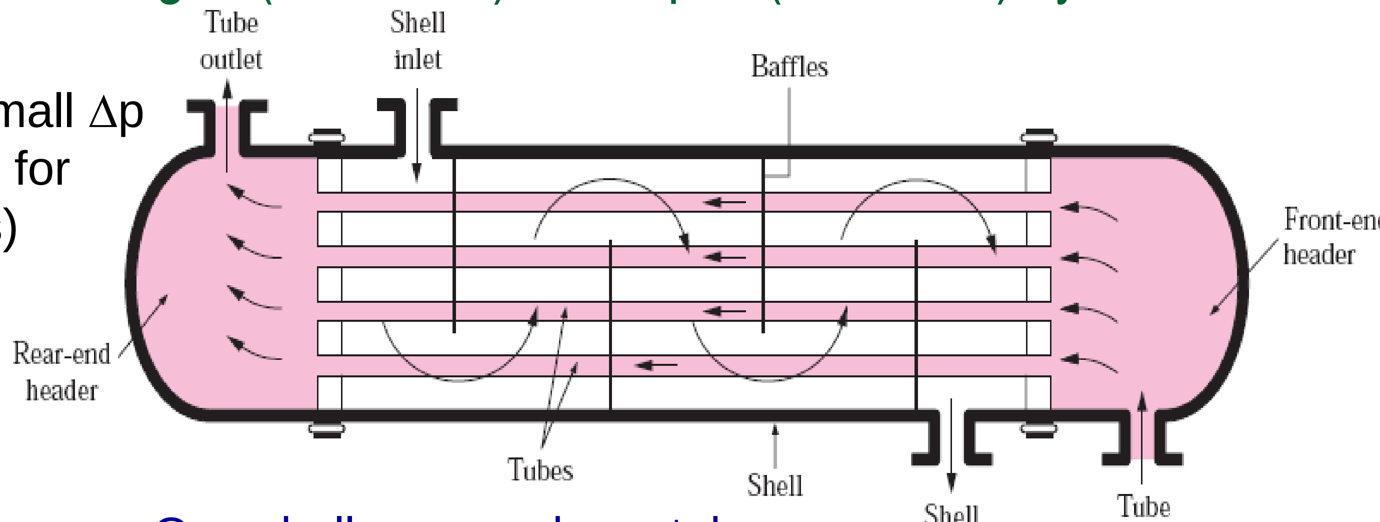


Cross flow
(one fluid mixed, one fluid unmixed)

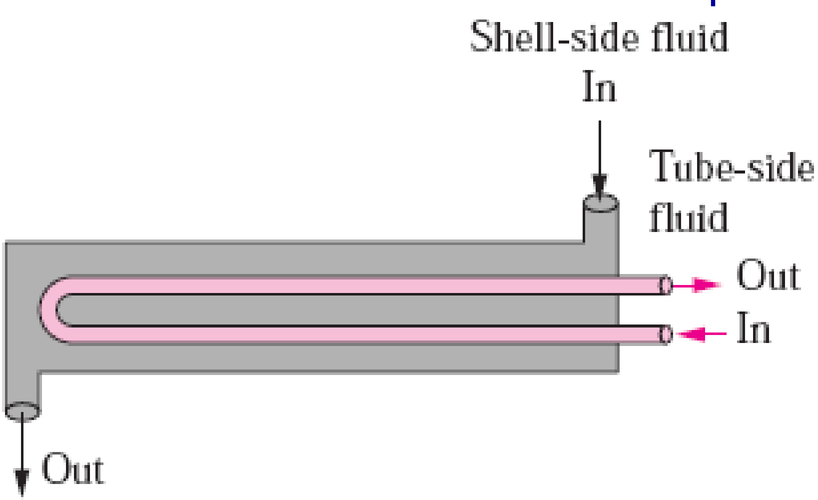
Shell and tube type heat exchanger

Generally deployed for gas (shell side) and liquid (tube side) systems

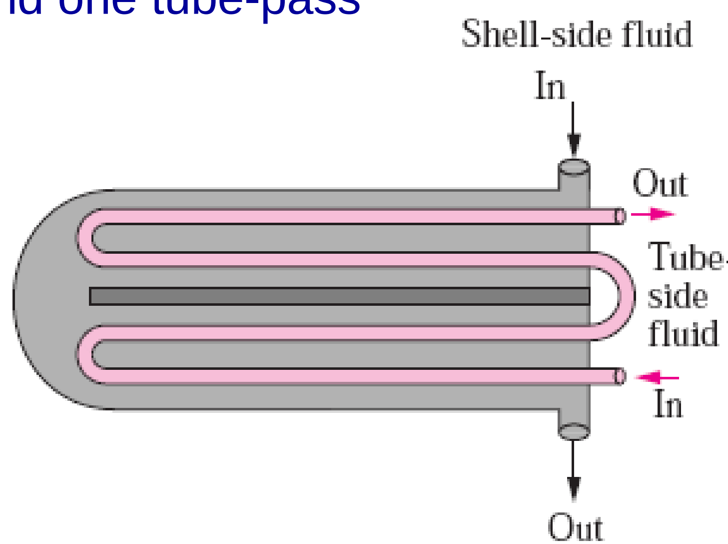
☺ Shell side offers small Δp
☹ Bulky (not suitable for compact applications)



One shell pass and one tube-pass

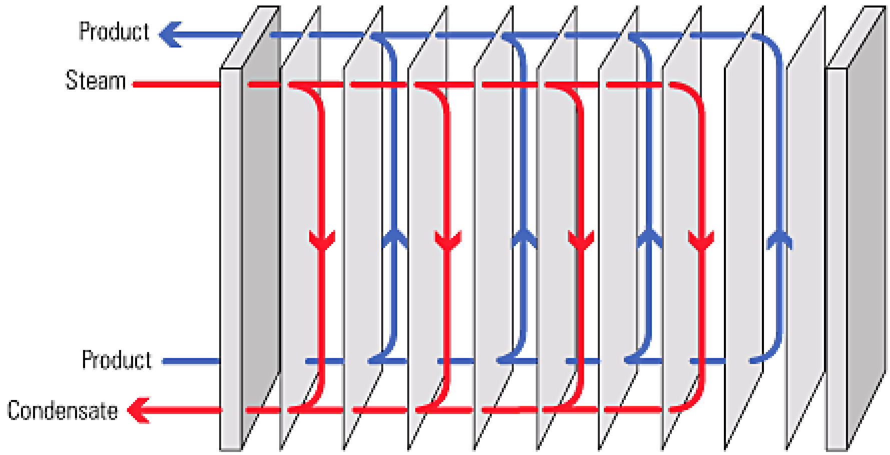


One shell-pass, two tube pass

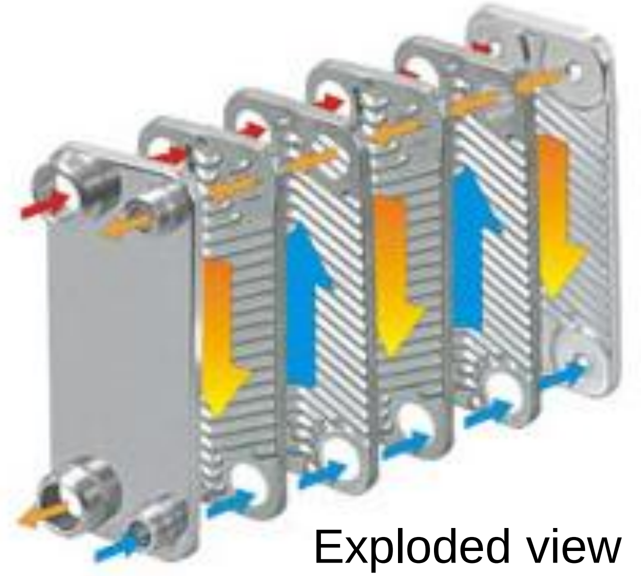


Two shell-pass, four tube pass

Plate type heat exchangers



Flow path



Exploded view

Compact view

The plate

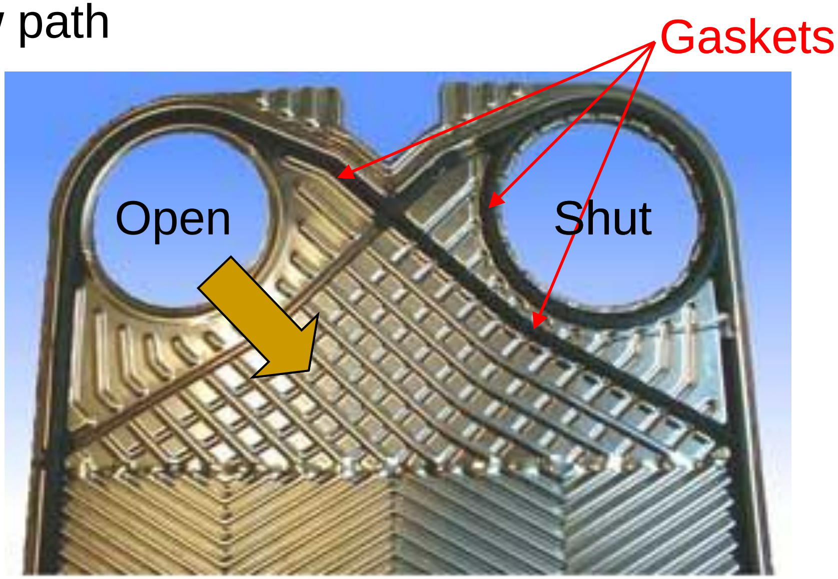
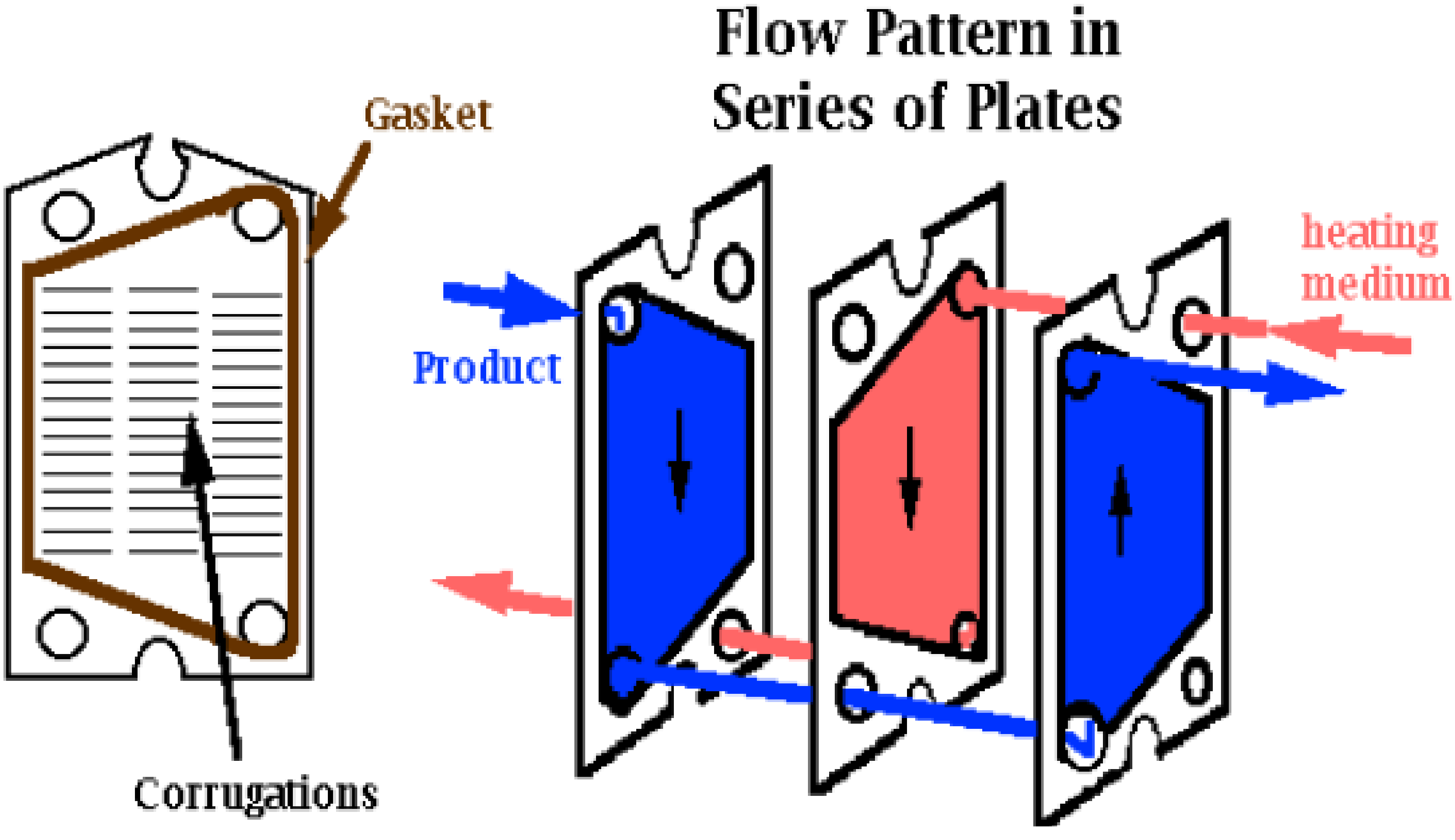


