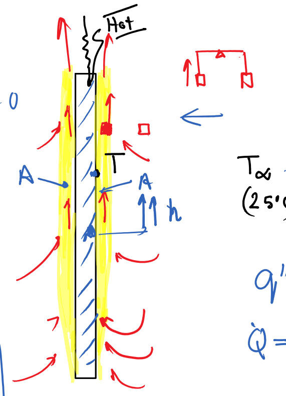
The free convection heat transfer coefficient on a thin hot vertical plate suspended in still air can be determined from observations of the change in plate temperature with time as it cools. Assuming the plate is isothermal and radiation exchange with its surroundings is negligible, evaluate the convection coefficient at the instant of time when the plate temperature is 225°C and the change in plate temperature with time (dT/dt) is −0.022 K/s. The ambient air temperature is 25°C and the plate measures 0.3 × 0.3 m with a mass of 3.75 kg and a specific heat of 2770 J/kg · K.

$\frac{\partial}{\partial x}, \frac{\partial}{\partial y} = 0$

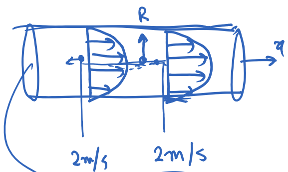


T_∞ (25°C)

$q'' = h(T - T_\infty)$ [W/m²]

$\dot{Q} = h \times (2A)(T - T_\infty)$

- uniform temp ⇒ $\frac{\partial T}{\partial x} = 0$ or $\frac{\partial T}{\partial y} = 0$,
- Steady " ⇒ $\frac{\partial T}{\partial t} = 0$ $\frac{\partial u}{\partial t} = 0$



Pitot Tube ⇒

$\frac{\partial u}{\partial x} = 0$

$\frac{\partial u}{\partial r} \neq 0$

Fully Developed Flow

Energy Balance (1st Law)

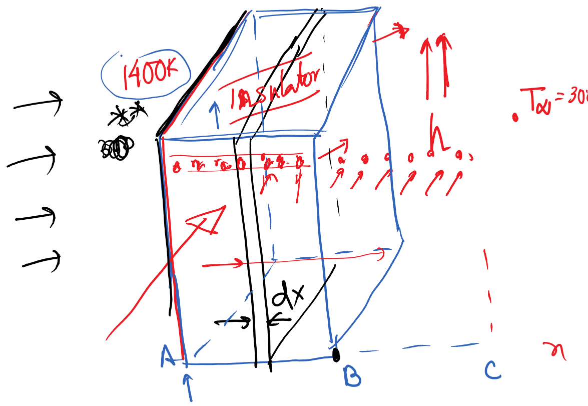
$\dot{Q} - \cancel{\dot{W}}^{0} = \frac{dE}{dt}$ $\frac{dE}{dt} = mc\frac{dT}{dt}$

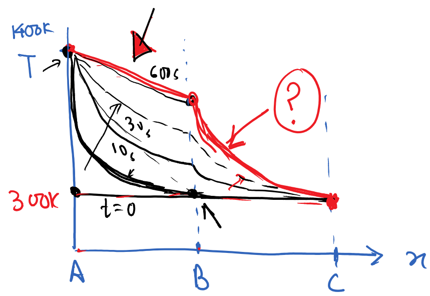
$\boxed{-h(2A)(T - T_\infty) = mc\frac{dT}{dt}}$

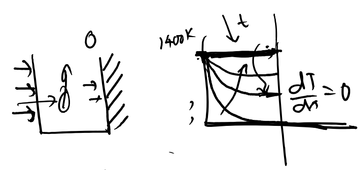
$h = -\frac{mc\frac{dT}{dt}}{2A(T - T_\infty)}$ (m = 3.75 kg, c = 2770 J/kg, $\frac{dT}{dt}$ = −0.022 K/s; A = 0.09 m², T = 225°C, T_∞ = 25°C)

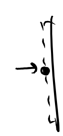
$= +\frac{3.75 \times 2770 \times 0.022}{2 \times .09 \times 200}$ $\frac{W}{m^2K}$

$=$





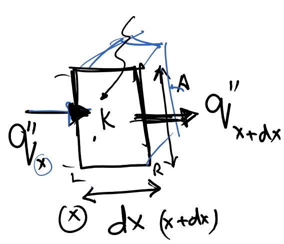




$q'' = 0$

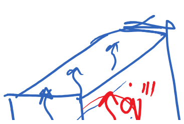
$-k\frac{dT}{dx} = 0$

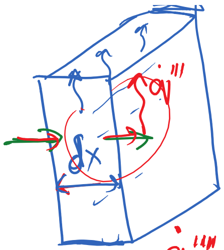
1D Generalized Heat Conduction Problem



$\delta\dot{Q} - \cancel{\delta\dot{W}} = \frac{d(\delta E)}{dt}$

$\boxed{\delta\dot{Q} = \frac{d}{dt}[\delta E]}$ — (1)





