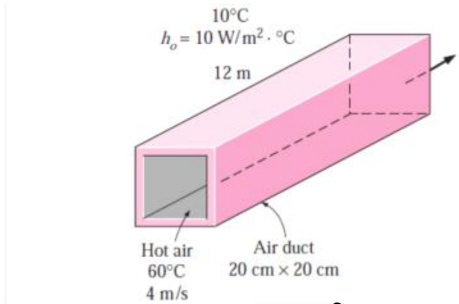
Exercise 1

Tuesday, April 13, 2021 10:30 PM

Hot air at 60 °C leaving the furnace of a house enters a 12-m-long section of a sheet metal duct of rectangular cross section 20 cm x 20 cm at an average velocity of 4 m/s. The thermal resistance of the duct is negligible, and the outer surface of the duct is exposed to the cold air at 10 °C in the basement, with a convection heat transfer coefficient of 10 W/m²K. Determine (a) the temperature at which the hot air will leave the basement and (b) the rate of heat loss from the hot air in the duct to the basement.



$D_h = 4A_c/P = \frac{4 \times 0.2 \times 0.2}{4 \times 0.2} m = 0.2 m$

$A_c = 0.04 m^2$, $A_s = 4 \times 0.2 \times 12 = 9.6 m^2$

Soln

Assumptions 1) Steady flow
2) Constant fluid properties

Step 1: Average bulk mean temp $= \frac{T_i + T_e}{2}$ →? (assume 10°C)
$= 35°C$

Step 2 Fluid properties @ 35°C

$\left.\begin{array}{l} \rho = 1.2\ kg/m^3 \quad C_p = 1.01\ kJ/kgK \\ k = 0.03\ W/mK, \quad P_r = 0.7 \end{array}\right\} \Rightarrow$

don't forget to convert it to J/kg K

Step 3: Flow $Re = \frac{\rho V D_h}{\mu} = \frac{C_p}{k} \times \frac{\rho V D_h}{P_r} = 46171$ $\quad \left[P_r = \mu C_p / k \Rightarrow \mu = \frac{k P_r}{C_p} \right]$

⇒ Turbulent flow ⇒ $L_e/D \approx 10$ ⇒ Entry length $L_e = .2 \times 10 = 2 m$

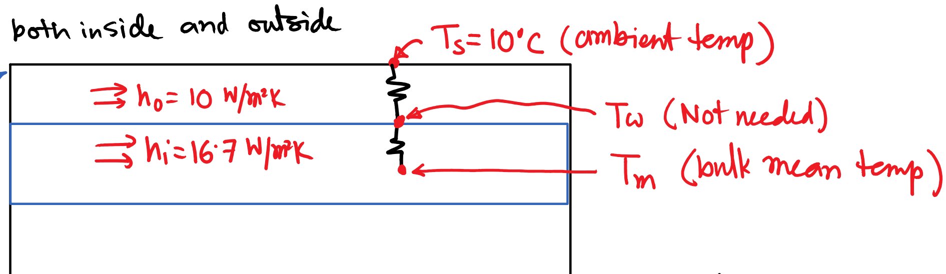
∴ We can assume, without incurring much error, that the flow in the pipe is thermally fully developed.

Step 4: $Nu = 0.023\ Re^{0.8}\ P_r^{0.3}$ ← Since this is "Cooling" of the internal fluid
$= 111.4$

Step 4: HTC for the inner flow $h_i = \frac{k Nu}{D} = \frac{0.03 \times 111.4}{0.2} = 16.7\ W/m^2K$

Step 5: To find the overall heat transfer, we must consider the HTCs both inside and outside

Here we treat the ambient as the outer boundary; as if the constant outermost wall temperature of 10°C



Overall HTC $U = \left(\frac{1}{h_i} + \frac{1}{h_o}\right)^{-1} = \left(\frac{1}{10} + \frac{1}{16.7}\right)^{-1} = 6.25\ W/m^2K$

Step 5: We can now apply the temperature profile for T_s = constant case for fully developed flow.

$\dot{m} = \rho A_c V_{av} = 1.2 \times 0.04 \times 4\ \frac{kg}{s}$
$= 0.192\ kg/s$

$T_e = T_s + (T_i - T_s) exp\left(-\frac{U A_s}{\dot{m} C_p}\right)$

$= 10 + (60 - 10) \times exp\left(-\frac{6.25 \times 9.6}{0.192 \times 1010}\right)$ — (A)

$= 10 + 50 \times 0.734$

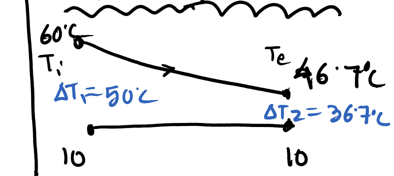
Ans(a) $\Rightarrow$ $= 46.7\ ^\circ C$

Ans(b) $\Rightarrow$ **Step 5:** $\dot{Q} = \dot{m} C_p (T_i - T_e) = 2579\ W$

$$\boxed{\text{Alternately } \dot{Q} = U A_s \, \text{lmTd} = 6.25 \times 9.8 \times 43 \ W = 2633\ W}$$

This 2% error crops up in rounding off in the exponential term in (A)