Plate fitted with gasket to
selectively restrict flow

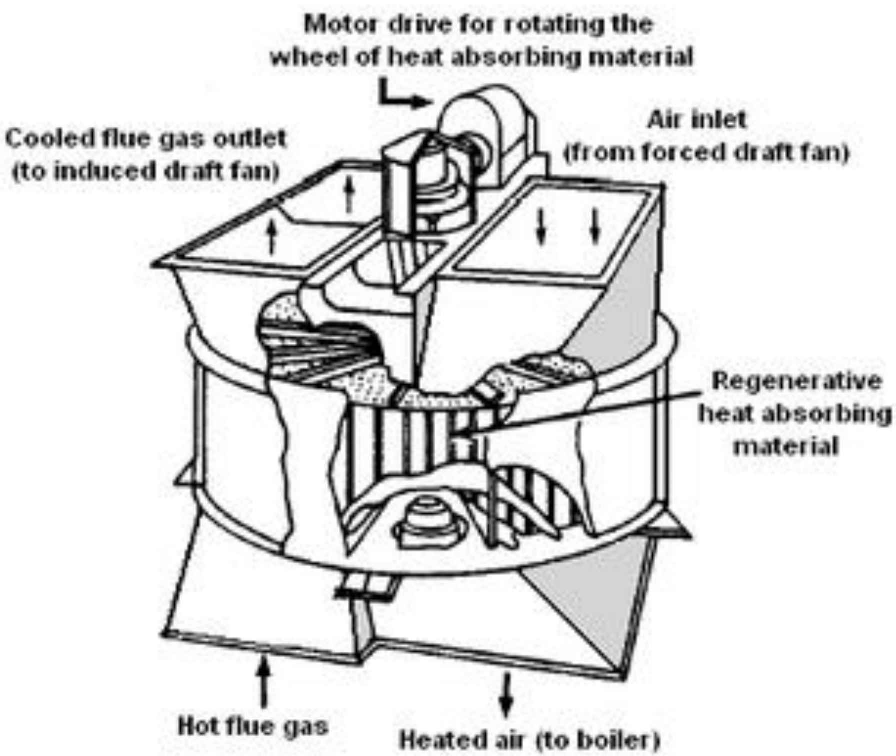
Flow path in plate heat exchanger

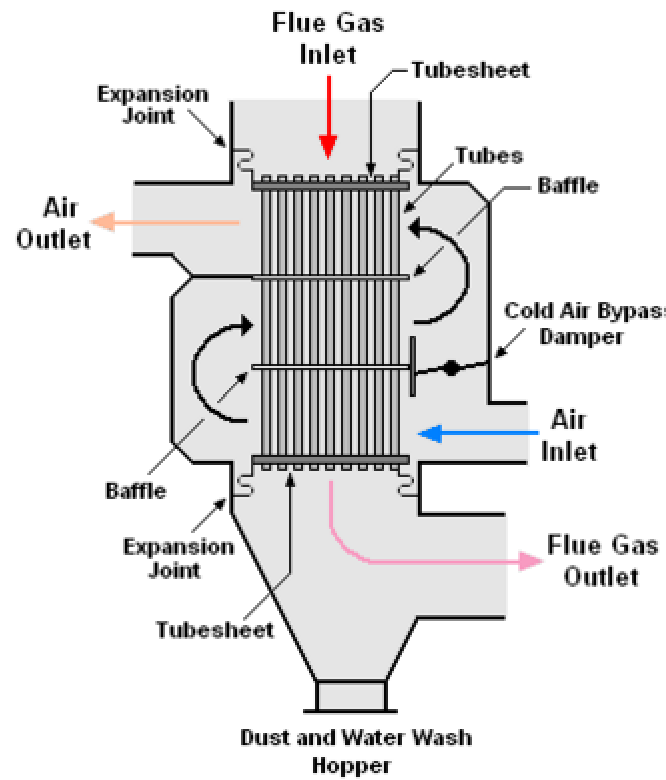


To learn more of how Plate Hex works:

https://www.youtube.com/watch?v=7TTF4aU3Pcs

Regenerative vs recuperative Hex





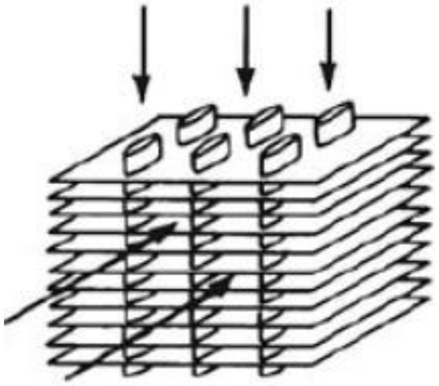
Regnerative Air Preheater

Recuperative Air Preheater

Requires an energy storing matrix

Direct energy transfer between the fluids

Compact heat exchangers

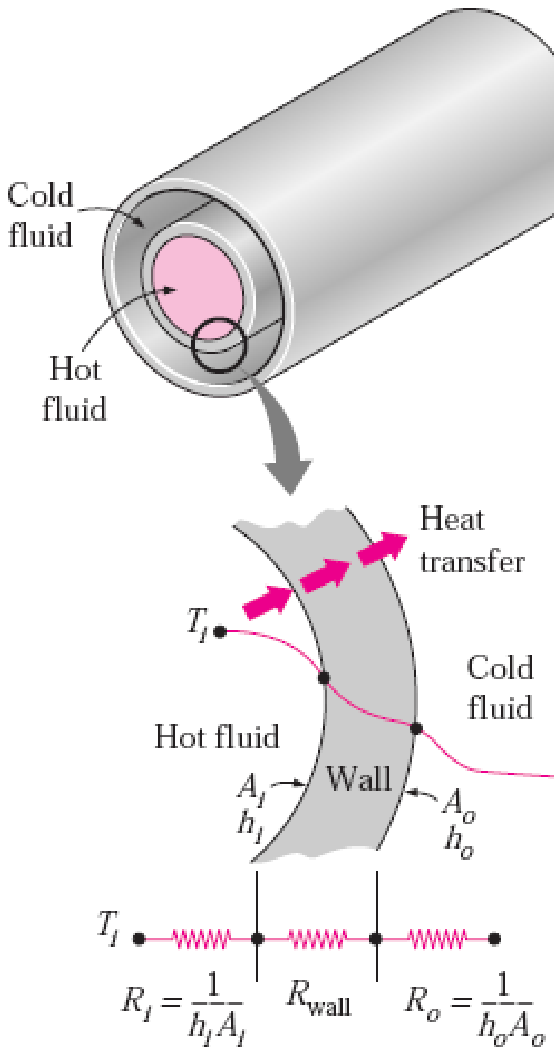


- Surface area to volume ratio

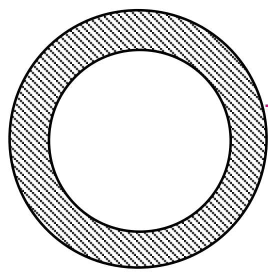
$$\beta = \frac{Heat\ Transfer\ Surface\ Area}{Heat\ Exchanger\ Volume}$$

- $\beta > 700\ m^{-1} \Rightarrow$ Compact heat exchanger
  - Car radiators: $\beta \sim 1000\ m^{-1}$
  - Glass ceramic gas turbine heat exchanger: $\beta \sim 6000\ m^{-1}$
  - Regenerative heat exchanger of Stirling engine: $\beta \sim 15000\ m^{-1}$
  - Human lung: $\beta \sim 20000\ m^{-1}$

Overall heat transfer coefficient (HTC)



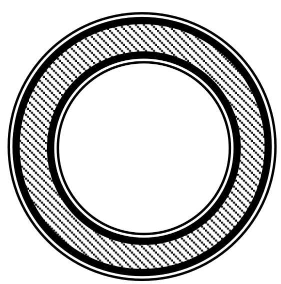
- Fouling factor: Additional thermal resistance due to physical/chemical/biological deposition of materials on the inner and/or outer heat transfer surface



Overall HTC without fouling

$$\frac{1}{UA_s} = \frac{1}{U_i A_i} = \frac{1}{U_o A_o} = R = \frac{1}{h_i A_i} + R_{\text{wall}} + \frac{1}{h_o A_o}$$

Where $R_{wall} = \frac{\ln(D_o/D_i)}{2\pi k L}$



Overall HTC with internal and external fouling

$$\frac{1}{UA_s} = \frac{1}{U_i A_i} = \frac{1}{U_o A_o} = R = \frac{1}{h_i A_i} + \frac{R_{f,\,i}}{A_i} + \frac{\ln(D_o/D_i)}{2\pi k L} + \frac{R_{f,\,o}}{A_o} + \frac{1}{h_o A_o}$$

$$A_i = \pi D_i L; \quad A_o = \pi D_o L$$

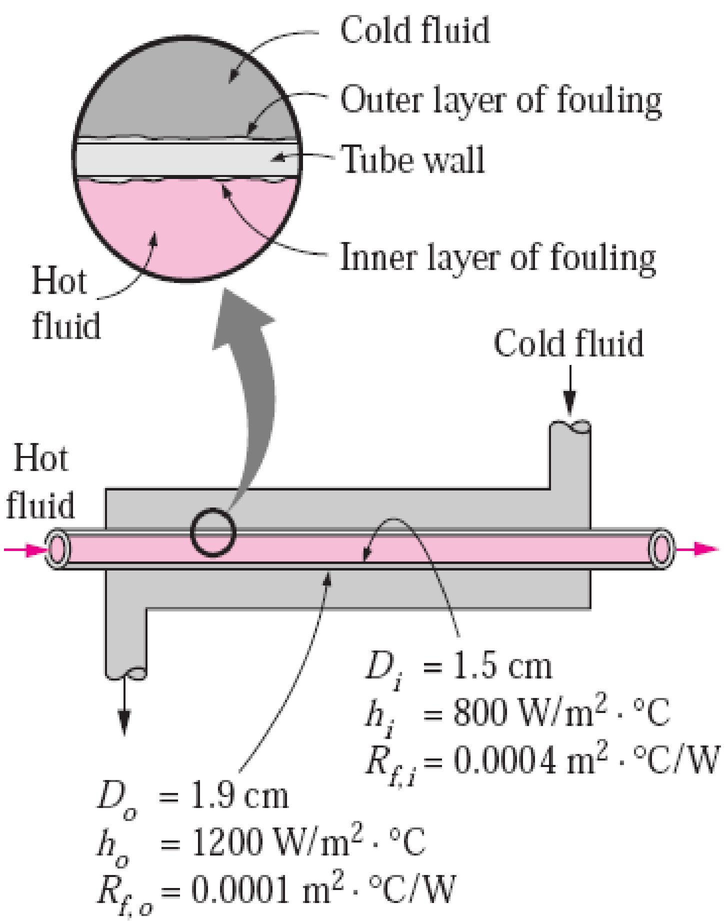
Some typical fouling factors

Fouling of ash on SH tubes

Fluid	R_f, $m^2 \cdot °C/W$
Distilled water, sea-water, river water, boiler feedwater:	
Below 50°C	0.0001
Above 50°C	0.0002
Fuel oil	0.0009
Steam (oil-free)	0.0001
Refrigerants (liquid)	0.0002
Refrigerants (vapor)	0.0004
Alcohol vapors	0.0001
Air	0.0004

Effect of fouling on overall HTC

A double-pipe (shell-and-tube) heat exchanger is constructed of a stainless steel (k = 15.1 W/m · °C) inner tube of inner diameter D_i = 1.5 cm and outer diameter D_o = 1.9 cm and an outer shell of inner diameter 3.2 cm. The convection heat transfer coefficient is given to be h_i = 800 W/m^2 · °C on the inner surface of the tube and h_o = 1200 W/m^2 · °C on the outer surface. For a fouling factor of $R_{f,i}$ = 0.0004 m^2 · °C/W on the tube side and $R_{f,o}$ = 0.0001 m^2 · °C/W on the shell side, determine (*a*) the thermal resistance of the heat exchanger per unit length and (*b*) the overall heat transfer coefficients, U_i and U_o based on the inner and outer surface areas of the tube, respectively.



