FREQUENCY RESPONSE ANALYSIS:

Sinusoidal signals → steady state behaviour 0 → finite

(B)

Characteristic eqn: $1 + GH(s) = 0$

$\Rightarrow |GH(s)| = 1$; $\angle GH(s) = -180°$ or $(2k+1)\pi$ for a just stable system ∴ Indicate Gain & Phase Margins to stability for freq. i.e. $s = j\omega$.

(D)

BODE PLOT : HENDRIK W. BODE b1905

$GH(j\omega) = |GH(\omega)| e^{j\phi(\omega)}$

$\therefore \ln GH(j\omega) = \ln|GH(\omega)| + j\phi(\omega)$ [usually in dB]

log scale ∴ cover large range of freq. ; log ∴ × & / to + and −.

$u = \log_{10}\omega \Rightarrow 10^u = \omega$

$u_2 - u_1 = \log_{10}\omega_2 - \log_{10}\omega_1 = \log_{10}\frac{\omega_2}{\omega_1}$ usually octaves $\omega_2 = 2\omega_1$ or decades $\omega_2 = 10\omega_1$

No. of decades : $\frac{\log_{10}\frac{\omega_2}{\omega_1}}{\log_{10}10} = \log_{10}\frac{\omega_2}{\omega_1} = x$ ⟵ $\frac{\omega_2}{\omega_1} = 10^x$, $= 2^y$ → $\log_2\frac{\omega_2}{\omega_1} = y$ → No. of octaves

Let $\omega = 10^u = 2^v$ ∴ $\log_{10}\omega = u$; $\log_2\omega = v$

$\Rightarrow v\log_{10}2 = u \Rightarrow v = \frac{u}{\log_{10}2} = \frac{\log_{10}\omega}{\log_{10}2} = \log_2\omega$

So 20 dB/decade $= 20\log_{10}\frac{\omega_2}{\omega_1} = 20 \times (\log_{10}2) \times \left(\log_2\frac{\omega_2}{\omega_1}\right) = 20 \times 0.301 \log_2\frac{\omega_2}{\omega_1}$ (CHAIN RULE) $= 6 \cdot y$

$= 6$ dB/octave

(A)

Ex $G(s) = \frac{1}{s+1}$ → low pass RC $R = 2\Omega$, $C = 0.5F$ with i/p $10\cos 4t$ V. v_0 across C ?

Laplace

$u(s) = \mathcal{L}[10\cos 4t] = \frac{10s}{s^2+16}$ ∴ $V_0(s) = \frac{10s}{(s+1)(s^2+16)} = \frac{-10/17}{s+1} + \frac{10/17\, s + 160/17}{s^2+16}$

$\therefore v_0(t) = -10/17\, e^{-t} + \left(\frac{10}{17}\cos 4t + \frac{40}{17}\sin 4t\right)$

(transient) (s.s.)

Sinusoidal Analysis

Let $s = j\omega$, ∴ $G(j\omega) = \frac{1}{1+j\omega}$ for $\omega = 4$ $= \frac{1-j4}{17} = \frac{1}{\sqrt{17}}\angle -76°$

$\therefore v_0(t) = \frac{10}{\sqrt{17}}\cos(4t - 76°)$ V $= \frac{10}{17}\cos 4t + \frac{40}{17}\sin 4t$.

(C)

Frequency Domain Analysis:

Mag. : $20\log|GH(s)| = 20\log 1 = 0$ dB → gives gain crossover freq. ω_c and PHASE MARGIN.

Phase : $\angle GH(s) = (2k+1)\pi = -180°$ → gives phase crossover freq. GAIN MARGIN

Gain Margin $= -20\log|GH(j\omega_p)|$ dB $= +20\log\left|\frac{1}{GH(j\omega_p)}\right|$ dB.

Phase Margin $= \angle GH(j\omega_g) + 180° \Rightarrow \angle GH(j\omega_g) - PM = -180°$

(Normally lagging systems ⇒ add^(+) phase

Both +ve ⇒ STABLE system.

