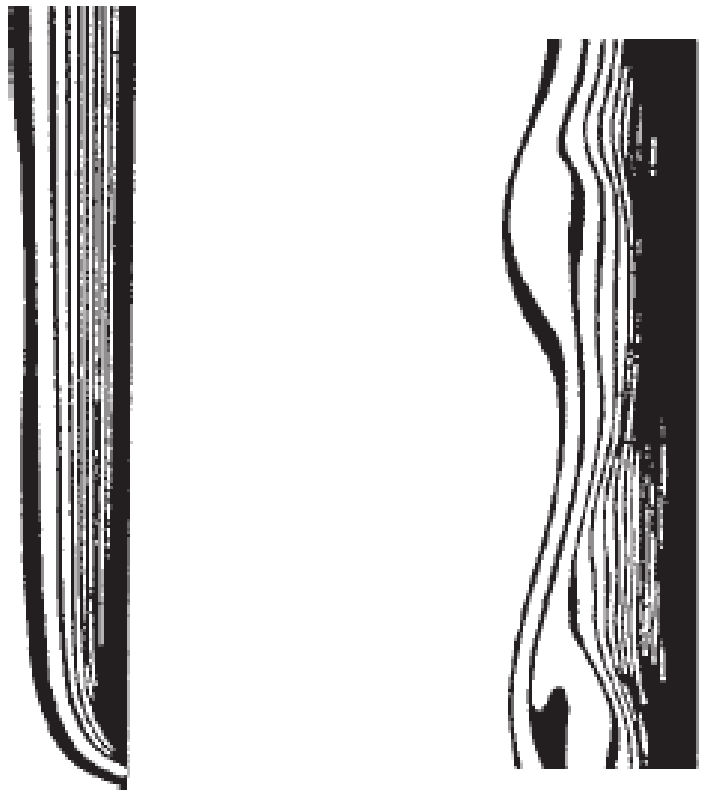
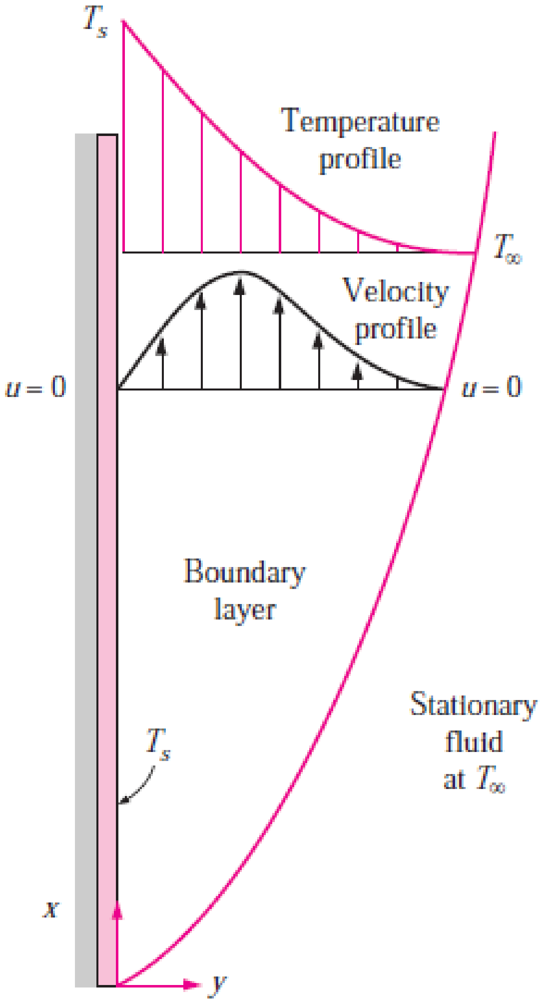
Free Convection