Hints:

$$R = \frac{1}{UA_s} = \frac{1}{U_i A_i} = \frac{1}{U_o A_o} = \frac{1}{h_i A_i} + \frac{R_{f,i}}{A_i} + \frac{\ln(D_o/D_i)}{2\pi kL} + \frac{R_{f,o}}{A_o} + \frac{1}{h_o A_o}$$

$$A_i = \pi D_i L = \pi(0.015\text{ m})(1\text{ m}) = 0.0471\text{ m}^2$$
$$A_o = \pi D_o L = \pi(0.019\text{ m})(1\text{ m}) = 0.0597\text{ m}^2$$

R= 0.0532 K/W, U_i = 399 W/m^{2}K, U_o = 315 W/m^{2}K

~ 19% of resistance contributed by fouling, ~5% due to the thermal resistance of the metal, the remaining ~76% is offered by the inner and outer convection resistances

Analysis of heat exchanger performance

- Questions we seek to answer…
  - Want to achieve a specified ΔT of a fluid stream: How much surface should we provide? [design]
    - Log Mean Temperature difference (LMTD) method is preferred
  - Have a specified heat exchanger: How much ΔT change will it produce in the hot and cold streams? [testing and performance]
    - Effectiveness (ε) - NTU method is preferred

Salient assumptions

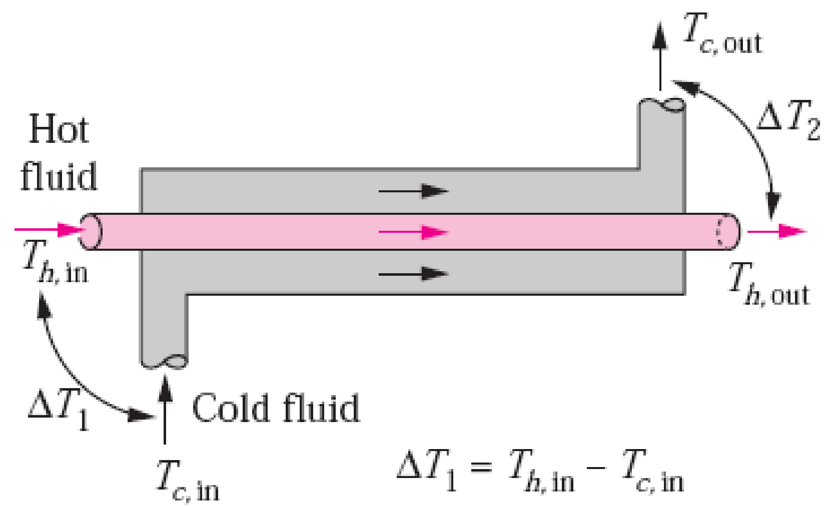
- Heat exchanger is insulated from the surrounding
- Axial conduction in the fluids are negligible as compared to the energy transaction between the two fluids
- Changes in potential and kinetic energy negligible
- Thermophysical properties remain constant
- Overall HTC remain constant

Heat transferred from hot fluid:

$$\dot{Q} = \dot{m}_h C_{ph}(T_{h,\text{ in}} - T_{h,\text{ out}})$$

Heat transferred to cold fluid:

$$\dot{Q} = \dot{m}_c C_{pc}(T_{c,\text{ out}} - T_{c,\text{ in}})$$



$$\Delta T_1 = T_{h,\text{ in}} - T_{c,\text{ in}}$$

$$\Delta T_2 = T_{h,\text{ out}} - T_{c,\text{ out}}$$

Parallel flow

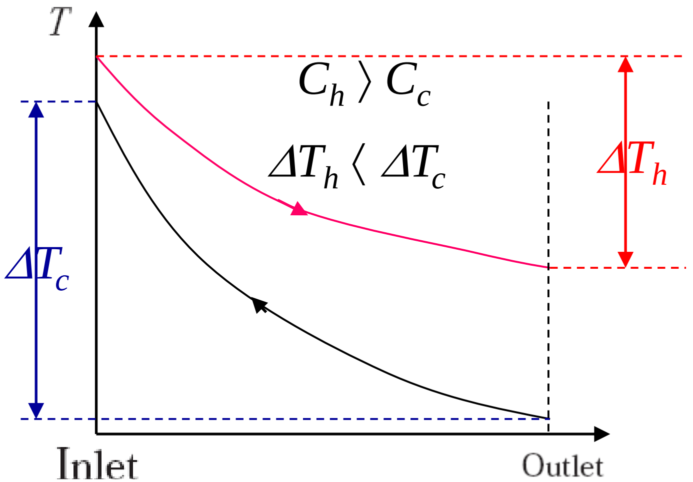
<u>Heat Capacity Rates</u>

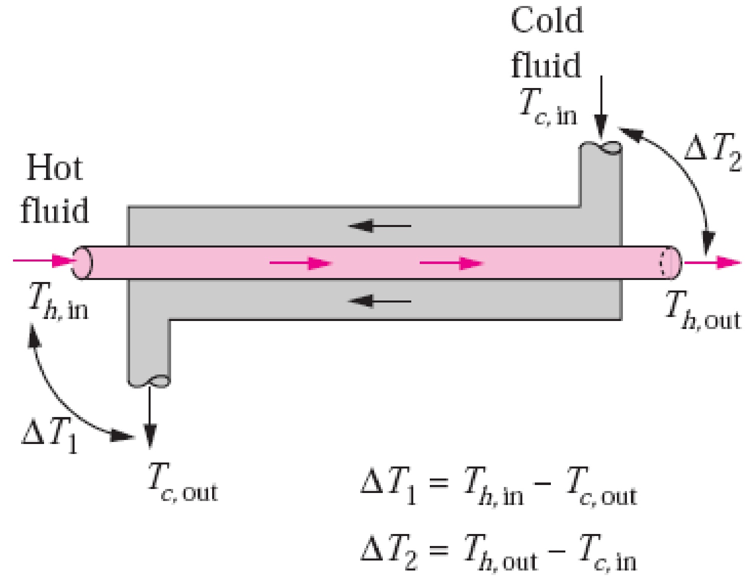
$$C_h = \dot{m}_h C_{ph}$$

$$C_c = \dot{m}_c C_{pc}$$

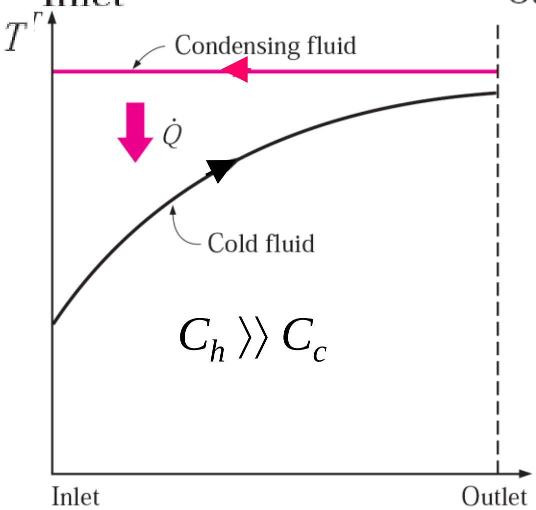
$$\dot{Q} = C_c(T_{c,\text{ out}} - T_{c,\text{ in}}) = C_h(T_{h,\text{ in}} - T_{h,\text{ out}})$$

Effect of C_h and C_c

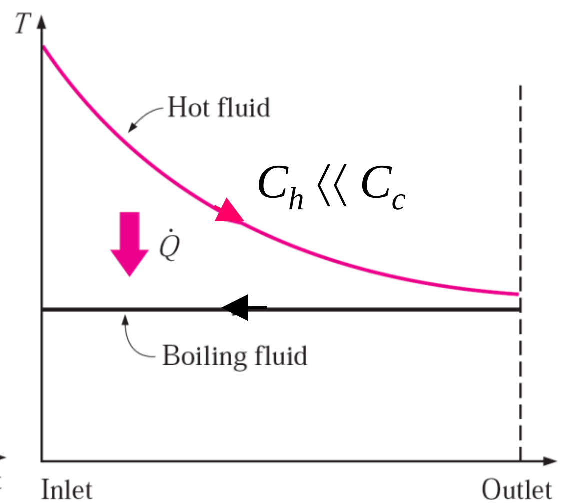




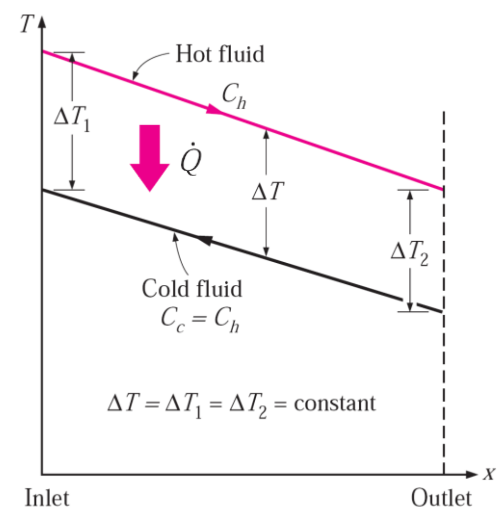
Counter flow



Condenser ($C_h \to \infty$)



Boiler ($C_c \to \infty$)



$C_h = C_c$

LMTD analysis of heat transfer

$$\delta \dot{Q} = -\dot{m}_h C_{ph}\, dT_h = \dot{m}_c C_{pc}\, dT_c \qquad (1)$$

For the hot stream: $dT_h = -\dfrac{\delta \dot{Q}}{\dot{m}_h C_{ph}}$ (2)

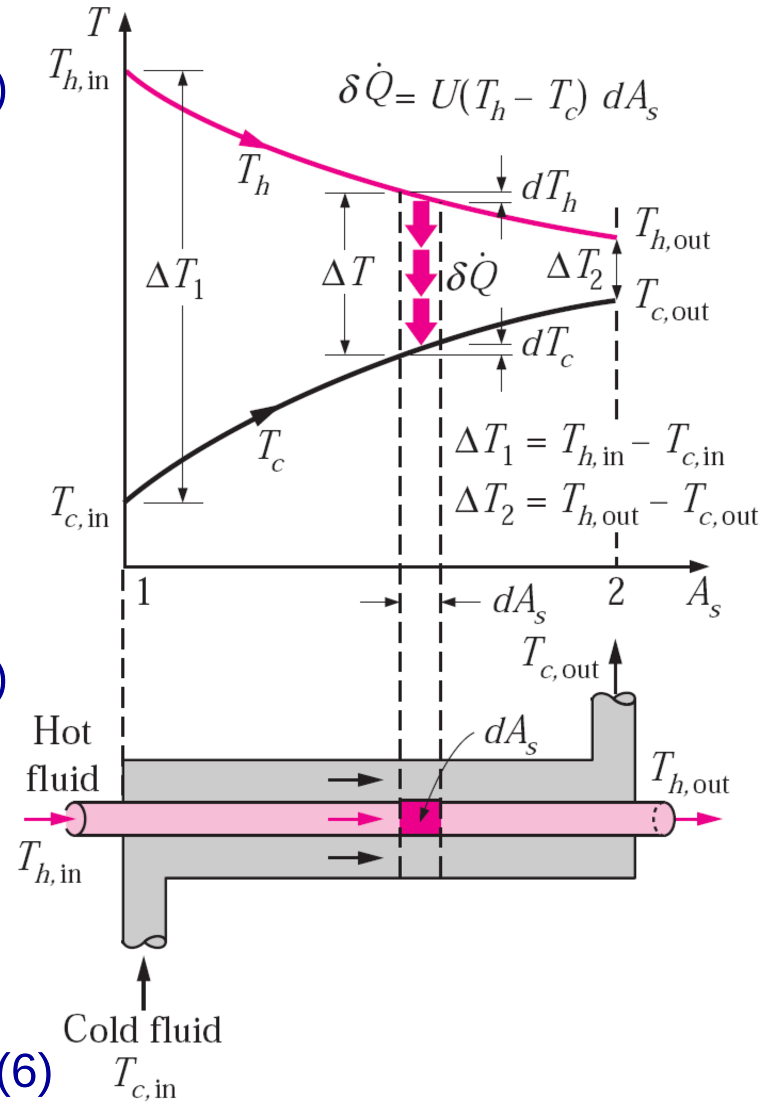
For the cold stream: $dT_c = \dfrac{\delta \dot{Q}}{\dot{m}_c C_{pc}}$ (3)

$$dT_h - dT_c = d(T_h - T_c) = -\delta \dot{Q}\left(\frac{1}{\dot{m}_h C_{ph}} + \frac{1}{\dot{m}_c C_{pc}}\right) \qquad (4)$$

But, $\delta \dot{Q} = U(T_h - T_c)\, dA_s$ (5) [convective heat trf.]

