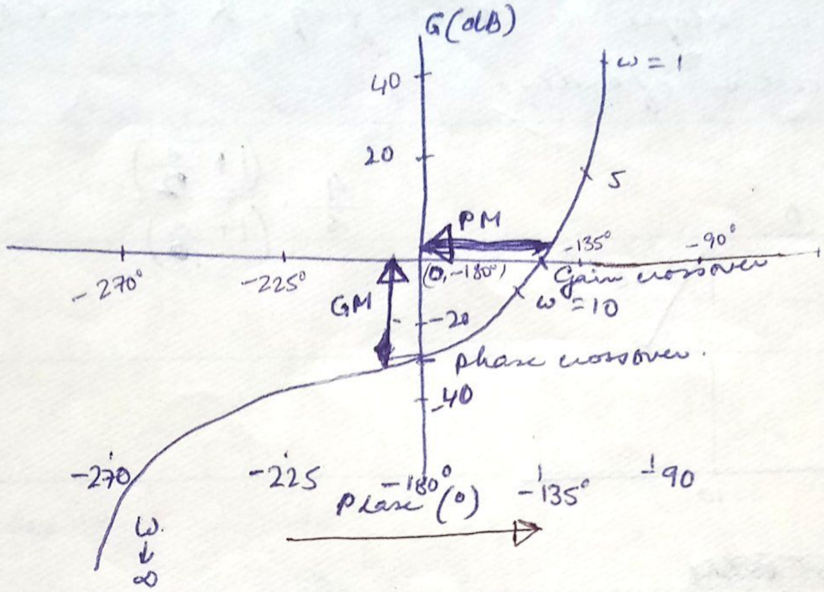
5 Log Magnitude vs. Phase Plot

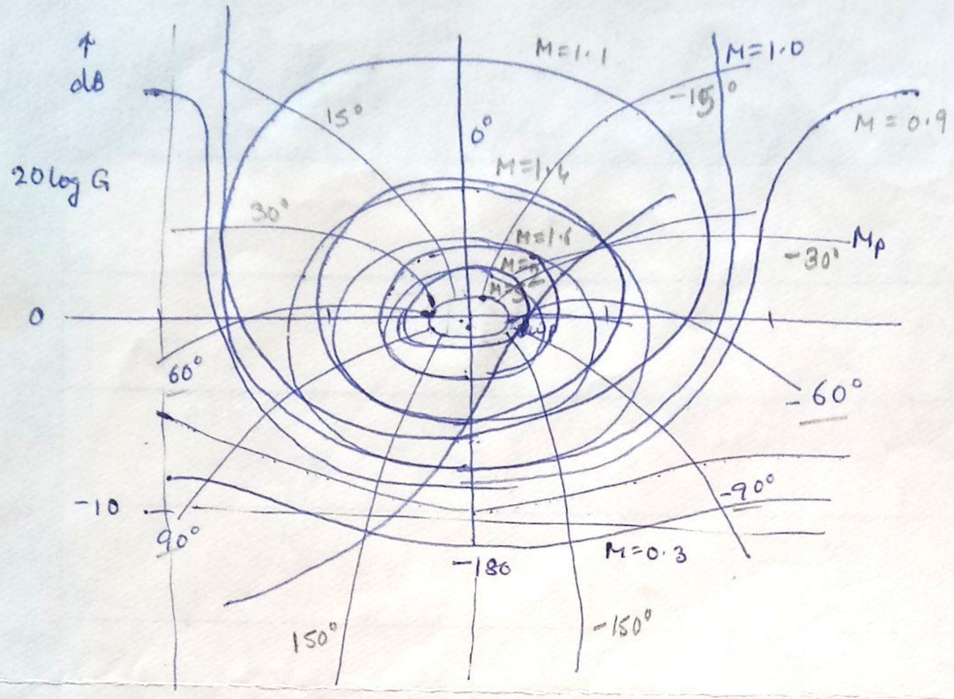
From Bode plot ⟶ crossover of mag. axis & crossover of $-180°$