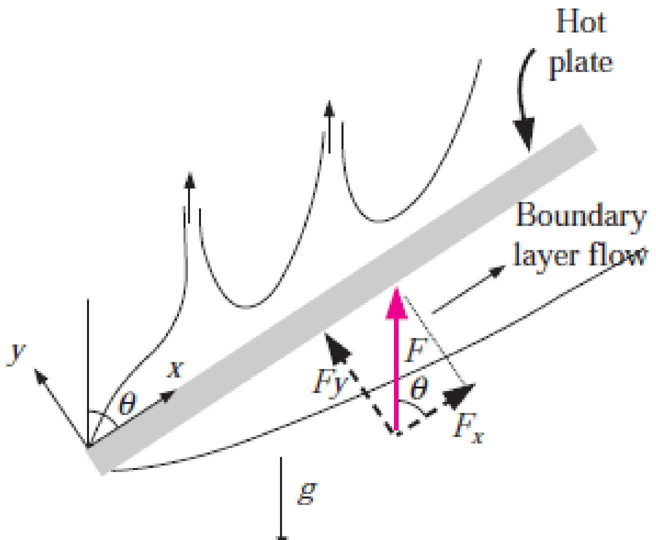
Free convection



(a) Laminar flow (b) Turbulent flow



Vertical Flat Plate



Inclined flat plate

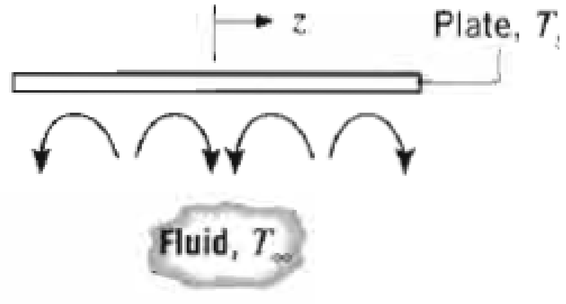


Plate facing down

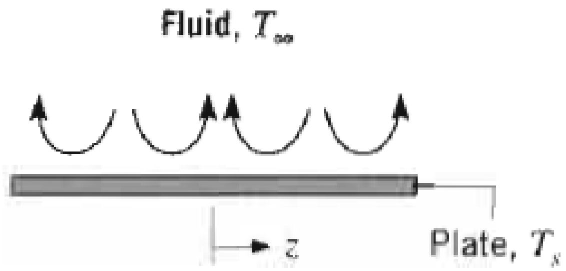
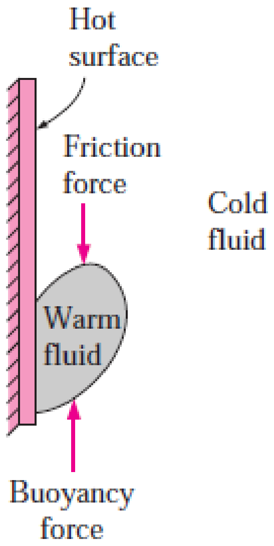


Plate facing up

Origin of free/ natural convection



Kinetic Energy: $\sim \rho U^2$

Realized from potential energy: $\sim g(\Delta\rho)L$

Within the Boundary Layer: $g(\Delta\rho)L \sim \rho U^2 \quad \Rightarrow \quad U \sim \sqrt{g\left(\frac{\Delta\rho}{\rho}\right)L}$

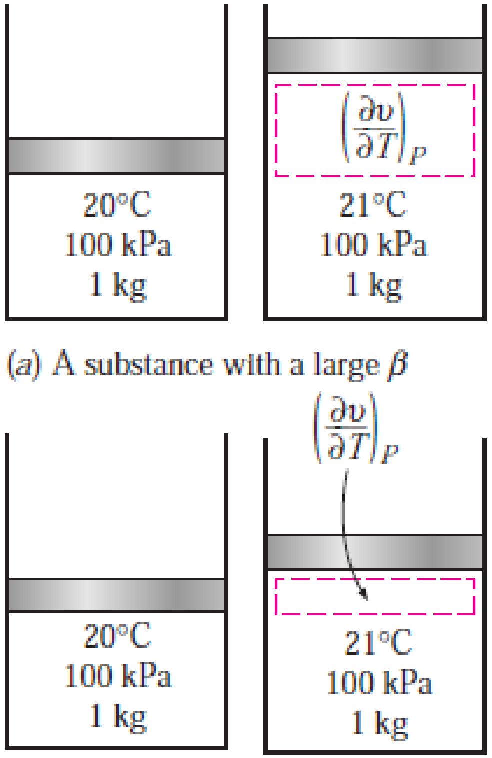
The equivalent Reynolds Number: $Re_{grav,L} \sim \frac{L\sqrt{gL}}{\nu} = \sqrt{\frac{g\left(\frac{\Delta\rho}{\rho}\right)L^3}{\nu^2}}$