Substituting (5) in (4), and rearranging

$$\frac{d(T_h - T_c)}{T_h - T_c} = -U\, dA_s\left(\frac{1}{\dot{m}_h C_{ph}} + \frac{1}{\dot{m}_c C_{pc}}\right) \qquad (6)$$



LMTD method (contd…)

Integrating (5) over the length (i.e., the total HT area) of the heat exchanger,

$$\ln\frac{T_{h,\,out}-T_{c,\,out}}{T_{h,\,in}-T_{c,\,in}} = -UA_s\left(\frac{1}{\dot{m}_h C_{ph}}+\frac{1}{\dot{m}_c C_{pc}}\right) \quad (6)$$

Reckoning $\dfrac{1}{\dot{m}_h C_{ph}} = \dfrac{\left(T_{h,in}-T_{h,out}\right)}{\dot{Q}}$ and $\dfrac{1}{\dot{m}_c C_{pc}} = \dfrac{\left(T_{c,out}-T_{c,in}\right)}{\dot{Q}}$

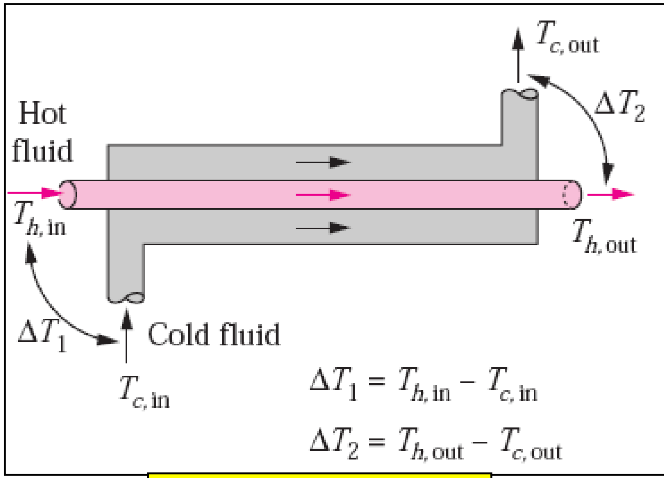
From (6)

$$ln\frac{\left(T_{h,out}-T_{c,out}\right)}{\left(T_{h,in}-T_{c,in}\right)} = -UA_s\frac{\left(T_{h,in}-T_{h,out}\right)+\left(T_{c,out}-T_{c,in}\right)}{\dot{Q}}$$

Rearranging,

$$\dot{Q} = -UA_s\frac{\left(T_{h,in}-T_{h,out}\right)+\left(T_{c,out}-T_{c,in}\right)}{ln\dfrac{\left(T_{h,out}-T_{c,out}\right)}{\left(T_{h,in}-T_{c,in}\right)}}$$

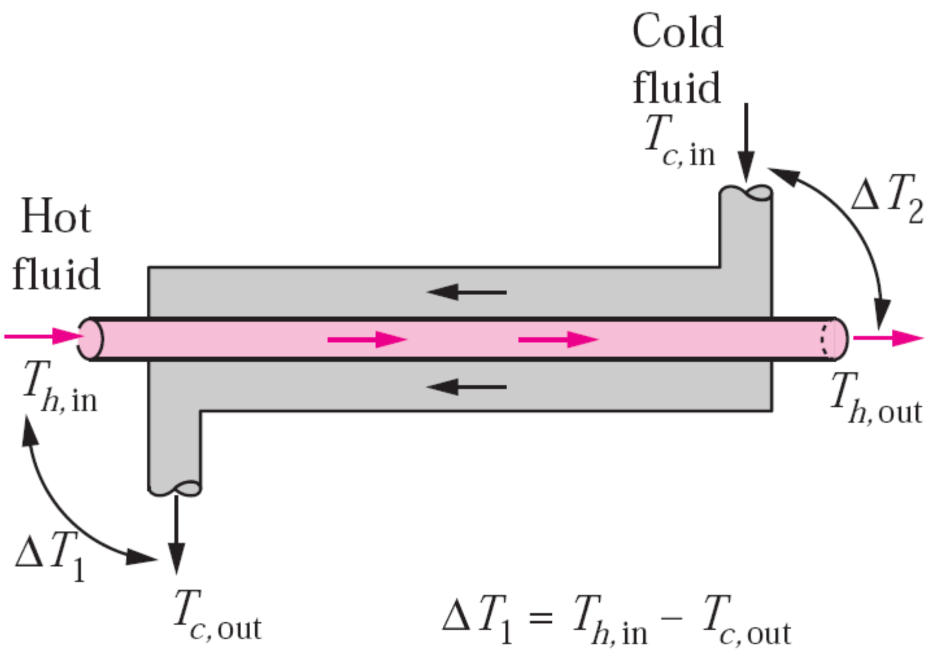
$$= UA_s\frac{\left(T_{h,in}-T_{c,in}\right)-\left(T_{h,out}-T_{c,out}\right)}{ln\dfrac{\left(T_{h,in}-T_{c,in}\right)}{\left(T_{h,out}-T_{c,out}\right)}} = UA_s \cdot LMTD\big|_{PF}$$



$$LMTD\big|_{PF} = \frac{\Delta T_1-\Delta T_2}{ln\dfrac{\Delta T_1}{\Delta T_2}}$$

For counterflow heat exchanger

- Similar analysis holds true…



$$\Delta T_1 = T_{h,\text{in}} - T_{c,\text{out}}$$

$$\Delta T_2 = T_{h,\text{out}} - T_{c,\text{in}}$$

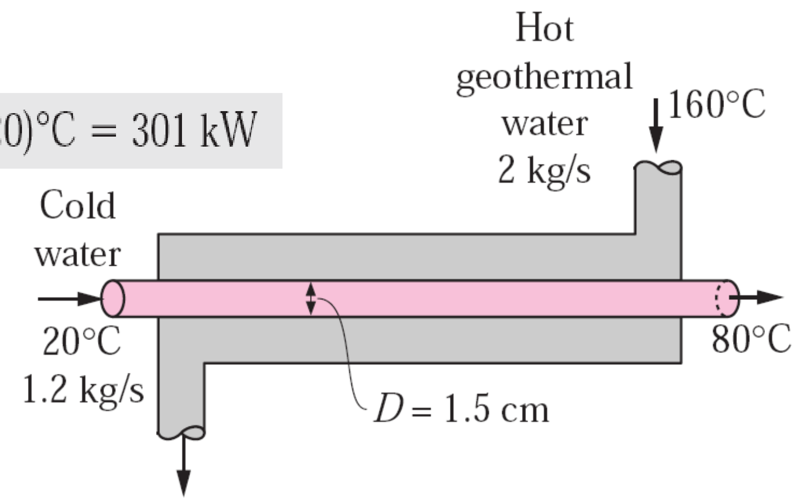
$$\dot{Q} = UA_s \cdot LMTD\Big|_{CF}$$

$$LMTD\Big|_{CF} = \frac{\Delta T_1 - \Delta T_2}{ln\dfrac{\Delta T_1}{\Delta T_2}}$$

EXAMPLE 23–4 Heating Water in a Counter-Flow Heat Exchanger

A counter-flow double-pipe heat exchanger is to heat water from 20°C to 80°C at a rate of 1.2 kg/s. The heating is to be accomplished by geothermal water available at 160°C at a mass flow rate of 2 kg/s. The inner tube is thin-walled and has a diameter of 1.5 cm. If the overall heat transfer coefficient of the heat exchanger is 640 W/m² · °C, determine the length of the heat exchanger required to achieve the desired heating.

Solution hints:



$$\dot{Q} = [\dot{m}C_p(T_{out} - T_{in})]_{water} = (1.2\text{ kg/s})(4.18\text{ kJ/kg}\cdot°\text{C})(80-20)°\text{C} = 301\text{ kW}$$

$$T_{out} = T_{in} - \frac{\dot{Q}}{\dot{m}C_p} = 160°\text{C} - \frac{301\text{ kW}}{(2\text{ kg/s})(4.31\text{ kJ/kg}\cdot°\text{C})} = 125°\text{C}$$

$$\Delta T_{lm} = \frac{\Delta T_1 - \Delta T_2}{\ln(\Delta T_1/\Delta T_2)} = \frac{80-105}{\ln(80/105)} = 92.0°\text{C}$$

$$\dot{Q} = UA_s\Delta T_{lm} \longrightarrow A_s = \frac{\dot{Q}}{U\Delta T_{lm}} = \frac{301{,}000\text{ W}}{(640\text{ W/m}^2\cdot°\text{C})(92.0°\text{C})} = 5.11\text{ m}^2$$

L = 108 m

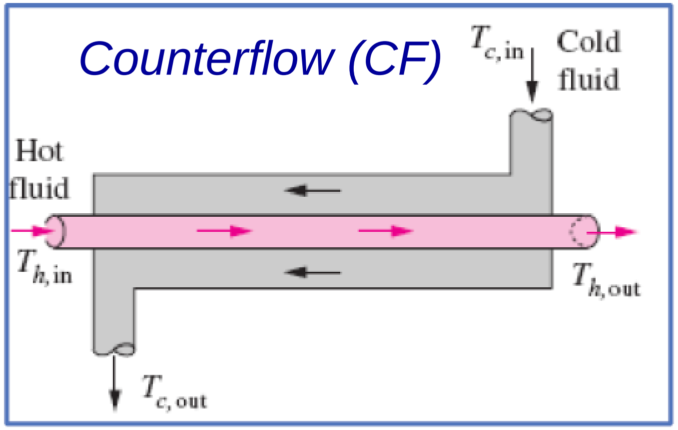
Logarithmic vs arithmetic mean temperature difference

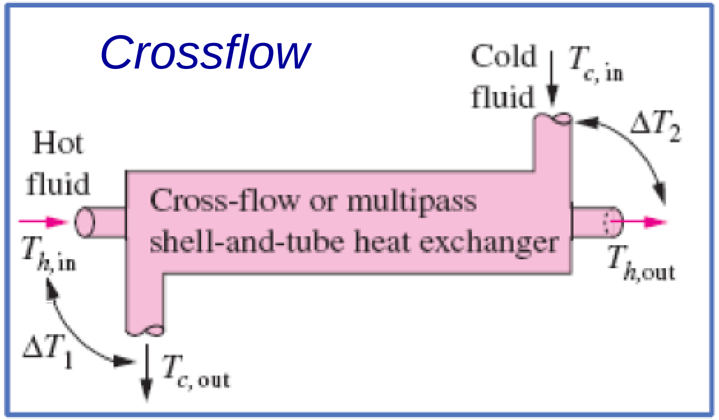
- The Arithmetic mean temperature difference $AMTD = \frac{1}{2}\left(\Delta T_1 + \Delta T_2\right)$
- The logarithmic mean temperature difference (LMTD) is an exact representation of the average temperature difference between the hot and cold fluids
- LMTD is always less than AMTD for parallel flow, and more than AMTD for counterflow
- when ΔT_1 and ΔT_2 differs by no more than 40%, the error in using AMTD is less than 1%. But the difference increases drastically at larger difference
- For a given $T_{h,in}$ $T_{c,in}$ $T_{h,out}$ $T_{c,out}$ the LMTD for counterflow is greater than that for parallel flow
- What is LMTD for $\Delta T_1 = \Delta T_2$?

Cross flow heat exchanger

The LMTD for cross flow is "somewhat" less than that of a counterflow HEX

So we assume, for the crossflow HEX $\Delta T_{lm} = F\,\Delta T_{lm,\,CF}$





The correction factor depends on

$$P = \frac{t_2 - t_1}{T_1 - t_1} \qquad R = \frac{T_1 - T_2}{t_2 - t_1} = \frac{(\dot{m}C_p)_{\text{tube side}}}{(\dot{m}C_p)_{\text{shell side}}}$$

The subscripts 1 and 2 represent the inlet and outlet, respectively..