Note: phase : x axis
log mag. : y axis



NICHOLS CHART:

Log magnitude vs phase plots



Non unity fb system

$$T(s) = \frac{G(s)}{1+GH(s)} = \frac{1}{H(s)}\,T_0(s)$$

$\therefore$ Plot for $T_0(s) \rightarrow$ use M & N circles for $G_0(s)$ to obtain $T_0(s)$

Then $T_0(s) \times \frac{1}{H(s)} \rightarrow$ use Bode plot $\rightarrow T(s)$

Freq. domain specs for design

2nd order prototype $\frac{\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2}$

$$M_p = \frac{1}{2\zeta\sqrt{1-\zeta^2}} \qquad 0<\zeta<0.707 \text{ (given } \zeta)$$

$$= 1 \qquad \zeta \geq 0.707$$

$\Rightarrow \zeta^2 = \frac{1}{2} - \frac{1}{2}\sqrt{\left(1-\frac{1}{M_p^2}\right)}$ for $M_p > 1$ (Given M_p)

M_p large $\Rightarrow$ usually large peak OVERSHOOT for STEP RESPONSE

$1.1 < M_p < 1.5 \iff 0.54 \cancel{>} \zeta \cancel{>} 0.36$ for stable system.

$\omega_p = \omega_n\sqrt{1-2\zeta^2} \Rightarrow$ system's speed of response

$BW = \omega_n\left[(1-2\zeta^2)+\sqrt{4\zeta^4-4\zeta^2+2}\right]^{1/2} \approx \omega_n\left[1-2\zeta^2+\frac{1}{2}(4\zeta^4-4\zeta^2+2)\right]^{1/2}$

$= \omega_n\left[2-4\zeta^2+2\zeta^4\right]^{1/2} = \omega_n(\sqrt{2}-\sqrt{2}\zeta^2)$

larger BW $\Rightarrow$ faster rise time, less noise rejection.

$\therefore$ higher freq. signals passed easily to o/p.

