Polar plot (NYQUIST PLOT) ω : 0 to ∞
ω : $-\infty$ to $+\infty$

plot of $|G(j\omega)|$ vs $\angle G(j\omega)$ in polar co-ord.

$\Rightarrow$ Re $G(j\omega)$ vs Im $G(j\omega)$ in Cartesian co-ord.

- $|G(j\omega)|$ and Re $G(j\omega)$ are even fns.
  $\angle G(j\omega)$ and Im $G(j\omega)$ are odd fns.

+ve phase angle is CCW.

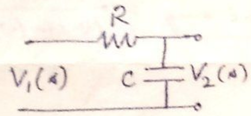
Disadv. : a) addn of poles & zeros to existing sys. req. recalculations

similar to root locus b) effects of addn. of individual poles/zeros not indicated

For cascaded T.F., use Bode plot & shift to polar plot.

Adv. freq. response over entire freq. range in one plot.

Ex

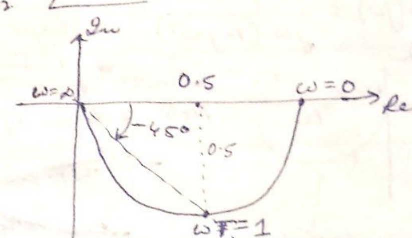


Low pass RC (LAG) ckt.

$$G(s) = \frac{V_2(s)}{V_1(s)} = \frac{1}{1+sT} \; ; \quad T = RC$$

$$\therefore G(j\omega) = \frac{1}{1+j\omega T} = \frac{1}{\sqrt{1+\omega^2T^2}} \angle -\tan^{-1}\omega T$$

ω	$\|G(j\omega)\|$	$\angle G(j\omega)$
0	1	0
$1/T$	0.707	$-45°$
∞	0	$-90°$

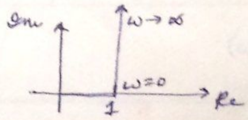


$\therefore$ Polar plot is a semicircle of rad. 0.5 and center at 0.5.

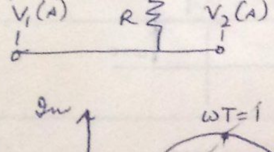
$$\left(X - \tfrac{1}{2}\right)^2 + Y^2 = \left(\tfrac{1}{2}\right)^2 \quad \text{where } X = \text{Re}\, G(j\omega) = \frac{1}{1+\omega^2T^2}, \; Y = \text{Im}\, G(j\omega) = \frac{-\omega T}{1+\omega^2 T^2}$$

O/p lags behind i/p $\forall\omega$: indication is plot in ($-j\omega$) plane.

Ex For $G(j\omega) = 1 + j\omega T \longrightarrow$ LEADS o/p.
ex. series RL ckt.
(NON CAUSAL)

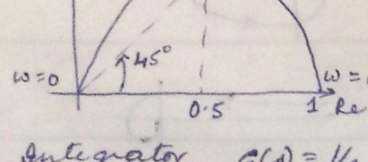


Ex High pass RC (LEAD) Network



$$G(s) = \frac{V_2(s)}{V_1(s)} = \frac{s}{s + 1/T} \qquad T = RC.$$

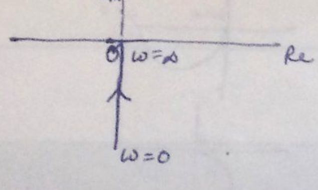
$$\Rightarrow G(j\omega) = \frac{j\omega}{j\omega + 1/T} = \left|\frac{\omega T}{\sqrt{1+\omega^2T^2}}\right| \angle 90° - \tan^{-1}\omega T$$



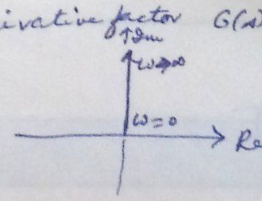
o/p voltage leads i/p at all times.

