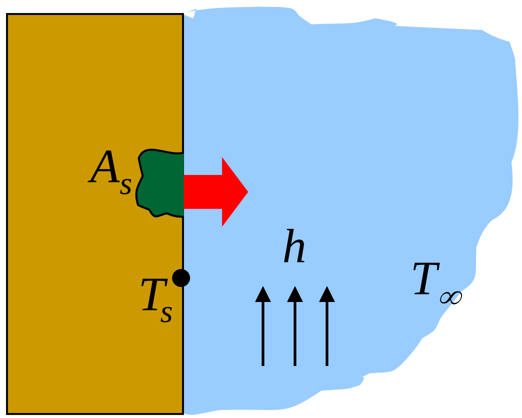
Heat Transfer from Extended Surfaces: Fins

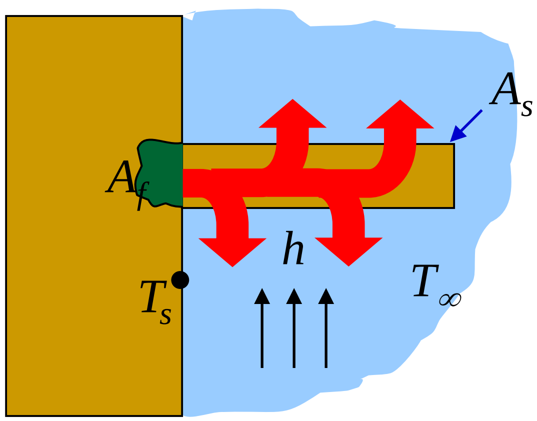
Ranjan Ganguly

Strategies of increasing heat transfer from a surface

$$\dot{Q}_{conv} = hA_s(T_s - T_\infty)$$

- Increase the temperature difference $(T_s - T_\infty)$ [may lead to surface overheating]
- Increase the heat transfer coefficient h [there is always a practical upper bound*]
- Increase area A_s



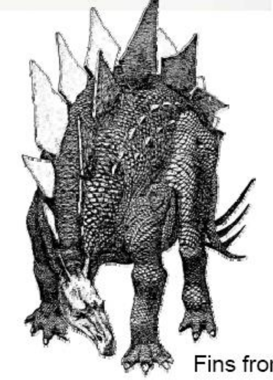


But the fin surface temperature is not T_s (because the fin material has a finite *k*)

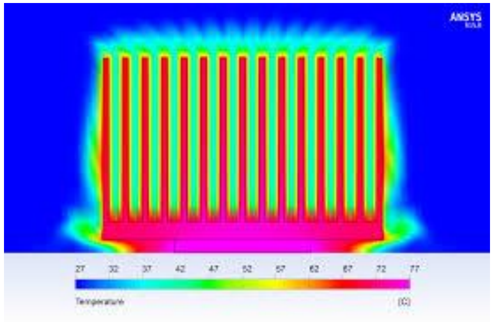
[* Would require installing a larger fan/ pump, would entail larger pressure drop, may not be economically viable]

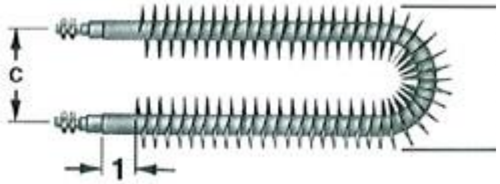
Fins used in engineering applications

- Electronics cooling
- Heat exchanger tubes
- Car radiators
- Engine cylinders
- AC…

