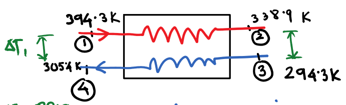
3. Oil flowing at the rate of 7258 kg/h with a mean heat capacity of 2.01 kJ/(kg K) is cooled from 394.3 K to 338.9 K in a counterflow heat exchanger by water entering at 294.3K and leaving at 305.4 K. Calculate the flow rate of the water and the overall heat transfer coefficient based on the inner area U_i if the inner area is 5.11 m^2.



$\dot{m}_o = 7258$ kg/h $= 2.016$ kg/s
$C_{p_o} = 2.01$ kJ/kgK
$\dot{m}_w = ?$, $C_w = 4.187$ kJ/kgK

$\Delta T_1 = 88.9$K
$\Delta T_2 = 44.6$K

To find : 1) $\dot{m}_w$
2) U_i

Sol: A) To find $\dot{m}_w$, we apply the energy balance, ie
Heat rejected by the hot fluid = Heat gained by the cold fluid

or $\dot{m}_w C_{pw}(T_{wo} - T_{wi}) = \dot{m}_o C_{p_o}(T_{oi} - T_{oo})$

water out, oil in

$\therefore \dot{m}_w = \dfrac{\dot{m}_o C_{po}(T_{oi} - T_{oo})}{C_{pw}(T_{wo} - T_{wi})}$

$= \dfrac{2.016 \times 2.01 \times (394.3 - 338.9)}{4.187 \times (305.4 - 294.3)}$

$= 4.83$ kg/s

B) To find U_i we use the formula

$\dot{Q} = U_i A_i \, LMTD$

$= U_i \, 5.11 \times 64.22$ — (A)

$U_i \to ?$

$\Delta T_1 = 88.9$
$\Delta T_2 = 44.6$

$lmtd = \dfrac{\Delta T_1 - \Delta T_2}{\ln\left(\frac{\Delta T_1}{\Delta T_2}\right)}$

$= \dfrac{44.3}{\ln\left(\frac{88.9}{44.6}\right)}$

$= 64.22$

How to calculate $\dot{Q}$?

$\dot{Q} = \dot{m}_o C_{p_o}(T_{oi} - T_{oo})$

$= 7258 \times 2.016 \times (394.3 - 338.9)$

$\boxed{\dot{Q} = 810.6 \times 10^3 \text{ W}}$ — (B)

$810.6 \times 10^3 = U_i \times 5.11 \times 64.22$

$\Rightarrow \boxed{U_i = 2470.2 \text{ W/m}^2\text{K}}$

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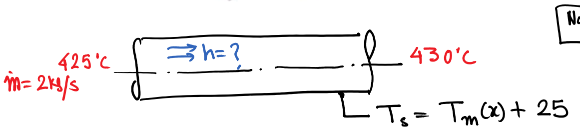
4. The liquid metal bismuth enters a tube having an inside diameter of 35 mm at 425°C and is heated to 430°C in the tube. The flow rate of the bismuth is 2.00 kg/s. The tube wall is maintained at a temperature of 25°C above the liquid bulk temperature. Calculate the tube length required. The physical propoerties of bismuth are as follows: k=15.6 W/(m K), C_p=149 J/(kg K), viscosity = 1.34 x 10^{-3} Pa s.



Note: This is not a problem of Heat Exchanger. It's problem of Forced Convection

So you should be able to identify the problem type

* Clearly, this is a case of q'' = const. where T_s differs from the bulk mean temp. by a constant amount.

$\dot{Q} = \dot{m} C_p (T_e - T_i)$

$= 2 \times 149 \times (430 - 425) = 1490$ W

To find L, we need to find h.
To find h, we need to find Nu

$Re = \dfrac{\rho V \times d}{\mu} = \left(\dfrac{4\dot{m}\,\rho}{\rho \times \pi d^2}\right) \times \dfrac{d}{\mu} = \dfrac{4 \times 2}{\pi \times 0.035 \times 1.34 \times 10^{-3}} = 54296$

$\Rightarrow$ Turbulent flow
So we need to look for the appropriate Nu-correlation

Here $Pr = \dfrac{\mu C_p}{k} = \dfrac{1.34 \times 10^{-3} \times 149}{15.6} = 0.0128$

So it's a very low Pr. flow $\Rightarrow$ Therefore, Dittus Boelter Eqn. does not apply!