MANAGEMENT REPORT

CONSTRUCTION OF BODE PLOT:

$$GH(s) = \frac{K_s \prod (s^2 + 2\zeta_k \omega_{nk} s + \omega_{nk}^2) \prod (sT_i + 1)}{s^N \prod (s^2 + 2\zeta_{n} \omega_{nn} s + \omega_{nn}^2) \prod (sT_l + 1)}$$

$$GH(j\omega) = \frac{K \prod (j\omega T_i + 1) \prod (1 + j2\zeta_k u_{nk} - u_{nk}^2)}{(j\omega)^N \prod (j\omega T_l + 1) \prod (1 + j2\zeta_n u_{nn} - u_{nn}^2)}$$

where $K = \frac{K_s \prod \omega_{nk}^2}{\prod \omega_{nn}^2}$ $\quad u_{nk} = \frac{\omega}{\omega_{nk}}$ $\quad u_{nn} = \frac{\omega}{\omega_{nn}}$

20 log

$\therefore$ Mag: $|GH(j\omega)|$ ~~dB~~ $= 20 \log |K| + \sum 20 \log |1 + j\omega T_i|$

$+ \sum 20 \log |1 + j2\zeta_k u_{nk} - u_{nk}^2|$

$- 20N \log |j\omega| - \sum 20 \log (1 + j\omega T_l)$

$- \sum 20 \log |1 + j2\zeta_n u_{nn} - u_{nn}^2|$

phase: $\angle GH(j\omega) = 0 + \sum \tan^{-1}(\omega T_i) + \sum \tan^{-1} \frac{2\zeta_k u_{nk}}{1 - u_{nk}^2}$

$- N \cdot \frac{\pi}{2} - \sum \tan^{-1}(\omega T_l) - \sum \tan^{-1} \frac{2\zeta_n u_{nn}}{1 - u_{nn}^2}$

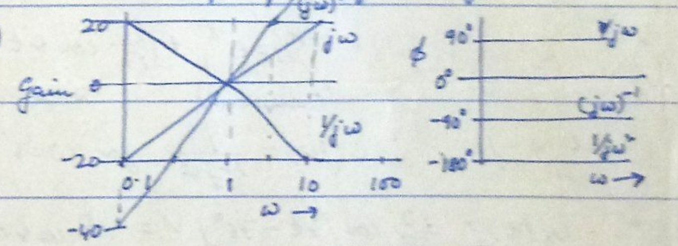
① Constant K $\quad 20 \log |K| =$ constant ; phase $= 0$.

② Zeros/poles at origin $(j\omega)^{\pm N}$ $\quad$ ← Ⓓ 20dB/dec = 6dB/octave

$\pm 20N \log (j\omega) = \pm 20N \log |\omega|$ dB

– straight line with $\pm 20N$ slope passing through 0dB at $\omega = 1$.

angle of $\pm N (\pi/2)$



③ Simple zero/pole $(1 + j\omega T)^{\pm 1}$ → Extra sheet

Let $GH(j\omega) = 1 + j\omega T$ $\quad \therefore$ Mag: $20 \log \sqrt{1 + \omega^2 T^2}$

For $\omega T \ll 1$, Mag $= 0$ $\quad \omega T \gg 1$, Mag $= 20 \log \omega + 20 \log T$

st. line with slope 20dB/dec

with origin at $1/T = \omega$.

Phase: $\tan^{-1} \omega T$ $\quad \therefore 45°$ at $\omega T = 1$ or $\omega = 1/T$

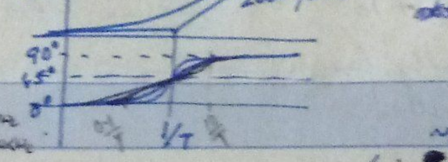
Let $\omega = K\omega_c$, $\omega > \omega_c$

$\omega = \frac{1}{K}\omega_c$, $\omega < \omega_c$

Both cases,

error $= 20 \log \sqrt{\frac{1 + K^2}{K^2}}$

$\therefore \omega T = \frac{\omega}{\omega_c} = \{ K \ldots, \frac{1}{K} \ldots$



$20 \log \sqrt{1 + \omega^2 T^2} - (20 \log 1)$ $\quad 0.5\omega_c \left(\frac{1}{T}\right)$

Error: $\pm 20 \log \sqrt{1^2 + \left(\frac{0.5\omega_c}{\omega_c}\right)^2} - (\pm 20 \log 1)$ $\quad \left[20 \log \sqrt{\frac{1^2 + \left(\frac{2\omega_c}{\omega_c}\right)^2}{\left(\frac{2\omega_c}{\omega_c}\right)^2}} \right]$

$= \pm 1$ dB at $0.5\omega_c$, $2\omega_c$

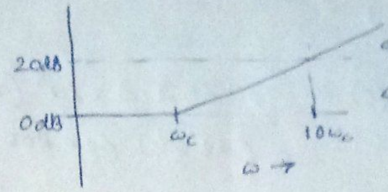
$\sim$ly ± 3 dB at ω_c

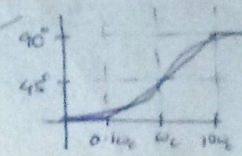
$\sim$ly ± 0.04 dB at $0.1\omega_c$, $10\omega_c$

