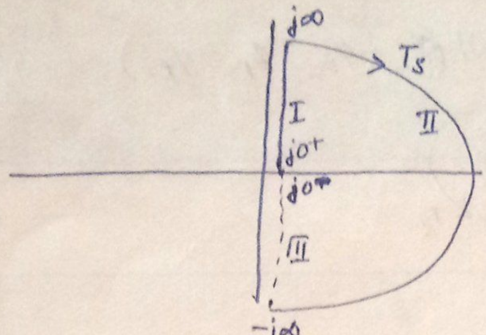
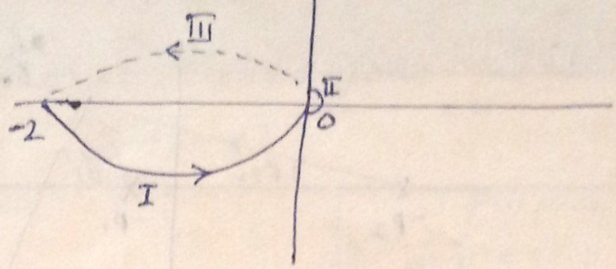
Ex 1 : $G(s)\,H(s) = \dfrac{K(s+2)}{(s+1)(s-1)}$ where $K = 1$.





I $GH(s) = \dfrac{s+2}{(s+1)(s-1)}$

$\therefore |GH(j\omega)| = \left|\dfrac{j\omega+2}{(j\omega+1)(j\omega-1)}\right|$

Note : θ varies from $+\pi/2$ to $-\pi/2$ through $0°$ on complete T_s semicircle of infinite radius

$\therefore \angle GH = \angle \tan^{-1}\omega/2 - \angle \tan^{-1}\omega - \angle \pi - \tan^{-1}\omega = -\pi + \tan^{-1}\omega/2$

for $\omega = 0$ to ∞ $\longrightarrow \angle GH \in [-\pi, -\pi/2]$ ~~for ϕ~~

II $s = \lim_{R\to\infty} R e^{j\phi}$ $\therefore \lim_{R\to\infty} GH(s)\Big|_{s=Re^{j\phi}} = \lim_{R\to\infty} \dfrac{1}{R} e^{-j\phi} = 0e^{-j\phi}$

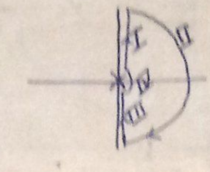
$\therefore$ ϵ radius circle CCW $\phi \in [\pi/2, -\pi/2]_{CW}$ $\angle GH \in [-\pi/2, \pi/2]_{CCW}$

III inverse plot of I.

$\therefore N = -1$, $P = +1$ $\therefore Z = N + P = 0$ $\therefore$ stable.

NYQUIST CONTOURS contd.

Ex 2. $GH(s) = \frac{K}{s(s+a)}$



I $j0^+$ to $+j\infty$

Polar plot for real freq.

Note: $-j\infty$ to $j0^-$: Inverse polar plot $Re(GH)$ even, $Im(GH)$ odd

$$GH(j\omega) = \frac{K(-\omega^2 - ja\omega)}{\omega^4 + a^2\omega^2} \Leftarrow \frac{K}{j\omega(j\omega+a)} = \frac{K(-j\omega)(a-j\omega)}{\omega^2(a^2+\omega^2)} \left(= \frac{K(-\omega^2 - ja\omega)}{\omega^4 + a^2\omega^2} \right)$$

Intersect on real axis when $Im\, GH(j\omega) = 0 \Rightarrow \omega = \infty$ $\because \frac{Ka\omega}{\omega^4 + a^2\omega^2} = \frac{Ka}{\omega(\omega^2+a^2)}$

IV $-j0^-$ to $+j0^+$ $\quad s = \lim_{\varepsilon \to 0} \varepsilon e^{j\theta} \quad \theta \in [-\pi/2, \pi/2]$ CCW