T and t represent the shell- and tube-side temperatures, respectively

Heat transfer rate:

$$\dot{Q} = UA_s F\,\Delta T_{lm,CF}$$

where

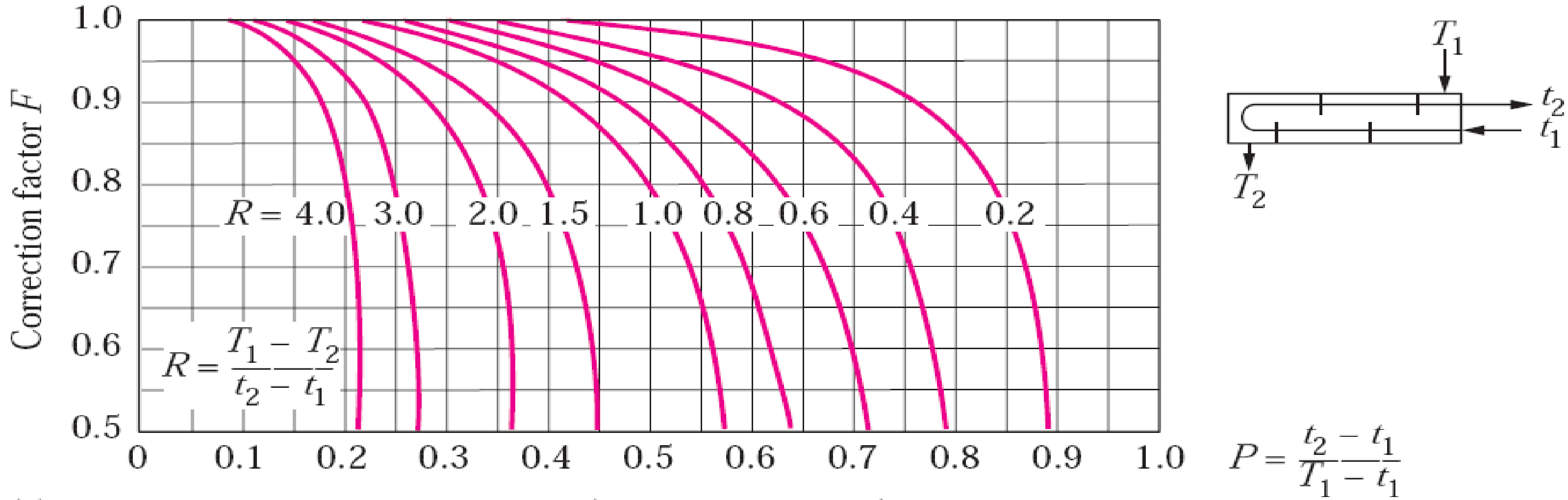
$$\Delta T_{lm,CF} = \frac{\Delta T_1 - \Delta T_2}{\ln(\Delta T_1/\Delta T_2)}$$

$$\Delta T_1 = T_{h,in} - T_{c,out}$$

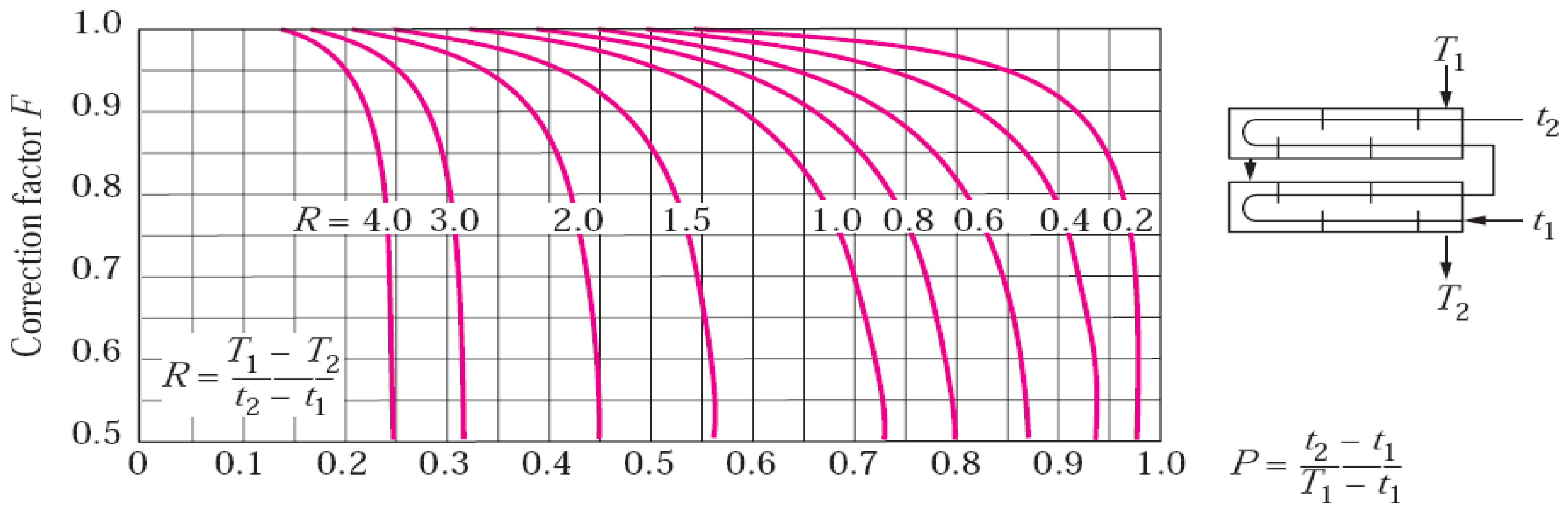
$$\Delta T_2 = T_{h,out} - T_{c,in}$$

☠ *The suffix CF stands for Counterflow, NOT cross flow*

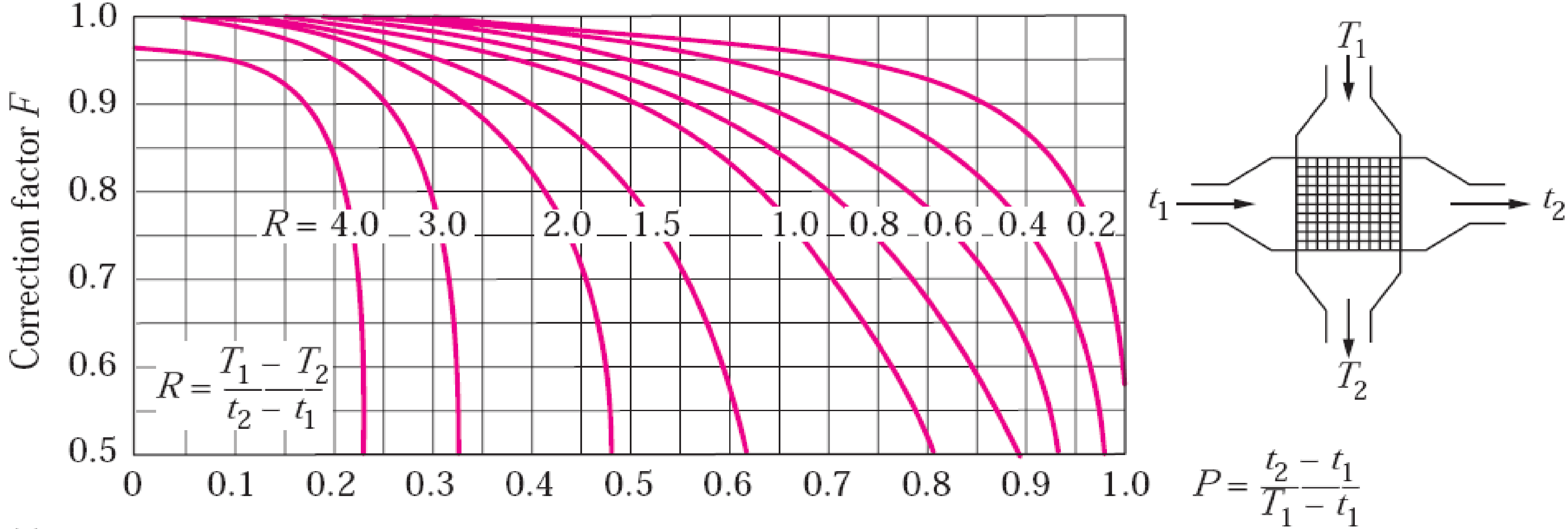
Bowman's chart for Correction Factor



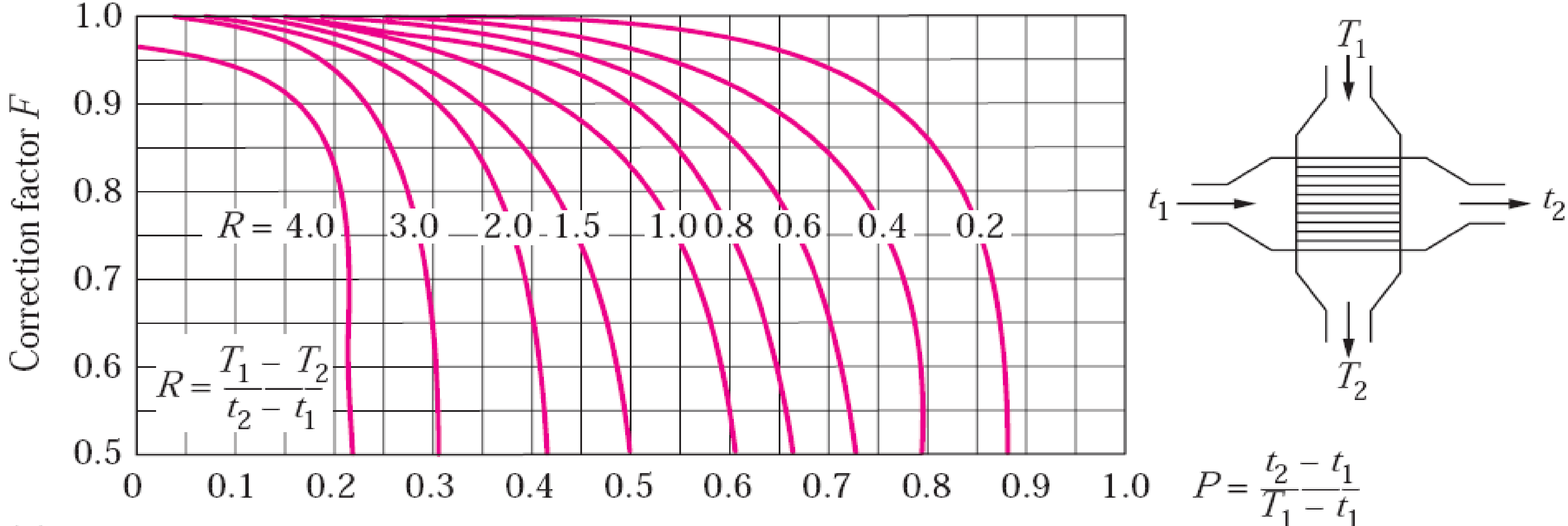
(*a*) One-shell pass and 2, 4, 6, etc. (any multiple of 2), tube passes



(*b*) Two-shell passes and 4, 8, 12, etc. (any multiple of 4), tube passes

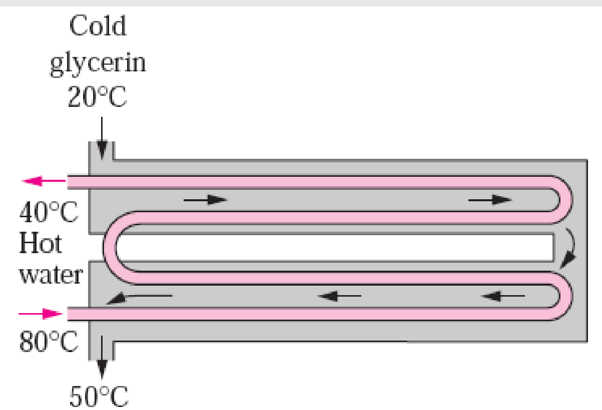


(c) Single-pass cross-flow with both fluids *unmixed*



(d) Single-pass cross-flow with one fluid *mixed* and the other *unmixed*

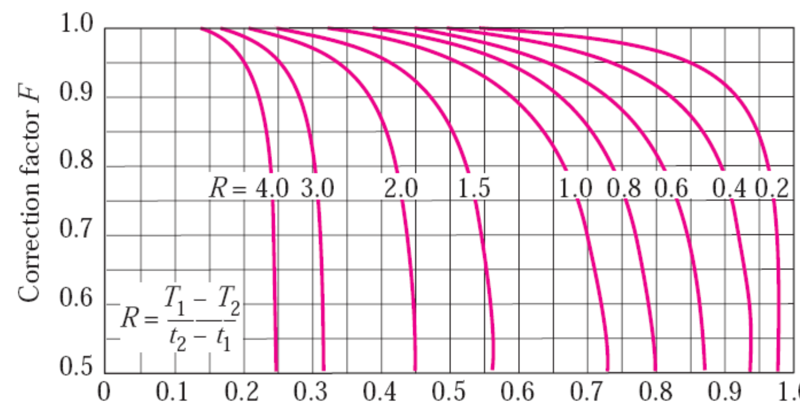
A 2-shell passes and 4-tube passes heat exchanger is used to heat glycerin from 20°C to 50°C by hot water, which enters the thin-walled 2-cm-diameter tubes at 80°C and leaves at 40°C (Fig. 23–21). The total length of the tubes in the heat exchanger is 60 m. The convection heat transfer coefficient is 25 $W/m^2 \cdot °C$ on the glycerin (shell) side and 160 $W/m^2 \cdot °C$ on the water (tube) side. Determine the rate of heat transfer in the heat exchanger (*a*) before any fouling occurs and (*b*) after fouling with a fouling factor of 0.0006 $m^2 \cdot °C/W$ occurs on the outer surfaces of the tubes.



