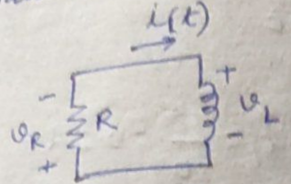
Simple RL and RC circuits:

(A)

A/ Source Free Ckts.: only stored energy considered.

Natural response / Transient response / Complementary fn.



$v_R + v_L = Ri + L\dfrac{di}{dt} = 0 \Rightarrow \dfrac{di}{i} = -\dfrac{R}{L}dt \ldots$

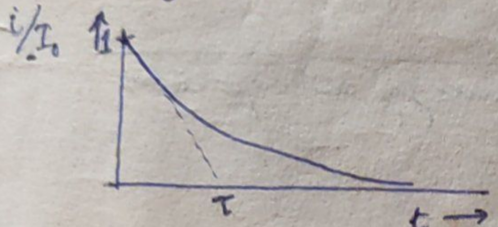
or Assume $i(t) = Ae^{s_1 t}$

$\therefore As_1 e^{s_1 t} + \dfrac{R}{L}Ae^{s_1 t} = 0 \Rightarrow \left(s_1 + \dfrac{R}{L}\right)Ae^{s_1 t} = 0.$

$\therefore$ Possible soln. $s_1 = -R/L$. $\because A = 0$ or $s_1 = -\infty \Rightarrow$ no response.

$\therefore i(t) = Ae^{-Rt/L}$ at $t = 0,\ i = I_0 \therefore A = I_0 \Rightarrow i(t) = I_0 e^{-Rt/L}$

$W_R = \int_0^\infty P_R\,dt = I_0^2 R\int_0^\infty e^{-2Rt/L}dt = \frac{1}{2}LI_0^2$ dissipated through resistor



initial rate of ~~decay~~ change $= \dfrac{d}{dt}\dfrac{i}{I_0}\Big|_{t=0} = -R/L$.

$\therefore$ ini rate of decay $= -(-R/L) = R/L$.

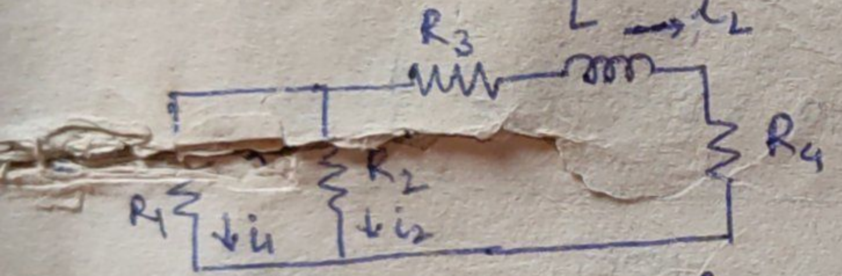
τ : Time constant $= L/R$

: time taken for i/I_0 to drop from unity to zero assuming constant rate of decay. $\boxed{\frac{R}{L}\tau = 1}$

$i(\tau) = e^{-1}I_0 = 0.368\,I_0$

3 to 5 time constants later $\rightarrow i(t) \ll I_0$ $\left(\overset{<5\%}{0.0498\,I_0} \longrightarrow \overset{<1\%}{0.0067\,I_0}\right)$.

Larger $L/R \Rightarrow$ response decays more slowly $\rightarrow$ greater energy storage and/or power flowing into R less for same I_0 $\therefore$ time more.



$$R_{eq} = R_3 + R_4 + \frac{R_1 R_2}{R_1 + R_2}$$

$$\tau = L/R_{eq}. \qquad i_L = i_L(0)\, e^{-t/\tau}$$

Say req. $i_2 = \left(\frac{R_1}{R_1 + R_2}\right)(-i_L)$.

