# Introduction to Convection

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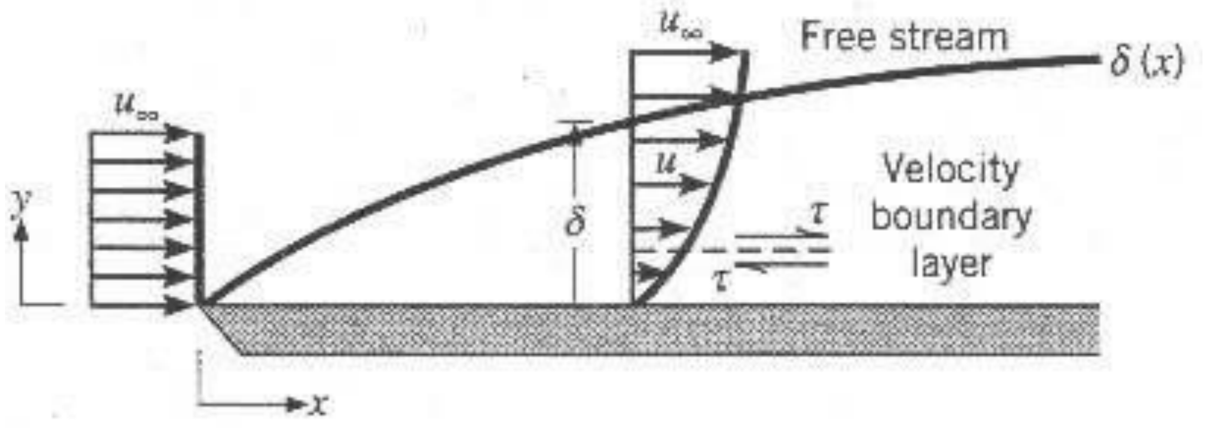
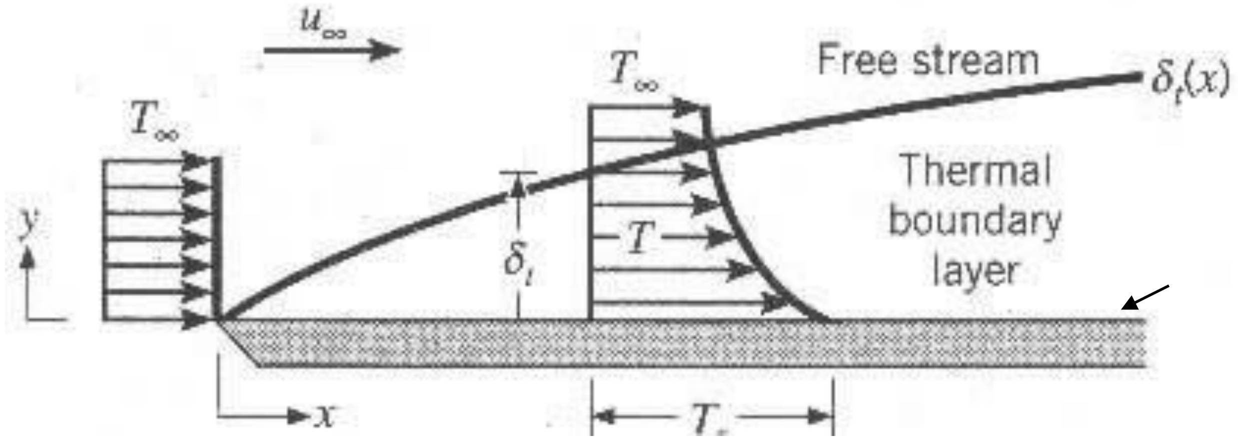
# Recall the convection overview




- Local wall shear stress is:

$$\tau = C_f \frac{1}{2} \rho u_\infty^2$$

where $C_f$ is the local friction factor

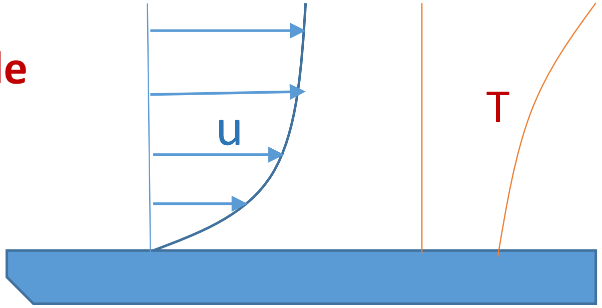



- Local heat flux is:

$$q'' = h(T_s - T_\infty)$$

where h is the ***local heat transfer coefficient***

**Example**




The velocity and temperature profiles for flow over a surface is given by:

$$u(y) = Ay + By^2 - Cy^3$$

$$T(y) = D + Ey + Fy^2 - Gy^3$$

Obtain the expressions of friction factor and heat transfer coefficients in terms of the constants A, B, C, D, E, F and G and the free stream velocity $U_\infty$ and $T_\infty$ , and the fluid properties ρ, μ, $C_p$ and k

