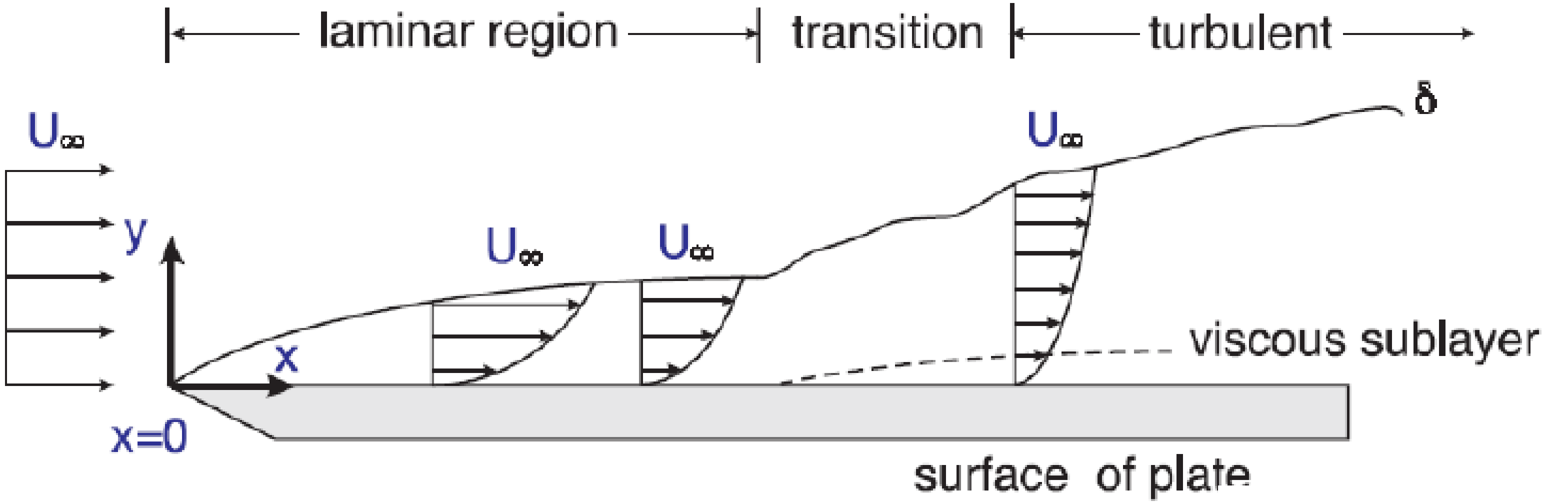
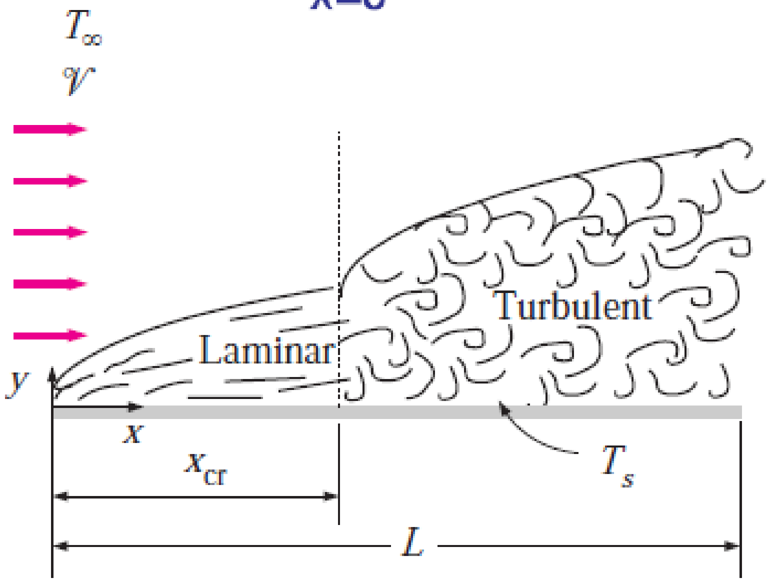
Forced Convection: External Flows

HTC for External Flow over a flat plate





Local Reynolds Number $\mathrm{Re}_x = \frac{\rho \mathcal{V} x}{\mu} = \frac{\mathcal{V} x}{\upsilon}$

Critical Re $\mathrm{Re}_{cr} = \frac{\rho \mathcal{V} x_{cr}}{\mu} = 5 \times 10^5$