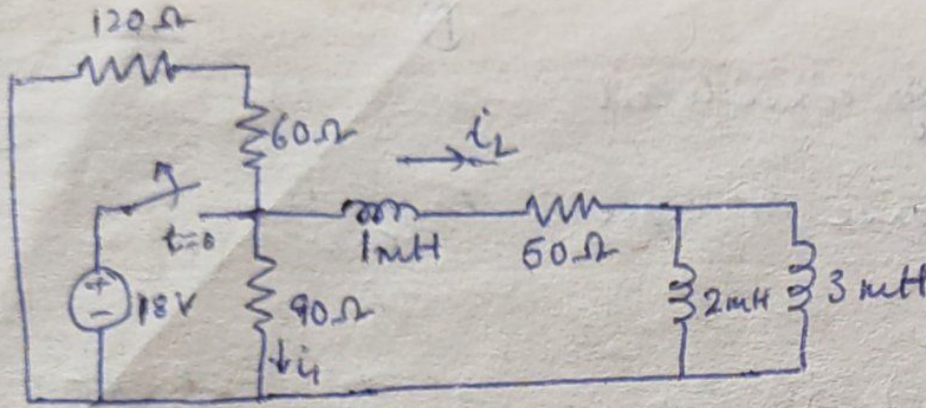
Say know $i_1(0^+)$ $\therefore i_2(0^+) = i_1(0^+) \cdot \frac{R_1}{R_2}$.

$$\therefore i_L(0^+) = -\left[i_1(0^+) + i_2(0^+)\right] = -\frac{R_1 + R_2}{R_2}\, i_1(0^+)$$

$$\therefore i_2 = i_1(0^+) \cdot \frac{R_1}{R_2}\, e^{-t/\tau}$$

<u>OR</u>. Say $i_2 = Ae^{-t/\tau}$, $\tau = L/R_{eq}$.

$$i_2(0^+) = A = i_1(0^+) \cdot \frac{R_1}{R_2}.$$



$$\tau = \frac{L_{eq}}{R_{eq}} = \frac{2.2\times10^{-3}}{110} = 20\mu s.$$

$$\therefore i_L = Ae^{-50000t}$$

For $t<0$ (source connected), $i_L = \frac{18}{50} = 0.36A$; $i_1 = \frac{18}{90} = 0.2A$

$t=0^+$ $\quad i_L = 0.36A$ (through inductor cannot change instantaneously).

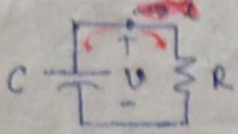
$$i_1(0^+) = -i_L(0^+)\cdot\frac{180}{270} \text{ (Current division) } = -0.24A$$

$$\therefore i_L = 0.36 \quad t<0; \quad = 0.36e^{-50000t} \quad t>0$$

$$i_1 = 0.2 \quad t<0; \quad -0.24e^{-50000t} \quad t>0.$$

Current through inductors 2 & 3 mH cons. current at $t\to\infty$.

$$i_2 = A_1 + A_2 e^{-t/\tau}$$



$v(0) = V_0$ → initial stored energy in capacitor.

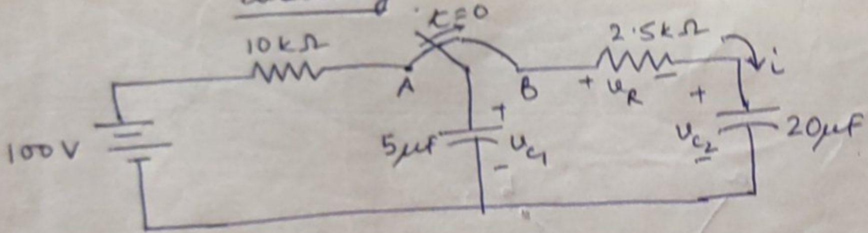
$$C\frac{dv}{dt} + \frac{v}{R} = 0. \quad \text{(node)}$$

$$v(t) = v(0)\, e^{-t/RC} = V_0 e^{-t/RC}. \qquad \tau = RC.$$

∴ Larger R & C → slower dissipation. ∵ $\left(\frac{v^2}{R}\right)$ → given voltage less dissipation..