**Sol:**

$$u(y) = Ay + By^2 - Cy^3$$

$$\frac{\partial u}{\partial y} = A + 2By - 3Cy^2$$

$$\tau_w = \mu \frac{\partial u}{\partial y}\bigg|_{y=0} = \mu A$$

$$C_f = \tau_w \Big/ \left(\frac{1}{2}\rho U_\infty^2\right) = \frac{2\mu A}{\rho U_\infty^2}$$

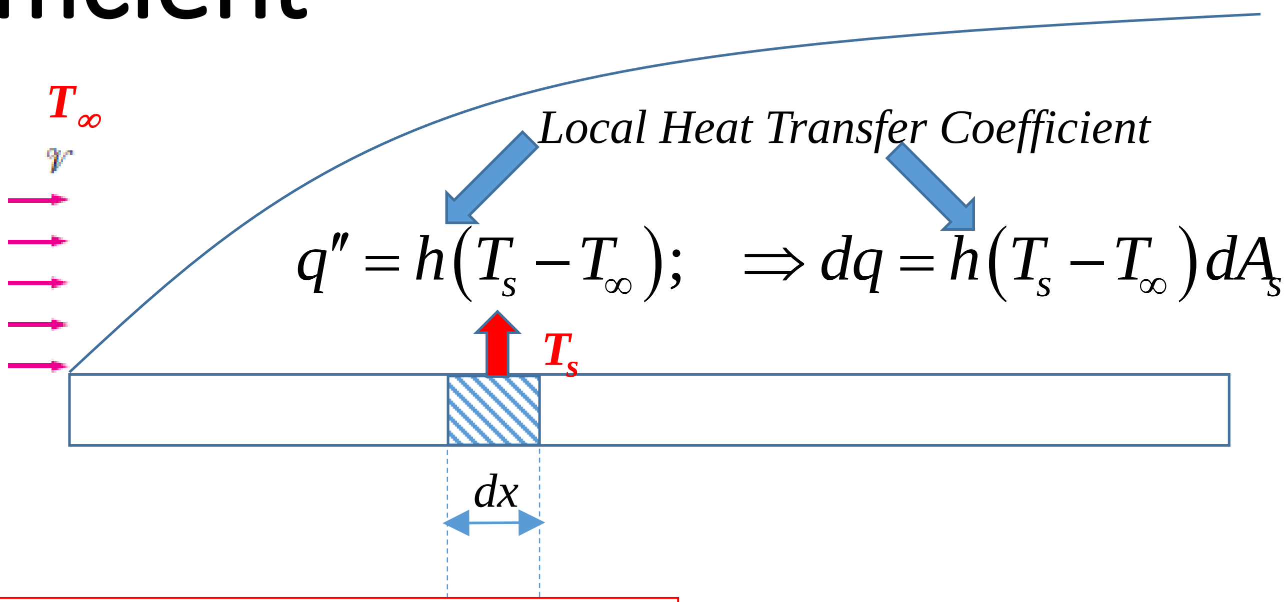
$$T(y) = D + Ey + Fy^2 - Gy^3 \; and \quad T_s = T(0) = D$$

