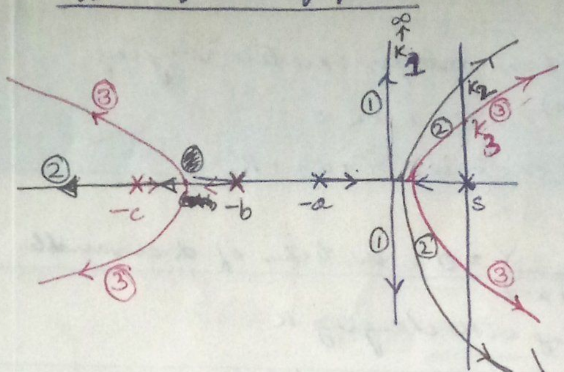
Effect of Adding poles.



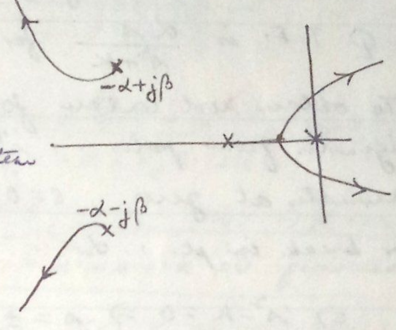
① $GH_1 = \dfrac{K}{s(s+a)}$ $a>0$

② $GH_2 = \dfrac{K}{s(s+a)(s+b)}$ $|b|>|a|$

③ $GH_3 = \dfrac{K}{s(s+a)(s+b)(s+c)}$ $|c|>|b|>|a|$

$\therefore K_1 = \infty > K_2 > K_3$

$GH_4 = \dfrac{K}{s(s+a)(s+\alpha \pm j\beta)}$



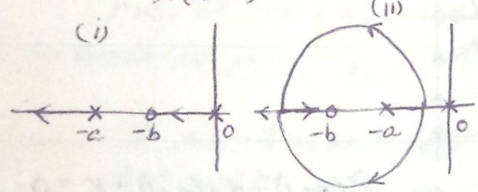
Addn. of pole $\equiv$ inc. ORDER of system
— decreases stability

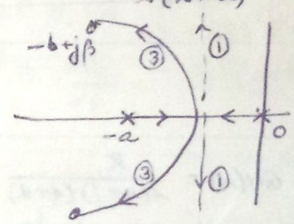
Effect of Adding Zeros:

$GH_1 = \dfrac{K}{s(s+a)}$ $a>0$

$GH_2 = \dfrac{K(s+b)}{s(s+a)}$

$GH_3 = \dfrac{K(s+b\pm j\beta)}{s(s+a)}$





Addn. of zero: stabilizes system
Q. — effect on transient? (loss of causality)?
complex conjugates ∴ UD — speed of response — ω_d?

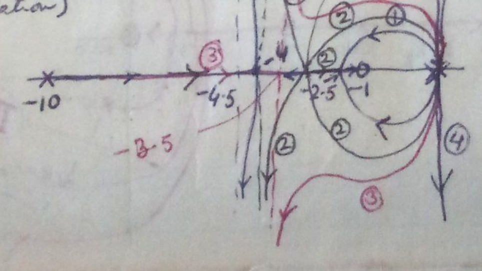
Effect of varying pole position:

$GH = \dfrac{K(s+1)}{s^2(s+a)}$ $\therefore K = -\dfrac{s^2(s+a)}{s+1}$ for root loci.

$\therefore \dfrac{dK}{ds} = \dfrac{-(s+1)(3s^2+2as)+s^2(s+a)}{(s+1)^2} = 0 \Rightarrow s(2s^2+(a+3)s+2a)=0$

$\Rightarrow s=0$ or $-\dfrac{(a+3)}{4} \pm \dfrac{\sqrt{a^2-10a+9}}{4}$ are possible break-in pts. or breakaway

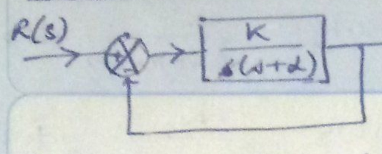
	a	∠asymptotes	intersection of asymptotes σ_A	Break-in/away pts.
①	10	±90°	−4.5	0, −2.5, −4
②	9	±90°	−4	0, −3
③	8	±90°	−3.5	0, no finite non-zero pt.
** ④	1	(pole zero cancellation) NOT reflected in root locus.		