$= \sqrt{2}\,\omega$

$= 2\omega$

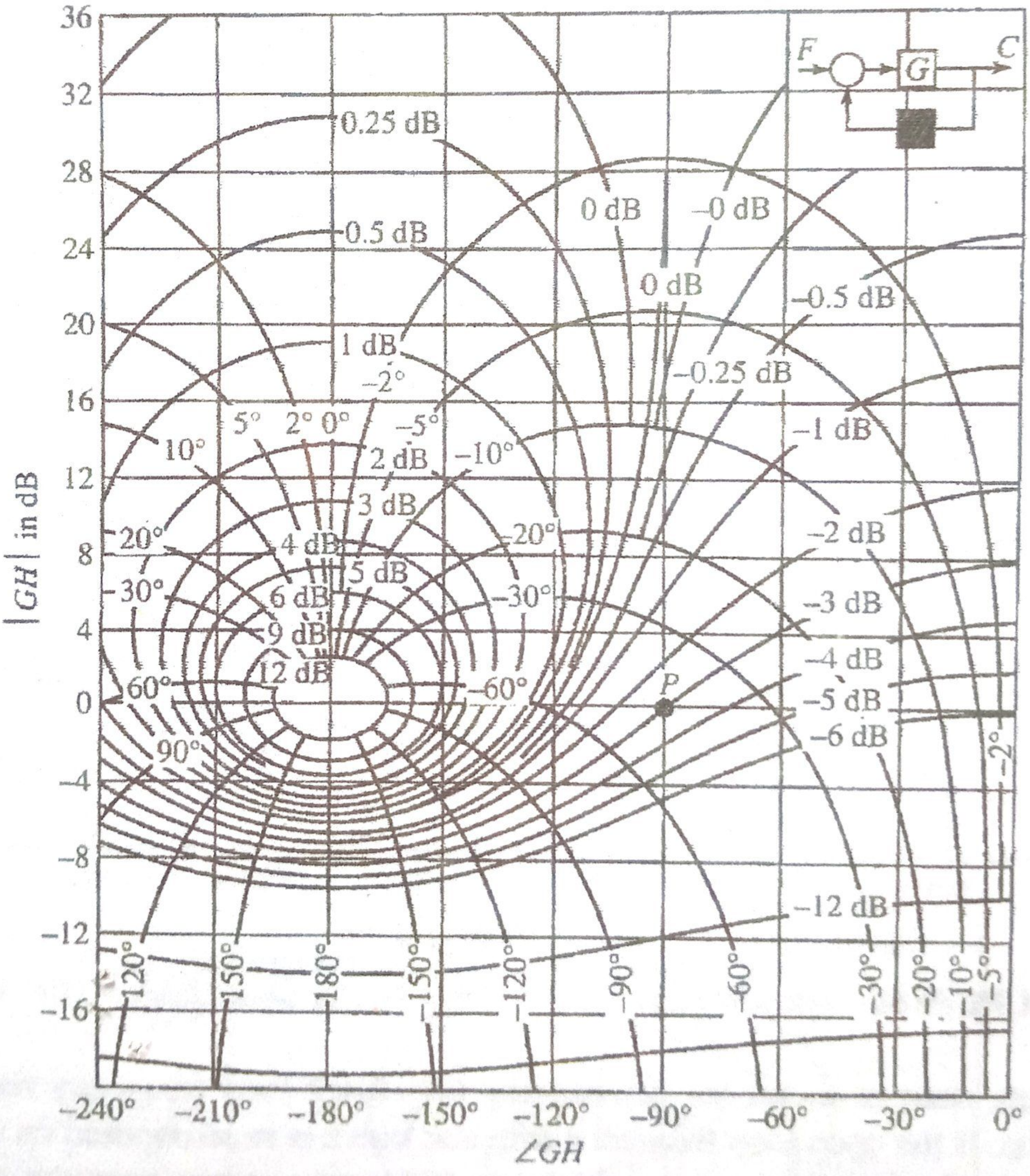
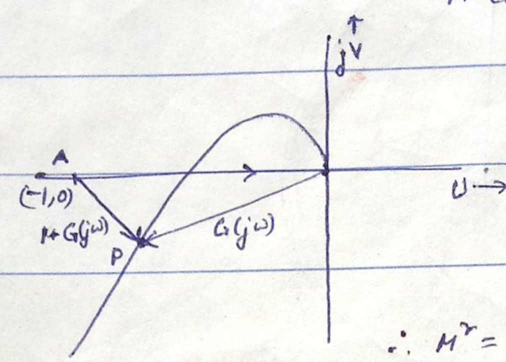


FIGURE 9.56 Nichols chart

CLOSED LOOP FREQUENCY RESPONSE:

Constant Magnitude Loci: M- circles

C.T.F. $= T(j\omega) = M(\omega)\, e^{j\alpha(\omega)} = \dfrac{G(j\omega)}{1+G(j\omega)}$ UNITY F/B.



$u = \text{Re}\, G(s)$
$V = \text{Im}\, G(s)$

$\therefore M = \left| \dfrac{U+jV}{1+U+jV} \right|$

$$\therefore M^2 = \frac{U^2+V^2}{(1+U)^2+V^2} \quad \text{— (1)}$$

(28)

$\Rightarrow (1-M^2)U^2 + (1-M^2)V^2 - 2M^2U = M^2$

Divide by $(1-M^2)$ & add $\left(\dfrac{M^2}{1-M^2}\right)^2$ to complete squares.

$$\therefore \left[U - \left(\frac{M^2}{1-M^2}\right)\right]^2 + V^2 = \left(\frac{M}{1-M^2}\right)^2$$