$$\frac{\partial T}{\partial y} = E + 2Fy - 3Gy^2$$

$$q''_w = -k \frac{\partial T}{\partial y}\bigg|_{y=0} = -kE$$

$$h = \frac{q''_w}{(T_s - T_\infty)} = \frac{-kE}{(D - T_\infty)}$$

# Concept of Average Heat Transfer Coefficient

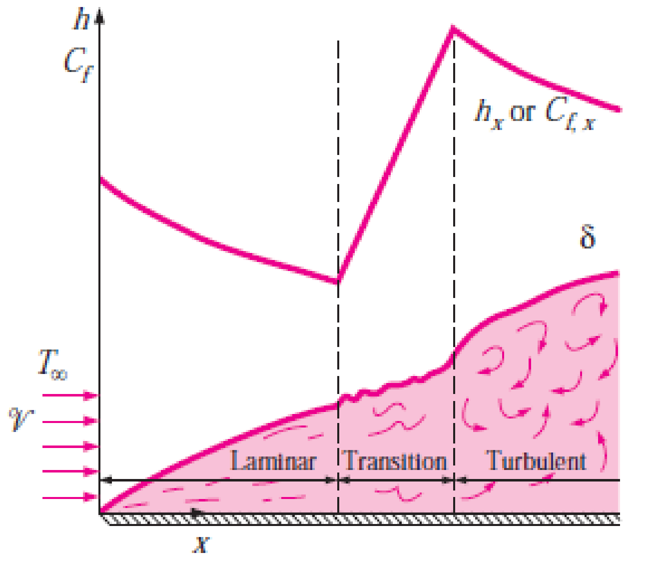


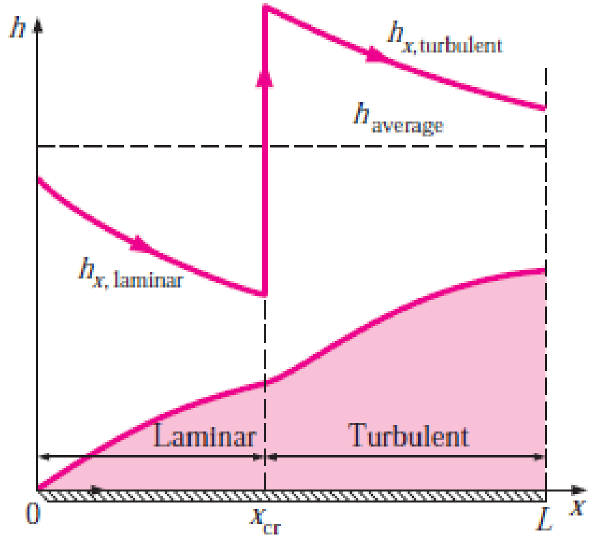

$$q'' = h\left(T_s - T_\infty\right); \quad \Rightarrow dq = h\left(T_s - T_\infty\right)dA_s$$