To 14 then To (15)

(23)

Root Contour :: More than one parameter varying.



$0 < K, \alpha < \infty$

ch. eqn. $s^2 + \alpha s + K = 0$

Rewrite as $1 + \alpha\left(\frac{s}{s^2+K}\right) = 0$ in terms of α as variable

with the root loci changing with changing K.

$\therefore$ Q T.F. is $\frac{\alpha s}{s^2+K}$ for any particular K.

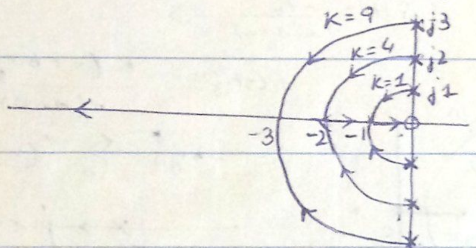
$\therefore$ To obtain root contour for various $K \longrightarrow$

originates from poles $\because s^2 + K = 0 \Rightarrow s = \pm j\sqrt{K}$

terminates at zeros $s = 0, -\infty$.

For break in pt. : $\alpha = -\frac{(s^2+K)}{s}$ $\therefore \frac{d\alpha}{ds} = \frac{-2s^2 + s^2 + K}{s^2} = 0$

$\Rightarrow s^2 - K = 0 \Rightarrow s = \pm\sqrt{K}$ possible $\therefore$ valid $s = -\sqrt{K}$.



24

Ex 6.15 $GH(s) = \frac{K}{s(s+1)(s+\alpha)}$ $\therefore$ ch. eqn. $s^2(s+1) + \alpha s(s+1) + K = 0$

$\therefore 1 + \frac{\alpha\, s(s+1)}{s^3+s^2+K} = 0$ $\therefore$ Q T.F. $\frac{s(s+1)}{s^3+s^2+K}$ II

Root contours originate ($\alpha = 0$) at Q poles of reduced ch. eqn.

$s^3 + s^2 + K = 0$. Rewrite as $1 + \frac{K}{s^2(s+1)} = 0$

I $\therefore$ Root locus of $\frac{K}{s^2(s+1)}$ provide start of root contour

$\sigma_A = -1/3$, $N = 3$, $\angle s = \pi, \pm\pi/3$

Root contours terminate at $s = 0, -1, -\infty$

$j\omega$ axis crossings : R-H array : $s^3 + s^2(\alpha+1) + \alpha s + K = 0$

s^3	1	α
s^2	$\alpha+1$	K
s^1	$\frac{\alpha^2+\alpha-K}{\alpha+1}$	
s^0	K	

$\therefore K > 0$

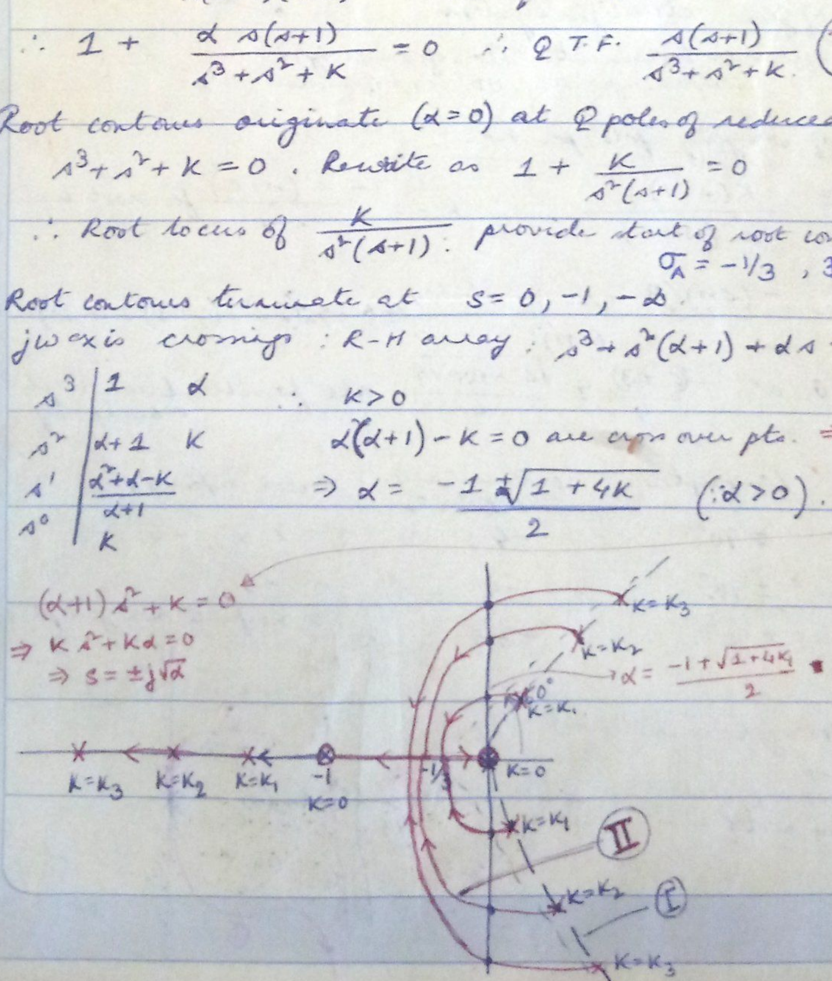
$\alpha(\alpha+1) - K = 0$ are cross over pts. $\Rightarrow (\alpha+1) = \frac{K}{\alpha}$