$10^5 < \mathrm{Re}_{cr} < 3\times10^6$ depending on the surface roughness and flow disturbances

$$Nu = \frac{hL}{k} = C\,\mathrm{Re}_L^m\,\mathrm{Pr}^n$$

Properties evaluated at $T_f = \frac{T_s + T_\infty}{2}$ Film Temperature

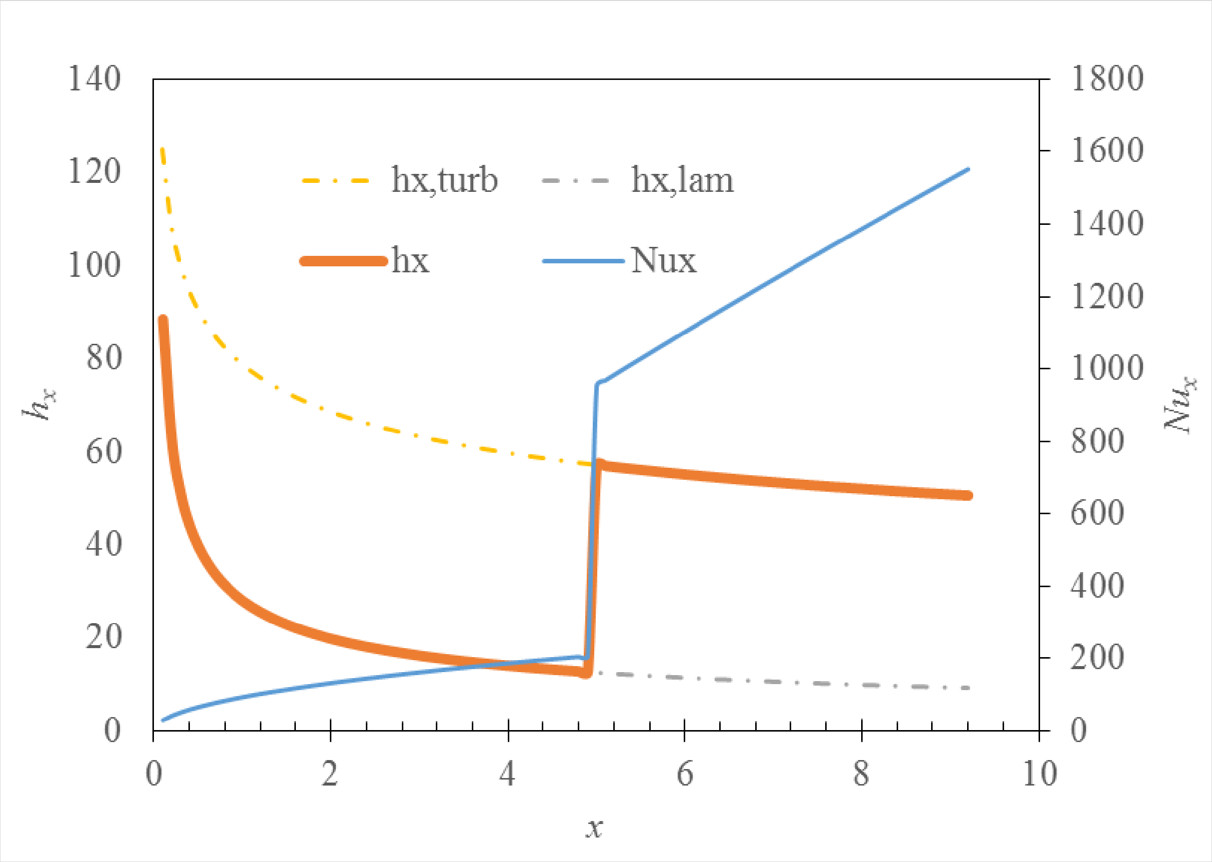
Local and average HTC and Nu for flat plates (Constant Wall Temperature)