$$\dot{Q} = UA_s F \Delta T_{lm,\,CF}$$

$$\Delta T_{lm,\,CF} = \frac{\Delta T_1 - \Delta T_2}{\ln(\Delta T_1/\Delta T_2)} = \frac{30 - 20}{\ln(30/20)} = 24.7°C$$

$$\left.\begin{aligned} P &= \frac{t_2 - t_1}{T_1 - t_1} = \frac{40 - 80}{20 - 80} = 0.67 \\ R &= \frac{T_1 - T_2}{t_2 - t_1} = \frac{20 - 50}{40 - 80} = 0.75 \end{aligned}\right\} F = 0.91$$

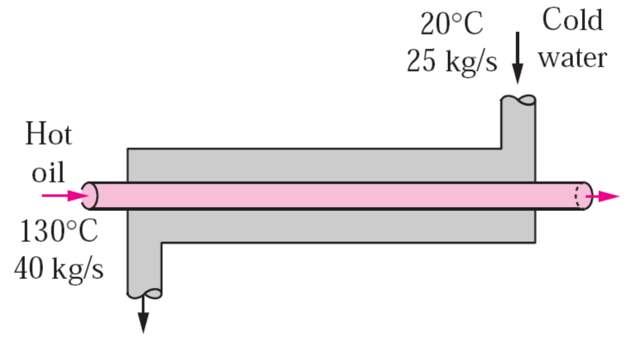


Rationale of LMTD method

- Suitable for determining the size of a heat exchanger to realize the prescribed outlet temperatures when the mass flow rates and the terminal temperatures are specified
- With LMTD method, the task is to **select** a heat exchanger that will meet the prescribed heat transfer requirements
  - Select the type of heat exchanger for the application
  - Determine any unknown inlet or outlet temperature and heat transfer rate using an energy balance
  - Calculate the LMTD (for cross flow use Bowman's chart for *F*)
  - Obtain (select or calculate) the value of overall HTC, i.e., *U*
  - Calculate the requisite heat transfer surface area, A_s
  - The task is completed by selecting a heat exchanger that has a heat transfer surface area equal to or larger than A_s
- Merit: Straightforward
- Limitation: Requires iterative solution if direct energy balance is not possible (e.g., only the two inlet stream temperatures specified)

Effectiveness-NTU method

- How to find the heat transfer rate and the outlet temperatures of the hot and cold fluids, provided the inlet temperatures, the heat capacity rates, and UA_s are specified?
- LMTD method would require iterative calculations

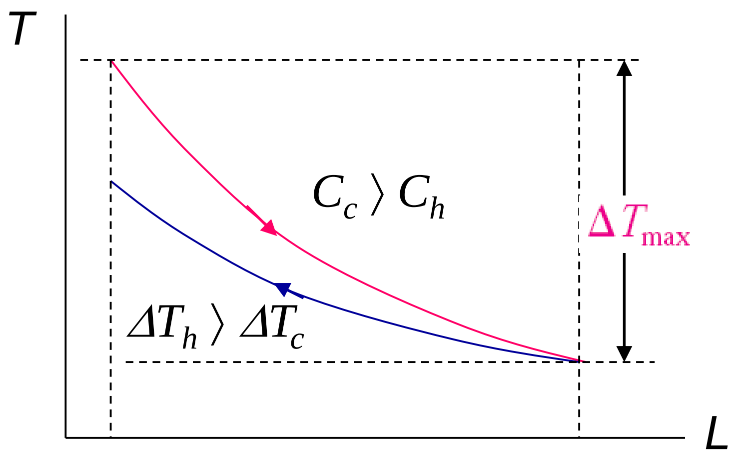


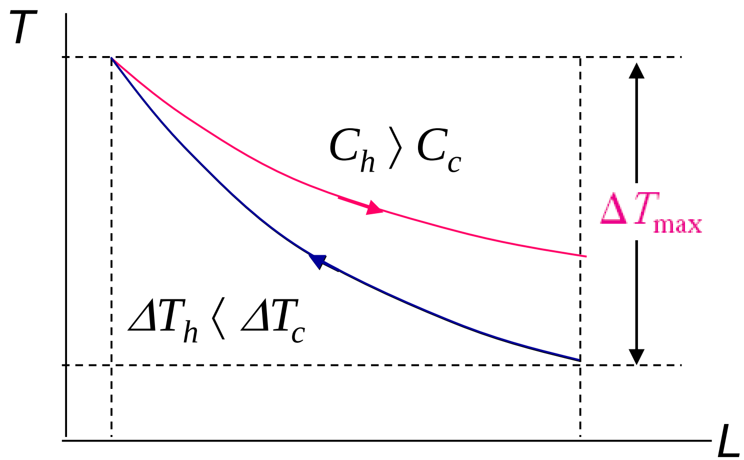
$$C_c = \dot{m}_c C_{pc} = 104.5 \text{ kW/°C}$$

$$C_h = \dot{m}_c C_{ph} = 92 \text{ kW/°C}$$

Heat Exchanger Effectiveness $\varepsilon = \dfrac{\dot{Q}}{\dot{Q}_{\max}} = \dfrac{\text{Actual heat transfer rate}}{\text{Maximum possible heat transfer rate}}$

$$\Delta T_{\max} = T_{h,\text{ in}} - T_{c,\text{ in}} \qquad \dot{Q}_{\max} = C_{\min}(T_{h,\text{ in}} - T_{c,\text{ in}})$$



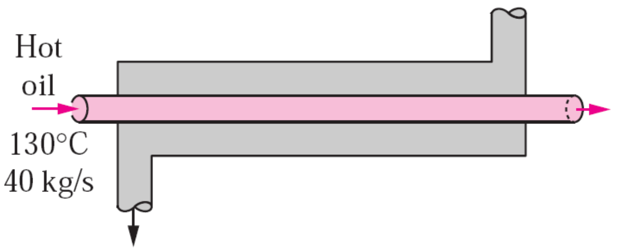


Fluid with the smaller C experience the maximum ΔT

NTU: the number of transfer unit

$$\text{NTU} = \frac{UA_s}{C_{\min}} = \frac{UA_s}{(\dot{m}C_p)_{\min}}$$

- Dimensionless “heat transfer area” or “thermal size factor”



Rationale of ε-NTU analysis

$C_c = \dot{m}_c C_{pc} = 104.5$ kW/°C

$C_h = \dot{m}_c C_{ph} = 92$ kW/°C

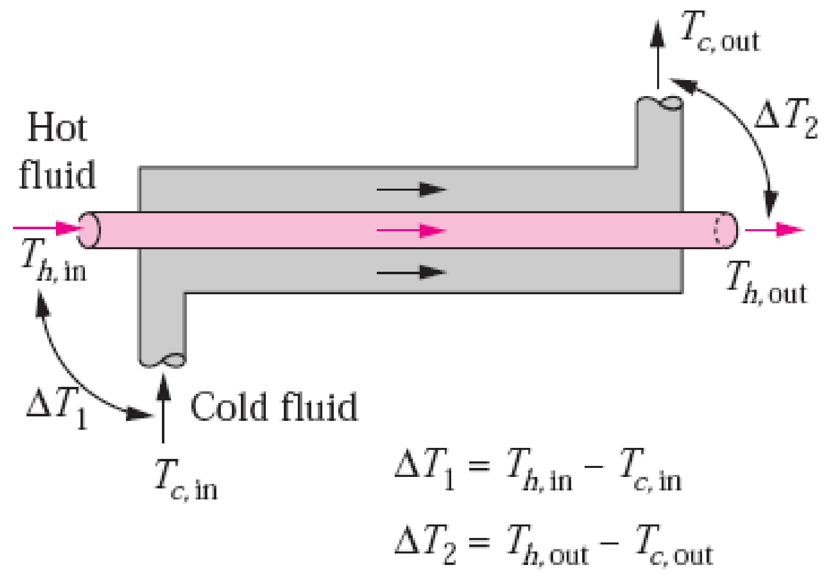
If ε is known in terms of the flow rates, thermophysical properties, and HEX geometry, then one can calculate the actual heat transfer as:

$$\dot{Q} = \varepsilon \dot{Q}_{\max} = \varepsilon C_{\min}(T_{h,\text{ in}} - T_{c,\text{ in}})$$

How to evaluate ε?

ε-NTU analysis for a Parallel Flow HEX

$$\ln \frac{T_{h,\,out} - T_{c,\,out}}{T_{h,\,in} - T_{c,\,in}} = -UA_s\left(\frac{1}{\dot{m}_h C_{ph}} + \frac{1}{\dot{m}_c C_{pc}}\right) \quad (6)$$



Also, from energy balance:

$$T_{h,\,out} = T_{h,\,in} - \frac{C_c}{C_h}(T_{c,\,out} - T_{c,\,in}) \quad (7)$$

Substituting in eq. (6) and adding and subtracting $T_{c,in}$ in the numerator on the left:

$$\ln \frac{T_{h,\,in} - T_{c,\,in} + T_{c,\,in} - T_{c,\,out} - \frac{C_c}{C_h}(T_{c,\,out} - T_{c,\,in})}{T_{h,\,in} - T_{c,\,in}} = -\frac{UA_s}{C_c}\left(1 + \frac{C_c}{C_h}\right)$$

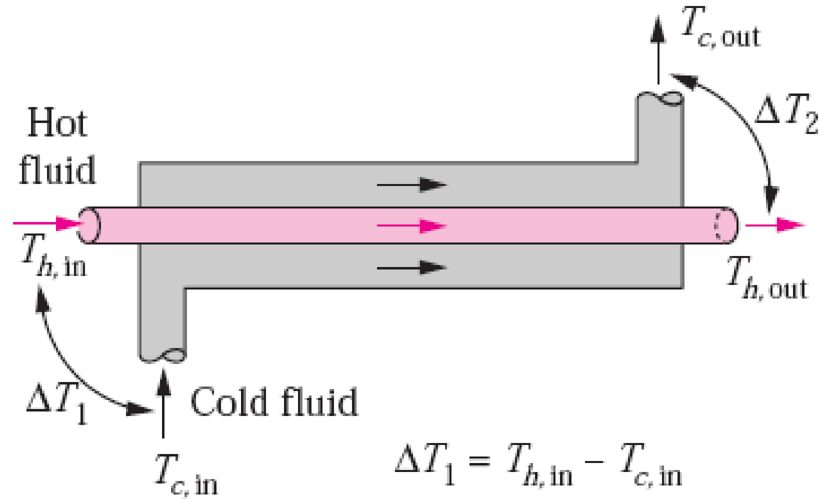
Simplifies to: $$\ln\left[1 - \left(1 + \frac{C_c}{C_h}\right)\frac{T_{c,\,out} - T_{c,\,in}}{T_{h,\,in} - T_{c,\,in}}\right] = -\frac{UA_s}{C_c}\left(1 + \frac{C_c}{C_h}\right) \quad (8)$$

ε-NTU analysis for a Parallel Flow HEX (continued…)

Also, from the definition of HEX effectiveness

$$\varepsilon = \frac{\dot{Q}}{\dot{Q}_{max}} = \frac{C_c(T_{c,\,out} - T_{c,\,in})}{C_{min}(T_{h,\,in} - T_{c,\,in})}$$

$$\Rightarrow \frac{T_{c,\,out} - T_{c,\,in}}{T_{h,\,in} - T_{c,\,in}} = \varepsilon\,\frac{C_{min}}{C_c}$$



$\Delta T_1 = T_{h,\,in} - T_{c,\,in}$

$\Delta T_2 = T_{h,\,out} - T_{c,\,out}$

Substituting back in (8)

$$\varepsilon_{parallel\ flow} = \frac{1 - \exp\left[-\frac{UA_s}{C_c}\left(1 + \frac{C_c}{C_h}\right)\right]}{\left(1 + \frac{C_c}{C_h}\right)\frac{C_{min}}{C_c}} \qquad (9)$$