$\Rightarrow \alpha = \frac{-1 \pm \sqrt{1+4K}}{2}$ ($\because \alpha > 0$).

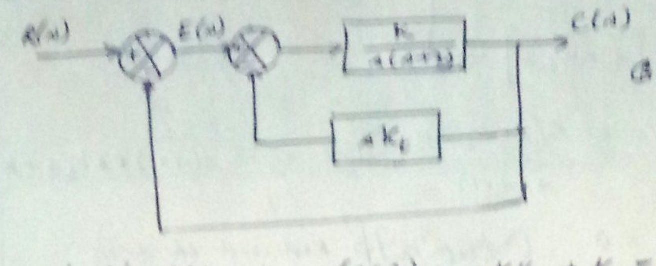
$(\alpha+1)s^2 + K = 0$

$\Rightarrow K s^2 + K\alpha = 0$

$\Rightarrow s = \pm j\sqrt{\alpha}$



Multiple loop systems :

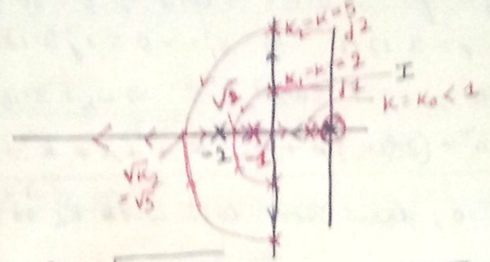


$$G(s) = \frac{\frac{K}{s(s+2)}}{1 + \frac{sKK_t}{s(s+2)}} = \frac{K}{s(s+2) + sKK_t}$$

$\therefore$ ch. eqn : $s(s+2) + sKK_t + K = 0$

$\therefore$ $1 + \frac{sK\,K_t}{s(s+2)+K} = 1 + \frac{\alpha s}{s^2+2s+K} = 0$ where $KK_t = \alpha$ II

Reduced ch. eqn : $s(s+2) + K = 0 \Rightarrow 1 + \frac{K}{s(s+2)} = 0$ I



Root contour

Poles : $s = \frac{-2 \pm \sqrt{4-4K}}{2} = -1 \pm \sqrt{1-K}$

Zeros : $s = 0, -\infty$

For Break in pt. : $\alpha = -\frac{(s^2+2s+K)}{s}$ $\therefore \frac{d\alpha}{ds} = \frac{-s(2s+2)+s^2+2s+K}{s^2} = 0$

$\Rightarrow s^2 - K = 0 \Rightarrow s = \sqrt{K}$ $\because K > 0$

System with Transportation lag : 26

$GH(s) = \frac{Ke^{-sT}}{s(s+2)}$ where T is transportation lag/delay

$e^{-sT} \approx 1 - sT$ for small delays.

Let $T = 1s$, then $GH(s) = \frac{K(1-s)}{s(s+2)} = \frac{-K(s-1)}{s(s+2)}$

$\therefore$ ~ to positive fb : $1 - G_1H_1(s) = 0$. $\Rightarrow G_1H_1(s) = \frac{K(s-1)}{s(s+2)}$

(i) $\angle$ condn. $\angle \frac{K(s-1)}{s(s+2)} = 2k\pi$

(ii) So Existence of root loci : for even (NOT ODD) no. of poles & zeros to rt. on real axis.

All other rules same $\because$ magnitude condn. dependent.

