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Forced Convection through pipes and ducts

(1) Laminar vs. turbulent flow

- Flow through tubes, transition Reynolds number Re_{D_t} is

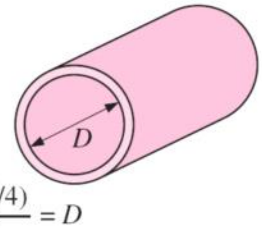
(2) Entrance vs. fully developed region

- Classification based on velocity and temperature profiles:
  - (i) Entrance region
  - (ii) Fully developed region

(3) Surface boundary conditions

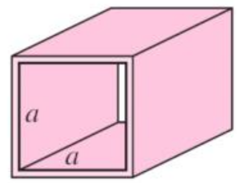
- Two common boundary conditions::
  - (i) Uniform surface temperature
  - (ii) Uniform surface heat flux

Circular tube:



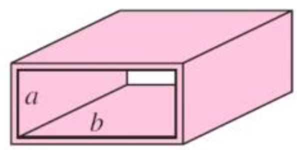
$$D_h = \frac{4(\pi D^2/4)}{\pi D} = D$$

Square duct:



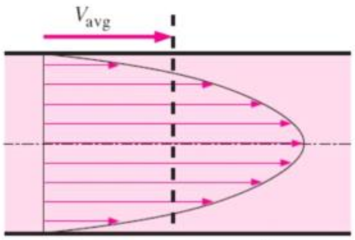
$$D_h = \frac{4a^2}{4a} = a$$

Rectangular duct:



$$D_h = \frac{4ab}{2(a+b)} = \frac{2ab}{a+b}$$

Average velocity and Reynolds Number



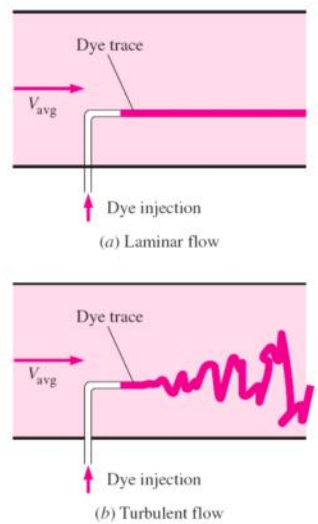
$$Re = \frac{\text{Inertial forces}}{\text{Viscous forces}} = \frac{V_{avg}D}{\nu} = \frac{\rho V_{avg}D}{\mu}$$

$$V_{avg} = \frac{\int_{A_c} \rho u(r)\, dA_c}{\rho A_c} = \frac{\int_0^R \rho u(r) 2\pi r\, dr}{\rho \pi R^2} = \frac{2}{R^2}\int_0^R u(r) r\, dr$$

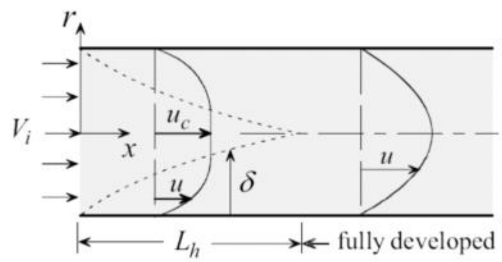
$Re \lesssim 2300$ laminar flow

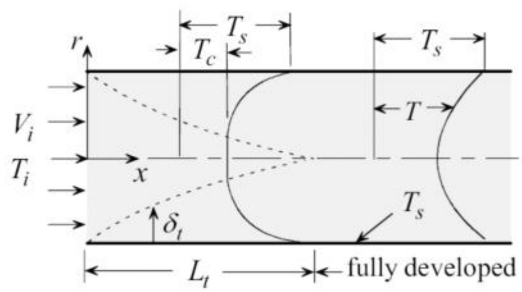
$2300 \lesssim Re \lesssim 4000$ transitional flow

$Re \gtrsim 4000$ turbulent flow



Hydrodynamic and thermal entry regions





Flow Field

(1) Entrance Region (Developing Flow, $0 \le x \le L_h$).

