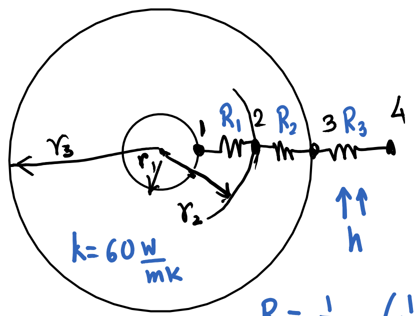
CLASS TEST (2021)

Friday, May 21, 2021 11:09 AM

1. A ceramic (k= 60 W/mK) hollow sphere having inner and outer diameters of 10 and 30 cm holds a molten alloy. The tank is exposed to a free convection environment with an ambient temperature of 30 °C. The outer surface temperature of the spherical tank is 100 °C, while a thermocouple embedded at 7.5 cm radius reads 250 °C. Estimate (i) the inner wall temperature of the tank and (ii) the convective heat transfer coefficient outside the tank. **8 marks**



$r_1 = 0.05$ m

$r_2 = 0.075$ m

$r_3 = 0.15$ m

$T_2 = 250°C$

$T_3 = 100°C$

$T_4 = 30°C$

$$R_1 = \frac{1}{4\pi k}\left(\frac{1}{r_1} - \frac{1}{r_2}\right) = \frac{100}{4\pi \cdot 60}\left(\frac{1}{5} - \frac{1}{7.5}\right) = 8.84 \times 10^{-3} \frac{K}{W}$$

$$R_2 = \frac{1}{4\pi k}\left(\frac{1}{r_2} - \frac{1}{r_3}\right) = \frac{100}{4\pi \cdot 60}\left(\frac{1}{7.5} - \frac{1}{15}\right) = 8.84 \times 10^{-3} \frac{K}{W}$$

$$\therefore \dot{Q} = \frac{T_3 - T_2}{R_2} = 16964.6 \text{ W}$$

Since $R_1 = R_2$, & $\dot{Q}$ is same through them, $\Delta T_1 = \Delta T_2 = 150°C$

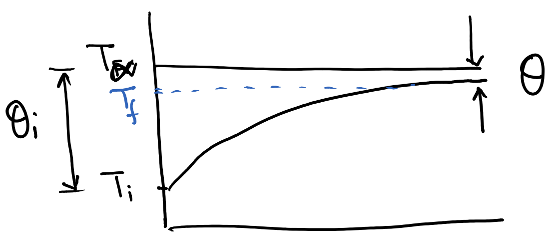
$\Rightarrow T_1 = T_2 + 150 = 400°C$

Outside:

$$\dot{Q} = hA(T_3 - T_4)$$

$$\Rightarrow h = \frac{\dot{Q}}{A \times (T_3 - T_4)} = \frac{16964.6}{4 \times \pi \times (0.15)^2 \times 70} = 857.1 \frac{W}{m^2K}$$

2. A thermocouple is suddenly inserted in a furnace to measure the temperature of the hot gas stream. The junction may be approximated as a sphere having a thermal conductivity of 25 W/mK, density 8400 kg/m^3, and specific heat of 0.4 kJ/kgK. The heat transfer coefficient between the junction and the gas is 560 W/m^{2}K. Calculate the diameter of the junction if the thermocouple should be able to read 95% of the applied temperature difference in 3 s. Assume the lumped capacitance model to hold good. **7 marks**



Considering lumped parameter

$$\theta(t) = \theta_i e^{-t/\tau}$$

For the thermocouple to read 95% of the applied temperature difference

$$\theta(t) = (1 - 0.95)\theta_i$$

$$\Rightarrow \frac{\theta(t)}{\theta_i} = 0.05 = e^{-t/\tau} \Rightarrow t/\tau = -\ln(0.05) = 2.996$$

Since $t = 3s$, $\tau = \frac{3}{2.996}$ s $= 1.001$ s

Now $\tau = RC = \frac{1}{hA} \times \rho C V = \frac{1}{h 4\pi r^2} \times \rho C \frac{4}{3}\pi r^3 = \frac{\rho C r}{3h}$

$$\Rightarrow r = \frac{3h\tau}{\rho C} = \frac{3 \times 560 \times 1.001}{8400 \times 400} = 5.007 \times 10^{-4} \text{ m}$$

$\therefore$ TC bead diameter is 1.001 mm

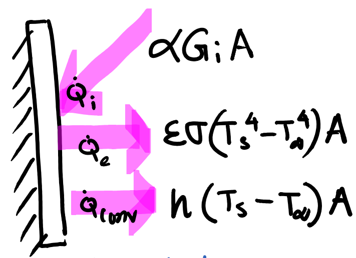
$\Rightarrow r = \frac{3h\tau}{\rho C} = \frac{3\times 80\times 1.001}{8400 \times 400} = 5.007\times 10^{-4}$ m