R-H

$s^2 + (2-K)s + K = 0$

s^2 1 K

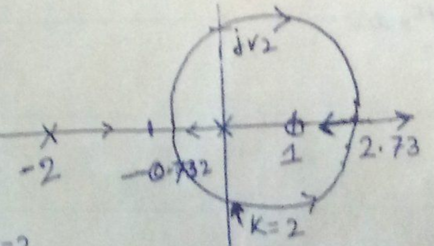
s^1 2-K

s^0 K

~~K=0~~ $2-K=0 \Rightarrow K=2$

$\Rightarrow s^2 + K = 0$

$\Rightarrow s = \pm j\sqrt{K} = \pm j\sqrt{2}$



Break away/in

$K = -\frac{s^2+2s}{1-s}$; $\frac{dK}{ds} = 0$

$\Rightarrow +(s-1)(2s+2) - (s^2+2s)$

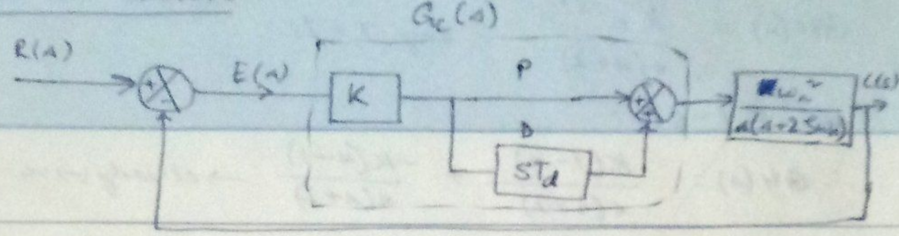
$= +2(s^2-1) - s^2 - 2s$

$= +s^2 - 2s - 2 = 0$

$\Rightarrow s = \frac{+2 \pm \sqrt{4+8}}{+2} = 1 \pm \sqrt{3}$

$= -0.732, 2.732$

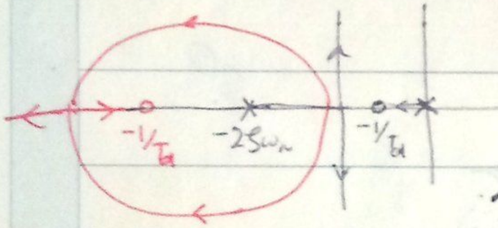
Derivative Control



∴ G w/o derivative control: $\frac{K\omega_n^2}{s(s+2\zeta\omega_n)}$

with derivative $\frac{K\omega_n^2(1+sT_d)}{s(s+2\zeta\omega_n)}$

→ addn. of zero.



$|1/T_d| > |2\zeta\omega_n|$ → then U.D. response

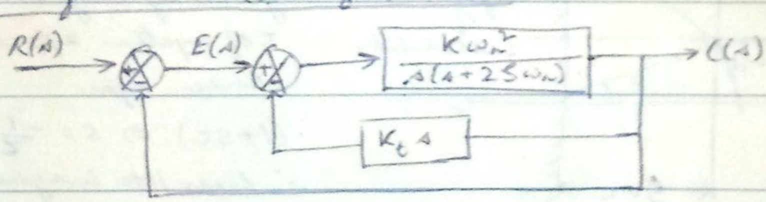
Else O.D. totally

If U.D. ($\zeta<1$), then overshoot but speed of response more.

22

**NOTE: Derivative control of error voltage (low voltage signal) in f/w path may req. power amplifier ⇒ K high ⇒ system O.D. ∴ slow

Rate feedback (Tacho f/b) Control:



$$\frac{C(s)}{R(s)} = \frac{K\omega_n^2}{s^2+(2\zeta\omega_n+KK_t\omega_n^2)s+K\omega_n^2}$$ ∴ Charac eqn. same

just replace T_d by K_t ∴ same change of damping ratio

*But rise time in derivative control faster due to added zero although same damping ratio. $[t_r = $ [illegible] $\tan^{-1}(\frac{\omega_d}{-\sigma})]$

wt. changes:

$$\frac{C(s)}{R(s)} = \frac{K(1+T_d s)\omega_n^2}{s^2+(2\zeta\omega_n+KT_d\omega_n^2)s+K\omega_n^2}$$ | DERIVATIVE CONTROL

But Rate f/b on o/p ∴ higher voltage level

Derivative control on f/w path ∴ error signal low

∴ use high pass RC (else derivative of noise too!)