$$q = \int_{A_s} q'' dA_s = (T_s - T_\infty)\int_{A_s} h dA_s$$

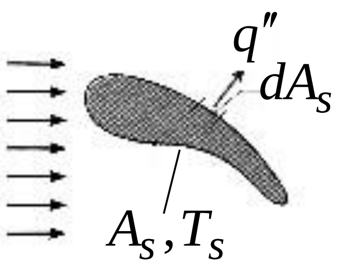
$$q = \bar{h} A_s (T_s - T_\infty)$$

$$\bar{h} = \frac{1}{A_s}\int_{A_s} h dA_s$$



# For any arbitrary shape



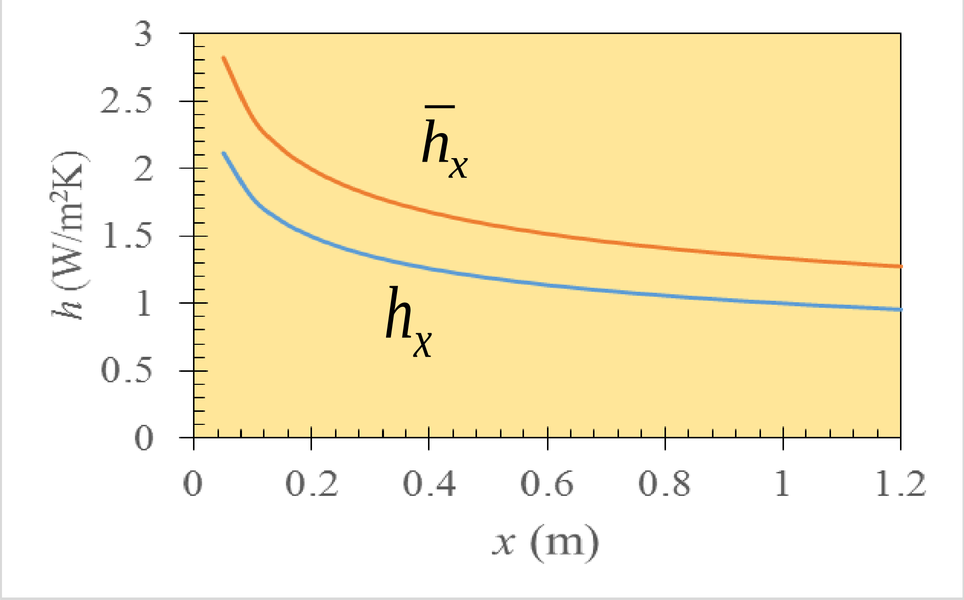

$$\bar{h} = \frac{1}{A_s}\int_{A_s} h\,dA_s$$ or, for unit width: $$\bar{h}_L = \frac{1}{L}\int_0^L h\,dx$$

$$\bar{h}_x = \frac{1}{A_s}\int_{A_s} h_x\,dA_s$$

Example:
*The local heat transfer coefficient for flow over a flat plate varies as* $h_x = Kx^{-n}$, $x$ = Distance from leading edge
Find the average heat transfer coefficient for a plate of length $x$. Compare it with the local heat transfer coefficient at the same location

$$\bar{h}_x = \frac{1}{x}\int_0^x h_x\,dx = \frac{1}{x}K\int_0^x x^{-n}\,dx$$

$$= \frac{K}{(-n+1)}\frac{1}{x}x^{(-n+1)} = \frac{Kx^{-n}}{(1-n)} = \frac{h_x}{1-n}$$



# Nondimensional number in flow: *Re*

Reynolds number: ratio of inertia forces to viscous forces in the fluid

$$\mathrm{Re} = \frac{\text{inertia forces}}{\text{viscous forces}} = \frac{\rho V \delta}{\mu} = \frac{V\delta}{\nu}$$

At large *Re* numbers, the inertia forces, which are proportional to the density and the velocity of the fluid, are large relative to the viscous forces; thus the viscous forces cannot prevent the random and rapid fluctuations of the fluid (turbulent regime).

# Nondimensional numbers for Convection

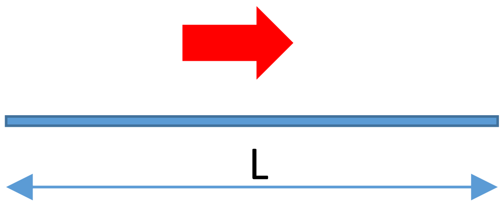
<u>Nusselt number</u>: non-dimensional heat transfer coefficient

$$Nu = \frac{h\delta}{k} = \frac{\dot{q}_{conv}}{\dot{q}_{cond}}$$

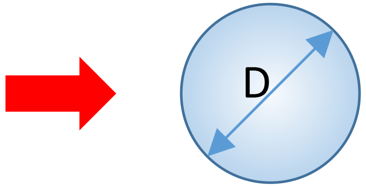
$$Nu = \frac{h}{k/L} = \frac{h\Delta T}{\frac{k}{L}\Delta T} = \frac{Convective\ heat\ flux}{Conductive\ heat\ flux}$$

where $\delta$ is the characteristic length, i.e. $D$ for the tube and $L$ for the flat plate. Nusselt number represents the enhancement of heat transfer through a fluid as a result of convection relative to conduction across the same fluid layer.