$20 \log \sqrt{1 + \omega^2 T^2} - 20 \log \omega T$ for $\omega > \omega_c$

$\angle$: $\pm 5.7°$ at $0.1/10\ \omega_c$; $\pm 26.6°$ at $0.5/2\ \omega_c$ for zero at $(1 + j\omega T)$

Simple zero and pole $(1+j\omega T)^{\pm 1}$





Let $GH(j\omega) = 1 + j\omega T$ Let $T = 1/\omega_c$

$= 1 + j\left(\frac{\omega}{\omega_c}\right)$

Mag: $20 \log \sqrt{1+\omega^2T^2} = 20 \log \sqrt{1+\left(\frac{\omega}{\omega_c}\right)^2}$ dB

Phase $\tan^{-1}\omega T = \tan^{-1}\frac{\omega}{\omega_c}$

$\omega T = \frac{\omega}{\omega_c} \ll 1$ $20 \log 1 = 0$ dB $\tan^{-1} 0 = 0°$

$\omega T = 1$ $10 \log 2 = 3.01$ dB $\tan^{-1} 1 = 45°$

$\omega T \gg 1$ $20 \log\left(\frac{\omega}{\omega_c}\right) = 20 \log \omega + 20 \log T$ $\tan^{-1}\frac{\omega}{\omega_c} \rightarrow \tan^{-1}\infty = 90°$

For $\frac{\omega}{\omega_c} = K$, $20 \log K$ $\therefore K = 10 \Rightarrow 20$dB.

$\therefore$ 20 dB/decade slope, origin at $\omega = \omega_c = \frac{1}{T}$

Approx. 0° upto $0.1\omega_c$
90° beyond $10\omega_c$
st. line from 0.1 to $10\omega_c$
with 45° at ω_c.

$y = 45\left[\log \frac{\omega}{\omega_c}\right] + (45) $ — at $\omega = \omega_c$ $\log 1 = 0$ origin

$= 45\left[\log\frac{\omega}{\omega_c} + 1\right]$

Error: $20 \log \sqrt{1+\left(\frac{\omega}{\omega_c}\right)^2}$ for $\omega \ll \omega_c$

$20 \log \sqrt{1+\left(\frac{\omega}{\omega_c}\right)^2} - 20 \log \frac{\omega}{\omega_c}$ for $\omega > \omega_c$

Let $\frac{\omega}{\omega_c} = K$ for $\omega > \omega_c$ $\therefore$ Error $= 20 \log \sqrt{\frac{1+K^2}{K^2}}$

— do —

For $\frac{\omega}{\omega_c} = \frac{1}{K}$

Phase error $\tan^{-1}\omega T - y$.

$\therefore K = 0.1$ or 10	$+0.043$ dB	$\pm 5.71°$	Phase error 0 at approx $K = 0.1586 = 1/K$
$K = 0.5$ or 2	$+0.97 \approx 1$ dB	$\mp 4.89°$	
$K = 1$	$+3.01$ dB	0	

Quadratic zero and pole: Mag $\propto \frac{1}{g(\omega)}$; At $\omega = \omega_n$, $G(j\omega_n) = -j \cdot \frac{1}{2\xi}$

$g(\omega) = \left[\frac{\omega^2 - \omega_n^2(1-2\xi^2)}{\omega_n^2}\right]^2 + 4\xi^2(1-\xi^2)$

$\omega_r = \omega_n\sqrt{1-2\xi^2}$ for resonant ω.

	$g(\omega)$	$\omega \ll \omega_n$	$\omega = \omega_n$	$G(j\omega_n)$	$\omega \gg \omega_n$	$\omega = \omega_r$	Value of ω_r
For $\xi = 1$	$g(\omega) = \left[\frac{\omega^2+\omega_n^2}{\omega_n^2}\right]^2 > 1$	$\rightarrow 1$	4	$-\frac{j}{2}$	$\rightarrow \infty$	NA	Not valid
$\xi = \frac{1}{\sqrt{2}} = 0.707$	$\left[\frac{\omega^2}{\omega_n^2}\right]^2 + \frac{4}{2}\cdot\frac{1}{2} = 1 + \left(\frac{\omega^2}{\omega_n^2}\right)^2$	$\rightarrow 1$	2	$-j/\sqrt{2}$	$\rightarrow \infty$	1	0
$\xi = \frac{1}{2} = 0.5$	$\left[\frac{2\omega^2-\omega_n^2}{2\omega_n^2}\right]^2 + \frac{4}{4}\cdot\frac{3}{4}$	$\rightarrow 1$	1.0	$-j$	$\rightarrow \infty$	0.75	$\frac{\omega_n}{\sqrt{2}}$
			0		$\rightarrow \infty$	0	ω_n
$\xi \ll 1$, $\xi = 0.05$	$\left[\frac{\omega^2-\omega_n^2(0.99975)}{\omega_n^2}\right]^2 + 0.0005$	$\rightarrow 1$	0.000500062	$-10j$	$\rightarrow \infty$	0.0005	$0.999875\,\omega_n$