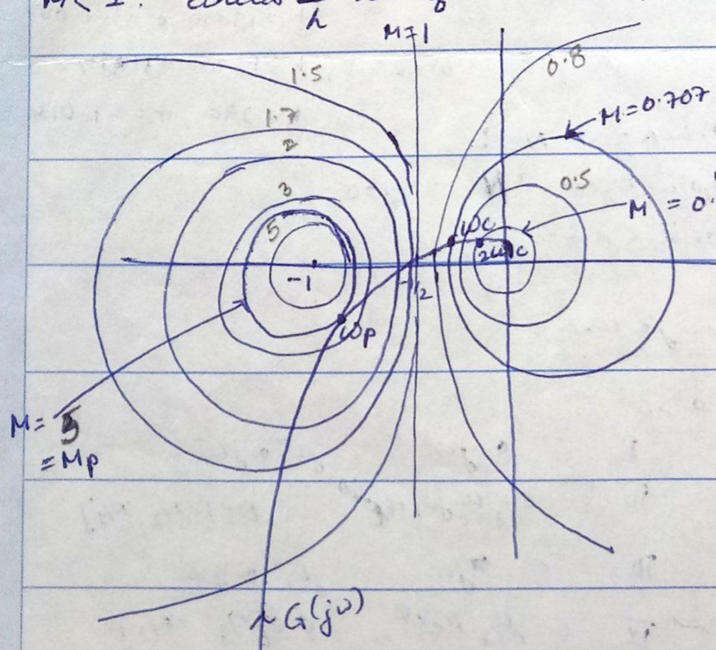
$\Rightarrow U = \pm\dfrac{M}{1-M^2} + \dfrac{M^2}{1-M^2} = \dfrac{M(1\pm M)}{(1+M)(1-M)} = \dfrac{-M}{1\pm M}$

$\therefore$ Circles of radius $\left|\dfrac{M}{1-M^2}\right|$ with centre at $\left(\dfrac{M^2}{1-M^2}, 0\right)$

$M > 1$: As M increases, radii of M circles decrease monotonically
centre shifts towards $(-1,0)$
at $M=\infty$, zero radius, centre at $(-1,0)$
$(U=-1,\ V=0)$

$M = 1$; radius ∞, centre at $U = -\infty$
intercept at $-1/2$ [Put $V=0$, $\therefore U = \dfrac{-M}{M\pm1}$ $\therefore$ for $M=1$, $U=-\infty, -\frac{1}{2}$
In (1), for $V=0$, $M=1$, $U^2 = (1+U)^2$
$\Rightarrow 2U+1=0$
$\Rightarrow U = -\frac{1}{2}$
$\therefore$ st. line ∥ to V axis is at $(-\frac{1}{2}, 0)$

$M < 1$: circles of dec. radius to rt. of $M=1$ line ; at $M=0$, radius is 0, centre 0.



ω_c = BW at $M = 0.707$ or 3dB.

M_p: largest M-circle at tangent to $G(j\omega)$.

CONSTANT PHASE ANGLE LOCI : N CIRCLES.

$$\angle e^{j\alpha} = \angle \frac{U+jV}{1+U+jV}$$

$$\Rightarrow \alpha = \tan^{-1}\left(\frac{V}{U}\right) - \tan^{-1}\frac{V}{1+U} \qquad \therefore \text{Let } \tan\alpha = N$$

$$\therefore N = \tan\left[\tan^{-1}\left(\frac{V}{U}\right) - \tan^{-1}\frac{V}{1+U}\right]$$