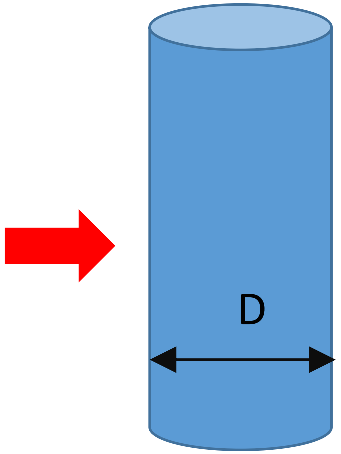
**What is the pertinent length-scale?**



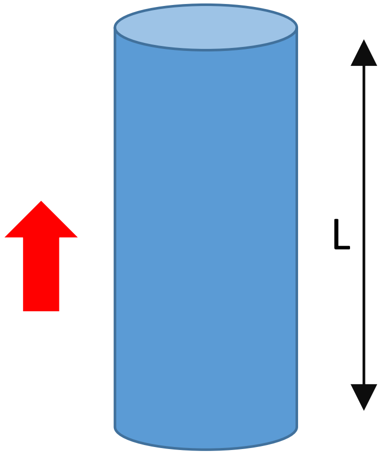
Flat Plate



Sphere



Cylinder



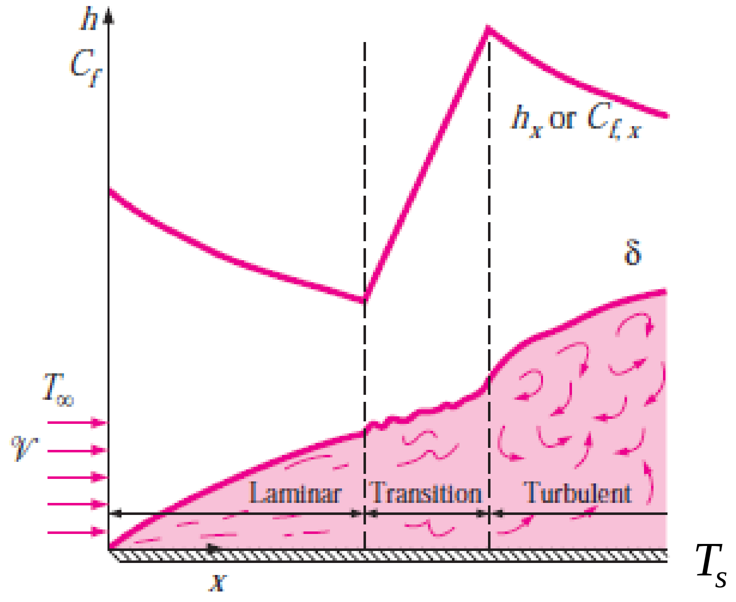
Cylinder

# Nusselt Number & non-dimensional temperature

Nondimensional Temperature: $\theta = \dfrac{(T - T_\infty)}{(T_s - T_\infty)}; \quad y^* = \dfrac{y}{L}$

$$h = \frac{-k_f \, \partial T / \partial y\big|_{y=0}}{(T_s - T_\infty)} = \frac{(T_s - T_\infty)}{(T_s - T_\infty)} \frac{k_f}{L} \frac{\partial \theta}{\partial y^*}\bigg|_{y^*=0} = \frac{k_f}{L} \frac{\partial \theta}{\partial y^*}\bigg|_{y^*=0}$$

$$\frac{\partial \theta}{\partial y^*}\bigg|_{y^*=0} = \frac{hL}{k} = Nu$$



# A few more words about the Nu

- Nusselt number can be looked upon as the nondimensional temperature gradient at the wall (over which convection is taking place)
- Nu can represented as the ratio of convective to conductive heat fluxes for a given configuration
- A Nusselt number of order unity would indicate a sluggish motion little more effective than pure fluid conduction: for example, laminar flow in a long pipe
- A large Nusselt number means very efficient convection: For example, turbulent pipe flow yields *Nu* of order 100 to 1000
- The useful thing about Nu is that for hydrodynamically and thermally similar flow conditions a single correlation / expression of Nu can be applied for various flow velocity, temperature difference and fluid properties

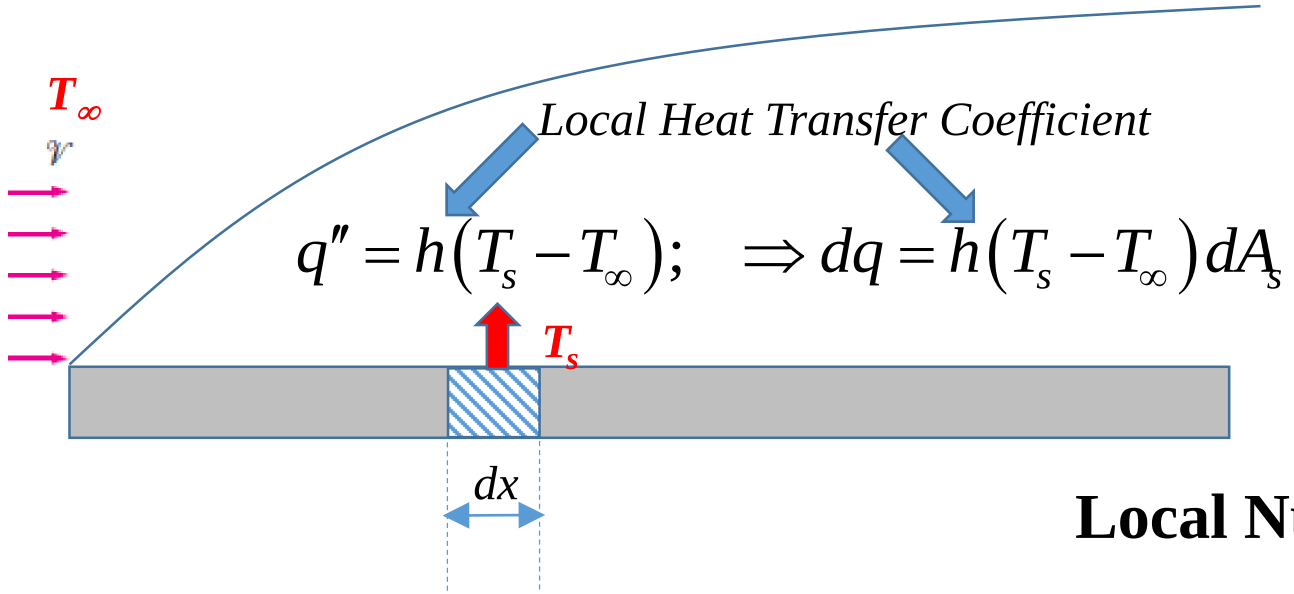
- What is the difference between *Nu* and *Bi*?