$\left[\left(\frac{\omega^2}{\omega_n^2}\right)^2 - 2(1-2\xi^2)\frac{\omega^2}{\omega_n^2} + (1-2\xi^2)^2\right] - (1-2\xi^2)^2$

$= 4\xi^4 + 4\xi^2$

$= \left[\frac{\omega^2 - \omega_n^2(1-2\xi^2)}{\omega_n^2}\right]^2 - 4\xi^2(1-\xi^2)$

At ω_r, $\frac{\omega_r}{\omega_n} = \sqrt{1-2\xi^2} \Rightarrow GH(j\omega_r) = \frac{1}{1-(1-2\xi^2) + j2\xi\sqrt{1-2\xi^2}}$

$= \frac{1}{2\xi(\xi + j\sqrt{1-2\xi^2})}$ $\therefore |GH(j\omega_r)| = \frac{1}{2\xi\sqrt{(1-\xi^2)}}$

At $\omega_0 = \omega_r = \omega_n\sqrt{1-2\xi^2}$

$u = \frac{\omega}{\omega_n} = \frac{\sqrt{1-2\xi^2}}{}$

~~$\therefore GH(j\omega_r) =$~~

~~$\frac{1}{\frac{1-2\xi^2}{1-2\xi^2} + j\frac{2\xi\sqrt{1-2\xi^2}}{(\sqrt{1-2\xi^2})^2}}$~~

~~$= \frac{1-2\xi^2}{2\xi(-\xi + j\sqrt{1-2\xi^2})}$~~

Gain Margin = $20 \log \frac{1}{|GH(j\omega_c)|}$ dB at phase crossover freq.
(by what factor can gain be inc.)

Phase Margin = $\angle GH(j\omega_g) + 180°$

∴ If stable, then G.M. > 1 (multiplying factor) or > 0 dB, PM +ve (amount of phase lag allowable that pushes sys. to instability).

→ ω_{ref} for pf

* Loading – ∥ paths – i_{req} increases
∴ Loading in terms of i
∴ Lagging 'LOAD' or 'i' w.r.t v
∴ Inductive load lagging pf.
→ usually inductive or lagging loads – coils, mass *

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1. Identify critical frequencies, magnitude & phase at these frequencies
2. Plot bode components while stating characteristics
3. Total plot — additive : semi log plot for magnitude & phase
4. Identify ω_g, ω_p, GM, PM from plot. Verify with tabular value
   Comment – % error, why & expectations.
5. Infer & stability.

f	M_n	$\angle_n$

$$\frac{K(s+150)}{s(s+20)(s^2+800s+64\times10^6)}$$

Bode plot :

$$GH(s) = \frac{13653.3 \times 10^8 (s+150)}{s(s+40)(s+800)(s^2+2000s+4\times10^6)}$$

$$\therefore GH(j\omega) = \frac{1600\left(1+\frac{j\omega}{150}\right)}{j\omega\left(1+\frac{j\omega}{40}\right)\left(1+\frac{j\omega}{800}\right)\left[1-\left(\frac{\omega}{2000}\right)^2+\frac{j\omega}{2000}\right]}$$

Four factors:

1) Constant $K = 1600 \triangleq 64dB = (20\log 1600)$: shift of origin, 0° phase

2) Pole at origin : $(j\omega)^{-1}$: Mag : $-20dB/dec$ through $\omega=1$, phase $= -90^\circ$

3) Simple poles and zeros :

	Mag	Phase
a) $\left(1+j\frac{\omega}{40}\right)^{-1}$	0dB till 40, then $-20dB/dec$	0 till 4, -45° at 40, -90° from 400
b) $\left(1+j\frac{\omega}{150}\right)$	150, $+20dB/dec$	till 15, 45° at 150, 90° from 1500
c) $\left(1+j\frac{\omega}{800}\right)^{-1}$	800, $-20dB/dec$	till 80, -45° at 800, -90° from 8000