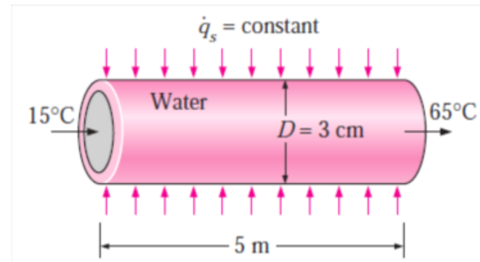
LMTD for this Case



$$\text{lmtd} = \frac{\Delta T_1 - \Delta T_2}{\ln \frac{\Delta T_1}{\Delta T_2}} = 43.0\ ^\circ C$$

Example 2

Water is to be heated from 15°C to 65°C as it flows through a 3-cm-internal-diameter 5-m-long tube (Fig. 19–36). The tube is equipped with an electric resistance heater that provides uniform heating throughout the surface of the tube. The outer surface of the heater is well insulated, so that in steady operation all the heat generated in the heater is transferred to the water in the tube. If the system is to provide hot water at a rate of 10 L/min, determine the power rating of the resistance heater. Also, estimate the inner surface temperature of the pipe at ~~the exit~~ $x = 2.5\,m$ & $x = 5\,m$.



Assumptions:

1. Steady flow
2. Constant q″
3. Measured temperatures represent T_m

Step 1:

Properties @ average Tm of 40 °C

$\rho = 992.1\ kg/m^3$	$C_p = 4179\ J/kg \cdot °C$
$k = 0.631\ W/m \cdot °C$	$Pr = 4.32$
$\upsilon = \mu/\rho = 0.658 \times 10^{-6}\ m^2/s$	

Step 2:

$$V = \frac{\dot{V}}{A_c} \quad \frac{(m^3/s)}{m^2} = 0.236\ m/s$$

$$\dot{V} = 10\ L/min = \frac{10^{-2}}{60}\ m^3/s = 1.67 \times 10^{-4}\ m^3/s$$

$$A_c = \pi D^2/4 = \frac{\pi \times (0.03)^2}{4} = 7.06 \times 10^{-4}\ m^2$$

$\therefore\ Re = 10750 \Rightarrow$ Turbulent flow

Entry Length $L_e \approx 10 \times D = 0.3\ m \Rightarrow L_e \ll L$

$\therefore$ we can assume TFD flow for the entire pipe

Step 3:

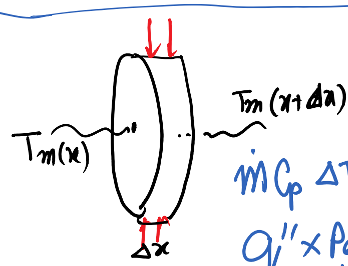
$$Nu = 0.023 \times Re^{0.8} \times Pr^{0.4} \quad (\because \text{This is a case of heating})$$

$$= 0.023 \times (10750)^{0.8} \times (4.32)^{0.4}$$

$$= 69.35$$

$$\Rightarrow h = \frac{k\,Nu}{D} = \frac{0.631 \times 69.35}{0.03} = 1458.7\ W/m^2K$$

Step 4: For $q'' =$ constant, we know $\dfrac{dT_m}{dx} = \dfrac{dT_s}{dx} = \dfrac{q'' \times P}{\dot{m}\, C_p}$



$\dot{m} C_p \Delta T_m$ = Rate of enthalpy rise in x-direction

$q'' \times$ Perimeter $\times \Delta x$ = Rate of Heat transfer into the differential element

$$\Rightarrow \dot{m} C_p \frac{dT_m}{dx} = q'' \times P \Rightarrow \frac{dT_m}{dx} = \frac{q'' P}{\dot{m} C_p} = \frac{q'' \times \pi D}{\dot{m} C_p}$$

$$\boxed{\Rightarrow T_m(x) = T_i + \frac{q'' \pi D}{\dot{m} C_p} = T_i + 10x}$$

⇒ T_m & T_s, both increase from the inlet (assuming fully developed flow to start from $x=0$) to the outlet at the same slope i.e., $\frac{q'' \times \pi D}{\dot{m} C_p}$ $\frac{K}{m}$

Step 5

$\dot{Q}$ = Heat added to water

$$= \dot{m} C_p (T_e - T_i) = \frac{992.1 \times 10^{-2}}{60} \times 4179 \times (65 - 15) \text{ W}$$

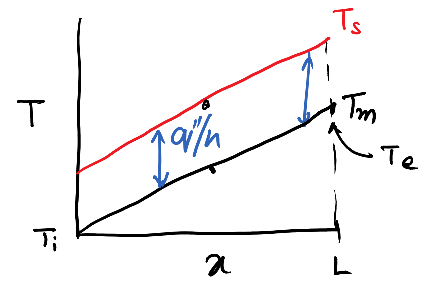
$$\boxed{= 34.55 \text{ kW}}$$ **Ans**

$$\Rightarrow q'' = \dot{Q}/\pi D L = 73.32 \text{ kW/m}^2$$

Step 5

At any x, $h = \frac{q''}{(T_s - T_m)}$

$$\Rightarrow T_s = T_m + \frac{q''}{h} = T_m(x) + 50.7 \text{ K}$$



at $x = 2.5$ m, $T_m = 40°C \Rightarrow T_s = 90.7°C$

at $x = 5$ m, $T_m = 65°C \Rightarrow T_s = 115.7°C$ **Ans**