	Laminar	Turbulent
Local	$\mathrm{Nu}_x = \frac{h_x x}{k} = 0.332\,\mathrm{Re}_x^{0.5}\,\mathrm{Pr}^{1/3}$ $\quad \mathrm{Pr} > 0.60$ $C_{f,x} = \frac{0.664}{\mathrm{Re}_x^{1/2}}$ $\delta = 5xRe_x^{-0.5}$; $\delta_t = \delta \cdot \mathrm{Pr}^{-1/3}$	$\mathrm{Nu}_x = \frac{h_x x}{k} = 0.0296\,\mathrm{Re}_x^{0.8}\,\mathrm{Pr}^{1/3}$ $\quad 0.6 \le \mathrm{Pr} \le 60$, $5 \times 10^5 \le \mathrm{Re}_x \le 10^7$ $C_{f,x} = \frac{0.0592}{\mathrm{Re}_x^{1/5}}$ $\delta = 0.37xRe_x^{-0.2}$; $\delta_t = \delta \cdot \mathrm{Pr}^{-1/3}$
Average	$\overline{Nu}_L = \frac{\bar{h}L}{k} = 0.664\,\mathrm{Re}_L^{1/2}\,\mathrm{Pr}^{1/3}$ $\quad \mathrm{Pr} \ge 0.6$ $\overline{C_f} = \frac{1.328}{\mathrm{Re}_L^{1/2}}$	$\overline{Nu}_L = \frac{\bar{h}L}{k} = 0.037\,\mathrm{Re}_x^{4/5}\,\mathrm{Pr}^{1/3}$ $\quad 0.6 \le \mathrm{Pr} \le 60$ $\quad 5\times10^5 \le \mathrm{Re}_L \le 10^7$ $\overline{C_f} = \frac{0.074}{\mathrm{Re}_L^{1/5}}$ $\quad 5\times10^5 \le \mathrm{Re}_L \le 10^7$

If $Nu_x = Px^m$, then the average Nusselt Number is

$$\overline{Nu_x} = \int_0^x \frac{Nu_x}{x} dx = \int_0^x \frac{Px^m}{x} dx = \frac{Px^m}{m} = \frac{1}{m} Nu_x$$

Laminar versus turbulent convection



U=0.1 m/s
ν=10^-6 m²/s
Pr=0.7
k= 0.03 W/mK

$$\mathrm{Nu}_x = \frac{h_x x}{k} = 0.332\,\mathrm{Re}_x^{0.5}\,\mathrm{Pr}^{1/3} \qquad \mathrm{Pr} > 0.60$$

$$\mathrm{Nu}_x = \frac{h_x x}{k} = 0.0296\,\mathrm{Re}_x^{0.8}\,\mathrm{Pr}^{1/3} \qquad \begin{array}{l} 0.6 \le \mathrm{Pr} \le 60 \\ 5 \times 10^5 \le \mathrm{Re}_x \le 10^7 \end{array}$$

Combined Laminar and Turbulent flow

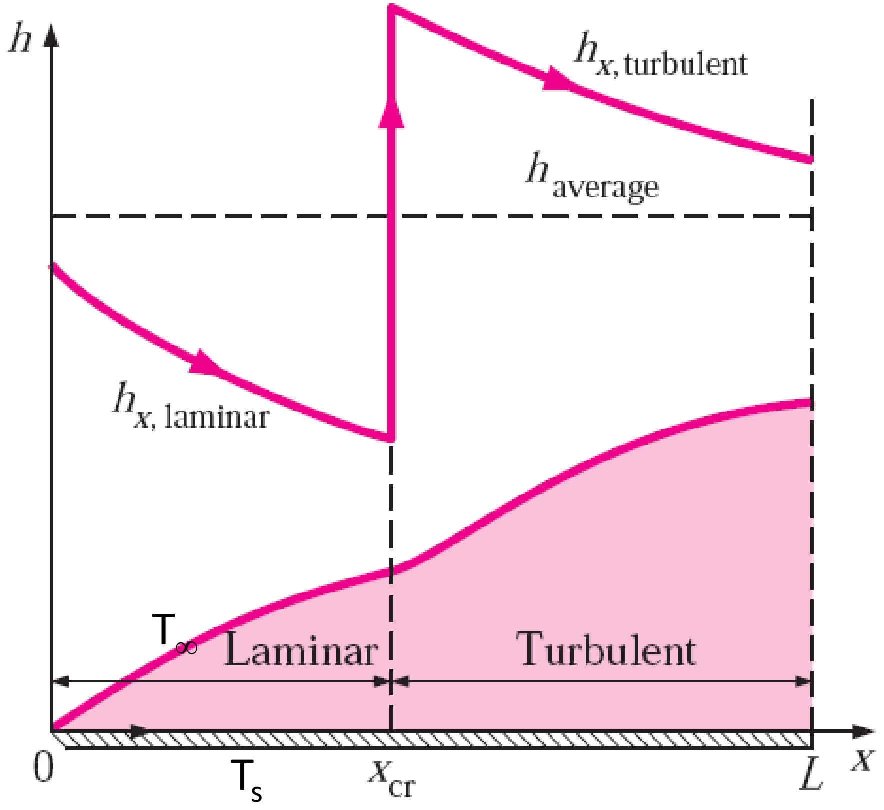
$$\overline{C_f} = \frac{1}{L}\left(\int_0^{x_{cr}} C_{f,x,Laminar}\,dx + \int_{x_{cr}}^{L} C_{f,x,Turbulent}\,dx\right)$$