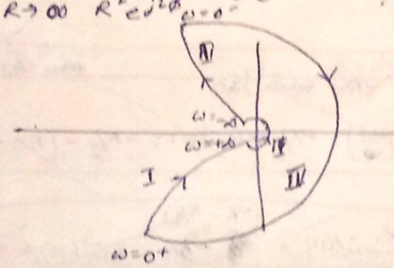
$\therefore GH|_s = \lim_{\varepsilon \to 0} \frac{K}{a\varepsilon e^{j\theta}} = \infty e^{-j\theta} \quad \therefore \angle GH \in [\pi/2, -\pi/2]$ CW

III $-j\infty$ to $j0^-$: inverse plot ($Re\, GH(j\omega)$ even; $Im\, GH(j\omega)$ odd fn.)
$\therefore$ Reflected about Re axis

IV $s = \lim_{R\to\infty} Re^{j\phi} \quad \phi \in [\pi/2, -\pi/2]$ CW

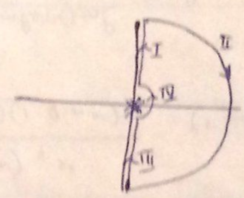
$\therefore GH|_s = \lim_{R\to\infty} \frac{K}{R^2 e^{j2\phi}} = 0 e^{-j2\phi} \quad \therefore \angle GH \in [-\pi, \pi]$ CCW

$\therefore$



$P = 0$
$N = 0$
$\therefore Z = P + N = 0$
$\therefore$ stable.

Ex 3. $GH(s) = \frac{K(T_2 s + 1)}{s^2 (T_1 s + 1)} \qquad T_1 > T_2$



I : $j0^+$ to $j\infty$

$$GH(j\omega) = \frac{K(jT_2\omega + 1)}{(j\omega)^2 (jT_1\omega + 1)}$$

$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$

$\angle GH(j\omega) = -180°$ at $\omega = \infty = (-\pi - \tan^{-1}\omega T_1 + \tan^{-1}\omega T_2)$

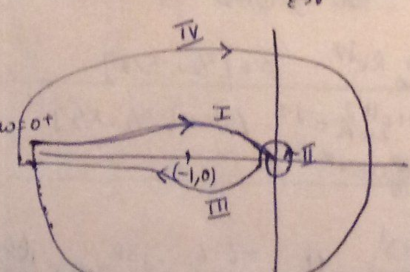
Polar plot. At some intermediate ω, $\angle$ becomes $-3\pi/2$ (CCW +ve)

II $j\infty$ to $-j\infty$ $\quad s = \lim_{R\to\infty} Re^{j\phi} \quad \phi \in [\pi/2, -\pi/2]$ CW

$\therefore GH|_s = 0 e^{-j2\phi} \quad \therefore \angle GH \in [-\pi$ to $\pi]$ CCW

III $-j\infty$ to $-j0^-$: Inverse plot

IV $-j0^-$ to $j0^+$: $\quad s = \lim_{\varepsilon\to 0} \varepsilon e^{j\theta} \quad \therefore GH|_s = \frac{K}{\varepsilon^2 e^{j2\theta}} = \infty e^{-j2\theta} \quad \theta \in [-\pi/2, \pi/2]$ CCW $\therefore \angle GH \in [\pi, -\pi]$ CW



$P = 0$
$N = 2$
$\therefore Z = 2 \quad \therefore$ unstable.