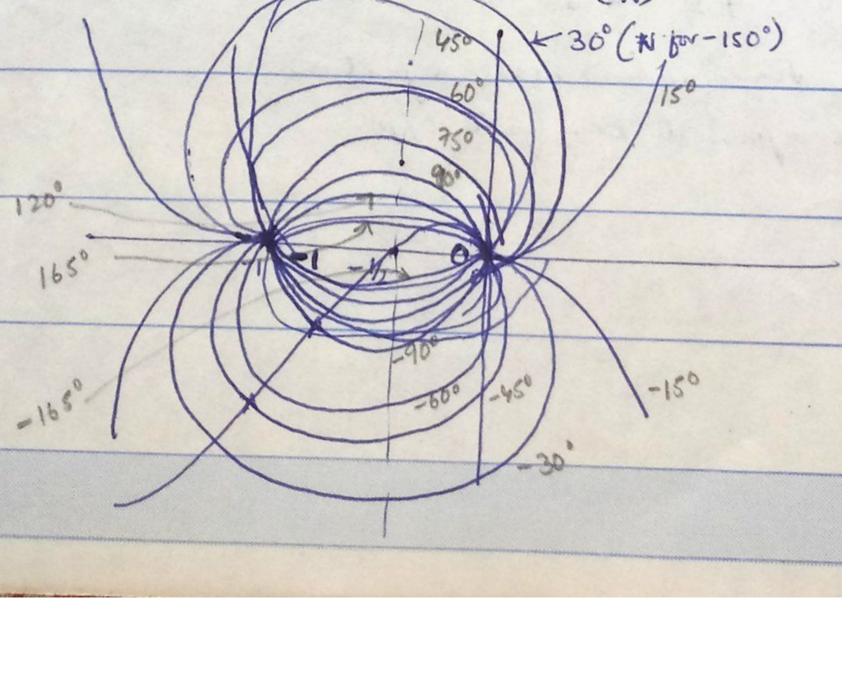
$$= \frac{\frac{V}{U} - \frac{V}{1+U}}{1 + \frac{V^2}{U(1+U)}} \Rightarrow U^2 + U + V^2 - \frac{1}{N}V = 0$$

for $V=0$, $U(1+U)=0$
$\Rightarrow U = 0, -1$ intersections $\forall N$.

Add $\left(\frac{1}{4} + \frac{1}{(2N)^2}\right)$ to both sides

$$\Rightarrow \left(U+\frac{1}{2}\right)^2 + \left(V - \frac{1}{2N}\right)^2 = \frac{1}{4} + \left(\frac{1}{2N}\right)^2$$

$\therefore$ CIRCLE of radius $\sqrt{1/4 + \left(\frac{1}{2N}\right)^2}$, centre $\left(-\frac{1}{2}, \frac{1}{2N}\right)$



For $N=\infty, \Rightarrow \alpha = 90°, -90°$
centre at $\left(-\frac{1}{2}, 0\right)$, rad $\frac{1}{2} = 0.5$

$N=1$, $\alpha = 45°, -135°$
centre at $\left(-\frac{1}{2}, \frac{1}{2}\right)$, rad $\frac{1}{\sqrt{2}} = 0.707$

$N=-1$, $\alpha = -45°, 135°$
centre at $\left(-\frac{1}{2}, -\frac{1}{2}\right)$, rad $\frac{1}{\sqrt{2}}$