$$\overline{h} = \frac{1}{L}\left(\int_0^{x_{cr}} h_{x,Laminar}\,dx + \int_{x_{cr}}^{L} h_{,x,Turbulent}\,dx\right)$$

$$\overline{Nu} = \frac{\overline{h}L}{k} = \left(0.037\,\mathrm{Re}_x^{4/5} - 871\right)\mathrm{Pr}^{1/3} \qquad 0.6 \le \mathrm{Pr} \le 60$$

$$5\times10^5 \le \mathrm{Re}_L \le 10^7$$

$$\overline{C_f} = \frac{0.074}{\mathrm{Re}_L^{1/5}} - \frac{1742}{\mathrm{Re}_L} \qquad 5\times10^5 \le \mathrm{Re}_L \le 10^7$$

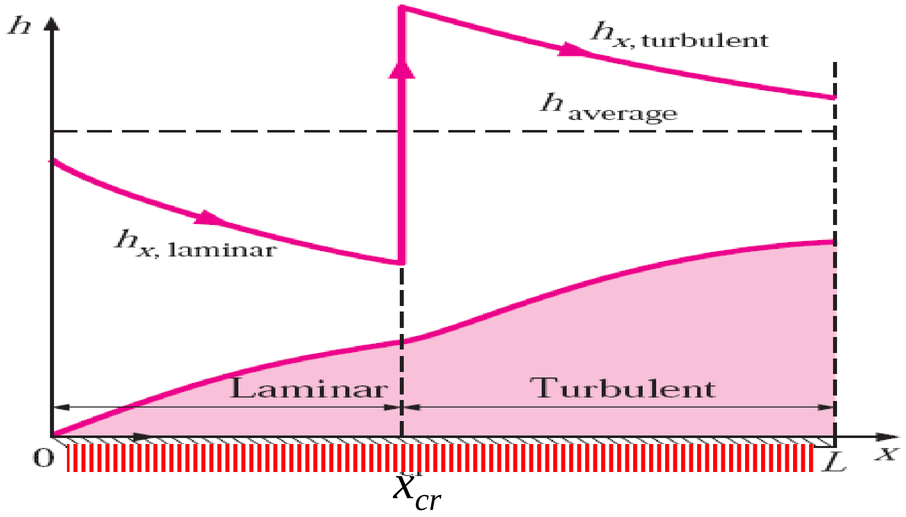


HTC correlations for other conditions

Constant Wall Heat Flux Condition

$$Nu_x = \frac{hx}{k} = 0.453\,\mathrm{Re}_x^{0.5}\,\mathrm{Pr}^{1/3} \quad \text{Laminar (isoflux plate)}$$

$$Nu_x = \frac{hx}{k} = 0.0308\,\mathrm{Re}_x^{0.8}\,\mathrm{Pr}^{1/3} \quad \text{Turbulent (isoflux plate)}$$

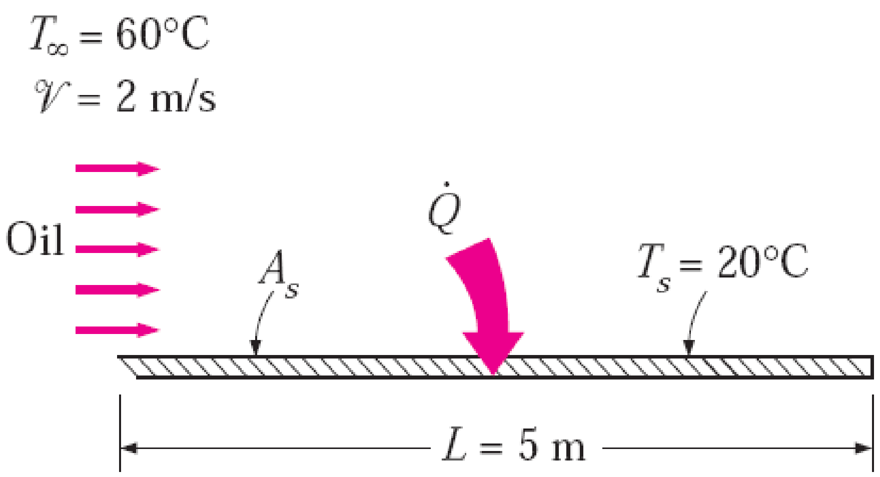


Low Pr fluids (Liquid Metals)

$$\mathrm{Nu}_x = 0.565(\mathrm{Re}_x\,\mathrm{Pr})^{1/2} \qquad \mathrm{Pr} < 0.05$$

Example

Engine oil at 60°C flows over the upper surface of a 5-m-long flat plate whose temperature is 20°C with a velocity of 2 m/s (Fig. 19–12). Determine the rate of heat transfer per unit width of the entire plate.