$\therefore$ TC bead diameter is 1.001 mm

3. A 0.5m by 0.5 m diffuse vertical flat plate receives concentrated solar radiation G_i = 10000 W/m^2, while its spectral absorptivity of the plate surface varies as: α = 0.9 for 0<λ<1 μm and 0 for 1μm <λ. <u>How much heat does the plate absorb?</u> The plate is thermally insulated at its back, and under a steady state it rejects the heat by radiation and free-convection from the front which is at a temperature of 327 °C. The free stream air and the surroundings are at 27 °C. <u>What is the emissivity of the surface</u>? Consider solar radiation to mimic the radiation from a blackbody maintained at 5000 K. Also assume the air properties at the ambient temperature of 300 K (k = 0.3 W/mK, C_p = 1 kJ/kgK, Pr = 0.7, ρ = 1.2 kg/m^3). Assume the following correlation for Nu for free convection on a vertical flat plate: Nu= 0.59 $Ra_L^{1/4}$. **10 marks**

There was a typo in the problem, $\alpha = 0.9$ & Not 0.4

No student was penalized who attempted the second part of the problem, but could not arrive at the final numerical value of ε. For those who succeeded in completing the first part only got 5 out of 10 marks.



To evaluate α, we reckon that at $\lambda = \mu m$, $\lambda T = 5000$ K.μm

$$\alpha = 0.9 f_{0-\lambda} + 0 \times f_{\lambda-\infty}$$

$$= 0.9 \times f_{\lambda T = 5000} + 0$$

$$= 0.9 \times 0.6337 = 0.57033$$

$\therefore$ $\boxed{\alpha G_i = 5703.3 \text{ W/m}^2}$ — (2) $\Rightarrow$ $\dot{Q}_i = A \times \alpha G_i = 0.25 \times 5703.3$ W $= 1425.8$ W

Under steady state

$$\dot{Q}_i = \dot{Q}_e + \dot{Q}_{conv}$$

$\uparrow$ emitted $\quad$ $\uparrow$ Free Convection

$$\therefore \alpha G_i A = \varepsilon \sigma A (T_s^4 - T_\infty^4) + hA(T_s - T_\infty)$$

or $\boxed{\alpha G_i = \varepsilon\sigma(T_s^4 - T_\infty^4) + h(T_s - T_\infty)}$ — (1)

<u>To evaluate h</u>:

$$\nu = \frac{Pr \cdot K}{\rho C_p} = \frac{0.7 \times 0.03}{1.2 \times 1000} = 1.75 \times 10^{-5} \text{ m}^2\text{/s}$$

Also $T_f = \frac{1}{2}(T_s + T_\infty) = \frac{1}{2}(600 + 300) = 450\text{K} \Rightarrow \beta = \frac{1}{450}$ K^{-1}

$$\Rightarrow Ra = \frac{g\beta\Delta T L^3}{\nu^2} \cdot Pr = \frac{9.81 \times \frac{1}{450} \times 300 \times (0.5)^3}{(1.75)^2 \times 10^{-10}} \times 0.7$$

$$= 1.87 \times 10^9$$

$$\Rightarrow Nu = 207.91 \Rightarrow h = \frac{k Nu}{L} = 12.47 \text{ W/m}^2\text{K} \text{ — (3)}$$

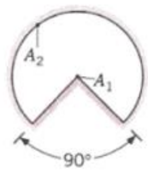
Hence from (1), (2) & (3)

$$5703.3 = \sigma\varepsilon(T_s^4 - T_\infty^4) + 12.47 \times 300$$

$$\text{or } \sigma\varepsilon(T_s^4 - T_\infty^4) = 1962.3$$

$$\Rightarrow \varepsilon = 0.285$$

4. Find the view factors F_{12} and F_{21} of the following two configurations: **5 marks**



Sphere lying on infinite plane

Per unit depth

$A_1 = 2R$

$A_2 = \frac{3}{4} \times 2\pi R$

By observation $\boxed{F_{12} = 1}$ ($\because F_{11} = 0$)

$\therefore$ By reciprocity relation

$A_1 F_{12} = A_2 F_{21} \Rightarrow F_{21} = \frac{A_1}{A_2} \cdot F_{12}$

$= \frac{\cancel{2}\cancel{R} \times 4}{3\cancel{6}\pi\cancel{R}} F_{12}$

$= \frac{4}{3\pi} \times 1$

By observation $\boxed{F_{12} = \frac{1}{2}}$

($\because$ the upper half of the sphere sees the sky only)

$F_{21} = \frac{A_1}{A_2} F_{12} = 0.5 \cdot \frac{A_1}{A_2}$

Since $A_1 \ll A_2$

$\boxed{F_{21} \to 0}$