… and we now expect the Nusselt Number will vary as $Nu \sim f\left(Re_{grav,L}, Pr\right)$

Grashof Number = $Re_{grav,L}{}^2$

Factors affecting the free convection

Volume expansion coefficient $$\beta = \frac{1}{\upsilon}\left(\frac{\partial \upsilon}{\partial T}\right)_P = -\frac{1}{\rho}\left(\frac{\partial \rho}{\partial T}\right)_P$$

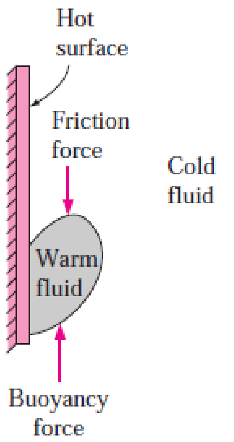


(a) A substance with a large β

(b) A substance with a small β

Grashof Number

$$Gr = \frac{g\beta\Delta T L^3}{\nu^2} = \frac{Buoyant\ Force}{Viscous\ Force}$$



g = gravitational acceleration, m/s^2

β = coefficient of volume expansion, 1/K ($\beta = 1/T$ for ideal gases)

T_s = temperature of the surface, °C

T_∞ = temperature of the fluid sufficiently far from the surface, °C

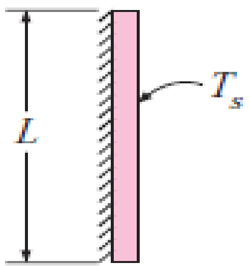
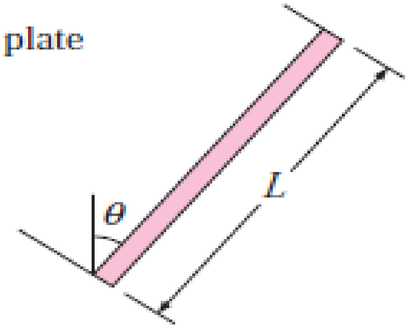
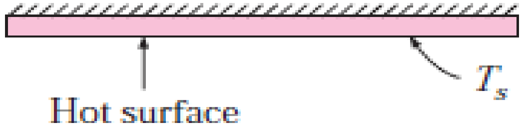
L_c = characteristic length of the geometry, m

υ = kinematic viscosity of the fluid, m^2/s

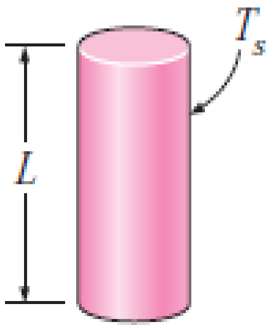
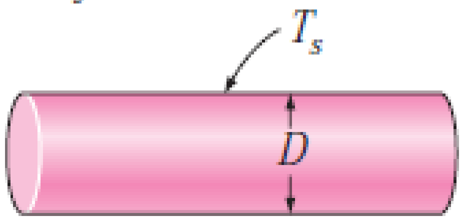
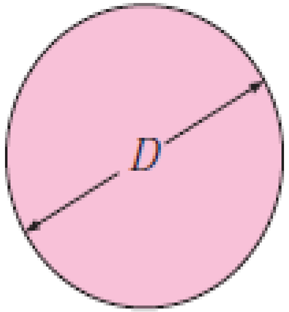
Nusselt Number Expression

$$Nu = \frac{hL_c}{k} = C\left(Gr_L \cdot \Pr\right)^n = CRa^n$$

Rayleigh Number $$Ra = \frac{g\beta\Delta T L^3}{\nu^2}\frac{\nu}{\alpha} = \frac{g\beta\Delta T L^3}{\nu\alpha}$$