Film Temperature $T_f = (T_s + T_\infty)/2 = (20 + 60)/2 = 40°C$

Properties at Film Temperature

$\rho = 876 \text{ kg/m}^3$ $\qquad$ $\Pr = 2870$

$k = 0.144 \text{ W/m} \cdot °C$ $\qquad$ $\upsilon = 242 \times 10^{-6} \text{ m}^2/\text{s}$

Flow Regime? $L = 5$ m

$$\text{Re}_L = \frac{\mathcal{V}L}{\upsilon} = \frac{(2 \text{ m/s})(5 \text{ m})}{0.242 \times 10^{-5} \text{ m}^2/\text{s}} = 4.13 \times 10^4$$

Laminar

Nu Correlation

$$\text{Nu} = \frac{hL}{k} = 0.664 \text{ Re}_L^{0.5} \Pr^{1/3} = 0.664 \times (4.13 \times 10^4)^{0.5} \times 2870^{1/3} = 1918$$

HTC

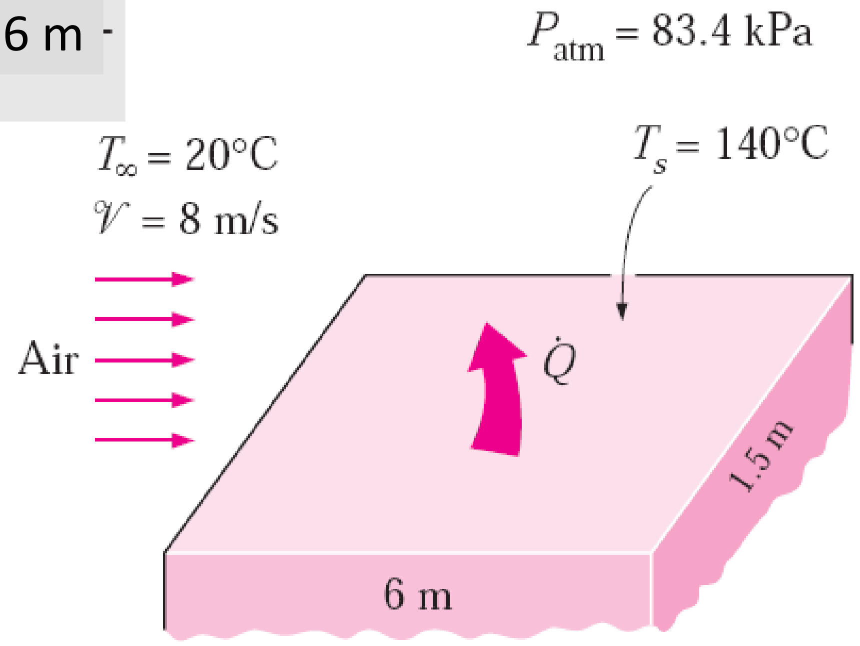
$$h = \frac{k}{L}\text{Nu} = \frac{0.144 \text{ W/m} \cdot °C}{5 \text{ m}}(1918) = 55.2 \text{ W/m}^2 \cdot °C$$

Total Heat Transfer

$$\dot{Q} = hA_s(T_\infty - T_s) = (55.2 \text{ W/m}^2 \cdot °C)(5 \times 1 \text{ m}^2)(60 - 20)°C = \mathbf{11{,}040 \text{ W}}$$

Example

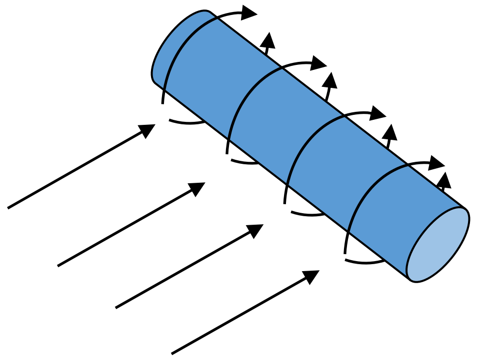
The local atmospheric pressure in Denver, Colorado (elevation 1610 m), is 83.4 kPa. Air at this pressure and 20°C flows with a velocity of 8 m/s over a 1.5-m × 6-m flat plate whose temperature is 140°C (Fig. 19–13). Determine the rate of heat transfer from the plate if the air flows parallel to the (*a*) 6 m -long side and (*b*) the 1.5-m side.