Ex (Prob 5-27)

Duality



① $v_{c_1}(0^-) = 100V$ (open ckt.)

$v_{c_2}(0^-) = 0V$

② $v_{c_1}(0^+) = 100V$

$v_{c_2}(0^+) = 0V$

$v_R(0^+) = 100V$.

③ $R_{eq} = 2.5\times10^3\,\Omega$

$$C_{eq} = \frac{20\times5\times10^{-6}\times\cancel{10^{-6}}}{25\times\cancel{10^{-6}}} = 4\times10^{-6}\,F$$

∴ $\tau = R_{eq}C_{eq} = \frac{1}{100}$ s.

④ $$i(t) = \frac{v_R(t)}{R} = \frac{100e^{-100t}}{2.5\times10^3} = 0.04\, e^{-100t}\ A.$$

⑤ $$v_{c_1}(t) = \frac{1}{C_1}\int(-i)\,dt + K.$$

$$= \frac{10^6}{5}\times\left(\frac{-4}{100}\right)\cdot\left(\frac{1}{-100}\right)\cdot e^{-100t} + K. \quad \text{~ly}$$

At $t=0^+$, $100 = 80e^{-100t}\Big|_{0^+} + K = 80 + K$.

∴ $K = 20$

⑥ $v_{c_2}(t) = 20 - 20e^{-100t}$

Note at $t=\infty$, $v_{c_2}(\infty) = 20$.

~ly $v_{c_1}(\infty) = 20$.

TANK CIRCUIT

⑦ Energy balance: Initial $\frac{1}{2}C_1 v_{c_1}^2(0^-) + \frac{1}{2}C_2 v_{c_2}^2(0^-) = \frac{1}{2}\times5\times10^{-6}\times10^4 + 0 = 25\,mJ + 0\,mJ$.

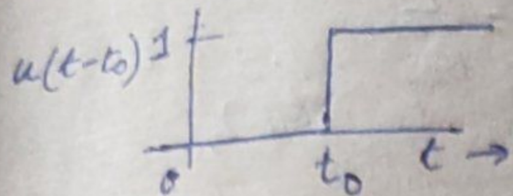
Final ~~dissipated~~: $\frac{1}{2}C_1V_{eq}^2 + \frac{1}{2}C_2V_{eq}^2 + \int_0^\infty \frac{V^2}{R}dt = \frac{1}{2}\times25\times10^{-6}\times(20)^2 + \frac{10^4}{2.5\times10^3}\int_0^\infty e^{-200t}dt$

$$= 5\times10^{-3} + \frac{1}{50} = 5\times10^{-3} + 20\times10^{-3}$$

$= 25\,mJ$.

(B) Application of Unit Step Forcing Function:

Forced response

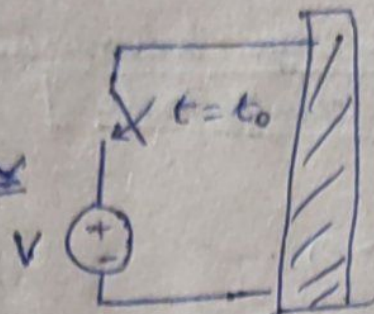


$$u(t-t_0) = \begin{cases} 0 & t < t_0 \\ 1 & t > t_0 \end{cases}$$

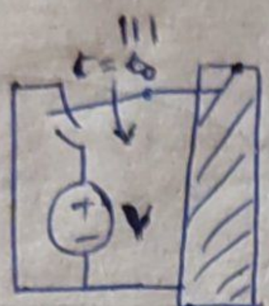
* Unit step fn. Not necessarily of t since only $u(x-x_0)$ say

$V u(t-t_0)$

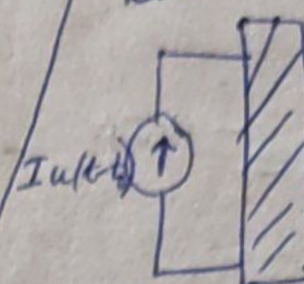
$\cong$



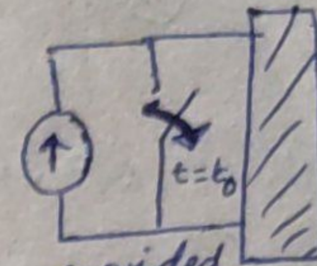
i.c. may not be met.