Ex Integrator $G(s) = 1/s$

$$\therefore G(j\omega) = \frac{1}{j\omega} = \frac{-j}{\omega} = \frac{1}{\omega} \angle -90°$$



Ex Derivative factor $G(s) = s$

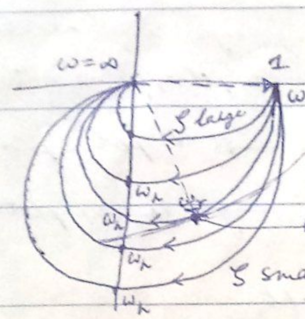


Ex. RLC series network with o/p across capacitor

$$G(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} \qquad \omega_n = \frac{1}{\sqrt{LC}}\ ,\ \zeta = \frac{R}{2}\sqrt{\frac{L}{C}}$$

$$\therefore G(j\omega) = \frac{1}{1 + j2\zeta(\omega/\omega_n) - (\omega/\omega_n)^2}$$

ω	$G(j\omega)$
0	$1\angle 0^\circ$
∞	$0\angle 180^\circ \rightarrow -\tan^{-1}\left(\frac{2\zeta}{(-\omega/\omega_n)}\right) \rightarrow -\tan^{-1}(0)$



At $\omega = \omega_n$, $G(j\omega_n) = 1/j2\zeta = -j\frac{1}{2\zeta}$

$\therefore$ on neg. real axis

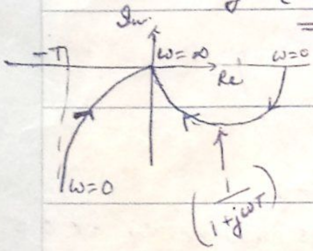
ω_r where distance from origin is max. $\omega_r = \omega_n\sqrt{1-2\zeta^2}$ $\quad |G(j\omega_r)| = \frac{1}{2\zeta\sqrt{1-\zeta^2}}$

Resonant mag. $= \frac{|\text{vector}(\omega_r)|}{|\text{vector}(\omega=0)|}$

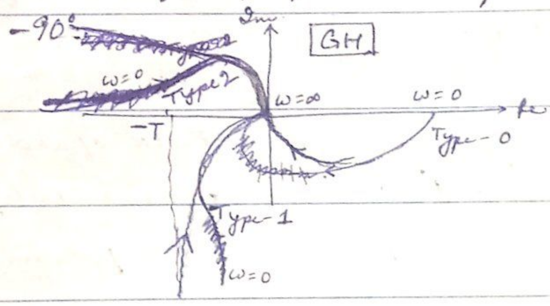
Ex $\quad G(s) = \frac{1}{s(sT+1)}$

$$G(j\omega) = \frac{1}{j\omega(1+j\omega T)} = \frac{1}{-\omega^2 T + j\omega} = \frac{-T}{(1+\omega^2T^2)} - \frac{j}{\omega(1+\omega^2T^2)}$$

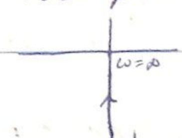
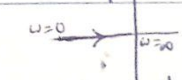
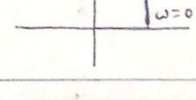
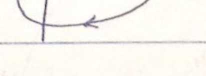
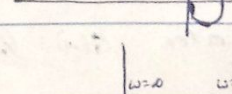
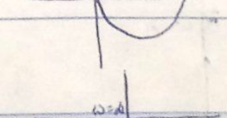
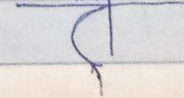
ω	$G(j\omega)$
0	$-T - j\infty = \infty\angle -90^\circ$
∞	$0\angle -180^\circ$



Note; Addn. of simple pole rotates polar plot by -90°

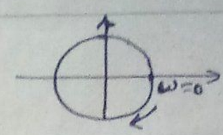


$$G(j\omega) = \frac{1}{(j\omega)^2(1+j\omega T)}$$
$$= \frac{-j}{\omega}\left[\frac{-T}{1+\omega^2T^2} - \frac{j}{\omega(1+\omega^2T^2)}\right]$$
$$= \frac{-1}{\omega^2(1+\omega^2T^2)} + j\frac{T}{\omega(1+\omega^2T^2)}$$
$$= -\infty + j\infty \text{ at } \omega = 0.$$

$G(j\omega)$	Pole-zero	Polar plot	Rem.
$\frac{1}{j\omega}$			
$\frac{1}{(j\omega)^2}$			
$1+j\omega T$			
$\frac{1}{(1+j\omega T_1)(1+j\omega T_2)(1+j\omega T_3)}$			
$\frac{1}{(j\omega)^2(1+j\omega T_1)(1+j\omega T_2)(1+j\omega T_3)}$			
$\frac{1}{1+j\omega T}$			
$\frac{1}{j\omega(1+j\omega T)}$			