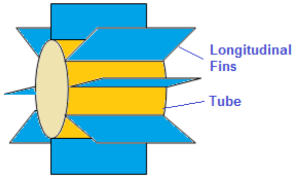


Fins from nature



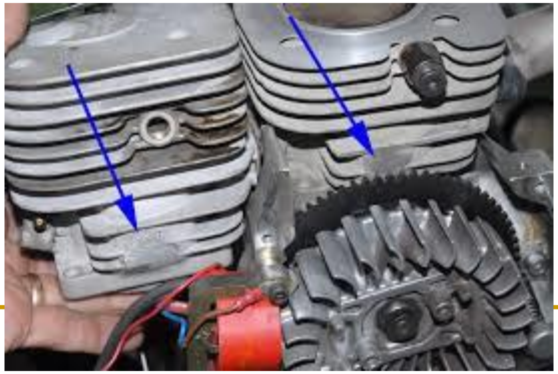


Radial fins



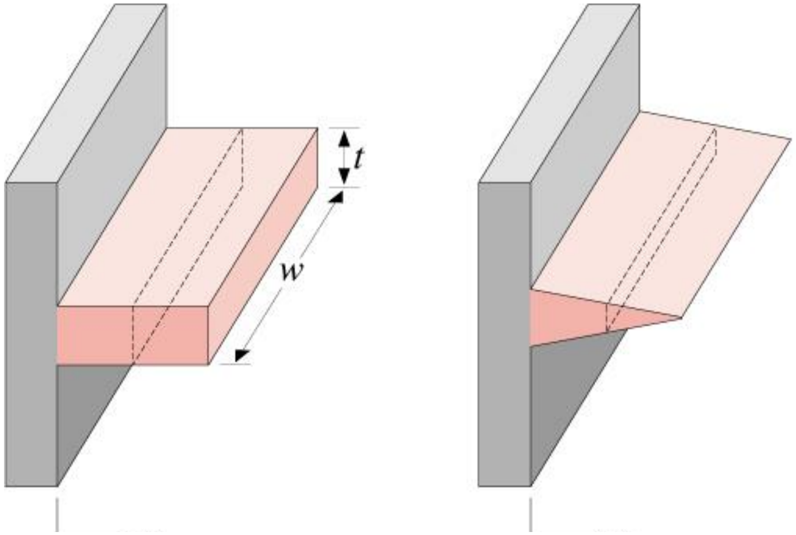
Longitudinal fins

Fins in car radiators

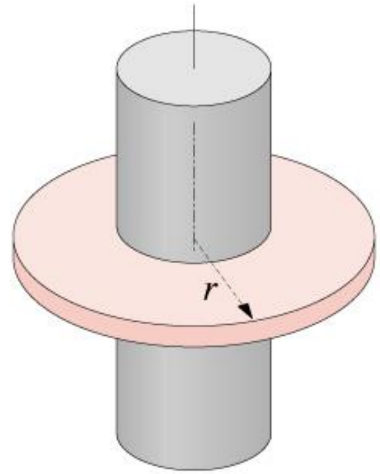


Fins on engine cylinders

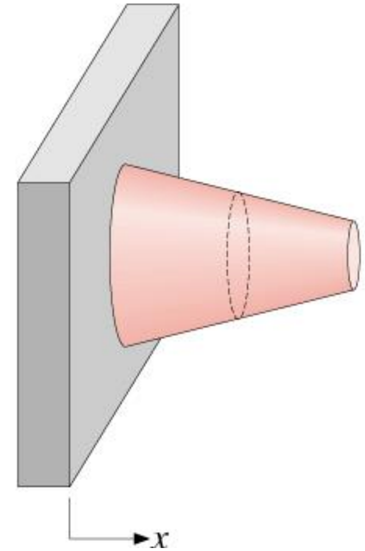
Different types of straight fins



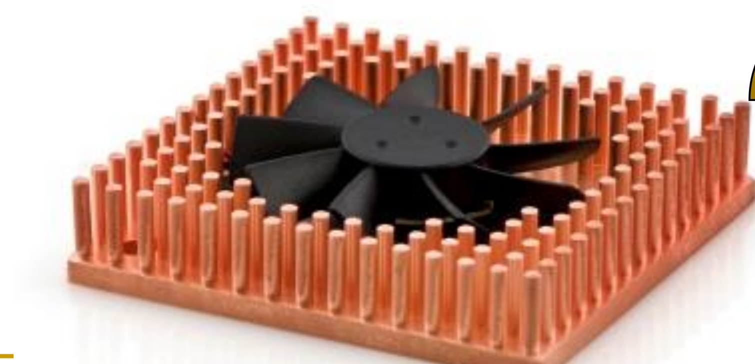
Longitudinal Fins
(a) uniform and (b) non-uniform cross sections