But

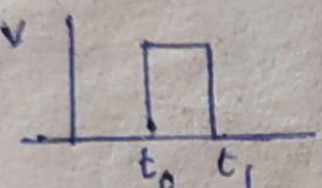


$\cong$

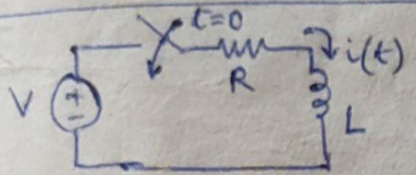


provided i.c. match

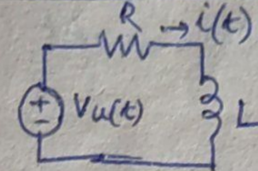
Note:



$$= V\left[u(t-t_0) - u(t-t_1)\right]$$



$\equiv$



KVL

$$Ri + L\frac{di}{dt} = Vu(t).$$

$$i(t) = 0 \qquad t < 0.$$

$t > 0$ $\quad Ri + L\frac{di}{dt} = V. \Rightarrow L\frac{di}{dt} + Ri = V.$

Form $\dfrac{di}{dt} + Pi = Q. \quad \Rightarrow di + Pi\,dt = Q\,dt.$

Multiply by $e^{\int P dt} = e^{Pt}$ [P being constant].

$$\therefore d(ie^{Pt}) = Qe^{Pt}dt.$$

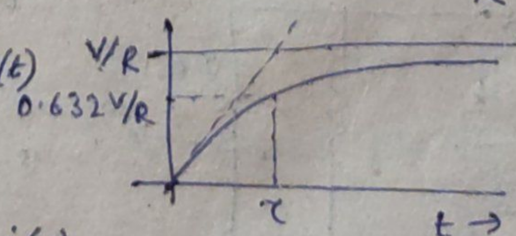
Integrate & transpose $\quad i = \underbrace{e^{-Pt}\int Q(t)e^{Pt}dt}_{\text{Particular fn. or forced response } i_f} + \underbrace{Ae^{-Pt}}_{\text{Natural response } i_n}$

For constant Q, $i_f = \dfrac{Q}{P}$

$$\therefore i(t) = \frac{Q}{P} + Ae^{-Pt} = i_f + i_n$$

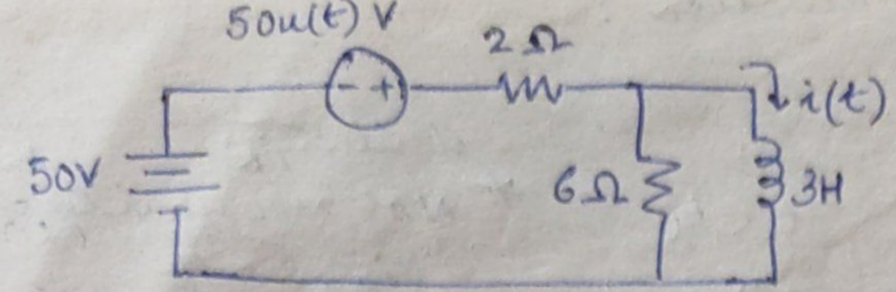
$\therefore \; i(t) = \dfrac{V}{R} + Ae^{-R/L\,t} \qquad 0 = A + \dfrac{V}{R} \;\therefore\; A = -\dfrac{V}{R}$ *(not necessarily ini. value of response).

$$\therefore i(t) = \frac{V}{R}\left(1 - e^{-Rt/L}\right)u(t)$$



Finally as R & L in series with battery $\therefore$ L short to dc $\therefore V/R = i(\infty)$.