- Name: *hydrodynamic entrance region.*
- Length: L_h (*hydrodynamic entrance length*).
- Streamlines are not parallel.
- Core velocity u_c increases with distance
- Pressure decreases with distance ($dp/dx < 0$).
- $\delta < D/2$.

(2) Fully Developed Flow Region. $x \ge L_h$

- Streamlines are parallel ($v_r = 0$).
- $\partial u / \partial x = 0$ for two-dimensional incompressible fluid.

Temperature Field

(1) Entrance Region (Developing Temperature, $0 \le x \le L_t$)

- Name: *Thermal entrance region*.
- Length: L_t (*thermal entrance length*).
- Core temperature T_c is uniform, $T_c = T_i$.
- $\delta_t < D/2$

(2) Fully Developed Temperature Region. $x \ge L_t$

- Temperature varies radially and axially, $\partial T / \partial x \neq 0$.

Hydrodynamic and thermal entry lengths

Hydrodynamic Entrance Length L_h

- Starting with external flow result

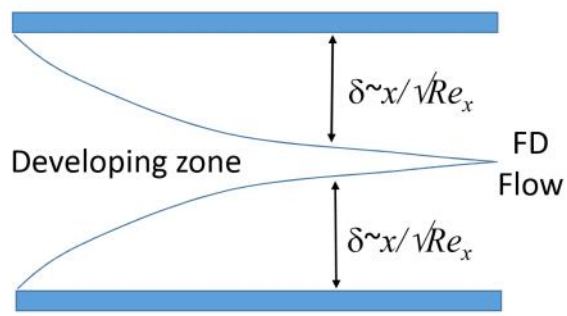
$$\frac{\delta}{x} \sim \frac{1}{\sqrt{Re_x}}$$

at $x = L_h$:

$$\delta \sim D \text{ and } Re_{L_h} = Re_D \frac{L_h}{D} \;\Rightarrow\; \left(\frac{L_h / D}{Re_D}\right)^{1/2} \sim 1$$

Thermal Entrance Length L_t

$$\frac{L_t}{L_h} \sim Pr$$



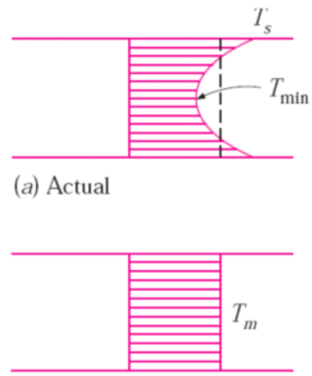
Concept of bulk mean temperature

Energy flow through a particular section:

$$\dot{E}_{\text{fluid}} = \dot{m}C_p T_m = \int_{\dot{m}} C_p T \delta \dot{m} = \int_{A_c} \rho C_p T \mathcal{V} \, dA_c$$

Bulk Mean Temperature:

$$T_m = \frac{\int_{\dot{m}} C_p T \delta \dot{m}}{\dot{m} C_p} = \frac{\int_0^R C_p T(\rho \mathcal{V} 2\pi r \, dr)}{\rho \mathcal{V}_m (\pi R^2) C_p} = \frac{2}{\mathcal{V}_m R^2} \int_0^R T(r, x) \, \mathcal{V}(r, x) \, r \, dr$$



(a) Actual

(b) Idealized

Nondimensional Temperature

$$\theta(x, r) = \frac{[T_s(x) - T(x, r)]}{[T_s(x) - T_m(x)]}$$

θ=0 at $r=R$

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Hydrodynamically & Thermally Fully developed flow

Hydrodynamically fully developed: $\frac{\partial \mathcal{V}(r,x)}{\partial x} = 0 \longrightarrow \mathcal{V} = \mathcal{V}(r)$