4) Quadratic pole : $1-\left(\frac{\omega}{2000}\right)^2+\frac{j\omega}{2000}$

$\therefore \xi = 0.5$

Mag : Asymptote : 0 till 2000, $-40dB/dec$ then

$G(j2000) = \frac{-j}{1} = -j \therefore$ Mag 1

$\omega_r = \omega_n\sqrt{1-2\xi^2} = \frac{2000}{\sqrt{2}} = 1414.$

$|G(j\omega_r)| = \frac{1}{\sqrt{\frac{10}{16}}} = \frac{4}{\sqrt{10}} = 1.265 = 2.042dB$

Phase : 0° till 200, -90° at 2000, -180° from 20000

$\therefore$ Critical Freq.	Mag. (Th.)	Ph. (Th.)
1		
4		
15		
40		
80		
150		
200		
400		
800		
1500		
2000		
8000		
20000		

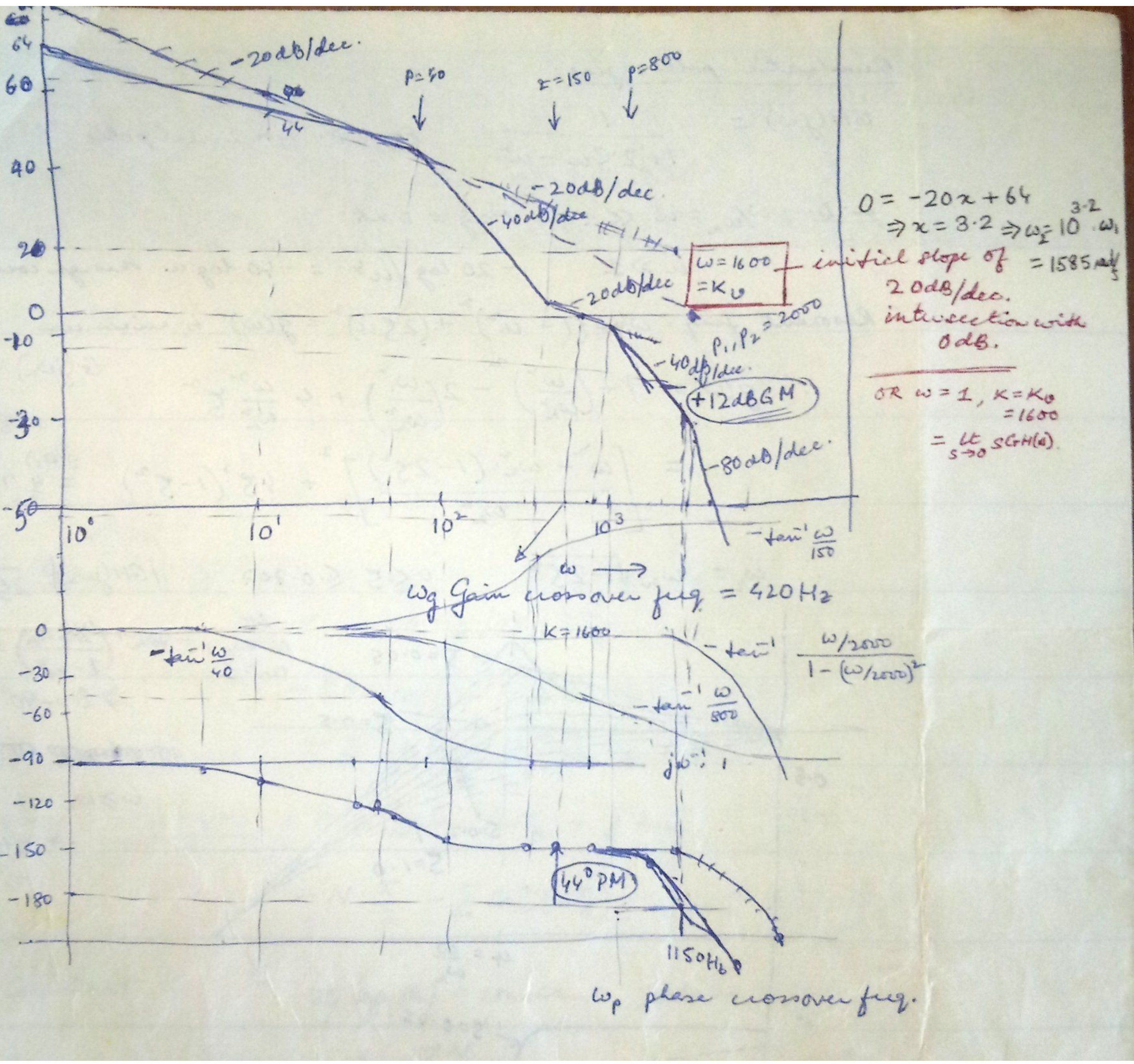
	GM +12dB	-180° ω_p (1150 Hz)
Theo.	1 ω_g (420Hz)	PM (44°)

Comparison of Theo and Practical ω_p, GM, ω_g, PM.