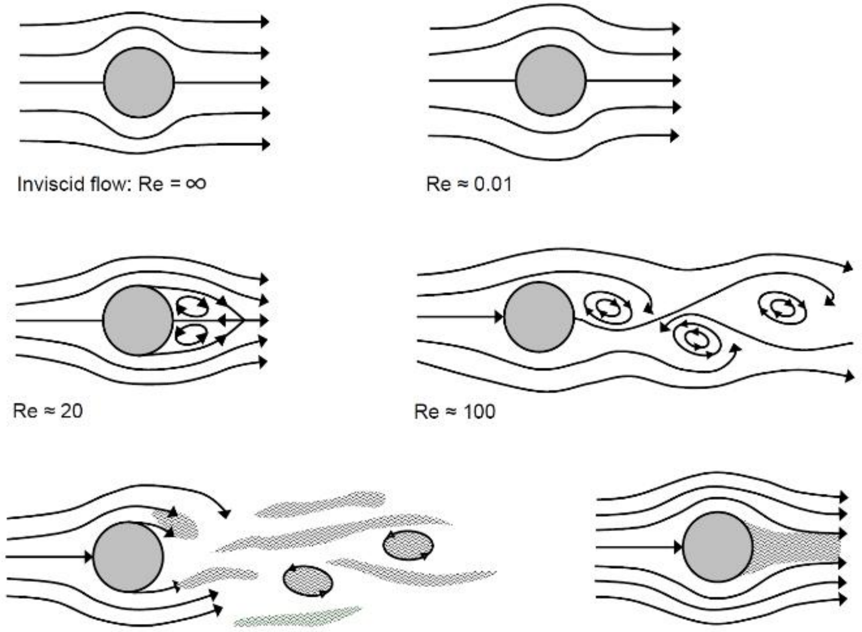


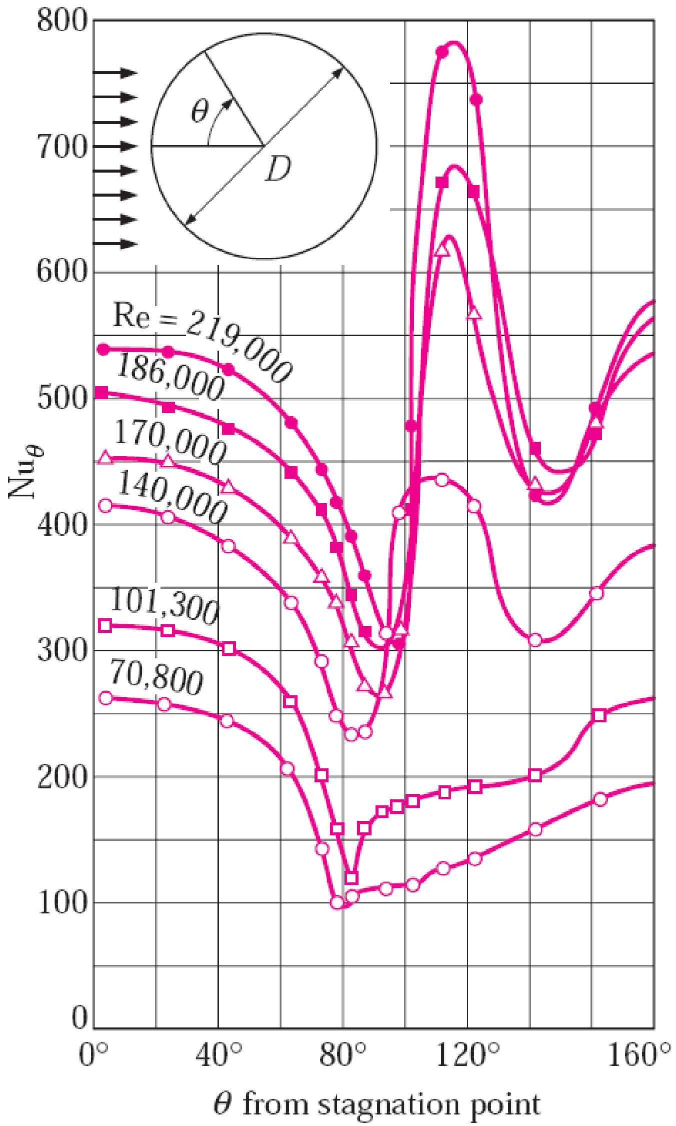
$T_f = (T_s + T_\infty)/2 = 80°C$

Hint: Kinematic viscosity varies inversely with density

External flows over cylinders





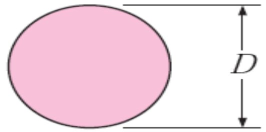
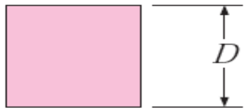
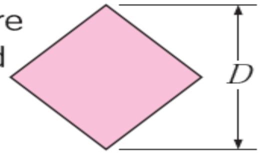
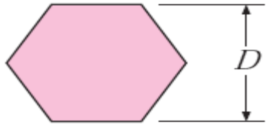
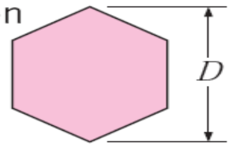
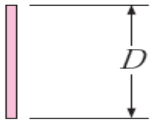
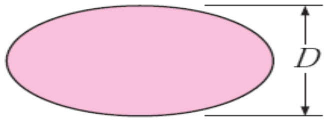


$$Nu_{cyl} = \frac{hD}{k} = 0.3 + \frac{0.62\, Re^{1/2}\, Pr^{1/3}}{[1 + (0.4/Pr)^{2/3}]^{1/4}}\left[1 + \left(\frac{Re}{282{,}000}\right)^{5/8}\right]^{4/5} \qquad Re\ Pr > 0.2$$

General correlations: All shapes

Compact Form $$\mathrm{Nu}_{cyl} = \frac{hD}{k} = C\,\mathrm{Re}^m\,\mathrm{Pr}^n$$

$$n = \frac{1}{3}$$

Cross section of the cylinder	Fluid	Range of Re	Nusselt number