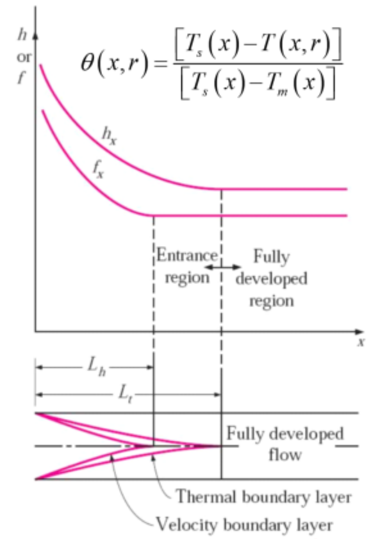
Thermally fully developed: $\frac{d\{\theta(x,y)\}}{dx} = \frac{\partial}{\partial x}\left[\frac{T_s(x) - T(r,x)}{T_s(x) - T_m(x)}\right] = 0$

$$\frac{\partial}{\partial r}\left(\frac{T_s - T}{T_s - T_m}\right)\Bigg|_{r=R} = \frac{-(\partial T/\partial r)\big|_{r=R}}{T_s - T_m} \neq f(x)$$

Now, we define the local heat transfer coefficient h_x as

$$\dot{q}_s = h_x(T_s - T_m) = k\frac{\partial T}{\partial r}\Bigg|_{r=R} \longrightarrow h_x = \frac{k(\partial T/\partial r)\big|_{r=R}}{T_s - T_m}$$

Therefore, for TFD flow, *h_x does not vary with x*



So, for T.F.D. flow, we don't need to worry about the Local HTC

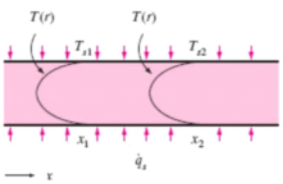
Thermally fully developed (TFD) flow: q" = constant

$$\dot{m}C_p dT_m = \dot{q}_s(pdx) \quad \rightarrow \quad \frac{dT_m}{dx} = \frac{\dot{q}_s p}{\dot{m}C_p} = constant \qquad \frac{dT_m}{dx} = \frac{dT_s}{dx}$$

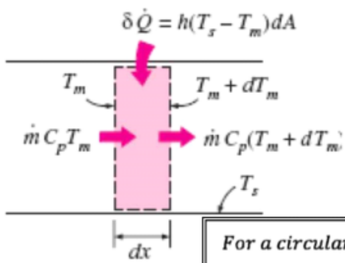
For a fully developed temperature profile,

$$\frac{\partial}{\partial x}\left(\frac{T_s - T}{T_s - T_m}\right) = 0 \quad \rightarrow \quad \frac{1}{T_s - T_m}\left(\frac{\partial T_s}{\partial x} - \frac{\partial T}{\partial x}\right) = 0 \quad \rightarrow \quad \frac{\partial T}{\partial x} = \frac{dT_s}{dx}$$

$$\frac{\partial T}{\partial x} = \frac{dT_s}{dx} = \frac{dT_m}{dx} = \frac{\dot{q}_s p}{\dot{m}C_p} = constant$$



The shape of the temperature profile remains unchanged in the fully developed region of a tube subjected to constant surface heat flux



For a circular tube, $\quad p = 2\pi R \quad$ *and* $\quad \dot{m} = \rho V_m A_c = \rho V_m(\pi R^2)$

$$\frac{\partial T}{\partial x} = \frac{dT_s}{dx} = \frac{dT_m}{dx} = \frac{2\dot{q}_s}{\rho V_m C_p R} = constant$$

TFD flow: constant wall heat flux case (contd...)