Liquid metals, T_s = constant: $\quad Nu = 4.8 + 0.0156\, Re^{0.85}\, Pr_s^{0.93}$

Liquid metals, $\dot{q}_s$ = constant: $\quad Nu = 6.3 + 0.0167\, Re^{0.85}\, Pr_s^{0.93}$

So $Nu = 6.3 + 0.0167 \times (54296)^{0.85} (0.0128)^{0.93}$

$= 9.37$

$\therefore\ h = kNu/d = 15.6 \times 9.37 / 0.035 = 4175.7$

Now we have h. So we use the relationship

$\dot{Q} = h \times A \times (T_s - T_m)$

$= 4175.7 \times \pi d L \times 25$

$\Rightarrow 1490 = (4175.7 \times \pi \times .035 \times 25) \times L$

$\Rightarrow L = 0.130\ m = 13\ cm$

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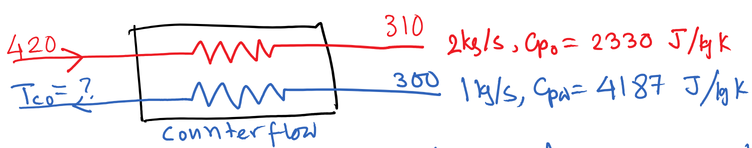
5. After a long time in service, a counterflow heat exchanger for cooling of turbine lube oil in a power plant is checked to ascertain if its performance has deteriorated due to fouling. In the test SAE 50 oil flowing at 2.0 kg/s is cooled from 420 K to 310 K by a water supply of 1.22 kg/s at 300 K. If the overall heat transfer surface is 3.33. m^2 and the design value of overall heat transfer coefficient is 11930 W/m^{2}K find the percentage degradation of the overall heat transfer coefficient from design value. C_p for SAE oil is 2330 J/kgK nd that fr water is 4187 J/kgK.

Design (new and clean) condition $\Rightarrow U = 11930\ W/m^2K$

Actual (Test) condition $\Rightarrow \dot{Q} = \dot{m} C_p (T_i - T_o)$

$= 2 \times 2330 \times (420 - 310) = 512600\ W$



To find out T_{co}, let's do energy balance

$\dot{m}_{oil}\, C_{p\,oil}\, (T_{o,i} - T_{o,e}) = \dot{m}_w\, C_{pw}\, (T_{wo} - T_{wi})$

$\Rightarrow 2 \times 2330 \times 110 = 1.2 \times 4187 \times \Delta T_w$

$\Rightarrow \Delta T_w = 1$

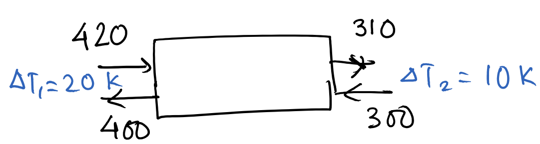
$\Rightarrow T_{co} = 100\ K$

Now $\dot{Q} = UA\ LMTD$

$= U \times 3.33 \times 14.42$

$\Rightarrow U = \dfrac{512600}{3.33 \times 14.42}$

$= 10669\ W/m^2K$



$LMTD = \dfrac{20 - 10}{\ln \frac{20}{10}} = 14.42$

$U_{new} = 11930\ W/m^2K$

$U_{Test} = 10669\ W/m^2K$

$\Rightarrow$ % degradation in overall HTC $= \dfrac{11930 - 10669}{\ldots} \times 100\ \%$

$U_{Test} = 10669 \ W/m^2K$

$\Rightarrow$ % degradation in overall HTC $= \dfrac{11930 - 10669}{11930} \times 100\ \%$

$= 10.56\ \%$

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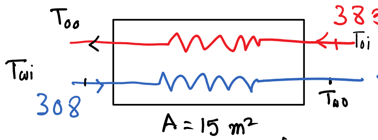
8. Water flowing at a rate of 0.667 kg/s enters a countercurrent, double-pipe heat exchanger at 308K and is heated by an oil stream entering at 383K at a rate of 2.85 kg/s (c_p=1.89 kJ/(kg K). The overall heat transfer coefficient of the heat exchanger is 300 W/(m^2 K) and the heat transfer area in the exchanger is 15.0 m^2. Calculate the heat-transfer rate and the exit water temperature.



$\dot m_o = 2.85$ kg/s, $C_p = 1.89$ kJ/kgK

$\dot m_w = 0.667$ kg/s, $C_p = 4.187$ kJ/kgK

$A = 15\ m^2$

$U = 300\ W/m^2K$

Here we do not know T_{wo} & T_{oo} !