Show that, for either $C_h = C_{min}$, or $C_c = C_{min}$, Eq. (9) reduces to

$$\varepsilon_{parallel\ flow} = \frac{1 - \exp\left[-\frac{UA_s}{C_{min}}\left(1 + \frac{C_{min}}{C_{max}}\right)\right]}{1 + \frac{C_{min}}{C_{max}}}$$

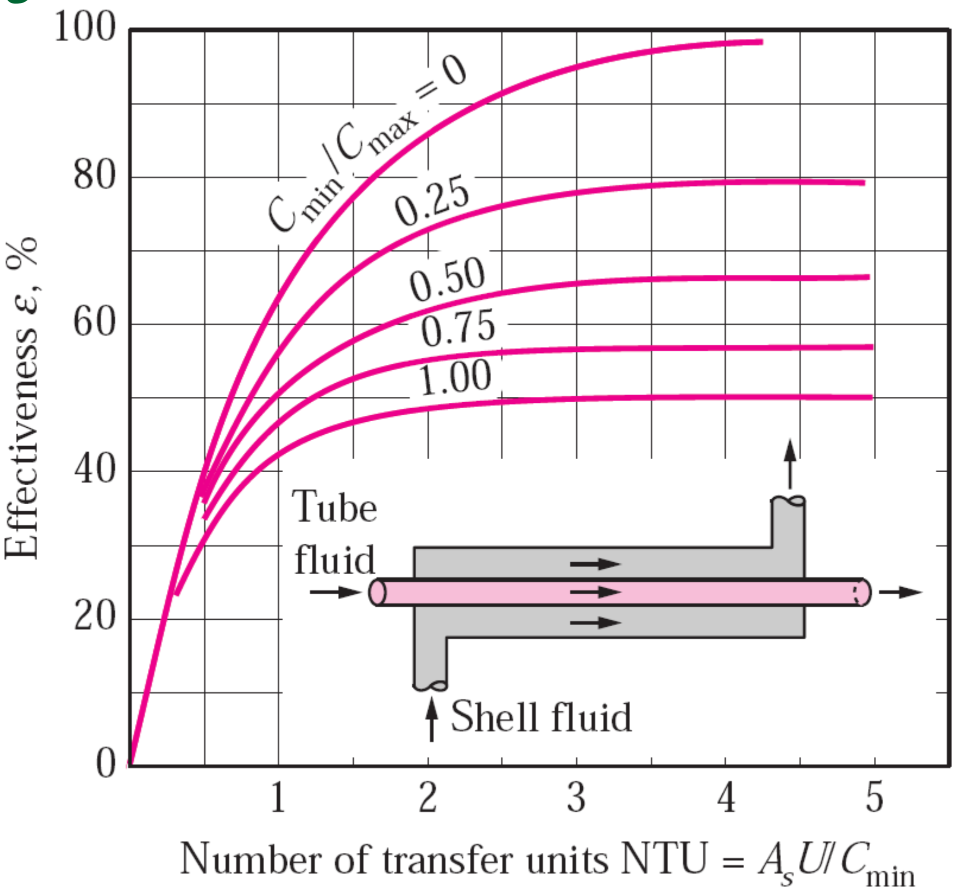
Kays-London chart

$$\varepsilon_{PF} = \frac{1 - exp\left[-NTU\left(1 + C_r\right)\right]}{1 + C_r}$$

$$C_r = \frac{C_{min}}{C_{max}}$$

Thus, for the parallel flow geometry

$$\varepsilon_{PF} = f\left[NTU, C_r\right]$$



- NTU is specified from the HEX design datasheet, where the HT surface area, HTC and the heat capacity rates are specified.
- C_r will vary with operating condition

Note: for one fluid experiencing phase change, $C_r = 0$,

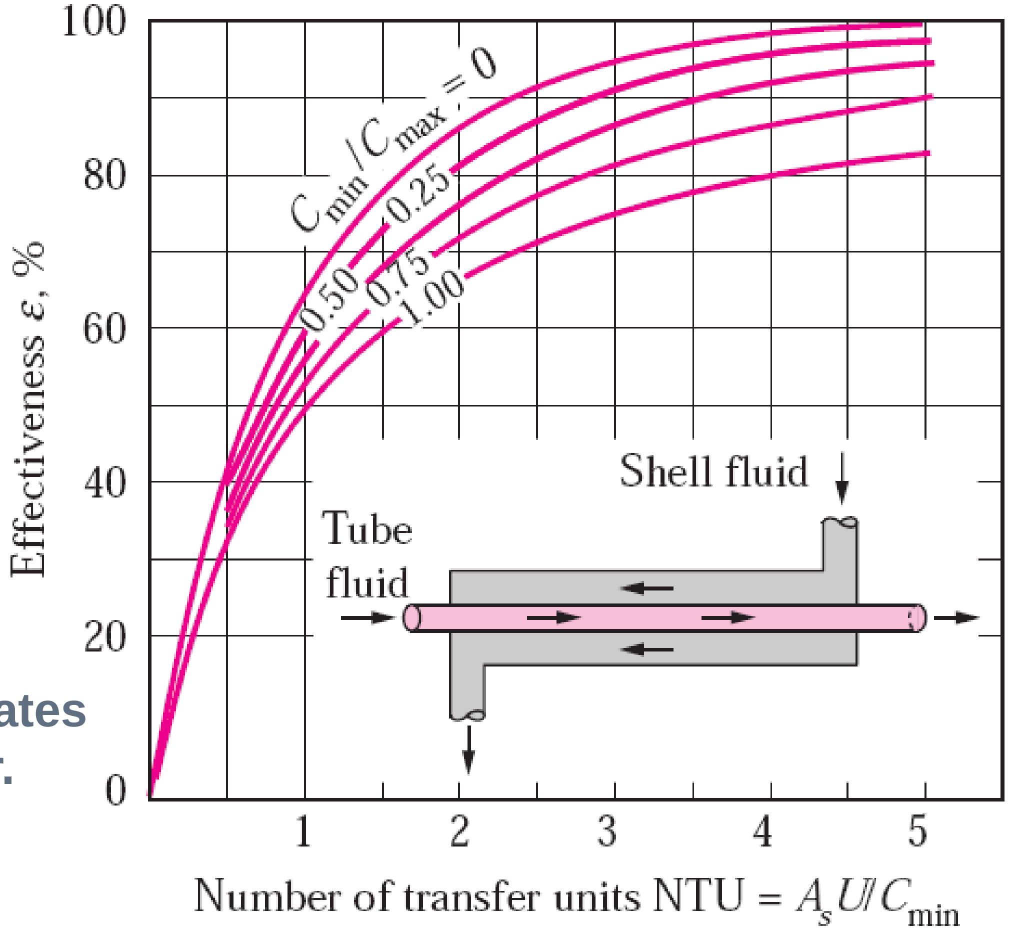
$$\varepsilon = 1 - exp\left[-NTU\right]$$

$$\varepsilon_{CF} = \frac{1-exp\left[-NTU\left(1-C_r\right)\right]}{1-C_r\,exp\left[-NTU\left(1-C_r\right)\right]} \quad \left(for\ C_r < 1\right)$$

$$= \frac{NTU}{1+NTU} \quad \left(for\ C_r = 1\right)$$

$$= 1-exp\left[-NTU\right]$$

$$\left(for\ C_r = 0\right)$$

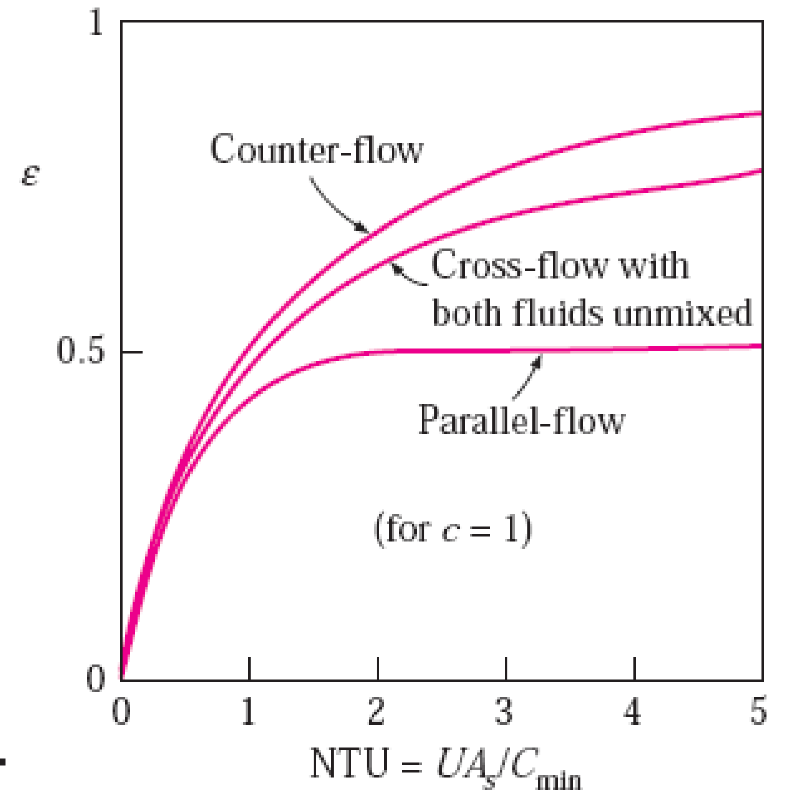


Larger NTU value indicates that the limit has reached for heat transfer whereas smaller value of NTU indicates more opportunities for heat transfer.

ε-NTU relations for different HEX

Effectiveness relations for heat exchangers: $\text{NTU} = UA_s/C_{min}$ and $c = C_{min}/C_{max} = (\dot{m}C_p)_{min}/(\dot{m}C_p)_{max}$ (Kays and London)

Heat exchanger type	Effectiveness relation
1 *Double pipe:*	
Parallel-flow	$\varepsilon = \dfrac{1 - \exp[-\text{NTU}(1+c)]}{1+c}$
Counter-flow	$\varepsilon = \dfrac{1 - \exp[-\text{NTU}(1-c)]}{1 - c\exp[-\text{NTU}(1-c)]}$
2 *Shell-and-tube:* One-shell pass 2, 4, . . . tube passes	$\varepsilon = 2\left\{1 + c + \sqrt{1+c^2}\,\dfrac{1+\exp[-\text{NTU}\sqrt{1+c^2}]}{1-\exp[-\text{NTU}\sqrt{1+c^2}]}\right\}^{-1}$
3 *Cross-flow (single-pass)*	
Both fluids unmixed	$\varepsilon = 1 - \exp\left\{\dfrac{\text{NTU}^{0.22}}{c}[\exp(-c\,\text{NTU}^{0.78}) - 1]\right\}$
C_{max} mixed, C_{min} unmixed	$\varepsilon = \dfrac{1}{c}(1 - \exp\{1 - c[1 - \exp(-\text{NTU})]\})$
C_{min} mixed, C_{max} unmixed	$\varepsilon = 1 - \exp\left\{-\dfrac{1}{c}[1 - \exp(-c\,\text{NTU})]\right\}$
4 *All heat exchangers with c = 0*	$\varepsilon = 1 - \exp(-\text{NTU})$



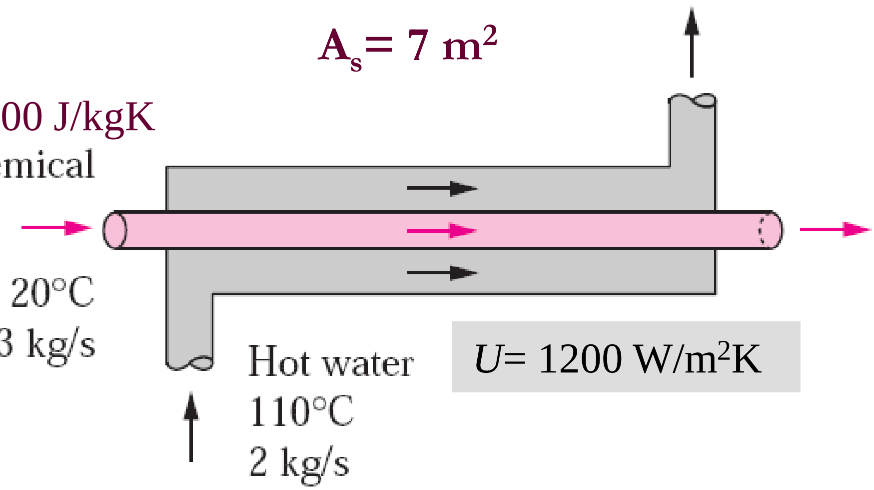
NTU- ε relations for different HEX

NTU relations for heat exchangers: $\text{NTU} = UA_s/C_{\min}$ and $c = C_{\min}/C_{\max} = (\dot{m}C_p)_{\min}/(\dot{m}C_p)_{\max}$ (Kays and London)