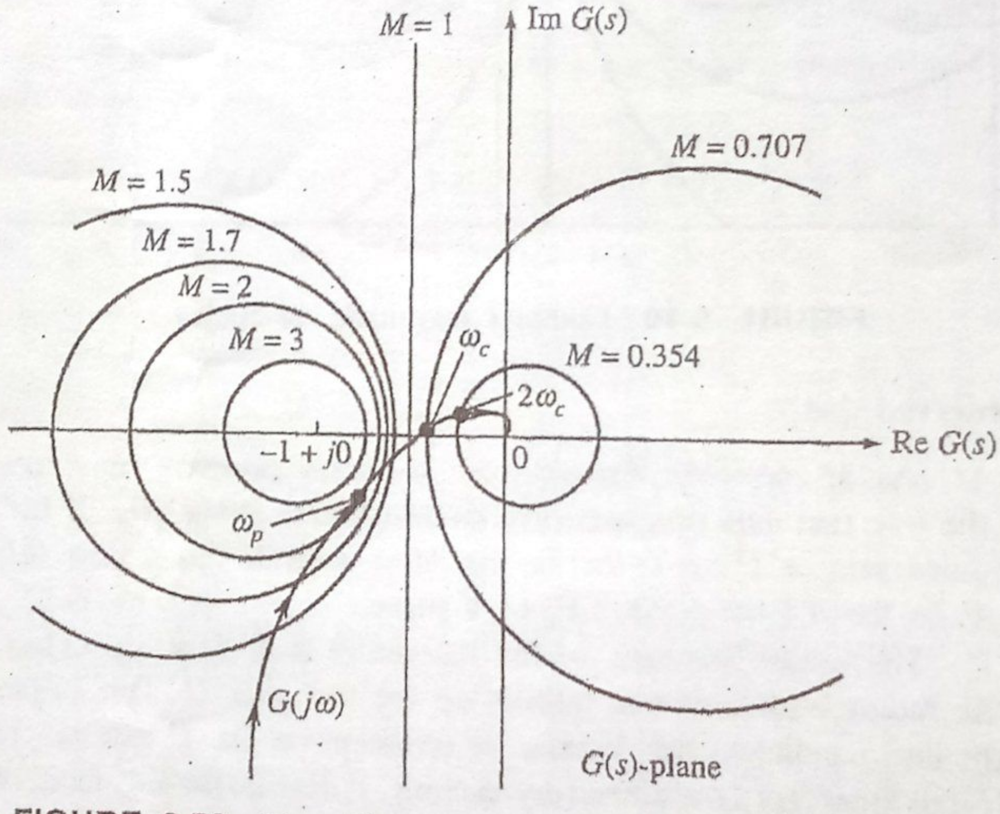


FIGURE 9.50 The $G(j\omega)$ locus with superimposed M-circles.

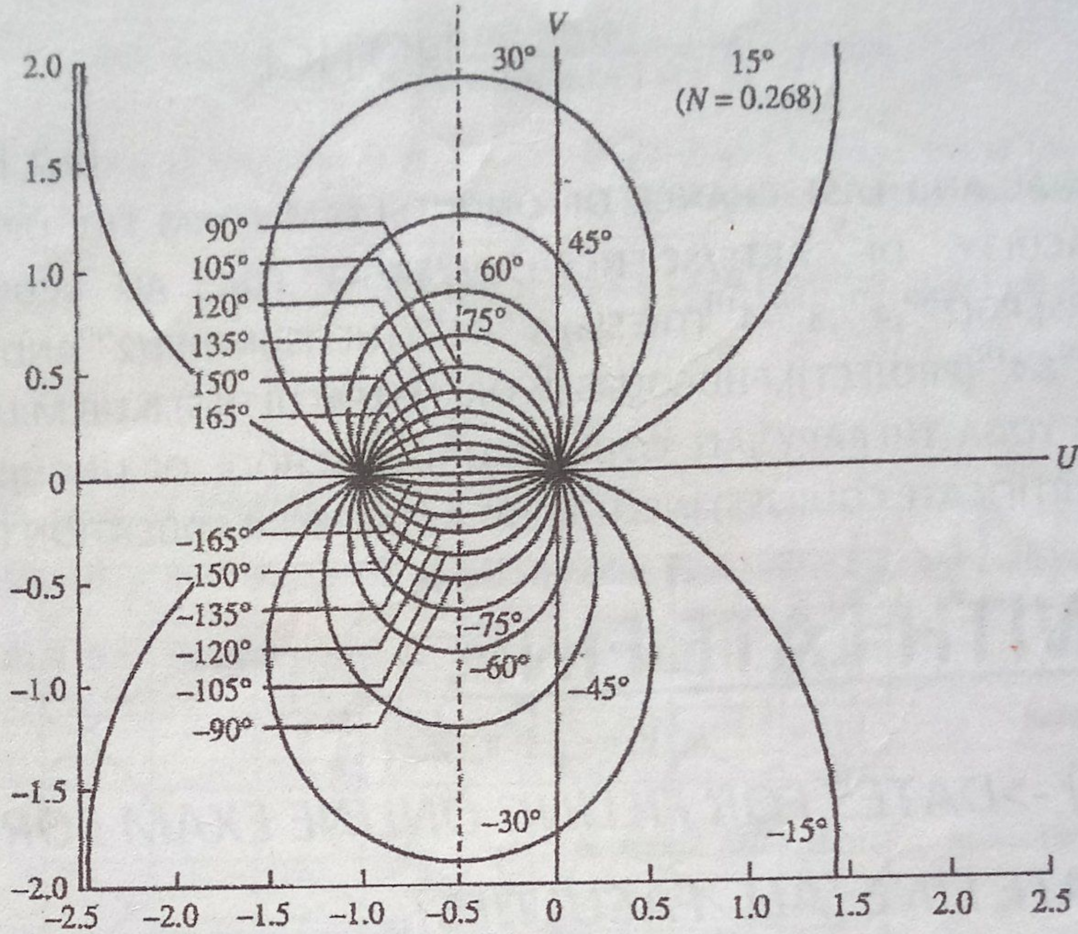


FIGURE 9.52 Constant *N*-circles.