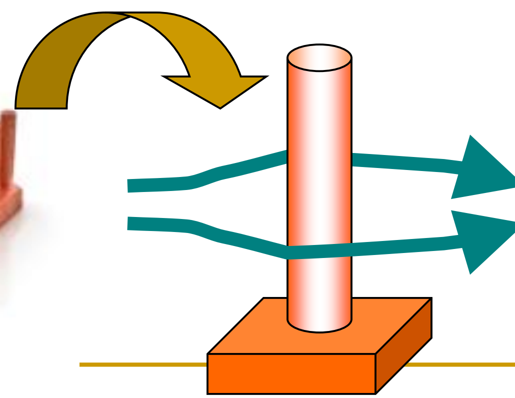


Annular fin



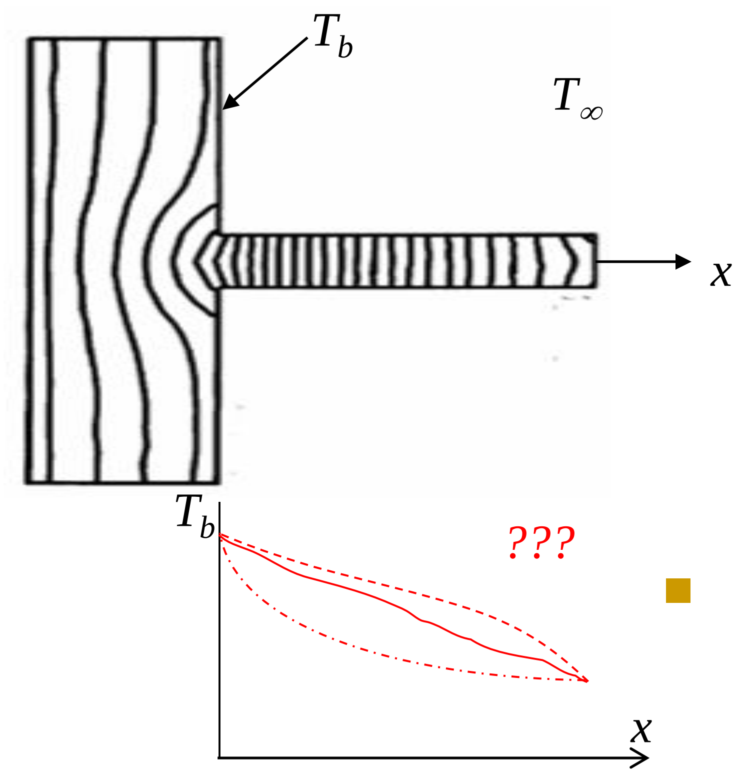
Pin fin of non-uniform cross section

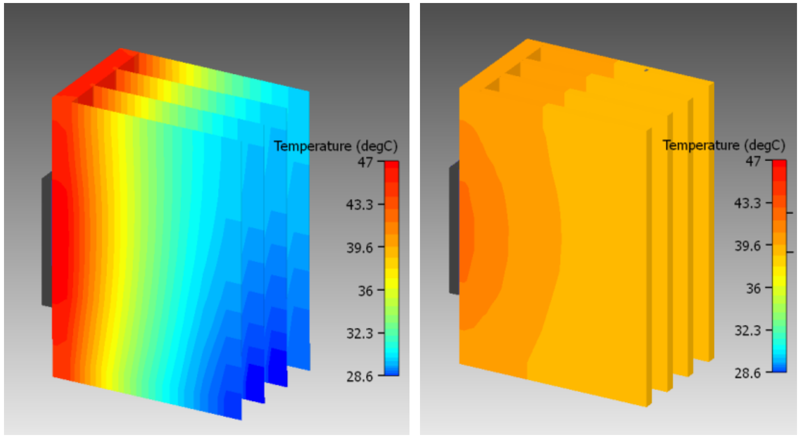




Pin fin of uniform cross section

Temperature distribution on the extended surface





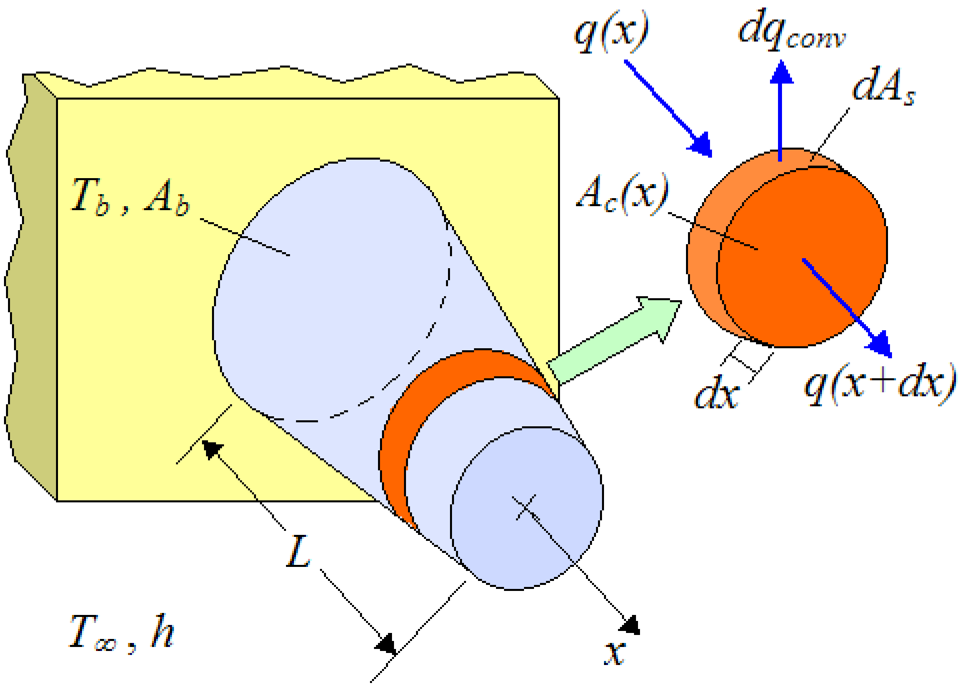
Fin with low *k* Fin with high *k*

- 1-D heat transfer
  - Temperature within the fin varies primarily in the *x* direction
  - Nearly homogeneous $T(x)$ within the fin at a particular cross-section

1-D analysis of pin fin

Assumptions:

- 1-D, steady state heat conduction
- Uniform k and h along the x



Energy balance across the slice

$$q(x) = q(x+dx) + dq_{conv} \qquad \text{(i)}$$

Fourier's Law of Heat Conduction

$$q(x) = -kA_c(x)\frac{dT}{dx}$$

Taylor series expansion

$$q(x+dx) = q(x) + \frac{d}{dx}\{q(x)\}dx$$

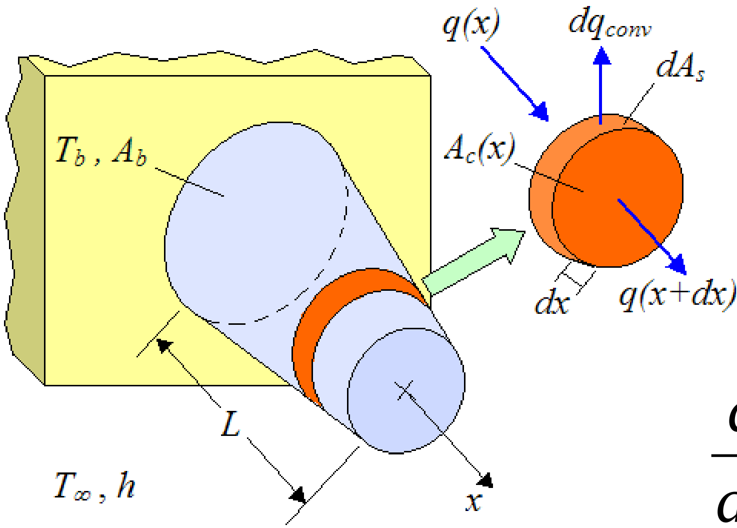
$$= q(x) + \frac{d}{dx}\left\{-kA_c(x)\frac{dT}{dx}\right\}dx \qquad \text{(ii)}$$

$$dq_{conv} = hdA_s(x)(T - T_\infty) \qquad \text{(iii)}$$

From (i), (ii) and (iii)

$$q(x) = q(x) + \frac{d}{dx}\left\{-kA_c(x)\frac{dT}{dx}\right\}dx + hdA_s(x)(T - T_\infty)$$

1-D analysis of pin fin, contd…



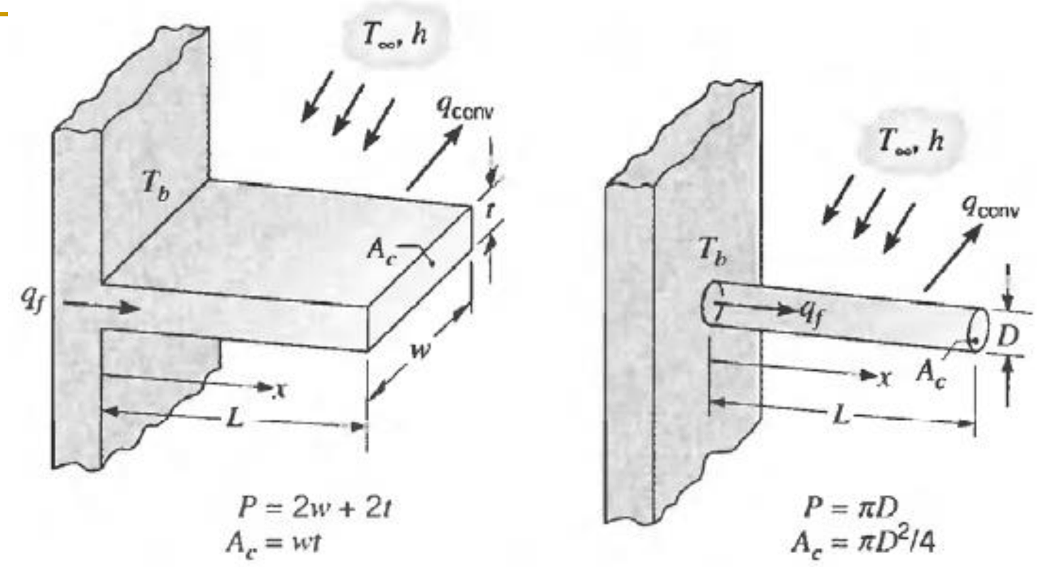
$$\frac{d}{dx}\left\{-kA_c(x)\frac{dT}{dx}\right\}+h\frac{dA_s(x)}{dx}(T-T_\infty)=0$$

$$\frac{d}{dx}\left\{A_c(x)\frac{dT}{dx}\right\}-\frac{h}{k}\frac{dA_s(x)}{dx}(T-T_\infty)=0$$

$$\left\{\frac{d^2T}{dx^2}\right\}+\frac{1}{A_c(x)}\left\{\frac{dA_c(x)}{dx}\right\}\left\{\frac{dT}{dx}\right\}-\frac{1}{A_c(x)}\frac{h}{k}\frac{dA_s(x)}{dx}(T-T_\infty)=0$$

For fins of uniform cross se

$$dA_s = P(x)dx = Pdx$$



$$\left\{\frac{d^2T}{dx^2}\right\} + \quad - \frac{1}{A_c}\frac{h}{k} \; P \quad (T - T_\infty) = 0$$

$$\left\{\frac{d^2T}{dx^2}\right\} - \frac{hP}{kA_c}(T - T_\infty) = 0$$

$$\theta = (T - T_\infty); \quad m = \sqrt{\frac{hP}{kA_c}}$$

$$\frac{d^2\theta}{dx^2} - m^2\theta = 0$$

θ can be looked up as the "excess temperature"; *m* as the ratio of convective to conductive HT

Fin Equation

General Equation

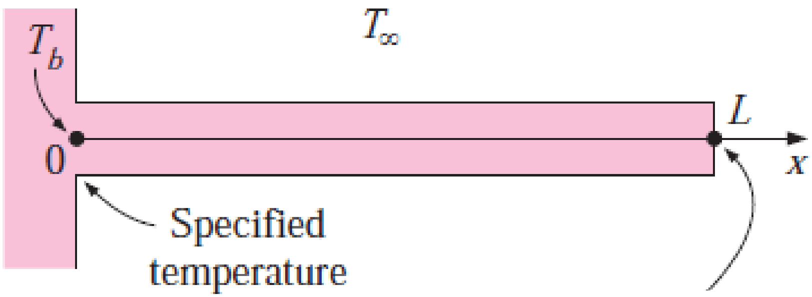
$$\theta(x) = C_1 e^{mx} + C_2 e^{-mx}$$

Boundary Conditions

At the Fin Base

$$\theta(0) = \theta_b = T_b - T_\infty$$

At the Fin Tip???