Geometry	Characteristic length L_c	Range of Ra	Nu
Vertical plate 	L	10^4–10^9 10^{20}–10^{13} Entire range	$\text{Nu} = 0.59\text{Ra}_L^{1/4}$ (20–19) $\text{Nu} = 0.1\text{Ra}_L^{1/3}$ (20–20) $\text{Nu} = \left\{0.825 + \frac{0.387\text{Ra}_L^{1/6}}{[1 + (0.492/\text{Pr})^{9/16}]^{8/27}}\right\}^2$ (20–21) (complex but more accurate)
Inclined plate 	L		Use vertical plate equations for the upper surface of a cold plate and the lower surface of a hot plate Replace g by $g\cos\theta$ for $\text{Ra} < 10^9$
Horizontal plate (Surface area A and perimeter p) (*a*) Upper surface of a hot plate (or lower surface of a cold plate)  (*b*) Lower surface of a hot plate (or upper surface of a cold plate) 	A_s/p	10^4–10^7 10^7–10^{11} 10^5–10^{11}	$\text{Nu} = 0.54\text{Ra}_L^{1/4}$ (20–22) $\text{Nu} = 0.15\text{Ra}_L^{1/3}$ (20–23) $\text{Nu} = 0.27\text{Ra}_L^{1/4}$ (20–24)

Free convection correlations, continued…

Vertical cylinder 	L		A vertical cylinder can be treated as a vertical plate when $D \geq \frac{35L}{Gr_L^{1/4}}$
Horizontal cylinder 	D	$Ra_D \leq 10^{12}$	$Nu = \left\{0.6 + \frac{0.387Ra_D^{1/6}}{[1 + (0.559/Pr)^{9/16}]^{8/27}}\right\}^2$ (20–25)
Sphere 	D	$Ra_D \leq 10^{11}$ ($Pr \geq 0.7$)	$Nu = 2 + \frac{0.589Ra_D^{1/4}}{[1 + (0.469/Pr)^{9/16}]^{4/9}}$ (20–26)

Example 1

A 6-m-long section of an 8-cm-diameter horizontal hot-water pipe shown in Fig. 20–13 passes through a large room whose temperature is 20°C. If the outer surface temperature of the pipe is 70°C, determine the rate of heat loss from the pipe by natural convection.

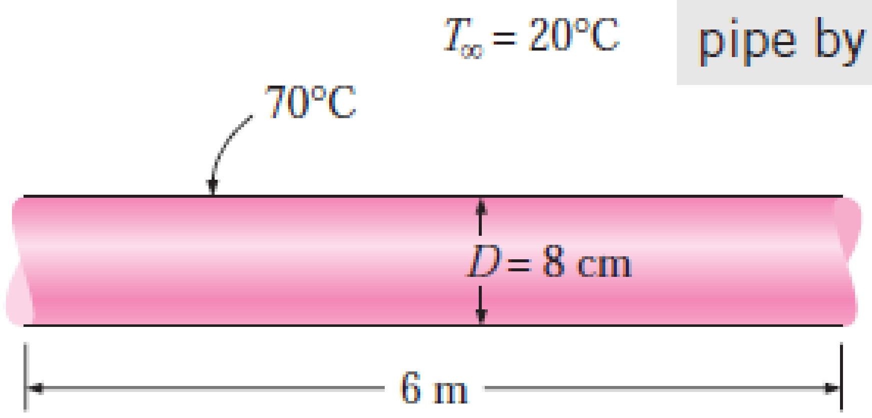
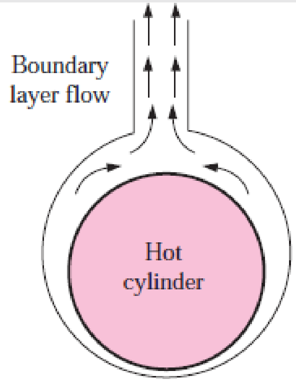