I $GH(j\omega) = \frac{K(j\omega + \ldots}{j\omega(j\omega \ldots}$

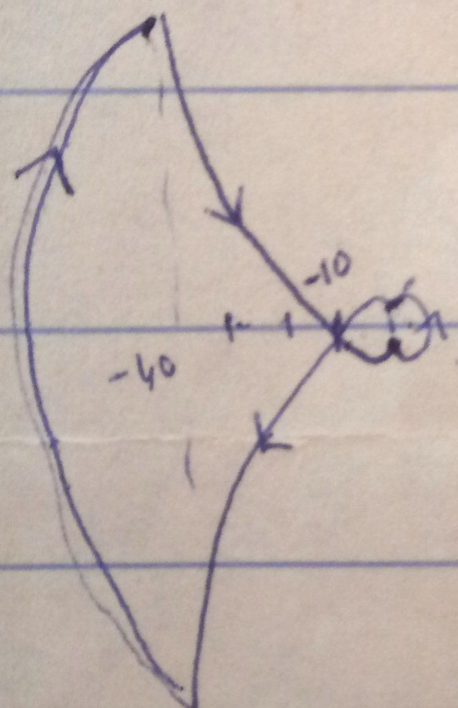
$\therefore \angle GH(j\omega) = - \ldots$

$= -3\pi/\ldots$

$(\tan^{-1} A + \tan^{-1} B) = \ldots$

$\therefore$ For $\text{Im}\, GH = 0 \ldots$

Ex. 6



II $s = \lim_{R\to\infty} Re^{j\phi}$

$GH(s) = \lim_{R\to\infty} \frac{10}{R} e^{-j\ldots}$

III IPP $\left[\frac{+\pi}{2}, \frac{+3\pi}{2}\right]$ c…

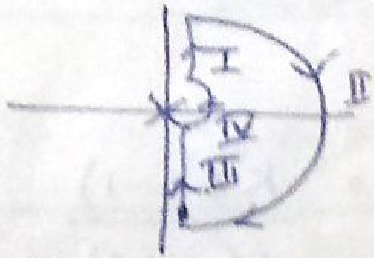
IV $s = \lim_{\varepsilon\to 0} \varepsilon e^{j\theta}$

$GH(s) = \frac{30}{-s}\Big|_{s=\ldots}$

$\therefore P = 1, N = -1$

Ex 7 $GH(s) = K(s-\ldots$

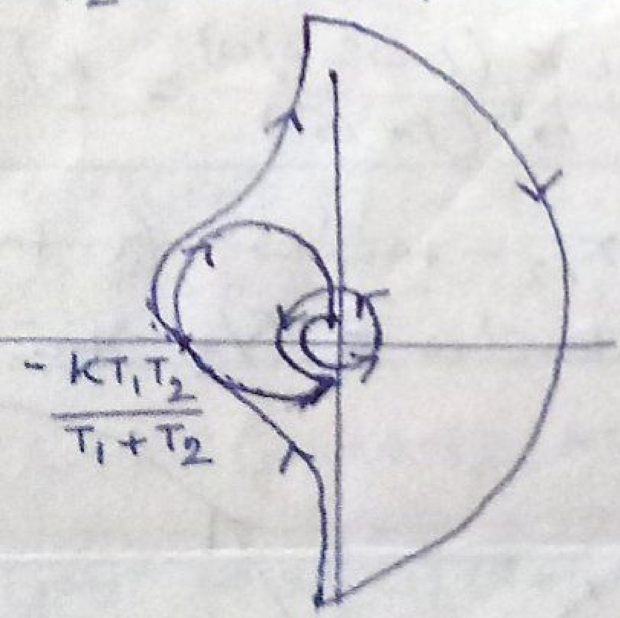
Ex 8



$$\frac{k}{\quad +1)(T_2 s+1)}$$

$$\frac{\quad}{\quad +1)(j\omega T_2+1)}$$

$$-\pi/2 - \tan^{-1}\omega T_1 - \tan^{-1}\omega T_2 \in [-\pi/2, -3\pi/2]$$



III IPP

IV $s = \lim_{\varepsilon \to 0} \varepsilon e^{j\theta}$

$\phi \in [-\pi/2, \pi/2]$

$GH(s) = \lim_{\varepsilon \to 0} \frac{K}{\varepsilon} e^{-j\theta} = \infty e^{-j\theta}$

$\therefore \angle GH(s) = [+\pi/2, -\pi/2)$