Fin Tip BC

A. ***Infinitely long Fin***
B. ***Prescribed fin tip temperature***
C. ***Adiabatic fin tip***
D. ***Convective BC at fin tip also***

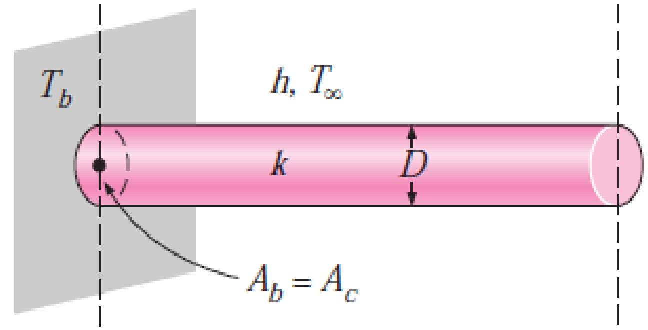
Infinitely long fin

$$\theta(x) = C_1 e^{mx} + C_2 e^{-mx}$$

Boundary Conditions:

$$x = 0 \Rightarrow \theta(0) = \theta_b$$

$$L \rightarrow \infty \Rightarrow \theta_L = (T_L - T_\infty) = \theta_L \rightarrow 0$$

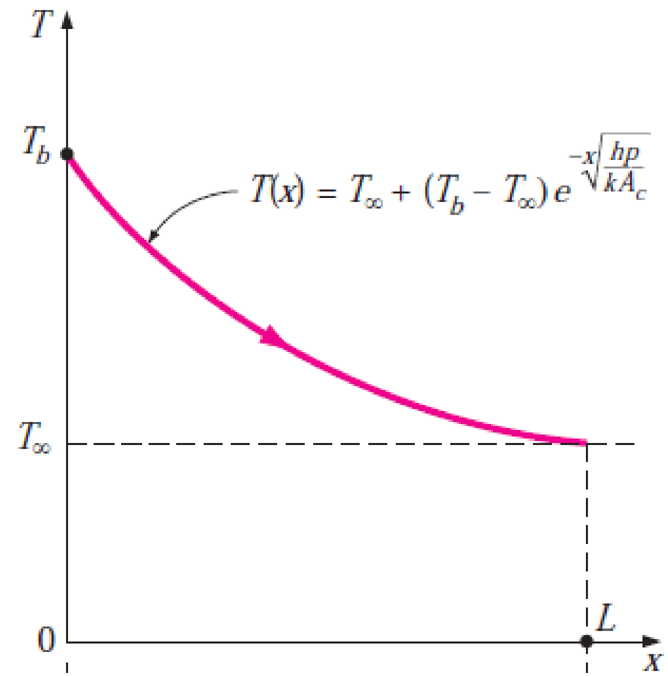


$(p = \pi D,\ A_c = \pi D^2/4$ for a cylindrical fin)

Imposing the two BCs

$$\theta(x) = \theta_b e^{-mx}$$

$$\frac{T(x) - T_\infty}{T_b - T_\infty} = e^{-mx} = e^{-x\sqrt{hp/kA_c}}$$



$$\dot{Q}_{\text{long fin}} = -kA_c \left.\frac{dT}{dx}\right|_{x=0} = \sqrt{hpkA_c}\,(T_b - T_\infty)$$

Prescribed fin tip temperature

$$\theta(0) = \theta_b \qquad \theta_b = C_1 + C_2$$

$$\theta(L) = \theta_L \qquad \theta_L = C_1 e^{mL} + C_2 e^{-mL}$$

$$\frac{\theta}{\theta_b} = \frac{(\theta_L/\theta_b)\sinh\, mx + \sinh\, m(L-x)}{\sinh\, mL}$$

$$\dot{Q}_{f,T_L} = -kA_c \left.\frac{dT}{dx}\right|_{x=0} = \sqrt{hPkA_c}\,\theta_b \frac{\left(\cosh mL - \theta_L/\theta_b\right)}{\sinh mL}$$