Circle 	Gas or liquid	0.4–4 4–40 40–4000 4000–40,000 40,000–400,000	$\mathrm{Nu} = 0.989\mathrm{Re}^{0.330}\,\mathrm{Pr}^{1/3}$ $\mathrm{Nu} = 0.911\mathrm{Re}^{0.385}\,\mathrm{Pr}^{1/3}$ $\mathrm{Nu} = 0.683\mathrm{Re}^{0.466}\,\mathrm{Pr}^{1/3}$ $\mathrm{Nu} = 0.193\mathrm{Re}^{0.618}\,\mathrm{Pr}^{1/3}$ $\mathrm{Nu} = 0.027\mathrm{Re}^{0.805}\,\mathrm{Pr}^{1/3}$
Square 	Gas	5000–100,000	$\mathrm{Nu} = 0.102\mathrm{Re}^{0.675}\,\mathrm{Pr}^{1/3}$
Square (tilted 45°) 	Gas	5000–100,000	$\mathrm{Nu} = 0.246\mathrm{Re}^{0.588}\,\mathrm{Pr}^{1/3}$
Hexagon 	Gas	5000–100,000	$\mathrm{Nu} = 0.153\mathrm{Re}^{0.638}\,\mathrm{Pr}^{1/3}$
Hexagon (tilted 45°) 	Gas	5000–19,500 19,500–100,000	$\mathrm{Nu} = 0.160\mathrm{Re}^{0.638}\,\mathrm{Pr}^{1/3}$ $\mathrm{Nu} = 0.0385\mathrm{Re}^{0.782}\,\mathrm{Pr}^{1/3}$
Vertical plate 	Gas	4000–15,000	$\mathrm{Nu} = 0.228\mathrm{Re}^{0.731}\,\mathrm{Pr}^{1/3}$
Ellipse 	Gas	2500–15,000	$\mathrm{Nu} = 0.248\mathrm{Re}^{0.612}\,\mathrm{Pr}^{1/3}$