FIGURE 20–13



$$\text{Ra}_D = \frac{g\beta(T_s - T_\infty)D^3}{\upsilon^2}\text{Pr} = \frac{(9.81\text{ m/s}^2)[1/(318\text{ K})](70-20\text{ K})(0.08\text{ m})^3}{(1.749\times10^{-5}\text{ m}^2\text{/s})^2}(0.7241) = 1.869\times10^6$$

$$T_f = (T_s + T_\infty)/2 = 45°C$$

$$k = 0.02699\text{ W/m}\cdot°C \qquad \text{Pr} = 0.7241$$

$$\upsilon = 1.749\times10^{-5}\text{ m}^2\text{/s} \qquad \beta = \frac{1}{T_f} = \frac{1}{318\text{ K}}$$

$$\text{Nu} = \left\{0.6 + \frac{0.387\text{Ra}_D^{1/6}}{[1+(0.559/\text{Pr})^{9/16}]^{8/27}}\right\}^2 = \left\{0.6 + \frac{0.387(1869\times10^6)^{1/6}}{[1+(0.559/0.7241)^{9/16}]^{8/27}}\right\}^2 = 17.40$$

$$\dot{Q} = hA_s(T_s - T_\infty) = (5.869\text{ W/m}^2\cdot°C)(1.508\text{ m}^2)(70-20)°C = \mathbf{443\text{ W}}$$

$$\dot{Q}_{\text{rad}} = \varepsilon A_s\sigma(T_s^4 - T_{\text{surr}}^4) = (1)(1.508\text{ m}^2)(5.67\times10^{-8}\text{ W/m}^2\cdot\text{K}^4)[(70+273\text{ K})^4 - (20+273\text{ K})^4] = 553\text{ W}$$

Example 2

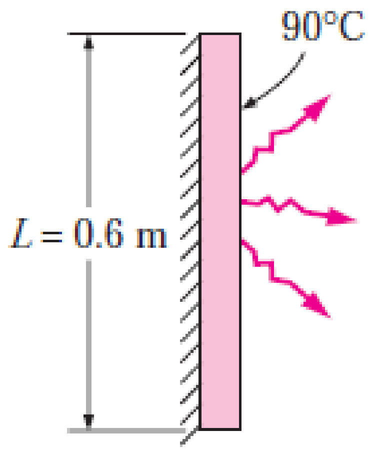
Consider a 0.6-m × 0.6-m thin square plate in a room at 30°C. One side of the plate is maintained at a temperature of 90°C, while the other side is insulated, as shown in Fig. 20–15. Determine the rate of heat transfer from the plate by natural convection if the plate is (*a*) vertical, (*b*) horizontal with hot surface facing up, and (*c*) horizontal with hot surface facing down.

$$T_f = (T_s + T_\infty)/2 = 60°\text{C}$$

$$k = 0.02808 \text{ W/m}\cdot°\text{C} \qquad \text{Pr} = 0.7202$$

$$\upsilon = 1.896 \times 10^{-5} \text{ m}^2/\text{s} \qquad \beta = \frac{1}{T_f} = \frac{1}{333 \text{ K}}$$

For vertically held plate:



(*a*) Vertical

$T_\infty = 30°\text{C}$ $L = 0.6$ m.

$$\text{Ra}_L = \frac{g\beta(T_s - T_\infty)L^3}{\upsilon^2}\text{Pr}$$

$$= \frac{(9.81 \text{ m/s}^2)[1/(333 \text{ K})](90 - 30 \text{ K})(0.6 \text{ m})^3}{(1.896 \times 10^{-5} \text{ m}^2/\text{s})^2}(0.722) = 7.656 \times 10^8$$

$$\text{Nu} = \left\{0.825 + \frac{0.387\text{Ra}_L^{1/6}}{[1 + (0.492/\text{Pr})^{9/16}]^{8/27}}\right\}^2$$

$$= \left\{0.825 + \frac{0.387(7.656 \times 10^8)^{1/6}}{1 + (0.492/0.7202)^{9/16}]^{8/27}}\right\}^2 = 113.4$$