Insulated fin tip

General Sol: $\theta(x) = C_1 e^{mx} + C_2 e^{-mx}$

BC 1: $\theta(0) = T_b - T_\infty \equiv \theta_b$

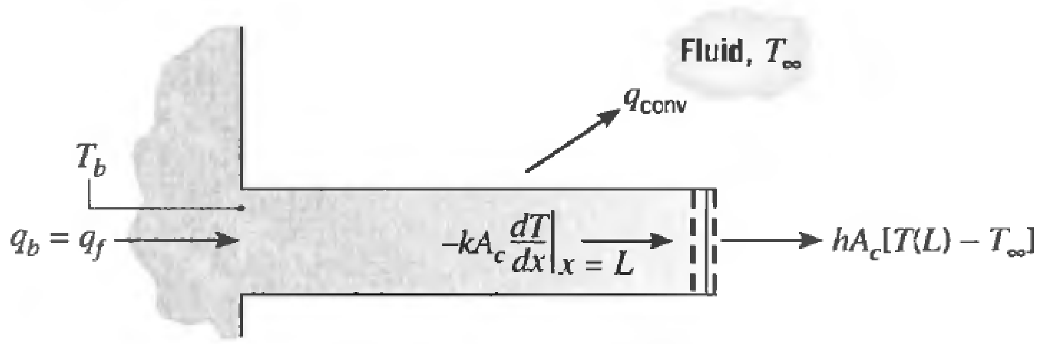
BC 2: $\frac{d\theta}{dx}\Big|_{x=L} = 0$

$$\frac{\theta}{\theta_b} = \frac{\cosh m(L-x)}{\cosh mL}$$

$$\dot{Q}_{f,adiabatic} = -kA_c \frac{dT}{dx}\bigg|_{x=0} = \sqrt{hPkA_c}\,\theta_b \tanh mL$$

When do you have insulated fin tip?

Convective fin tip boundary condition



q denotes $\dot{Q}$

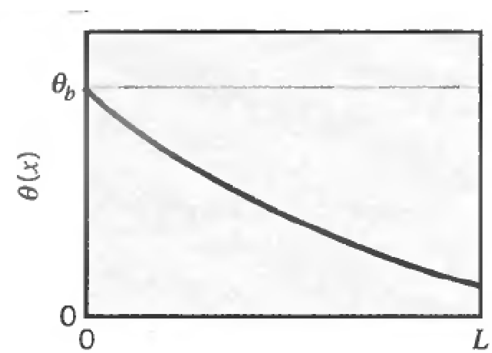
BCs

$$T(0) = T_\infty$$

$$\theta(0) = T_b - T_\infty \equiv \theta_b$$

$$hA_c\left[T(L) - T_\infty\right] = -kA_c \frac{dT}{dx}\Big|_{x=L}$$

$$h\theta(L) = -k\frac{d\theta}{dx}\Big|_{x=L}$$

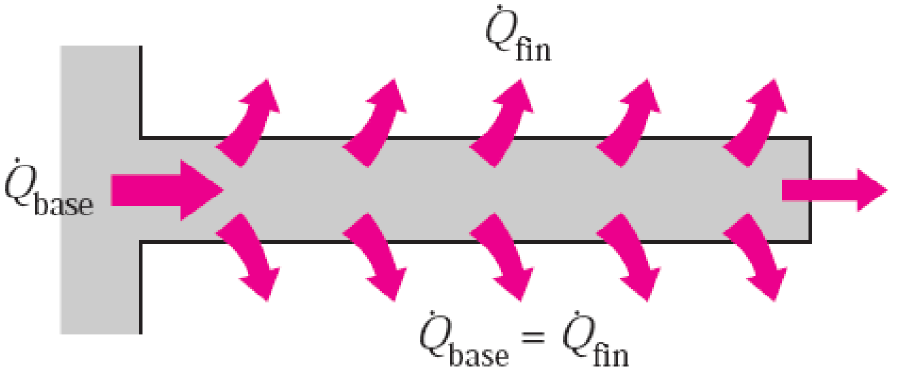


$$\frac{\theta}{\theta_b} = \frac{\cosh m(L-x) + (h/mk)\sinh m(L-x)}{\cosh mL + (h/mk)\sinh mL}$$

$$\dot{Q}_{f,convective} = -kA_c \frac{dT}{dx}\Big|_{x=0} = \sqrt{hPkA_c}\,\theta_b \frac{\left\{\sinh mL + \left(h/mk\right)\cosh mL\right\}}{\left\{\cosh mL + \left(h/mk\right)\sinh mL\right\}}$$

Fin Performance

Heat in at base = heat lost from fin surface



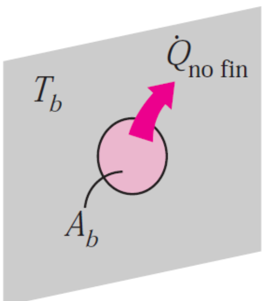
$$\dot{Q}_{base} = -kA_c \left.\frac{dT}{dx}\right|_{x=0}$$

$$\dot{Q}_{\text{fin}} = \int_{A_{\text{fin}}} h[T(x) - T_\infty]\, dA_{\text{fin}} = \int_{A_{\text{fin}}} h\theta(x)\, dA_{\text{fin}}$$

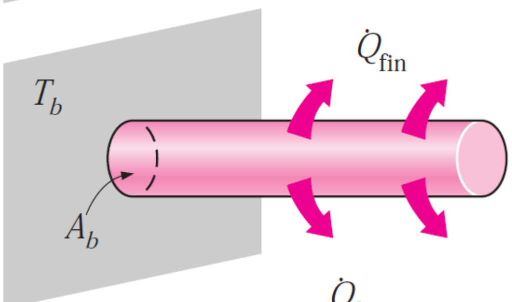
Heat transfer by fin can be determined by either method