$$Nu = \frac{hL}{k_f}; \quad Bi = \frac{hL}{k_{solid}}$$

Helps us to evaluate convective heat transfer for thermally and hydrodynamically similar flows from known correlations

Helps us to check if the conduction within the solid is fast enough (w.r.t. the convective transport) so that the temperature within the solid is nearly uniform (and we can use lumped capacitance model)

# Local and average Nusselt Number




$$q'' = h\left(T_s - T_\infty\right); \quad \Rightarrow dq = h\left(T_s - T_\infty\right) dA_s$$

$$\bar{h}_x = \frac{1}{x}\int_0^x h_x dx$$

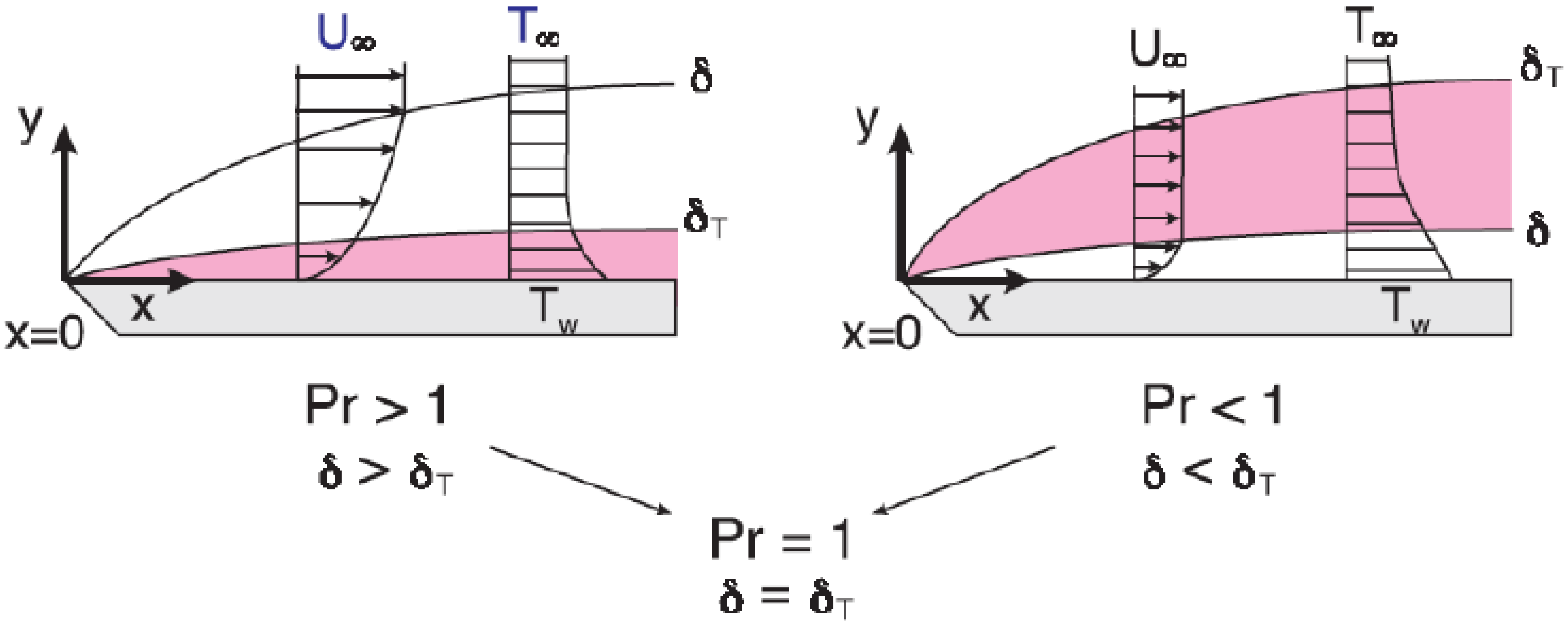
**Local Nusselt Number:** $Nu_x = \dfrac{hx}{k}$

