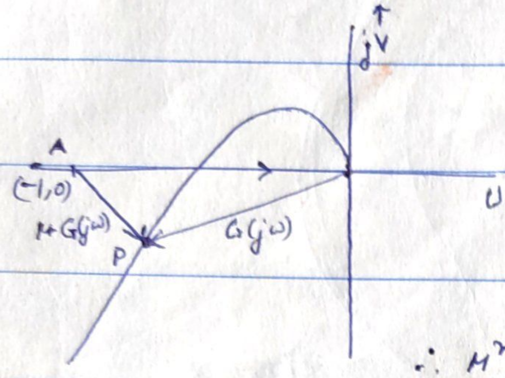
CLOSED LOOP FREQUENCY RESPONSE :

Constant Magnitude Loci : M-circles

C.T.F. $= T(j\omega) = M(\omega)\, e^{j\alpha(\omega)} = \dfrac{G(j\omega)}{1+G(j\omega)}$ UNITY F/B.



$U = Re\, G(s)$
$V = Im\, G(s)$

$\therefore M = \left| \dfrac{U+jV}{1+U+jV} \right|$

$\therefore M^2 = \dfrac{U^2+V^2}{(1+U)^2+V^2}$ —— ①

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$\Rightarrow (1-M^2)U^2 + (1-M^2)V^2 - 2M^2 U = M^2$

Divide by $(1-M^2)$ & add $\left(\dfrac{M^2}{1-M^2}\right)^2$ to complete squares.

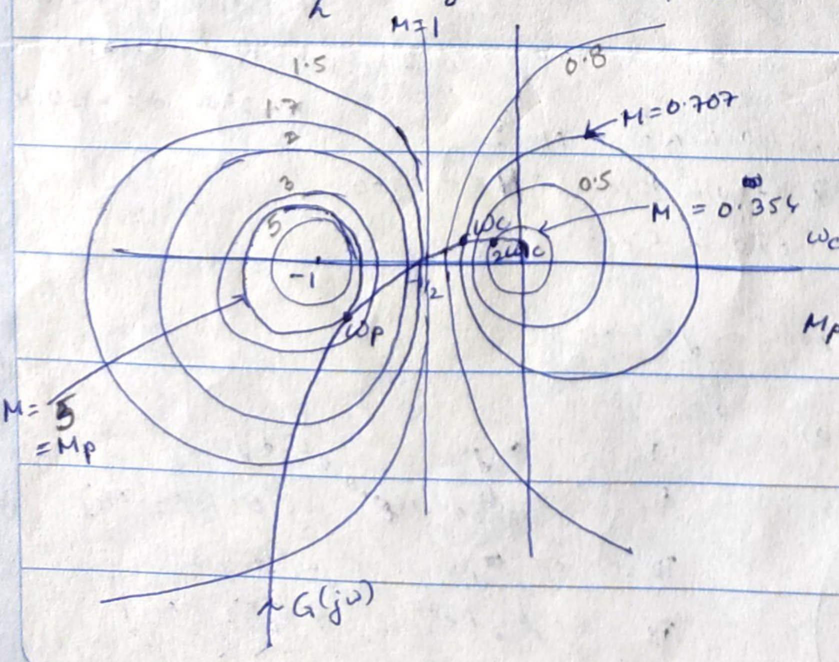
$\therefore \left[U - \left(\dfrac{M^2}{1-M^2}\right)\right]^2 + V^2 = \left(\dfrac{M}{1-M^2}\right)^2$ $\Rightarrow U = \pm \dfrac{M}{1-M^2} + \dfrac{M^2}{1-M^2} = \dfrac{\pm M(1\pm M)}{(1+M)(1-M)} = \dfrac{-M}{1\pm M}$

$\therefore$ Circles of radius $\left|\dfrac{M}{1-M^2}\right|$ with centre at $\left(\dfrac{M^2}{1-M^2}, 0\right)$

$M > 1$: As M increases, radii of M circles decrease monotonically
centre shifts towards $(-1,0)$
at $M=\infty$, zero radius, centre at $(-1,0)$
$\left(\begin{matrix}U=-1\\V=0\end{matrix}\right)$