Fin Effectiveness

$$\varepsilon_{\text{fin}} = \frac{\dot{Q}_{\text{fin}}}{\dot{Q}_{\text{no fin}}} = \frac{\dot{Q}_{\text{fin}}}{hA_b\,(T_b - T_\infty)} = \frac{\text{Heat transfer rate from the fin of } \textit{base area } A_b}{\text{Heat transfer rate from the surface of } \textit{area } A_b}$$



$$\varepsilon_{longfin} = \frac{\dot{Q}_{fin}}{\dot{Q}_{nofin}} = \frac{\sqrt{hPkA_c}\,(T_b - T_\infty)}{hA_b(T_b - T_\infty)} = \sqrt{\frac{kP}{hA_c}}$$



$$\varepsilon_{\text{fin}} = \frac{\dot{Q}_{\text{fin}}}{\dot{Q}_{\text{no fin}}}$$

What is the right effectiveness?

$\varepsilon_{fin} = 1$ Does not affect the heat transfer at all.

$\varepsilon_{fin} < 1$ Fin act as insulation (if low k material is used)

$\varepsilon_{fin} > 1$ Enhancing heat transfer (use of fins justified if $\varepsilon_{\text{fin}}>2$)

Factors leading to high fin effectiveness

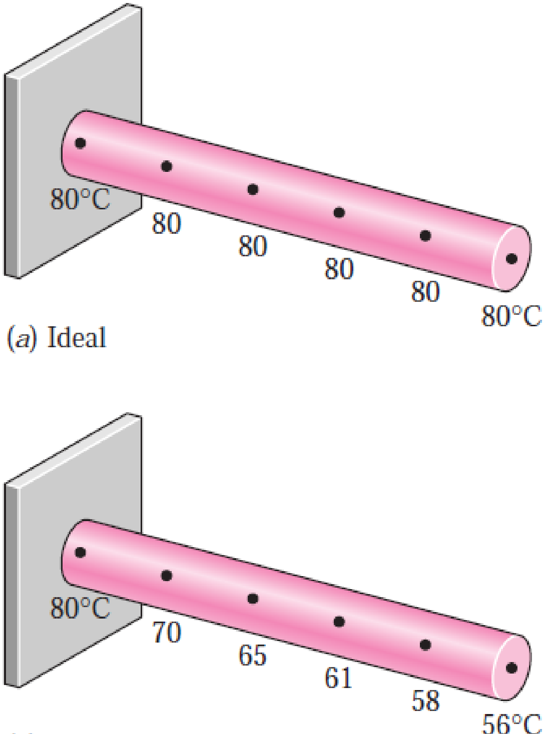
1. k should be as high as possible, (copper, aluminum, iron).
   Aluminum is preferred: low cost and weight, resistance to corrosion.
2. p/A_c should be as high as possible. (Thin plate fins and slender pin fins)
3. Most effective in applications where h is low. (Use of fins justified if when the medium is gas and heat transfer is by natural convection).

Fin Efficiency

$$\eta_{\text{fin}} = \frac{\dot{Q}_{\text{fin}}}{\dot{Q}_{\text{fin, max}}} = \frac{\text{Actual heat transfer rate from the fin}}{\begin{array}{c}\text{Ideal heat transfer rate from the fin}\\ \text{if the entire fin were at base temperature}\end{array}}$$

$$\dot{Q}_{\text{fin}} = \eta_{\text{fin}}\,\dot{Q}_{\text{fin, max}} = \eta_{\text{fin}}\,hA_{\text{fin}}\,(T_b - T_\infty)$$

$$\eta_{\text{long fin}} = \frac{\dot{Q}_{\text{fin}}}{\dot{Q}_{\text{fin, max}}} = \frac{\sqrt{hPkA_c}\,(T_b - T_\infty)}{hA_{\text{fin}}\,(T_b - T_\infty)} = \frac{1}{L}\sqrt{\frac{kA_c}{hP}} = \frac{1}{mL}$$