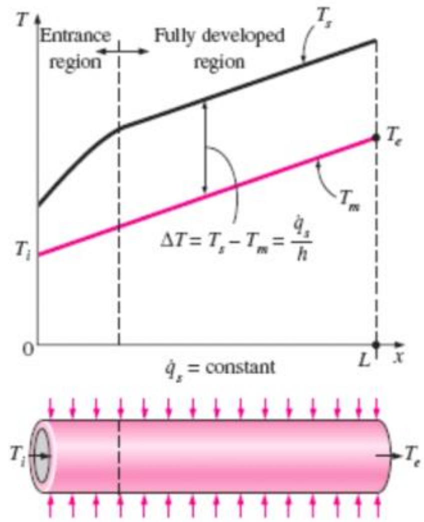
$$\dot{Q} = \dot{q}_s A_s = \dot{m} C_p (T_e - T_i)$$

$$T_e = T_i + \frac{\dot{q}_s A_s}{\dot{m} C_p}$$

$$\dot{q}_s = h(T_s - T_m) \quad \rightarrow \quad T_s = T_m + \frac{\dot{q}_s}{h}$$

In fully developed region:
(as h is constant)

$$\frac{dT_m}{dx} = \frac{dT_s}{dx}$$



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TFD flow: T_s = constant

Energy balance gives:

$$\dot{m}C_p dT_m = h(T_s - T_m)dA_s$$

$$dT_m = -d(T_s - T_m)$$ ← constant

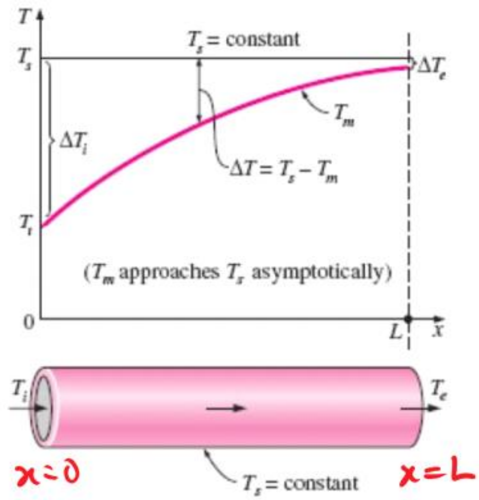
$$\frac{d(T_s - T_m)}{T_s - T_m} = -\frac{hp}{\dot{m}C_p}dx$$

Integrating from x = 0 to x = L

$$\ln\frac{T_s - T_e}{T_s - T_i} = -\frac{hA_s}{\dot{m}C_p} \qquad A_s = pL$$

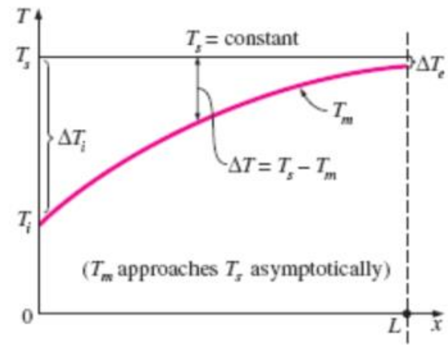
$$T_e = T_s - (T_s - T_i)\exp(-hA_s/\dot{m}C_p)$$

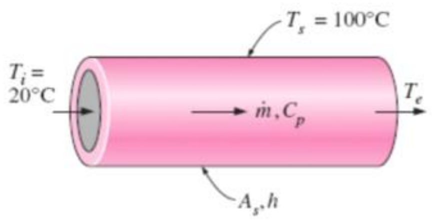
Possible to find out temperature at any x by replacing A_s = px



TFD flow: T_s = C (contd…)

Note that the temperature difference between the fluid and the surface decays exponentially in the flow direction, and the rate of decay depends on the magnitude of the exponent $hA_x / m.C_p$



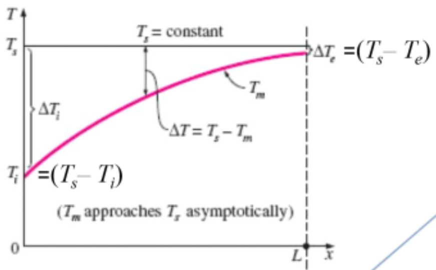


NTU = $hA_s / \dot{m}C_p$	T_e, °C
0.01	20.8
0.05	23.9
0.10	27.6
0.50	51.5
1.00	70.6
5.00	99.5
10.00	100.0