**Average Nusselt Number:**

$$\overline{Nu}_x = \frac{\bar{h}_x x}{k} = \frac{x}{xk}\int_0^x h_x dx = \frac{1}{k}\int_0^x \frac{kNu_x}{x} dx = \int_0^x \frac{Nu_x}{x} dx$$

# Prandtl Number

$$\Pr = \frac{\textit{Momentum Diffusivity}}{\textit{Thermal Diffusivity}} = \frac{\nu}{\alpha} = \frac{\mu C_p}{k}$$




Indicates the relative thickness of hydrodynamic to thermal boundary layers

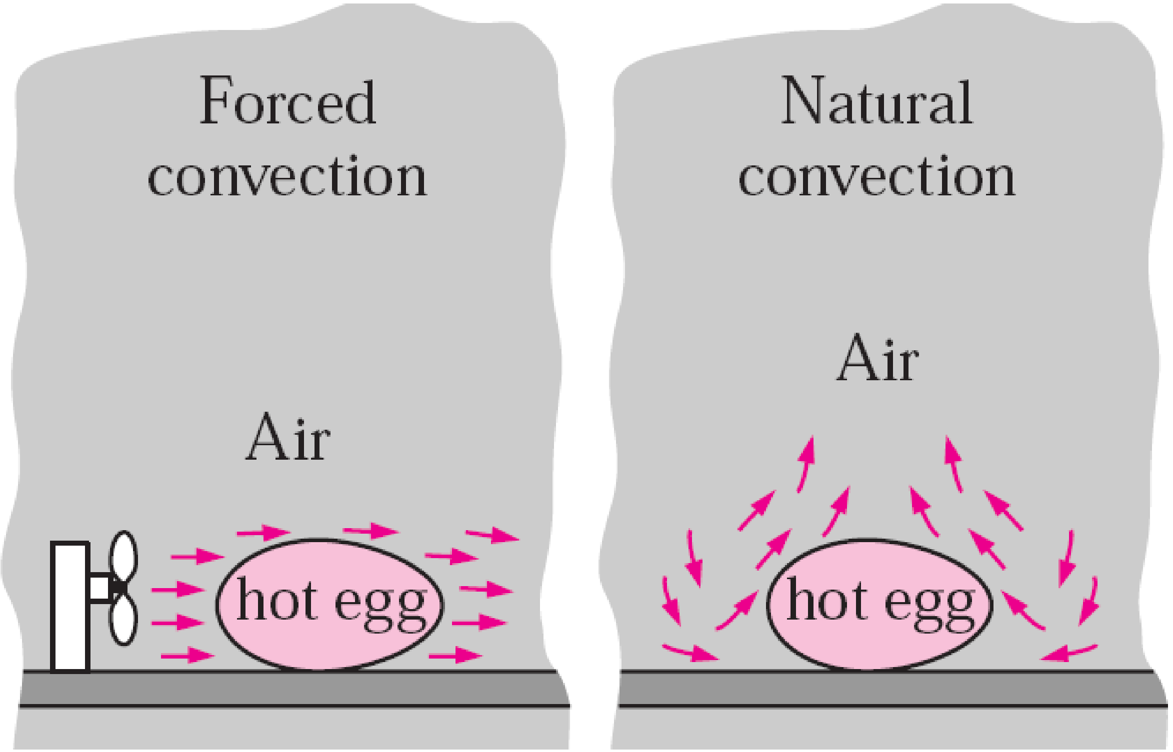
$$\Pr = \left(\frac{\delta}{\delta_T}\right)^n$$

$$\Pr = \frac{\nu}{\alpha} = \frac{\text{Ability of a fluid to transport momentum by molecular means}}{\text{Ability of that fluid to transport energy by molecular means}}$$