$\dot{q}''' =$ Volumetric Heat generation (W/m^3)

$$\boxed{\delta \dot{Q} = \frac{d}{dt}[\delta E]} \quad (1)$$

$$-\dot{Q}_R + \dot{Q}_L + \dot{Q}_{gen} = -q''_{x+dx} A + q''_x A + \dot{q}''' \times A\delta x$$

$$= -A\left[q''_{x+dx} - q''_x\right] + \dot{q}''' A\delta x$$

$$= -A\left[\cancel{q''_x} + \frac{\partial q''_x}{dx}\delta x - \cancel{q''_x}\right] + \dot{q}''' A\delta x$$

$$= -A\delta x \frac{\partial q''_x}{dx} + \dot{q}''' A\delta x$$

LHS of eqn (1)

$$q''_{x+dx} = q''_x + \frac{\partial}{\partial x}(q''_x)\,\delta x + \frac{1}{2}\frac{\partial^2 q''_x}{\partial x^2}(\delta x)^2 + \ldots$$

δx is small $\Rightarrow$ higher order terms may be ignored

$$q''_{x+dx} = q''_x + \frac{\partial q''_x}{\partial x}\,dx$$

$$\frac{\partial}{\partial t}[(\delta E)] = \frac{\partial}{\partial t}[\delta m c T] = \delta m c \frac{\partial T}{\partial t} = \rho c (A\delta x)\left(\frac{\partial T}{\partial t}\right)$$

RHS of (1)

Eq. 1

$$\dot{q}'''\cancel{A\delta x} - \cancel{A\delta x}\frac{\partial q''}{\partial x} = \rho c \cancel{A\delta x}\frac{\partial T}{\partial t}$$

or

$$\boxed{\rho c \frac{\partial T}{\partial t} = -\frac{\partial q''}{\partial x} + \dot{q}'''}$$

Governing eqn for 1-D heat Conduction

$$\frac{kg}{m^3}\,\frac{J}{kg\,K}\,\frac{K}{s} \quad W/m^3$$

$$\boxed{\rho c \frac{\partial T}{\partial t} = -\frac{\partial q''}{\partial x} + \dot{q}'''} \quad (2)$$

Rate of Change of internal energy per unit volume (W/m^3)

Net Heat transfer to the control volume

Volumetric Heat generation (W/m^3)

If H.T. to/from the L/R walls are due to conduction then

$$\boxed{q'' = -k\frac{\partial T}{\partial x}} \quad (3) \quad \left[\text{Fourier's Law of heat conduction}\right]$$

Substituting (3) in (2)

$$\rho c \frac{\partial T}{\partial t} = -\frac{\partial}{\partial x}\left(-k\frac{\partial T}{\partial x}\right) + \dot{q}'''$$

or

$$\boxed{\rho c \frac{\partial T}{\partial t} = \frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right) + \dot{q}'''} \quad (4) \quad \left[\frac{W}{m^3}\right]$$

Generalized 1-D Heat Conduction Eqn.

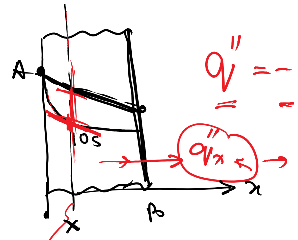
Extending the Generalized Heat Conduction Eqn. to 2D or 3D

... Heat Conduction

Extending the Generalized Heat Conduction ...

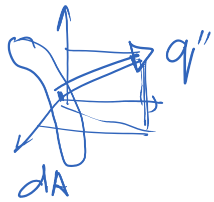
Fourier's Law of Heat Conduction

$$q''_x = -k\frac{\partial T}{\partial x}$$



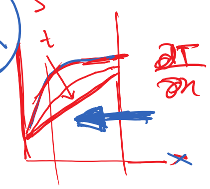
$$q''_x = -k\frac{\partial T}{\partial x}$$

$$\vec{q}'' = q''_x\hat{i} + q''_y\hat{j} + q''_z\hat{k}$$



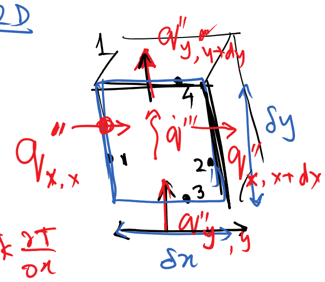
$$\vec{q}'' = -k\left(\frac{\partial T}{\partial x}\hat{i} + \frac{\partial T}{\partial y}\hat{j} + \frac{\partial T}{\partial z}\hat{k}\right)$$

$$\left.\begin{aligned} q''_x &= -k\frac{\partial T}{\partial x} \\ q''_y &= -k\frac{\partial T}{\partial y} \\ q''_z &= -k\frac{\partial T}{\partial z}\end{aligned}\right\}$$



$$\underset{-ve}{q''_x} = -k\underset{+ve}{\frac{\partial T}{\partial x}}$$

In 2D



$$q''_x = -k\frac{\partial T}{\partial x}$$

Net heat transfer $\delta\dot{Q}_{net}$

$$= \dot{Q}_1 - \dot{Q}_2 + \dot{Q}_3 - \dot{Q}_4 + \delta\dot{Q}_{gen}$$

$$= -\left[\frac{\partial}{\partial x}(q''_x)\,\delta x\right]\delta y - \left[\frac{\partial}{\partial y}(q''_y)\cdot\delta y\right]\delta x + \dot{q}'''\,\delta x\,\delta y\,1$$

$$= -\left(\frac{\partial q''_x}{\partial x} + \frac{\partial q''_y}{\partial y}\right)\delta x\,\delta y + \dot{q}'''\,\delta x\,\delta y \quad \leftarrow$$