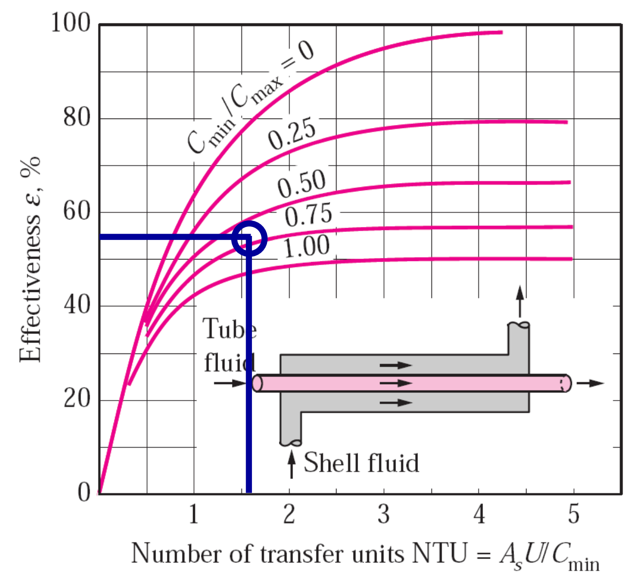
Heat exchanger type	NTU relation
1 *Double-pipe:*	
Parallel-flow	$\text{NTU} = -\dfrac{\ln[1-\varepsilon(1+c)]}{1+c}$
Counter-flow	$\text{NTU} = \dfrac{1}{c-1}\ln\left(\dfrac{\varepsilon-1}{\varepsilon c-1}\right)$
2 *Shell and tube:* One-shell pass 2, 4, . . . tube passes	$\text{NTU} = -\dfrac{1}{\sqrt{1+c^2}}\ln\left(\dfrac{2/\varepsilon-1-c-\sqrt{1+c^2}}{2/\varepsilon-1-c+\sqrt{1+c^2}}\right)$
3 *Cross-flow* (*single-pass*): $C_{\max}$ mixed, $C_{\min}$ unmixed	$\text{NTU} = -\ln\left[1+\dfrac{\ln(1-\varepsilon c)}{c}\right]$
$C_{\min}$ mixed, $C_{\max}$ unmixed	$\text{NTU} = -\dfrac{\ln[c\ln(1-\varepsilon)+1]}{c}$
4 *All heat exchangers with* $c = 0$	$\text{NTU} = -\ln(1-\varepsilon)$

Example 1: ε-NTU analysis

Calculate the outlet temperatures of both the fluids, and the heat transfer



- $C_{min} = 5400 \text{ J/kgK},$
  $C_{max} = 8360 \text{ J/kgK},$
  $C_r = 0.646$
- $\text{NTU} = \dfrac{UA_s}{C_{min}} = \dfrac{1200 \times 7}{5400} = 1.556$
- $\varepsilon_{PF} = \dfrac{1 - exp\left[-NTU\left(1 + C_r\right)\right]}{1 + C_r} = 0.561$
- $Q_{max} = C_{min}\left(T_{hi} - T_{ci}\right) = 5400 \times 90 = 486000 \text{ W}$
- $Q_{actual} = \varepsilon Q_{max} = 0.561 \times 486000 = 272646 \text{ W}$

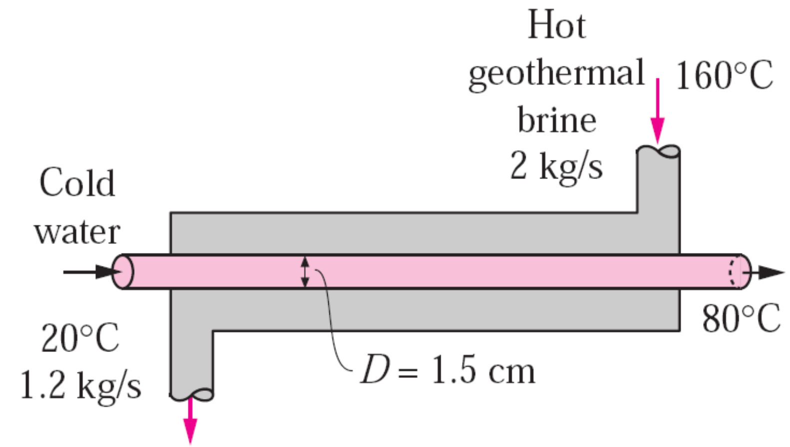


$T_{h,o}$ = 110 - 32.61 = 77.39 °C

$T_{c,o}$ = 20 + 50.49 = 70.49 °C

Example 2: ε-NTU analysis

Solve the same problem (example discussed for the LMTD technique using ε-NTU analysis:



☞ $$C_h = \dot{m}_h C_{ph} = (2\ \text{kg/s})(4.31\ \text{kJ/kg}\cdot{}^\circ\text{C}) = 8.62\ \text{kW/}^\circ\text{C}$$
$$C_c = \dot{m}_c C_{pc} = (1.2\ \text{kg/s})(4.18\ \text{kJ/kg}\cdot{}^\circ\text{C}) = 5.02\ \text{kW/}^\circ\text{C}$$

☞ $$c = C_{\min}/C_{\max} = 5.02/8.62 = 0.583$$

☞ $$\dot{Q}_{\max} = C_{\min}(T_{h,\text{ in}} - T_{c,\text{ in}}) = (5.02\ \text{kW/}^\circ\text{C})(160 - 20)^\circ\text{C} = 702.8\ \text{kW}$$

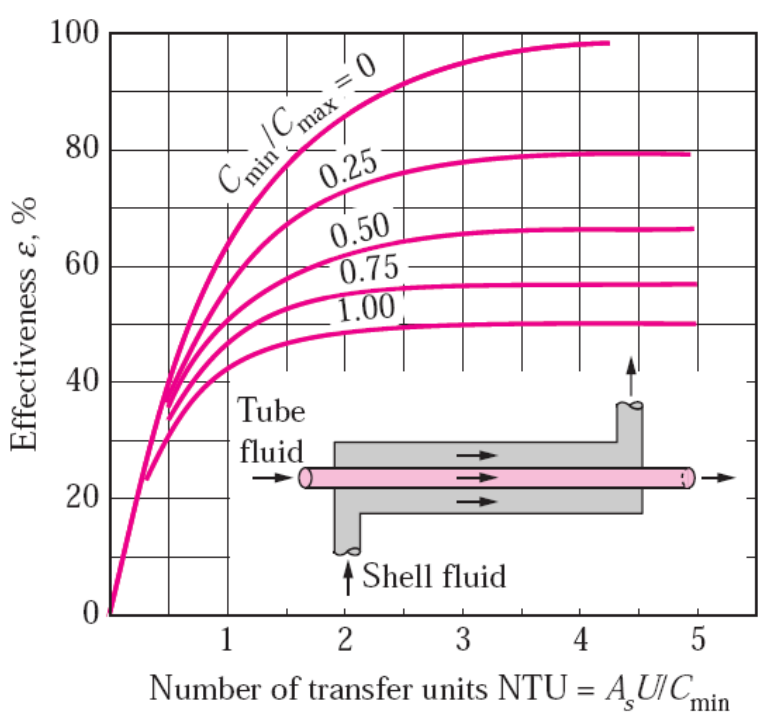
☞ $$\dot{Q} = [\dot{m}C_p(T_{\text{out}} - T_{\text{in}})]_{\text{water}} = (1.2\ \text{kg/s})(4.18\ \text{kJ/kg}\cdot{}^\circ\text{C})(80 - 20)^\circ\text{C} = 301.0\ \text{kW}$$

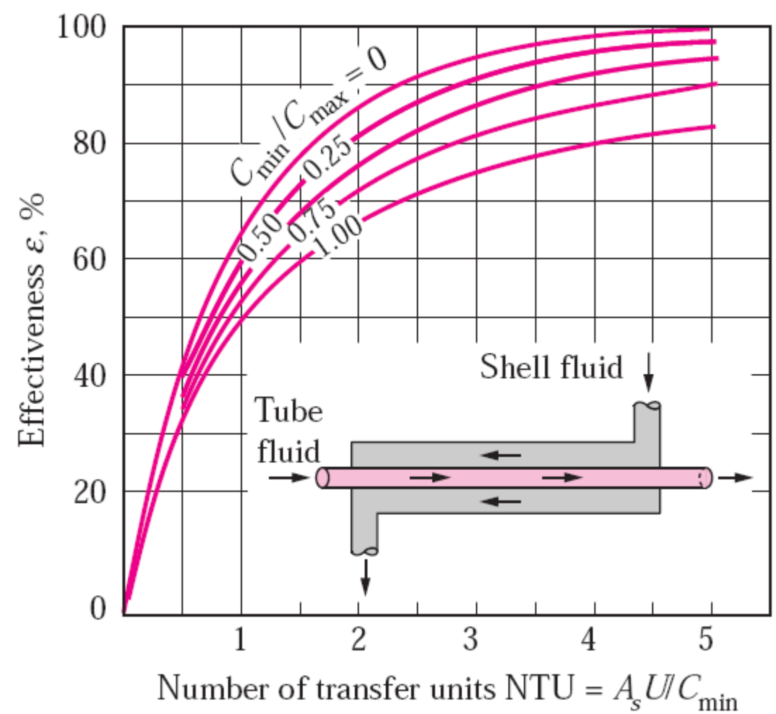
☞ $$\text{NTU} = \frac{1}{c-1}\ln\left(\frac{\varepsilon - 1}{\varepsilon c - 1}\right) = \frac{1}{0.583 - 1}\ln\left(\frac{0.428 - 1}{0.428 \times 0.583 - 1}\right) = 0.651$$

$$\Rightarrow A_s = \frac{\text{NTU}\ C_{\min}}{U} = 5.11\ \text{m}^2$$

L = 108 m

Comparison: PF vs CF





At $C_r = 0$, both parallel and counterflow HEX have the same ε

$$\varepsilon_{PF} = \varepsilon_{CF} = 1 - exp\left[-NTU\right]$$

Observations from the effectiveness relations and charts

- The value of the effectiveness ranges from 0 to 1. It increases rapidly with NTU for small values (up to about NTU = 1.5) but rather slowly for larger values. Therefore, the use of a heat exchanger with a large NTU (usually larger than 3) and thus a large size cannot be justified economically, since a large increase in NTU in this case corresponds to a small increase in effectiveness.
- For a given NTU and capacity ratio $c = C_{min} / C_{max}$, the *counter-flow* heat exchanger has the *highest* effectiveness, followed closely by the cross-flow heat exchangers with both fluids unmixed. The lowest effectiveness values are encountered in parallel-flow heat exchangers.
- The effectiveness of a heat exchanger is independent of the capacity ratio c for NTU values of less than about 0.3.
- The value of the capacity ratio c ranges between 0 and 1. For a given NTU, the effectiveness becomes a *maximum* for $c = 0$ (e.g., boiler, condenser) and a *minimum* for $c = 1$ (when the heat capacity rates of the two fluids are equal).

Selection of Heat Exchangers

The uncertainty in the predicted value of *U* can exceed 30 percent. Thus, it is natural to tend to overdesign the heat exchangers.

Heat transfer enhancement in heat exchangers is usually accompanied by *increased pressure drop*, and thus *higher pumping power*.

Therefore, any gain from the enhancement in heat transfer should be weighed against the cost of the accompanying pressure drop.

Usually, the *more viscous fluid is more suitable for the shell side* (larger passage area and thus lower pressure drop) and *the fluid with the higher pressure for the tube side*

The proper selection of a heat exchanger depends on several factors:

- **Heat Transfer Rate**
- **Cost**
- **Pumping Power**
- **Size and Weight**
- **Type**
- **Materials**

The *rate of heat transfer* in the prospective heat exchanger

$$\dot{Q} = \dot{m}c_p(T_{in} - T_{out})$$

The annual cost of electricity associated with the operation of the pumps and fans

Operating cost = pumping power × hours of operation × tariff

Reference:

- Heat Transfer, a Practical Approach by Y. Çengel
- ASPEN Plus literature

- Interesting videos:
  - https://www.youtube.com/watch?v=OyQ3SaU4KKU
  - https://www.youtube.com/watch?v=M_jOsTWVIH8
- Applications of Heat Exchangers in industry
  - https://youtu.be/WAiTFp54xZQ