This dimensionless parameter is called the number of transfer units, denoted by NTU, and is a measure of the effectiveness of the heat transfer systems

TFD flow: T_s = C (contd…):

Logarithmic mean temperature difference (LMTD = ΔT_{ln})



$$\dot{m} C_p = -\frac{hA_s}{\ln[(T_s - T_e)/(T_s - T_i)]}$$

$$\dot{Q} = \dot{q}_s A_s = \dot{m} C_p (T_e - T_i)$$

At the same time, $\dot{Q} = hA_s \Delta T_{\ln}$

$$(T_i - T_e) = (T_s - T_e) - (T_s - T_i)$$

$$\ln \frac{T_s - T_e}{T_s - T_i} = -\frac{hA_s}{\dot{m} C_p}$$

$$\Delta T_{\ln} = \frac{T_i - T_e}{\ln[(T_s - T_e)/(T_s - T_i)]} = \frac{\Delta T_e - \Delta T_i}{\ln(\Delta T_e / \Delta T_i)}$$

* We will see more about *LMTD* and *NTU* later on, while discussing Heat Exchangers

How long does the entrance length persist?

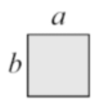
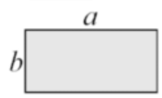
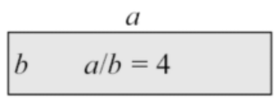
$$\frac{L_h}{D_e} = C_h Re_{De}$$

$$\frac{L_t}{D_e} = C_t Pr Re_D$$

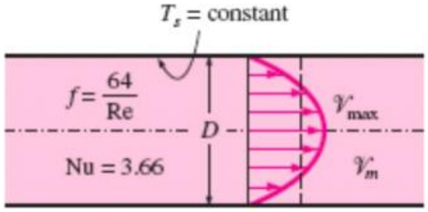
Turbulent flow: $L = L_h = L_t$

$$\frac{L}{D} \approx 10$$

Laminar Entrance length coefficients C_h and C_t [1]

geometry	C_h	C_t	
		uniform surface flux	uniform surface temperature
	0.056	0.043	0.033
 $a/b = 1$	0.09	0.066	0.041
 $a/b = 2$	0.085	0.057	0.049
 $a/b = 4$	0.075	0.042	0.054
	0.011	0.012	0.008

Nusselt number for fully developed pipe flows



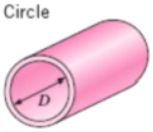
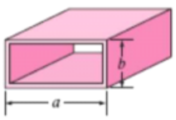
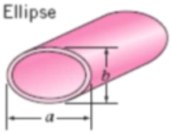
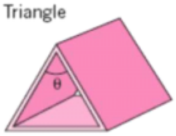
Fully developed laminar flow

A similar analysis can be performed for fully developed laminar flow in a circular tube for the case of constant surface temperature T_s. The solution procedure in this case is more complex as it requires iterations, but the Nusselt number relation obtained is equally simple

Circular tube, laminar (T_s = constant): $$Nu = \frac{hD}{k} = 3.66$$

(q" = constant) $$Nu = \frac{hD}{k} = 4.36$$

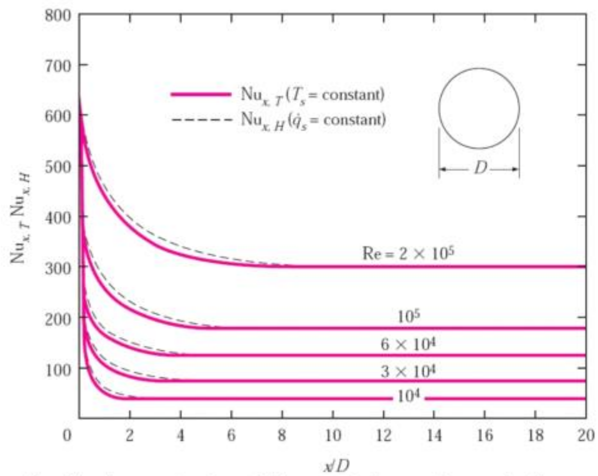
Nusselt number and friction factor for fully developed laminar flow in tubes of various cross sections ($D_h = 4A_c/p$, $Re = \mathcal{V}_m D_h/\nu$, and $Nu = hD_h/k$)