First Law

$$\delta\dot{Q}_{net} = \frac{\partial}{\partial t}(\delta m\,cT)$$

$$\delta m = \rho c\frac{\partial T}{\partial t}\times\delta x\,\delta y\,1$$

$$\rho c\left(\frac{\partial T}{\partial t}\right)\delta x\,\delta y\,1 = -\left(\frac{\partial q''_x}{\partial x} + \frac{\partial q''_y}{\partial y}\right)\delta x\,\delta y + \dot{q}'''\,\delta x\,\delta y$$

or

$$\boxed{\rho c\frac{\partial T}{\partial t} = -\left(\frac{\partial q''_x}{\partial x} + \frac{\partial q''_y}{\partial y}\right) + \dot{q}'''}$$

$$\rightarrow \boxed{\rho c\frac{\partial T}{\partial t} = -\nabla\cdot\vec{q}'' + \dot{q}'''}$$

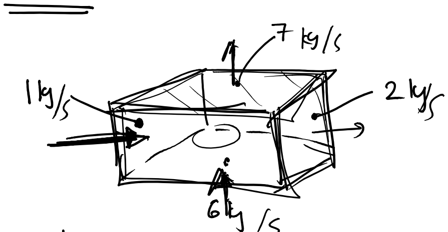
Conservation of energy in Differential form

Divergence of Any vector

$\equiv$ Net Efflux of the vector

$$\vec{q}'' = q''_x\hat{i} + q''_y\hat{j}$$

$$\Rightarrow \boxed{\vec{\nabla}\cdot\vec{q}'' = \frac{\partial q''_x}{\partial x} + \frac{\partial q''_y}{\partial y}}$$



$$\vec{\nabla}\cdot\vec{V} =$$

Net efflux of mass = mass out − mass in

$\nearrow m''$ ...

Net efflux of mass = mass out − mass in

$$\vec{\nabla}\cdot\vec{m}'' = (7+2) - (6+1) = \underline{2 \text{ kg/s}}$$

$$-\vec{\nabla}\cdot(\rho\vec{u}) = \frac{\partial \rho}{\partial t} \qquad = -\frac{dM}{dt}$$

$$\vec{m}'' = \boxed{\rho\, C_n\, d\vec{A}}$$

or $\boxed{\dfrac{\partial \rho}{\partial t} + \vec{\nabla}\cdot(\rho\vec{u}) = 0}$ ⟸ Conservation of mass

or $\cancel{\dfrac{\partial \rho}{\partial t}}^{\,0} + \rho(\vec{\nabla}\cdot\vec{u}) = 0$ For incompressible fluid

$\Rightarrow \boxed{\vec{\nabla}\cdot\vec{u} = 0}$ → Continuity Eqn.

For Conduction

$$\boxed{\vec{q}'' = -k\vec{\nabla}T}$$ Fourier's Law of heat conduction

$$\boxed{\rho C\frac{\partial T}{\partial t} = -\vec{\nabla}\cdot\vec{q}'' + \dot{q}'''}$$ — Gen. Energy eqn.

$$\boxed{\rho C\frac{\partial T}{\partial t} = +\vec{\nabla}\cdot(k\vec{\nabla}T) + \dot{q}'''}$$ Generalized Conduction eqn.

$$\boxed{\rho C\frac{\partial T}{\partial t} = \frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right) + \frac{\partial}{\partial y}\left(k\frac{\partial T}{\partial y}\right) + \frac{\partial}{\partial z}\left(k\frac{\partial T}{\partial z}\right) + \dot{q}'''}$$

→ Generalized Conduction eqn. in 3-D cartesian System

If k = Constant [i.e. $k \neq f(x), g(y)$ or $h(z)$]

$$\rho C\frac{\partial T}{\partial t} = k\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2}\right) + \dot{q}'''$$

or $$\boxed{\rho C\frac{\partial T}{\partial t} = k\underbrace{\nabla^2 T}_{\text{Laplacian}} + \dot{q}'''}$$

$$\left[\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right]$$

For steady heat conduction $\dfrac{\partial}{\partial t} = 0$

$$\Rightarrow \quad 0 = k\nabla^2 T + \dot{q}'''$$

For steady heat conduction with no volumetric heat generation

$$k\nabla^2 T = 0$$

or $\boxed{\nabla^2 T = 0}$ or $\boxed{\dfrac{\partial^2 T}{\partial x^2} + \dfrac{\partial^2 T}{\partial y^2} + \dfrac{\partial^2 T}{\partial z^2} = 0}$