**Typical Pr Range**

Liquid metals: ~$10^{-2}$ (k is very high); gases : 0.7 – 1; water: 5—7; Oils: 50 – 100 ( $\mu$ is very high)

# Types of Convection: Forced and free




- External pumping needed
- Nu is a function of Re and Pr

- No External pumping needed
- Nu is a function of Gr and Pr

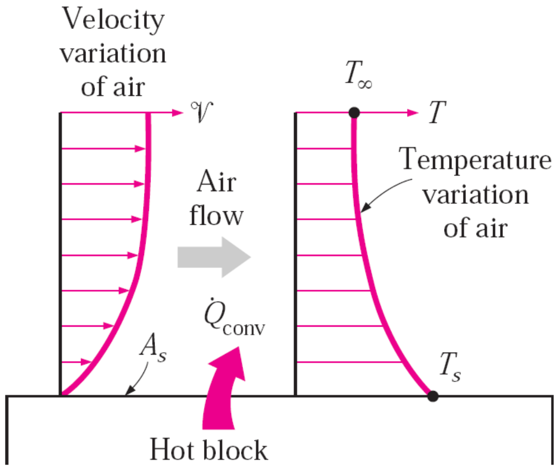
Mixed Convection = Both forced and free convections are significant

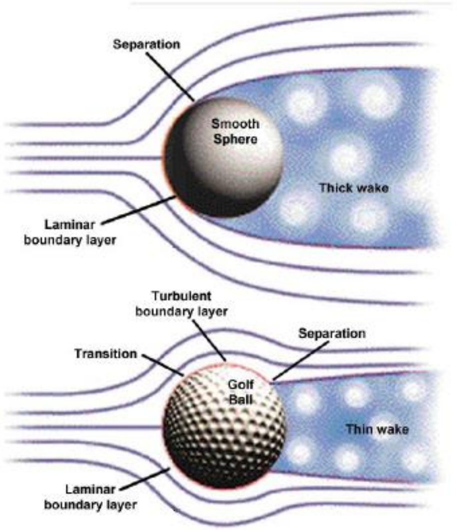
Gr = Grashof Number

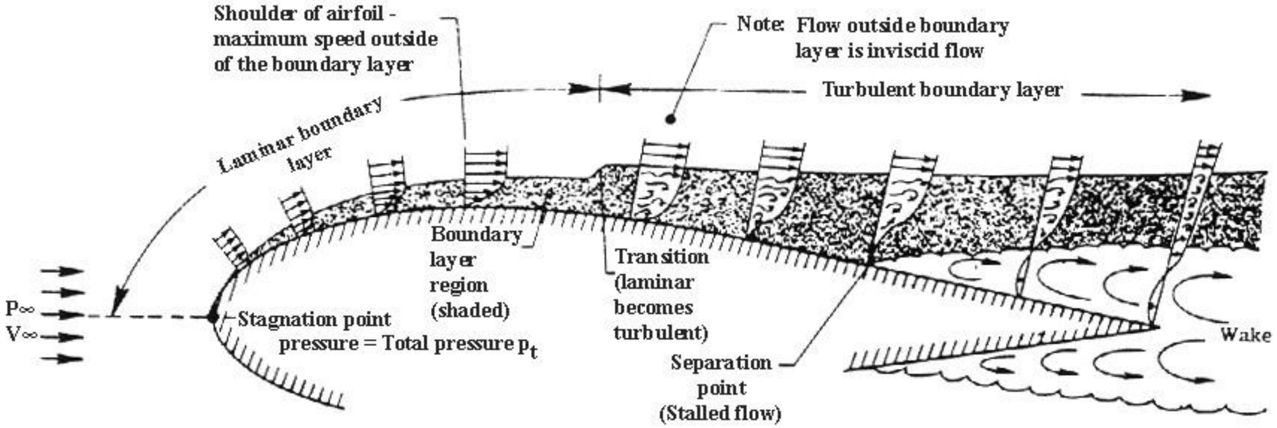
$$Gr = \frac{g\beta\Delta TL^3}{\nu^2} = \frac{Gravitational\ inertia}{Viscous}$$

# Types of flows: Internal and External

External




Internal