$M = 1$: radius ∞, centre at $U = -\infty$
intercept at $-1/2$ [Put $V=0$, $\therefore U = \dfrac{-M}{M\pm 1}$ $\therefore$ for $M=1$, $U = -\infty, -\frac{1}{2}$
$\therefore$ st. line $\parallel$ to V axis at $(-1/2, 0)$ ~~W~~ In ①, for $V=0$, $M=1$, $U^2 = (1+U)^2$
$\Rightarrow 2U+1=0$
$\Rightarrow U = -1/2$

$M < 1$: circles of dec. radius to rt. of $M=1$ line ; at $M=0$, radius is 0, centre 0.



ω_c = BW at $M = 0.707$ or 3dB.

M_p: largest M-circle at tangent to $G(j\omega)$.

CONSTANT PHASE ANGLE LOCI : N CIRCLES.

$$\angle e^{j\alpha} = \angle \frac{U+jV}{1+U+jV}$$

$$\Rightarrow \alpha = \tan^{-1}\left(\frac{V}{U}\right) - \tan^{-1}\frac{V}{1+U} \qquad \therefore \text{Let } \tan\alpha = N$$

$$\therefore N = \tan\left[\tan^{-1}\left(\frac{V}{U}\right) - \tan^{-1}\frac{V}{1+U}\right]$$

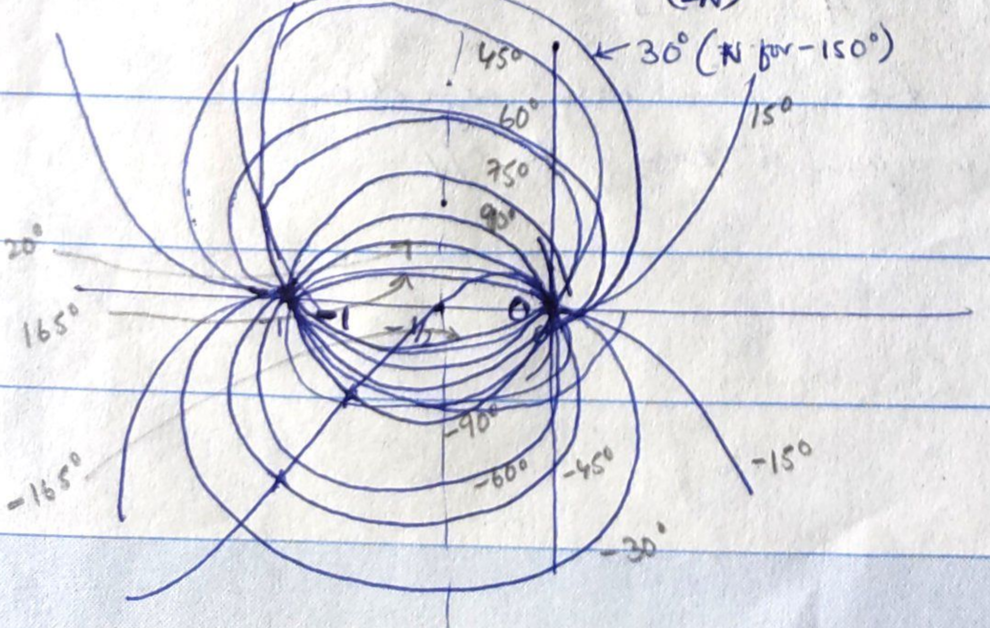
$$= \frac{\frac{V}{U} - \frac{V}{1+U}}{1 + \frac{V^2}{U(1+U)}} \Rightarrow U^2 + U + V^2 - \frac{1}{N}V = 0$$

for $V=0$, $U(1+U)=0$
$\Rightarrow U = 0, -1$ intersections $\forall N$.