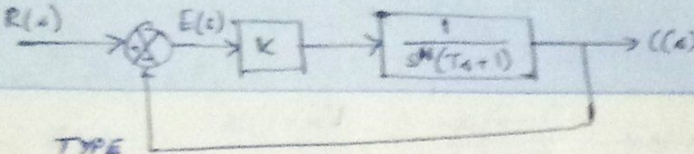
ex. o/p of potentiometer is discontinuous

∴ the jumps differentiated more.

+ power amplifier ∴ K↑ ∴ O.D

Integral controller

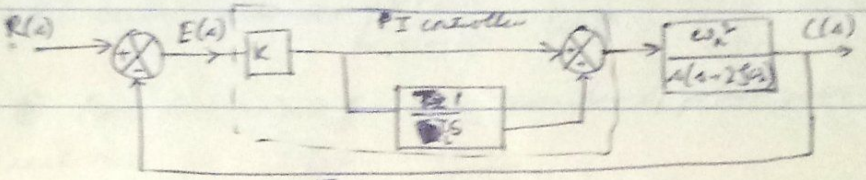
To reduce offset in response to step i/ps.



TYPE

For $N=0$; $K_p = \lim_{s\to 0} GH(s) = K$ $\therefore e_{ss} = \frac{1}{1+K}$ = finite.

$N \neq 0$; $K_p = \infty$ $\therefore e_{ss} = 0$



$$\therefore G(s) = \frac{K\omega_n^2\left(1+\frac{1}{T_i s}\right)}{s(s+2\zeta\omega_n)} = \frac{K\omega_n^2(T_i s+1)}{T_i s^2(s+2\zeta\omega_n)} \nearrow \frac{s+1/T_i}{s}$$

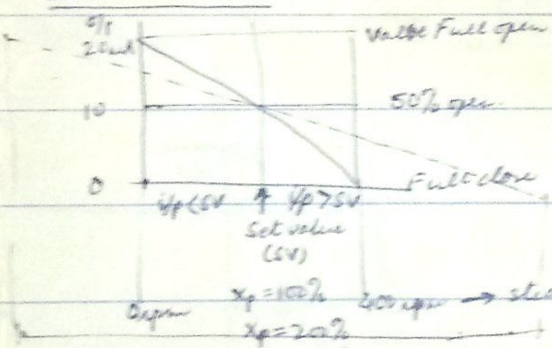
$\therefore$ Eq. to adding zero at $-1/T_i$ and a pole at 0.

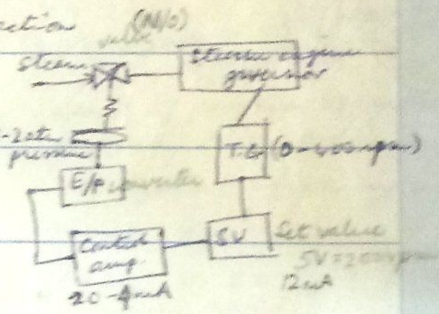
$\therefore$ inc. system order by one $\therefore$ ss error improved

$\therefore$ For PID controller : $G_c(s) = K\left(1 + T_d s + \frac{1}{T_i s}\right)$

21

CONTROL VALVE with reverse control action (N/O)





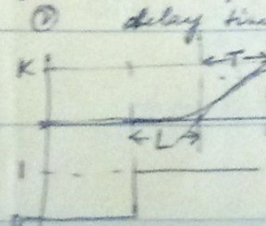
200 rpm → steam engine governor speed.

Proportional band C/V 0% ⇒ ON OFF control.

$$\therefore G_c(s) = K_p\left(1 + \frac{1}{T_i s} + T_d s\right)$$

Rules		K_p	T_i	T_d	
Z-N ①	P	$0.5K_c$	∞	0	K_c critical gain; T : period of oscillation
	PI	$0.45K_c$	$0.83T$	0	
	PID	$0.6K_c$	$0.5T$	$0.125T$	

Z-N ② unit step transient as S curve, then identify delay time L, Time constant T and gain K (steady state)



$$\frac{Y(s)}{U(s)} = \frac{Ke^{-Ls}}{Ts+1}$$

	$K_p = 1/x_p$	T_i	T_d
P	T/L	∞	0
PI	$0.9T/L$	$L/0.3$	0
PID	$1.2T/L$	$2L$	$0.5L$

$\Delta E/\Delta u = A$ for parameter change.