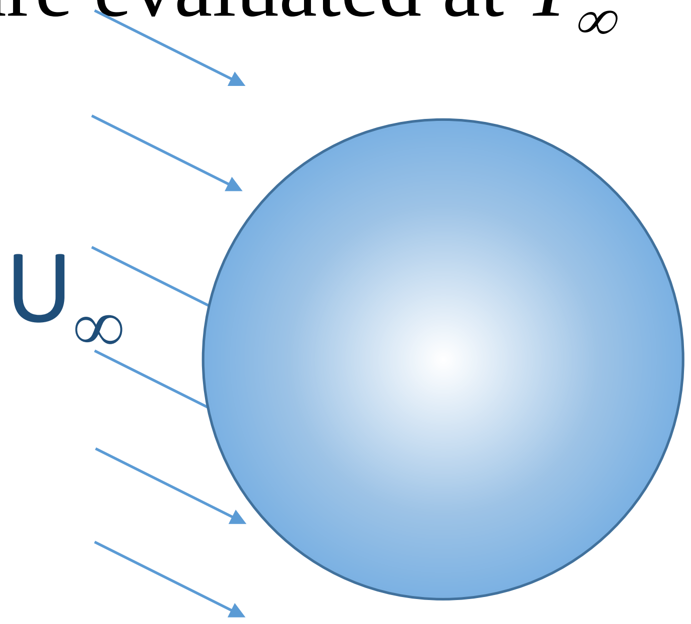
External flows over Spheres

- Expression of Nu: $\mathrm{Nu}_{\mathrm{sph}} = \frac{hD}{k} = 2 + [0.4\,\mathrm{Re}^{1/2} + 0.06\,\mathrm{Re}^{2/3}]\,\mathrm{Pr}^{0.4}\left(\frac{\mu_\infty}{\mu_s}\right)^{1/4}$

valid for $3.5 \leq \mathrm{Re} \leq 80{,}000$ and $0.7 \leq \mathrm{Pr} \leq 380$.

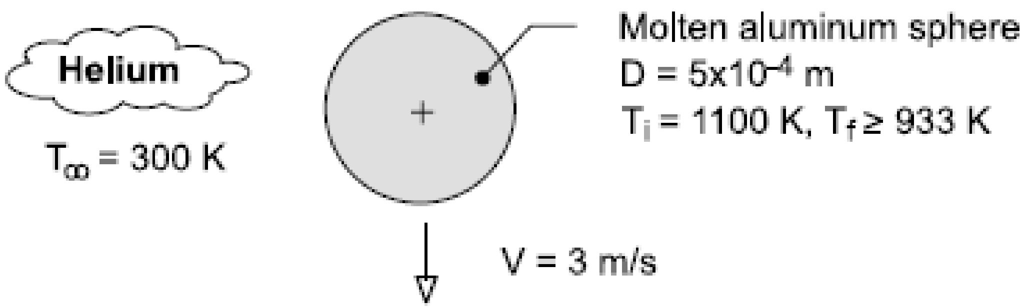
Fluid properties (except μ_s) are evaluated at T_∞

For U_∞=0, Nu = 2.0
(Minimum Nu over a sphere in a quiescent flow)



HW: Obtain the same expression from steady heat conduction equations

Cooling of a sphere



Calculate time required to cool from 1100 to 933 K

Assumptions: (i) Lumped capacitance model valid (?)
(ii) Negligible radiation
(iii) Constant properties

$$t = \frac{\rho \forall c}{\bar{h} A_s} \ln \frac{\theta_i}{\theta} = \frac{\rho c D}{6\bar{h}} \ln \frac{T_i - T_\infty}{T_f - T_\infty}$$

$$Bi = \bar{h}(D/6)/k = 4 \times 10^{-4}$$

$$t = \frac{(2500\,kg/m^3)1200\,J/kg\cdot K(0.0005m)}{6 \times 975\,W/m^2\cdot K} \ln\left(\frac{800}{633}\right) = 0.06\,s$$

What is the average HTC?

Properties

Helium $(T_\infty = 300K): \nu = 122 \times 10^{-6} m^2/s, \;\; \mu = 199 \times 10^{-7} N\cdot s/m^2$

$k = 0.152\,W/m\cdot K, \;\; Pr = 0.68.$

Helium $(T_s = 1000K): \mu_s = 446 \times 10^{-7} N\cdot s/m^2$

Aluminum:

$\rho = 2500\,kg/m^3, \; c = 1200\,J/kg\cdot K, \; k = 200\,W/m\cdot K.$

Reynolds number

$$Re_D = VD/\nu = 3\,m/s\left(5 \times 10^{-4} m\right)/122 \times 10^{-6} m^2/s = 12.3$$

Heat Transfer Correlation (Whitekar)

$$Nu_{sph} = \frac{hD}{k} = 2 + [0.4\,Re^{1/2} + 0.06\,Re^{2/3}]\,Pr^{0.4}\left(\frac{\mu_\infty}{\mu_s}\right)^{1/4}$$

$$\bar{h} = 975\,W/m^2\cdot K$$