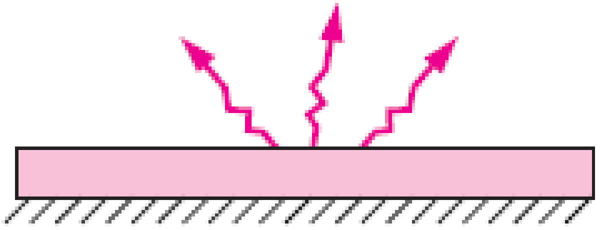
$$\dot{Q} = 115 \text{ W}$$

Example 2 (contd...)

$$L_c = \frac{A_s}{p} = \frac{L^2}{4L} = \frac{L}{4} = \frac{0.6\text{ m}}{4} = 0.15\text{ m}$$

$$\text{Ra}_L = \frac{g\beta(T_s - T_\infty)L_c^3}{\upsilon^2}\text{Pr}$$

$$= \frac{(9.81\text{ m/s}^2)[1/(333\text{ K})](90 - 30\text{ K})(0.15\text{ m})^3}{(1.896 \times 10^{-5}\text{ m}^2/\text{s})^2}(0.7202) = 1.196 \times 10^7$$

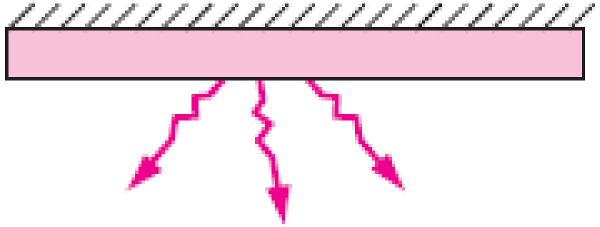


(b) Hot surface facing up

For horizontal plate facing hot side up:

$$\text{Nu} = 0.54\text{Ra}_L^{1/4} = 0.54(1.196 \times 10^7)^{1/4} = 31.76$$

$$\dot{Q} = 128\text{ W}$$



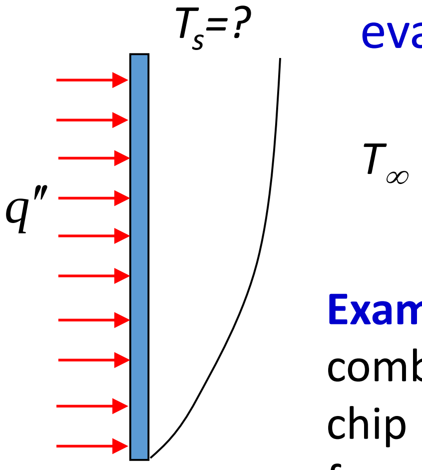
(c) Hot surface facing down

For horizontal plate facing hot side down:

$$\text{Nu} = 0.27\text{Ra}_L^{1/4} = 0.27(1.196 \times 10^7)^{1/4} = 15.86$$

$$\dot{Q} = 64.2\text{ W}$$

Modified Grashof Number for q″= constant



Since ΔT is unknown here we need iterative solution. Istead, it would help if the Gr is evaluated in term of the heat flux …

Since q″~k ΔT/L, we use $$Gr^* = \frac{g\beta\left(q''L/k\right)L^3}{\nu^2} = \frac{g\beta q''L^4}{k\nu^2}$$

Correlations with Gr* will be different, though…

Example: A 30 cm× 30 cm circuit board contains 121 square chips on one side is to be cooled by combined natural convection and radiation by mounting it on a vertical surface in a room at 25°C. Each chip dissipates 0.05 W of power, and the emissivity of the chip is 0.7. Assuming that the heat transfer from the back side of the circuit board to be negligible, and the surrounding surfaces are also at the room temperature, determine the surface temperature of the chip.