Ex



$i = i_f + i_x$

$i_x = A e^{-t/2}$ $\because \tau = \frac{L}{R_{eq}} = \frac{3}{1.5} = 2$

$t > 0$. $(6 \| 2)$

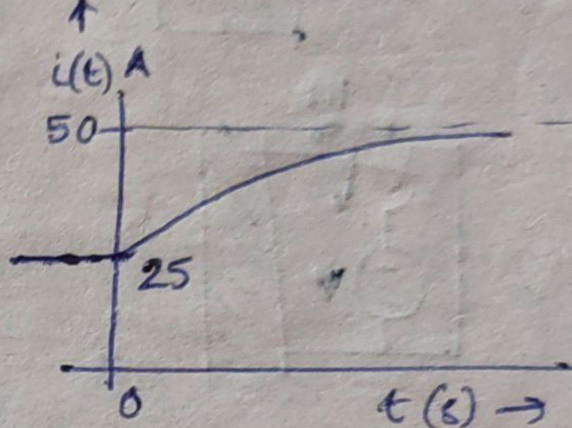
$i_f = \frac{100}{2} = 50\,A$ [no voltage across inductor at $t = \infty$ $\therefore$ short to dc]

$\therefore \quad i = 50 + A e^{-0.5t} \qquad t > 0$

$i_L(0^-) = \frac{50}{2} = 25$. $\quad \therefore \; 25 = 50 + A \quad \therefore A = -25$.

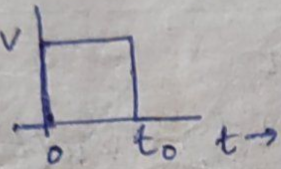
$\therefore \; i = 50 - 25 e^{-0.5t}\,A \quad t > 0$

$= 25 \; A \qquad t < 0$.



or $\; i(t) = 25 + 25\left(1 - e^{-0.5t}\right)u(t)\; A$.

Say apply



to RL series ckt.

$$\therefore\ v(t) = V\,u(t) - V\,u(t-t_0).$$

$$i(t) = i_1(t) + i_2(t).$$

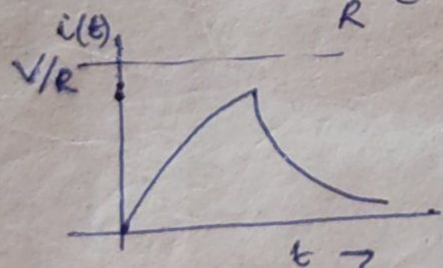
$$i_1(t) = \frac{V}{R}\left(1 - e^{-Rt/L}\right) \quad t>0$$

$$i_2(t) = -\frac{V}{R}\left(1 - e^{-R(t-t_0)/L}\right) \quad t>t_0.$$

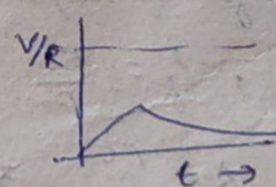
$$\therefore\ i(t) = \frac{V}{R}\left(1-e^{-Rt/L}\right) \quad 0<t<t_0$$

$$= \frac{V}{R}\left(1-e^{-Rt/L}\right) - \frac{V}{R}\left(1-e^{-R(t-t_0)/L}\right) \quad t>t_0.$$