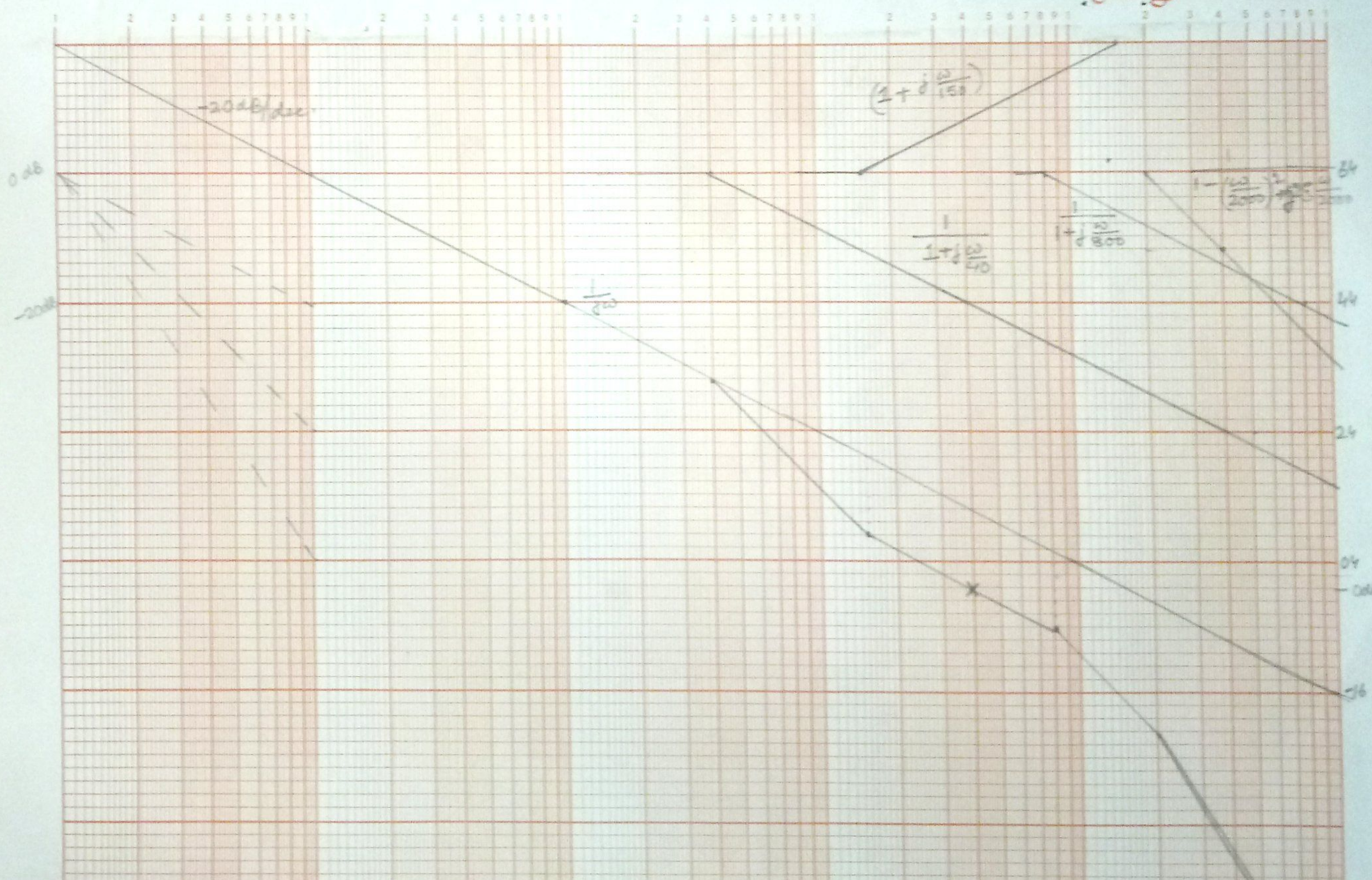
Gain Margin : $20\log\frac{1}{|GH(j\omega_p)|}$ dB at ω_p : If stable, then +ve GM $\left(\because \frac{1}{|GH(\omega_p)|} > 1\right)$

Phase Margin : $\angle GH(j\omega_g) - (-180^\circ)$ If stable, then +ve PM (amount of lag allowable)

Infer & stability.