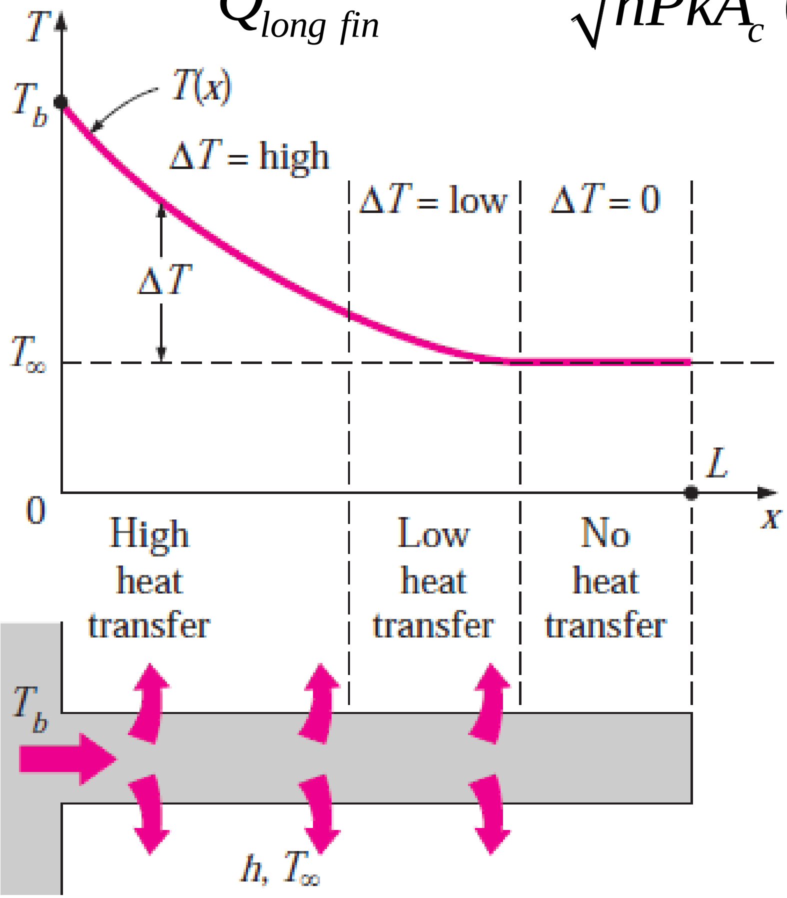


(a) Ideal

(b) Actual

Proper length of a Fin

$$\frac{Q_{fin}}{Q_{long\ fin}} = \frac{\sqrt{hPkA_c}\,(T_b - T_\infty)\tanh mL}{\sqrt{hPkA_c}\,(T_b - T_\infty)} = \tanh mL$$

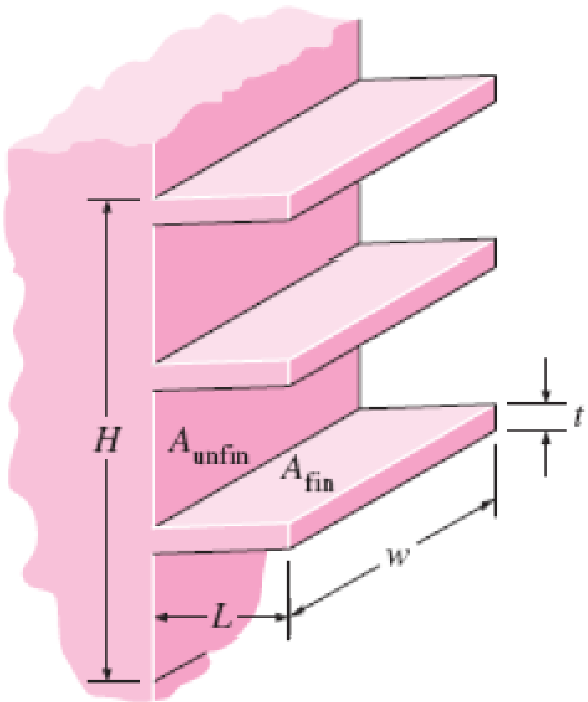


The variation of heat transfer from a fin relative to that from an infinitely long fin

mL	$\frac{\dot{Q}_{fin}}{\dot{Q}_{long\ fin}} = \tanh mL$
0.1	0.100
0.2	0.197
0.5	0.462
1.0	0.762
1.5	0.905
2.0	0.964
2.5	0.987
3.0	0.995
4.0	0.999
5.0	1.000

mL> 5 is not necessary

Overall fin efficiency



$$A_{\text{no fin}} = w \times H$$
$$A_{\text{unfin}} = w \times H - 3 \times (t \times w)$$
$$A_{\text{fin}} = 2 \times L \times w + t \times w \text{ (one fin)}$$
$$\approx 2 \times L \times w$$

When determining the rate of heat transfer from a finned surface, we must consider the unfinned portion of the surface as well as the fins. Therefore, the rate of heat transfer for a surface containing n fins can be expressed as

$$\dot{Q}_{total,fin} = \dot{Q}_{unfin} + \dot{Q}_{fin}$$
$$= hA_{unfin}(T_b - T_\infty) + \eta_{fin} hA_{fin}(T_b - T_\infty)$$
$$= h(A_{unfin} + \eta_{fin} A_{fin})(T_b - T_\infty)$$

We can also define an **overall effectiveness for a finned surface as the ratio** of the total heat transfer from the finned surface to the heat transfer from the same surface if there were no fins

$$\varepsilon_{fin,overall} = \frac{\dot{Q}_{total,fin}}{\dot{Q}_{total,nofin}} = \frac{h(A_{unfin} + \eta_{fin} A_{fin})(T_b - T_\infty)}{hA_{nofin}(T_b - T_\infty)}$$

Fin Effectiveness vs Efficiency

$$\varepsilon_{\text{fin}} = \frac{\dot{Q}_{\text{fin}}}{\dot{Q}_{\text{no fin}}} = \frac{\dot{Q}_{\text{fin}}}{hA_b\,(T_b - T_\infty)} = \frac{\eta_{\text{fin}}\,hA_{\text{fin}}\,(T_b - T_\infty)}{hA_b\,(T_b - T_\infty)} = \frac{A_{\text{fin}}}{A_b}\,\eta_{\text{fin}}$$