$$Nu_x = 0.60(Gr_x^* Pr)^{1/5} \quad \text{for } 10^5 < Gr_x^* Pr < 10^{11} \text{ (laminar)}$$

$$h_x \sim \frac{1}{x}(Gr_x^*)^{0.2} \sim \frac{1}{x}(x^4)^{0.2} \sim x^{-0.2}$$

$$Nu_x = 0.568(Gr_x^* Pr)^{0.22} \quad \text{for } 2 \times 10^{13} < Gr_x^* Pr < 10^{16} \text{ (turbulent)}$$

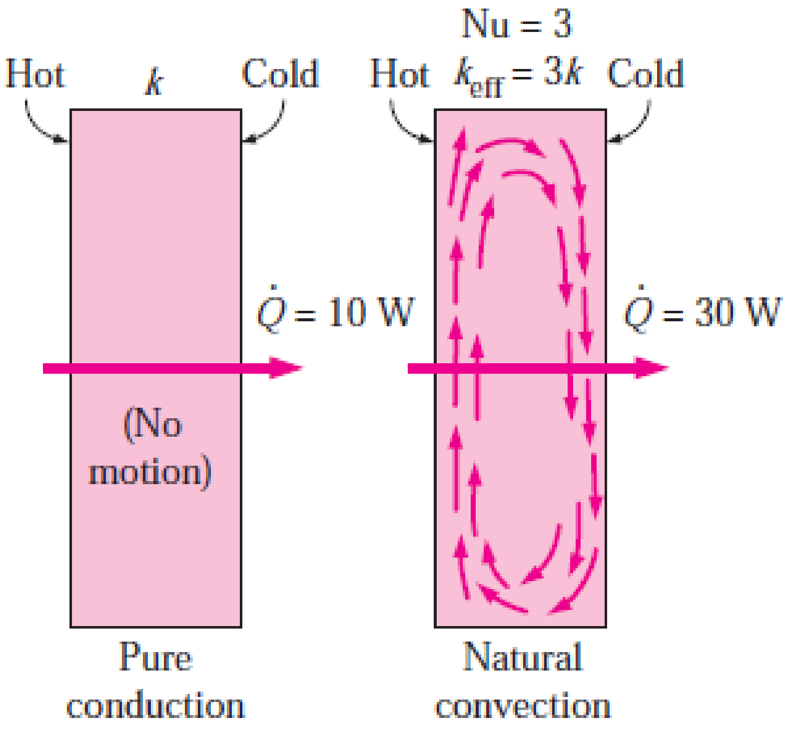
$$h_x \sim \frac{1}{x}(Gr_x^*)^{0.22} \sim \frac{1}{x}(x^4)^{0.22} \sim x^{-0.12}$$

$$Nu_m = \frac{1}{1-0.2}[Nu_x]_{x=L} = 1.25[Nu_x]_{x=L} \quad \text{for } 10^5 < Gr_x^* Pr < 10^{11}$$

$$Nu_m = \frac{1}{1-0.12}[Nu_x]_{x=L} = 1.136[Nu_x]_{x=L} \quad \text{for } 2 \times 10^{13} < Gr_x^* Pr < 10^{16}$$

Natural Convection inside Enclosures (internal flow)