So we cannot calculate $\dot Q$

$$\dot Q = \underset{\text{known}}{U}\ \underset{\text{known}}{A}\ \text{LMTD} \longrightarrow \text{Not known}$$

So we need to make a guess of any one temperature

Before we make a guess, we take a look at the $\dot m C_p$ products

For oil stream $(\dot m C_p)_{oil} = 2.85 \times 1.89$ kW/K $= 5.3865$ kW/K

$(\dot m C_{p\omega}) = 0.667 \times 4.187$ " $= 2.7927$ kW/K

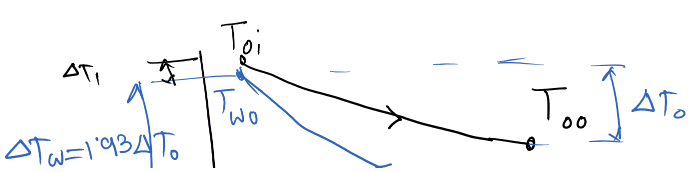
Since $(\dot m C_p)_o (T_{oi} - T_{oo}) = (\dot m C_p)_\omega (T_{wo} - T_{wi})$

$\Rightarrow \quad 5.387 (\Delta T_{oil}) = 2.793 (\Delta T_\omega)$

temp. drop of oil ↗ ↖ temp rise of water

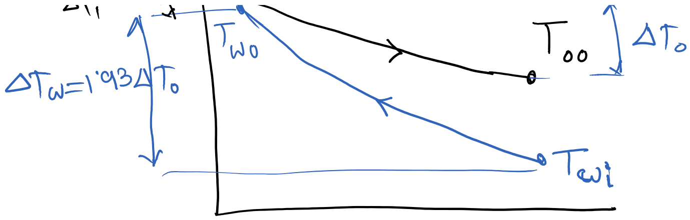
$$\Rightarrow \boxed{\Delta T_\omega = 1.93\, \Delta T_o}$$

We have a counterflow Hex. if it were an ideal counterflow Hex, we then would have seen that the water temp. rises



So it is legitimate to assume that T_{wo} is close to T_{oi}

Let us assume that at the hot end



that T_{wo} is close to T_{oi}

Let us assume that at the hot end

$\Delta T_1 = 5°C$

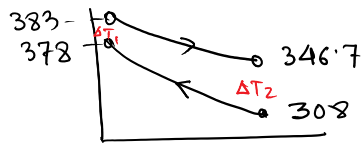
$\Rightarrow \quad T_{wo} = T_{oi} - 5 = 378°C$

$\therefore \quad \dot{Q} = 2.793 \times (378 - 308) = 195.5 \text{ kW}$

Since $\dot{Q} = (\dot{m}c_p)_{oil}\,(T_{oi} - T_{oo})$

$195.5 = 5.387\,(383 - T_{oo}) \Rightarrow T_{oo} = 346.7 \text{ K}$

So we have



$$\Rightarrow \quad LMTD = \frac{\Delta T_1 - \Delta T_2}{\ln \Delta T_1/\Delta T_2} = \frac{5 - 38.7}{\ln \frac{5}{38.7}}$$

$$= 16.47 \text{ K}$$

So with this first guess of (LMTD), we get

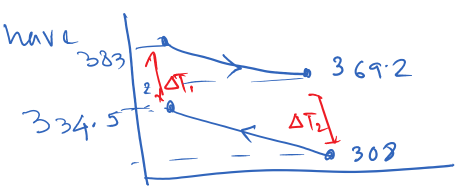
$$\dot{Q}_{(1)} = U.A.LMTD_{(1)} = \frac{300 \times 15 \times 16.47}{1000} \text{ kW} = 74.115$$

With this value of $\dot{Q}_{(1)}$, we calculate the 2nd guess of T_{wo} & T_{oo}

$74.115 = 2.793 \times (T_{wo} - 308) \Rightarrow T_{wo} = 334.5 \text{ K}$

& $74.115 = 5.387 \times (383 - T_{oo}) = 369.2$

Therefore we now have



$$lmtd_{(2)} = \frac{61.2 - 48.5}{\ln \frac{61.2}{48.5}} = 54.6$$

$\Delta T_1 = 48.5$

$\Delta T_2 = 61.2$

$$\Rightarrow \dot{Q}^{(3)} = 300 \times 15 \times 54.6 = 245.71$$

... So the solution is diverging....

Can we have a better method?

Yes! — So we adopt the ϵ-NTU method.