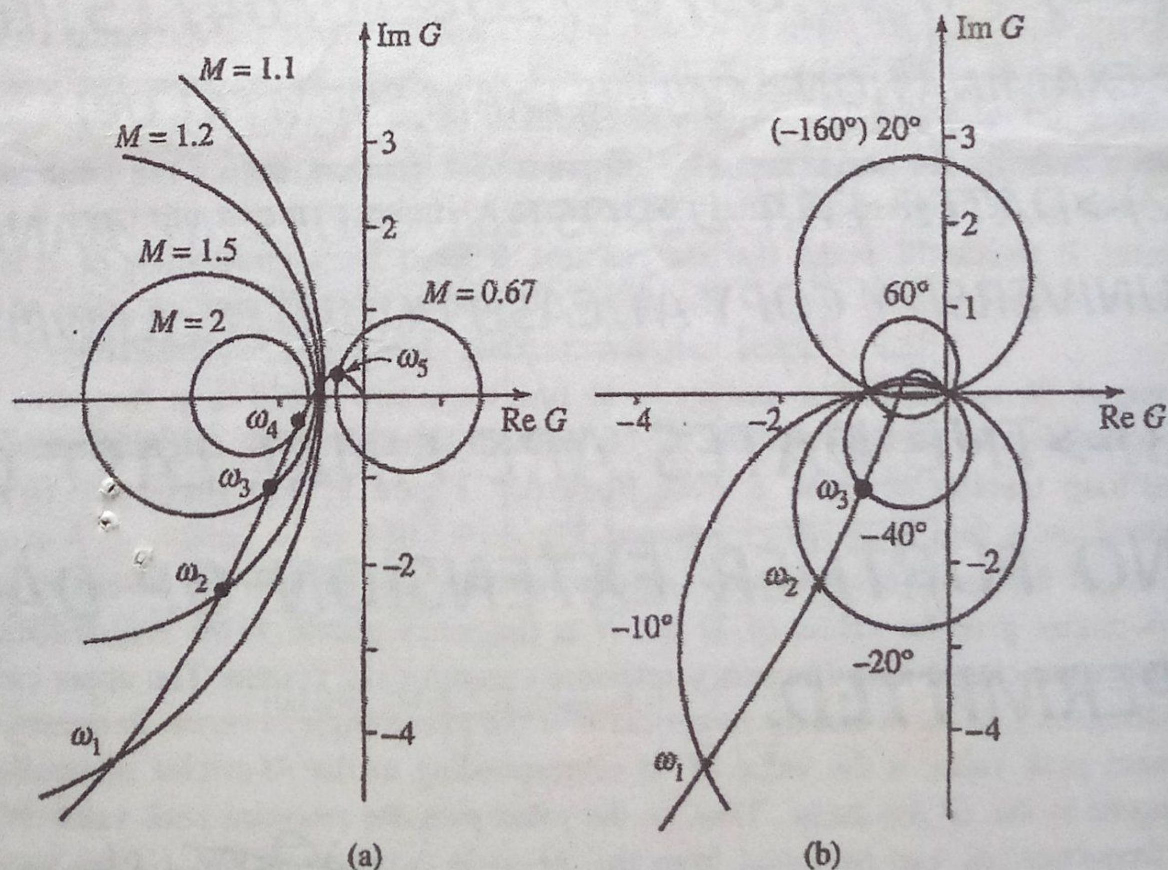


FIGURE 9.53(a) and (b) (a) $G(j\omega)$ locus superimposed on a family of *M*-circles, (b) $G(j$ superimposed on a family of *N*-circles.