$$\dot{Q} = hA_s(T_1 - T_2) = k\text{Nu}A_s\frac{T_1 - T_2}{L_c}$$

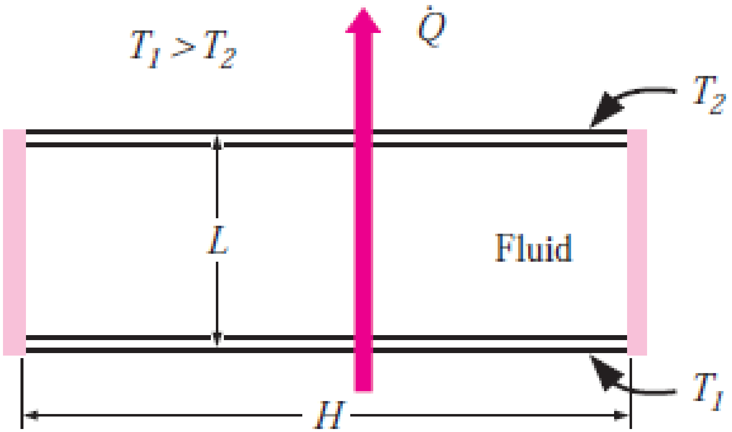


Convection in Horizontal Enclosures

Nu = 1 for hotter plate on top

Nu >1 for colder plate on top

$$\text{Ra}_L > 1708$$

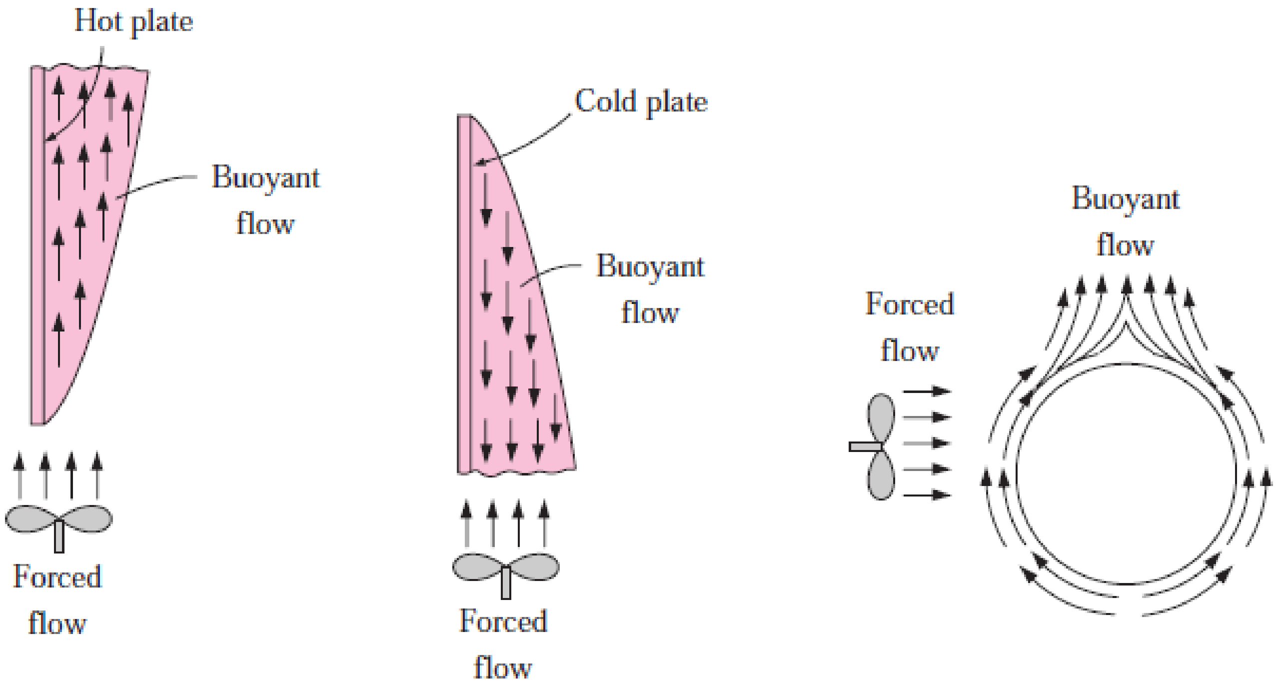


$$\text{Nu} = 0.195\text{Ra}_L^{1/4} \qquad 10^4 < \text{Ra}_L < 4 \times 10^5$$

$$\text{Nu} = 0.068\text{Ra}_L^{1/3} \qquad 4 \times 10^5 < \text{Ra}_L < 10^7$$

$$0.5 < \text{Pr} < 2$$

Mixed convection



(a) Assisting flow (b) Opposing flow (c) Transverse flow

Forced convection

$$Gr/Re^2 < 0.1$$

Free convection

$$Gr/Re^2 > 10$$

Mixed convection

$$0.1 < Gr/Re^2 < 10$$

$$Nu_{combined} = (Nu_{forced}^n \pm Nu_{natural}^n)^{1/n}$$

$$3 < n < 4$$

Mixed convection regimes