-20dB/dec.
p=40
z=150
p=800
-20dB/dec.
-40dB/dec
ω=1600
=K_v
initial slope of 20dB/dec.
intersection with 0dB.
OR ω=1, K=K_v =1600 = lt s→0 sGH(s).
0 = -20x + 64
⇒ x = 3.2 ⇒ ω_2 = 10^3.2 ω_1 = 1585 rad/s
-20dB/dec
p1, p2 = 2000
-40dB/dec.
+12dBGM
-80dB/dec.
10^0
10^1
10^2
10^3
-tan⁻¹ ω/150
ω →
ω_g Gain crossover freq. = 420Hz
K=1600
-tan⁻¹ ω/40
-tan⁻¹ (ω/2000)/(1-(ω/2000)²)
-tan⁻¹ ω/800
j ω⁻¹
44° PM
1150Hz
ω_p phase crossover freq.

MS
2/11/13.



Magnitude

smooth curve through the calculated points. See Figure 8.11.

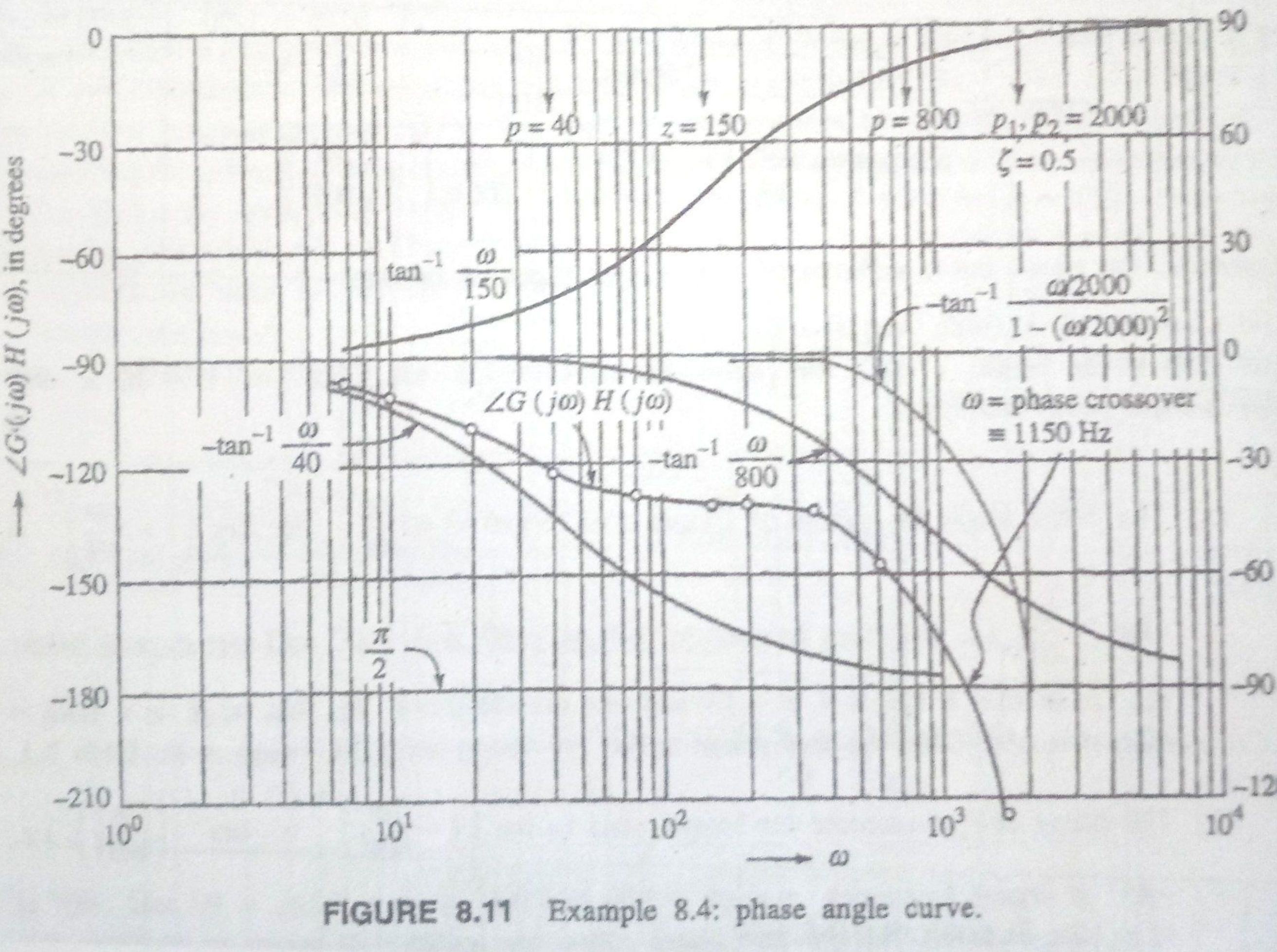


FIGURE 8.11 Example 8.4: phase angle curve.

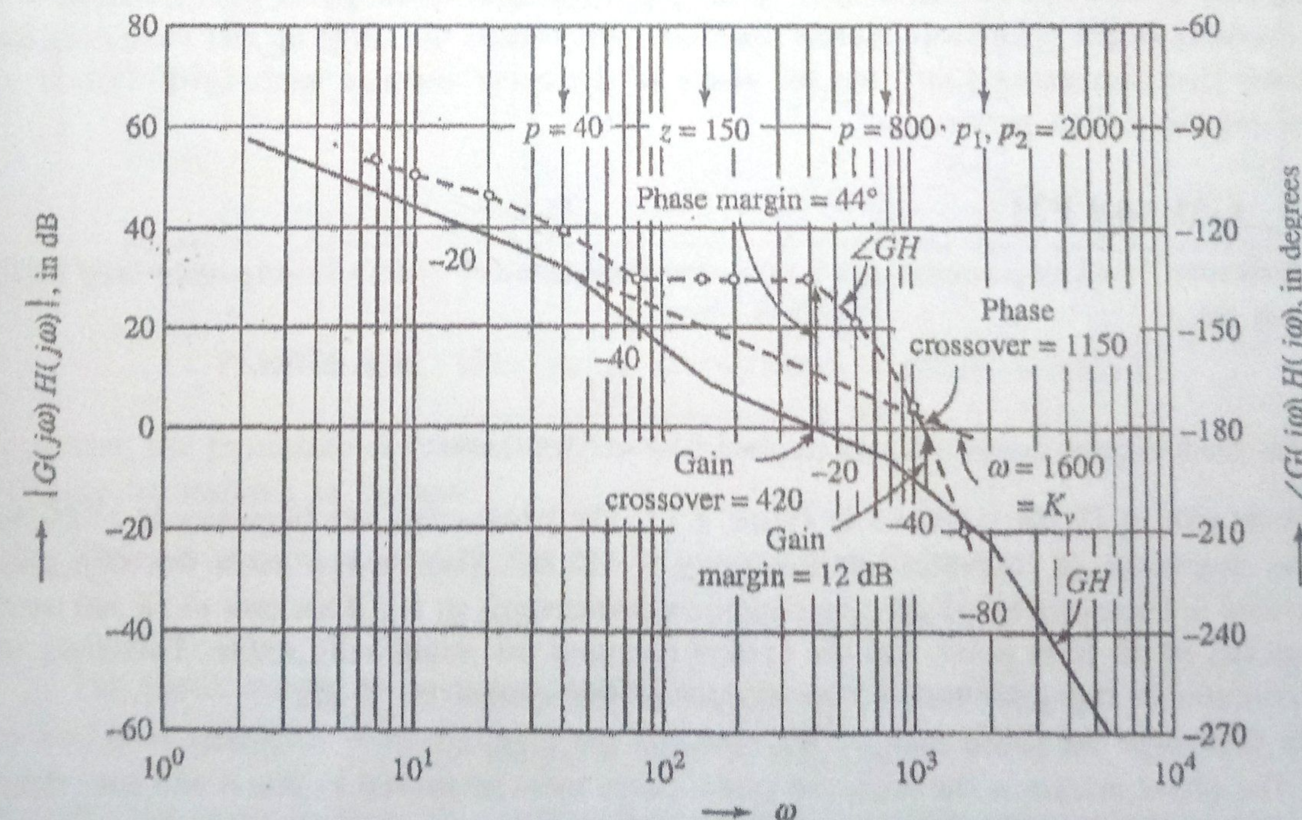


FIGURE 8.12 Example 8.5: magnitude and phase curves.