Tube Geometry	a/b or θ°	Nusselt Number		Friction Factor f
		T_s = Const.	$\dot{q}_s$ = Const.	
Circle 	—	3.66	4.36	64.00/Re
Rectangle 	a/b			
	1	2.98	3.61	56.92/Re
	2	3.39	4.12	62.20/Re
	3	3.96	4.79	68.36/Re
	4	4.44	5.33	72.92/Re
	6	5.14	6.05	78.80/Re
	8	5.60	6.49	82.32/Re
	∞	7.54	8.24	96.00/Re
Ellipse 	a/b			
	1	3.66	4.36	64.00/Re
	2	3.74	4.56	67.28/Re
	4	3.79	4.88	72.96/Re
	8	3.72	5.09	76.60/Re
	16	3.65	5.18	78.16/Re
Triangle 	θ			
	10°	1.61	2.45	50.80/Re
	30°	2.26	2.91	52.28/Re
	60°	2.47	3.11	53.32/Re
	90°	2.34	2.98	52.60/Re
	120°	2.00	2.68	50.96/Re

Laminar Flows

Nu and friction factor for non-circular tubes

Variation of local Nusselt number along a tube in turbulent flow



Note: beyond the entry region, the Nu is constant and does not depend on whether constant heat flux or T_s boundary condition prevails

For turbulent flows

For fully developed turbulent flow in smooth tubes, a simple relation for the Nusselt number can be obtained by substituting the simple power law relation $f = 0.184\ Re^{-0.2}$ for the friction factor into

$$Nu = 0.125\, f\, Re Pr^{1/3}$$

$$Nu = 0.023\ Re^{0.8}\ Pr^{1/3} \qquad \begin{pmatrix} 0.7 \le Pr \le 160 \\ Re > 10{,}000 \end{pmatrix}$$

The accuracy of this equation can be improved by modifying it as

$$Nu = 0.023\ Re^{0.8}\ Pr^{n}$$

Remember!

where n = 0.4 for heating and 0.3 for cooling of the fluid flowing through the tube. This equation is known as the Dittus–Boelter equation [Dittus and Boelter (1930),

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Other correlations

Gnielinski (1976) (More accurate)

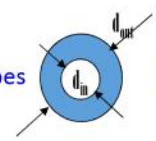
$$\mathrm{Nu} = \frac{(f/8)(\mathrm{Re} - 1000)\,\mathrm{Pr}}{1 + 12.7(f/8)^{0.5}(\mathrm{Pr}^{2/3} - 1)} \qquad \begin{pmatrix} 0.5 \le \mathrm{Pr} \le 2000 \\ 3 \times 10^3 < \mathrm{Re} < 5 \times 10^6 \end{pmatrix}$$

Liquid metals, T_s = constant: $\mathrm{Nu} = 4.8 + 0.0156\,\mathrm{Re}^{0.85}\,\mathrm{Pr}_s^{0.93}$

Liquid metals, $\dot{q}_s$ = constant: $\mathrm{Nu} = 6.3 + 0.0167\,\mathrm{Re}^{0.85}\,\mathrm{Pr}_s^{0.93}$

All the turbulent equations are applicable for non-circular ducts: Replace D by D_h [= 4A/P]

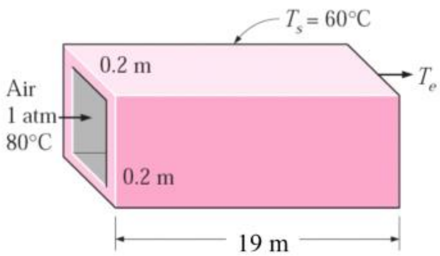
Flow through concentric tubes



D_h = Annular area / wetter perimeter = $(d_{out} - d_{in})$

Example

Hot air at atmospheric pressure and 80°C enters an 19-m-long uninsulated square duct of cross section 0.2 m × 0.2 m that passes through the attic of a house at a rate of 0.15 m³/s (Fig. 19–37). The duct is observed to be nearly isothermal at 60°C. Determine the exit temperature of the air and the rate of heat loss from the duct to the attic space.