$$= \frac{V}{R}\,e^{-Rt_0/L}\left(e^{Rt_0/L} - 1\right).$$

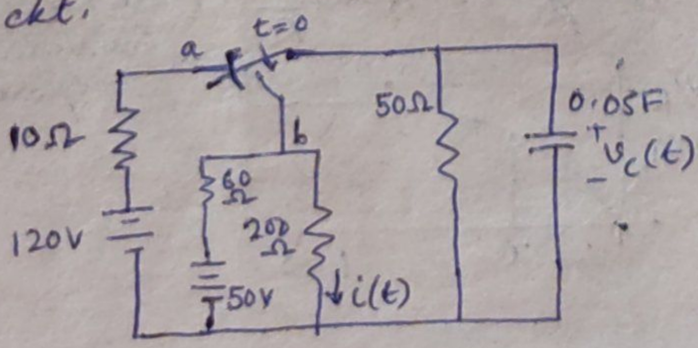


or



depending on τ.

Ex RC ckt.



$$v_c(0^+) = v_c(0^-) = \frac{50}{50+10} \times 120 = 100\,V.$$

$$v_c = v_{c_f} + v_{c_n}.$$

$$v_{c_n} = A e^{-t/R_{eq}C}.$$

$$R_{eq} = \frac{1}{\frac{1}{50} + \frac{1}{200} + \frac{1}{60}} = 24\,\Omega$$

$$\therefore\ v_{c_n} = A e^{-t/1.2}$$

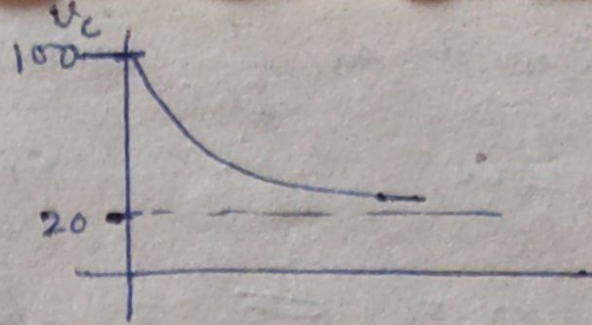
(Cap. open)

$$v_{c_f} = \underset{(v_s)}{50} \times \frac{(50\times 200)/250}{60 + \frac{50\times 200}{250}} = 20V.$$

$$\therefore\ v_c = \left(20 + A e^{-t/1.2}\right) V$$

So $100 = 20 + A$. $\therefore A = 80$

$\therefore v_c = 20 + 80\, e^{-t/1.2}$ V $\quad t > 0$.



Say req. $i(t)$.

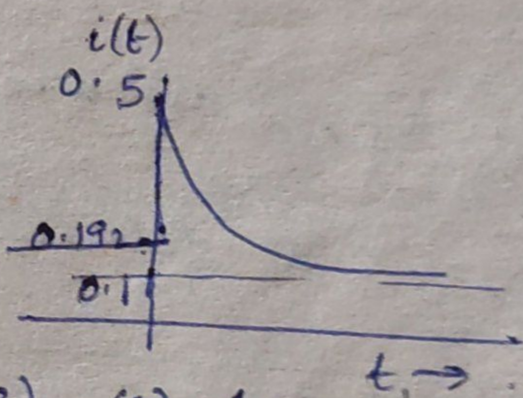
$i(0^-) = \dfrac{50}{260}$ A $= 0.192$ A.

$i_f = \dfrac{50}{200+50} \times \dfrac{50}{60 + \dfrac{50 \times 200}{250}} = 0.1$ A.

$i = 0.1 + Ae^{-t/1.2}$

req. $i(0^+) \longrightarrow v_c(0^+) = 100$ V. $\quad i(0^+) = \dfrac{100}{200}$ A $= 0.5$ A (all with cap.) 200 Ω resistance

$\therefore A = 0.4$ $\quad$ ~~অর্থ 0.5 + 0.192 = 0.1 + A ⇒ A = 0.592 ?~~



$\therefore i(t) = 0.192$ A $\quad t < 0$

$= 0.1 + 0.4\, e^{-t/1.2} \quad t > 0$.

or $i(t) = 0.192 + \left(-0.092 + 0.4\, e^{-t/1.2}\right) u(t)$ A.