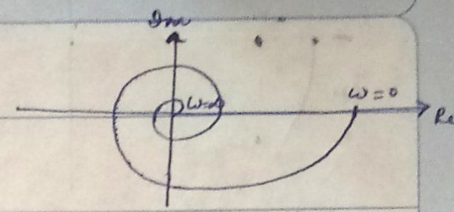
Ex Transportation lag $G(s) = e^{-sT}$

$G(j\omega) = 1 \angle -\omega T$



Ex. $G(s) = \dfrac{\exp(-sL)}{1+sT}$

$\therefore G(j\omega) = \dfrac{1}{\sqrt{1+\omega^2T^2}} \angle -\omega L - \tan^{-1}\omega T$



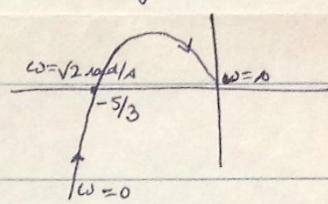
Ex $G(s) = \dfrac{10}{s(s+1)(s+2)}$ $\quad G(j\omega) = \dfrac{10(-j\omega)(1-j\omega)(2-j\omega)}{\omega^2(1+\omega^2)(2+\omega^2)}$

$= \dfrac{10\left[-3\omega^2 - j\omega(2-\omega^2)\right]}{\omega^2(1+\omega^2)(2+\omega^2)}$

ω	$G(j\omega)$
0	$\infty \angle -90^\circ$
∞	$0 \angle -270^\circ$
$\sqrt{2}$	$\frac{5}{3} \angle -180^\circ$

Re $G(j\omega) = 0$ at $\omega = \infty$

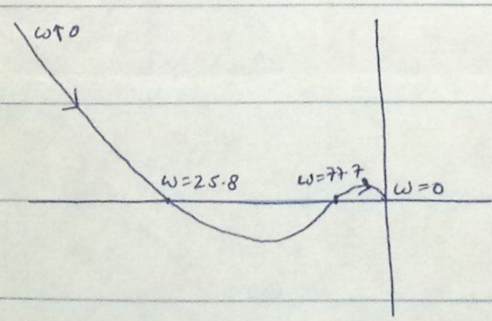
Im $G(j\omega) = 0$ at $\omega^2 = 2$.



Ex $GH(s) = \dfrac{100K(s+5)(s+40)}{s^3(s+100)(s+200)}$

ω	$\lvert GH(j\omega)\rvert$	$\angle GH(j\omega)$
0	∞	-270°
$0<\omega<25.8$	finite	$-180^\circ < \theta < -270^\circ$
25.8	$(-3.497\times10^{-4})K$	-180°
	finite	$-180^\circ < \theta < -90^\circ$
77.7	$(-5.3\times10^{-5})K$	-180°
∞	0	-270°

Im $GH(j\omega) = 0$.
at $\omega = 25.8$ rad/s & 77.7 rad/s

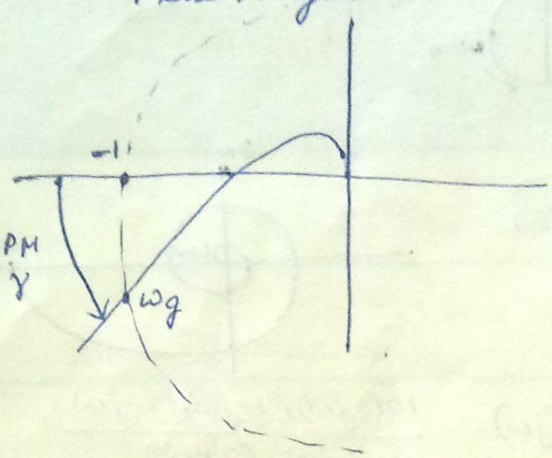


$\dfrac{1-j\omega T}{j^2\omega^2(1+\omega^2T^2)} = \dfrac{-1}{\omega^2(1+\omega^2T^2)} + j\dfrac{T}{\omega(1+\omega^2T^2)}$

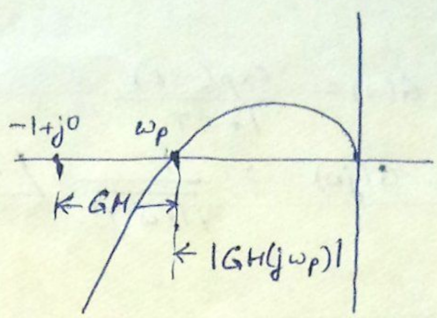
$= -\infty + j\infty$

Relative stability : GM & PM.

Phase Margin



Gain Margin



$$\therefore \quad GM = 20 \log_{10} \frac{1}{|GH(j\omega_p)|} \text{ dB.}$$

For MP systems, both GM & PM +ve ⇒ CL stable **SUFFICIENT CONDN.**