How to calculate the Bulk mean temperature?

$$\rho = 0.9994 \text{ kg/m}^3 \qquad C_p = 1008 \text{ J/kg}\cdot°\text{C}$$
$$k = 0.02953 \text{ W/m}\cdot°\text{C} \qquad \text{Pr} = 0.7154$$
$$\upsilon = 2.097 \times 10^{-5} \text{ m}^2\text{/s}$$

$$L_h \approx L_t \approx 10D = 10 \times 0.2 \text{ m} = 2 \text{ m}$$

We can consider FD flow

$$D_h = \frac{4A_c}{p} = \frac{4a^2}{4a} = a = 0.2 \text{ m}$$

$$\mathcal{V}_m = \frac{\dot{V}}{A_c} = \frac{0.15 \text{ m}^3\text{/s}}{(0.2 \text{ m})^2} = 3.75 \text{ m/s}$$

$$\text{Re} = \frac{\mathcal{V}_m D_h}{\upsilon} = \frac{(3.75 \text{ m/s})(0.2 \text{ m})}{2.097 \times 10^{-5} \text{ m}^2\text{/s}} = 35{,}765$$

flow is turbulent

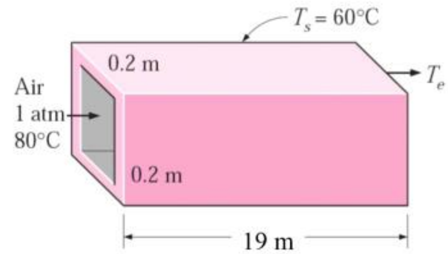
$$\text{Nu} = \frac{hD_h}{k} = 0.023 \text{ Re}^{0.8} \text{ Pr}^{0.3} = 0.023(35{,}765)^{0.8} (0.7154)^{0.3} = 91.4$$

Example (contd...)

$$h = \frac{k}{D_h}\,\text{Nu} = \frac{0.02953\ \text{W/m}\cdot{}^\circ\text{C}}{0.2\ \text{m}}\,(91.4) = 13.5\ \text{W/m}^2\cdot{}^\circ\text{C}$$

$$A_s = pL = 4aL = 4\times(0.2\ \text{m})(19\ \text{m}) = 15.2\ \text{m}^2$$

$$\dot{m} = \rho\dot{V} = (1.009\ \text{kg/m}^3)(0.15\ \text{m}^3/\text{s}) = 0.151\ \text{kg/s}$$



$$T_e = T_s - (T_s - T_i)\exp(-hA_s/\dot{m}C_p)$$

$$= 60^\circ\text{C} - [(60-80)^\circ\text{C}]\exp\left[-\frac{(13.5\ \text{W/m}^2\cdot{}^\circ\text{C})(15.2\text{m}^2)}{(0.151\ \text{kg/s})(1008\ \text{J/kg}\cdot{}^\circ\text{C})}\right]$$

$$= 65.1\ {}^\circ\text{C}$$

$$\Delta T_{\ln} = \frac{T_i - T_e}{\ln\frac{T_s - T_e}{T_s - T_i}} = \frac{80 - 65.1}{\ln\frac{60-65.1}{60-80}} = -10.9\ {}^\circ\text{C}$$

$$\dot{Q} = hA_s\Delta T_{\ln} = (13.5\ \text{W/m}^2\cdot{}^\circ\text{C})(15.2\text{m}^2)(-10.9\ {}^\circ\text{C}) = -2237.4\text{W}$$

Alternately:

$$\dot{Q} = \dot{m}C_p(80 - 65.1) = 2268W$$

Mismatch in the two results is less than 1.5%