Add $\left(\frac{1}{4} + \frac{1}{(2N)^2}\right)$ to both sides

$$\Rightarrow \left(U+\frac{1}{2}\right)^2 + \left(V - \frac{1}{2N}\right)^2 = \frac{1}{4} + \left(\frac{1}{2N}\right)^2$$

$\therefore$ CIRCLE of radius $\sqrt{1/4 + \left(\frac{1}{2N}\right)^2}$, centre $\left(-\frac{1}{2}, \frac{1}{2N}\right)$



For $N = \infty$, $\Rightarrow \alpha = 90°, -90°$
centre at $\left(-\frac{1}{2}, 0\right)$, rad $\frac{1}{2} = 0.5$

$N = 1$, $\alpha = 45°, -135°$
centre at $\left(-\frac{1}{2}, \frac{1}{2}\right)$, rad $\frac{1}{\sqrt{2}} = 0.707$

$N = -1$, $\alpha = -45°, 135°$
centre at $\left(-\frac{1}{2}, -\frac{1}{2}\right)$, rad $\frac{1}{\sqrt{2}}$

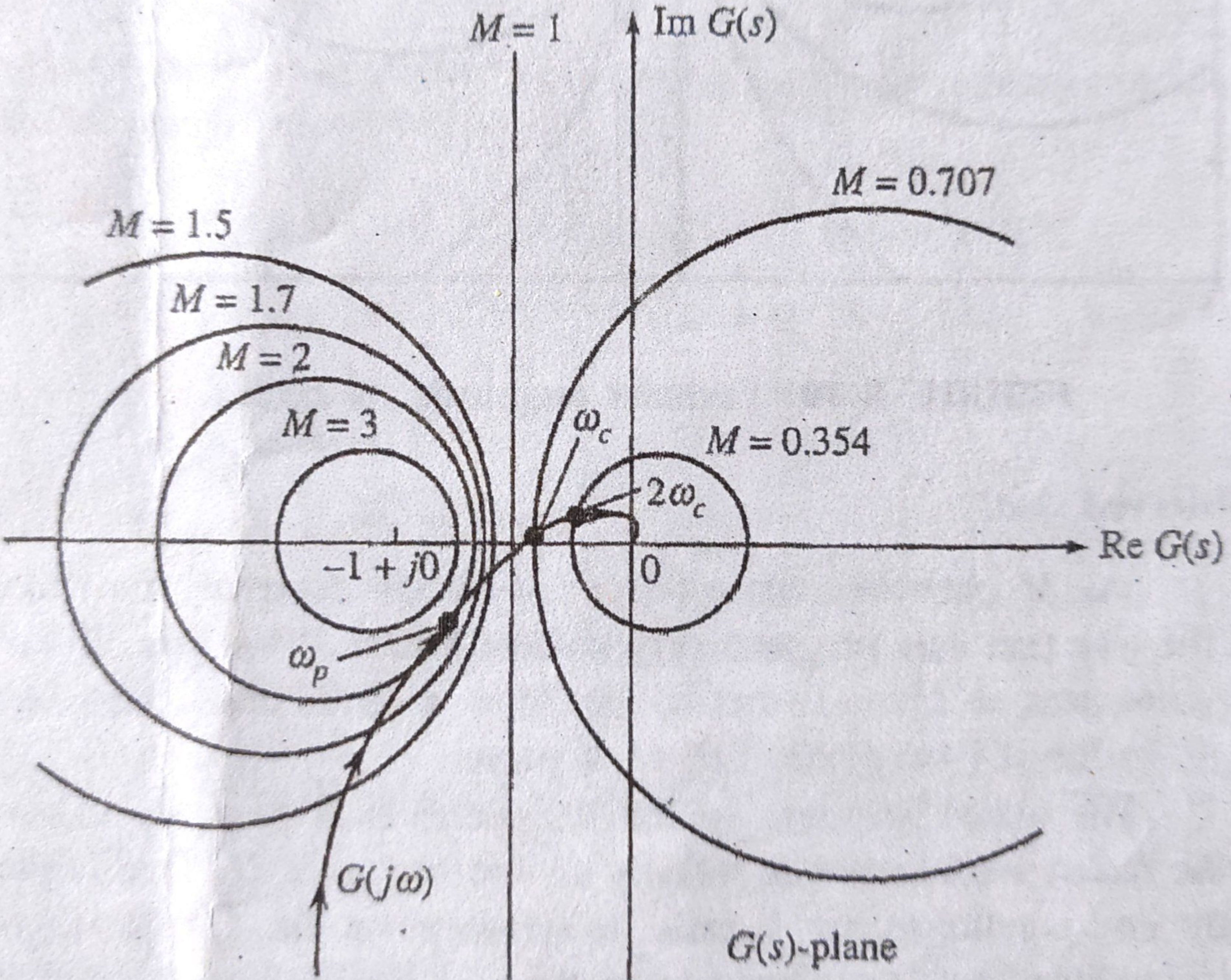


FIGURE 9.50 The $G(j\omega)$ locus with superimposed M-circles.

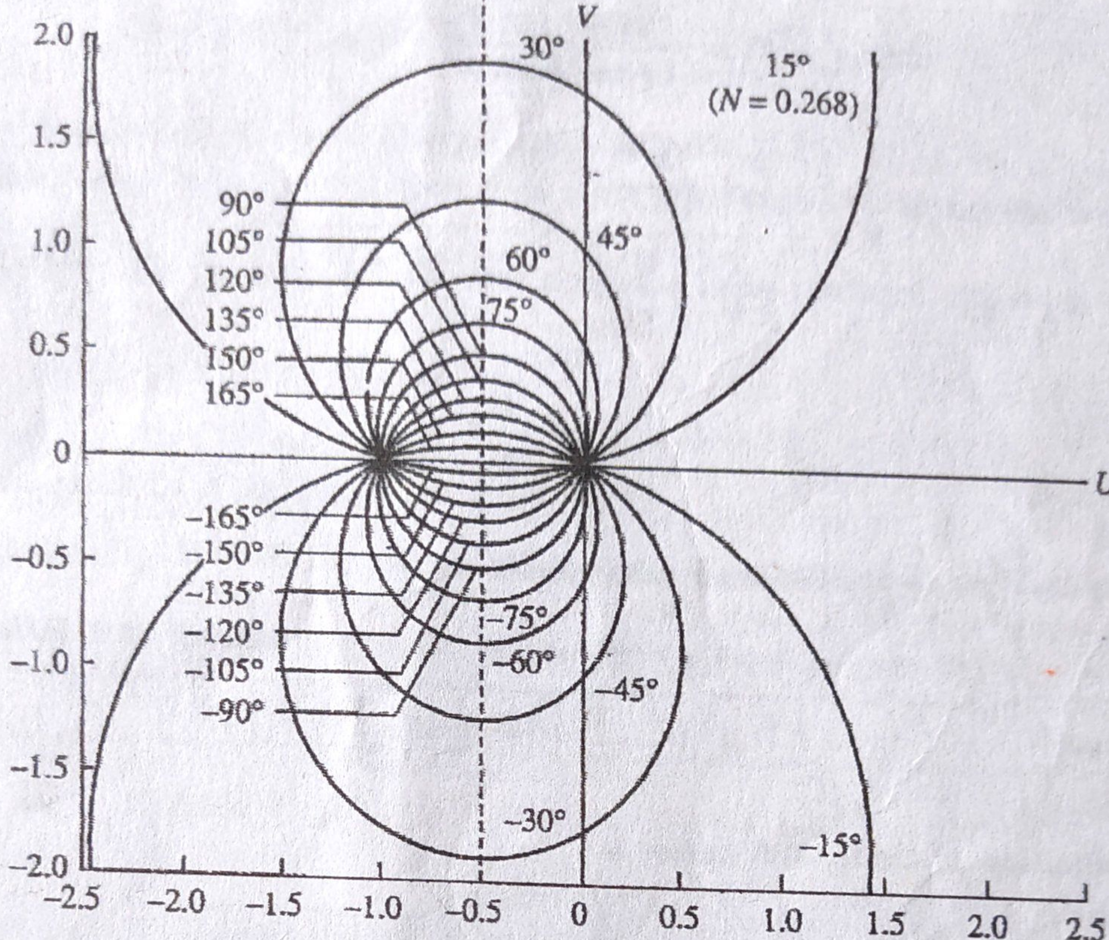


FIGURE 9.52 Constant *N*-circles.

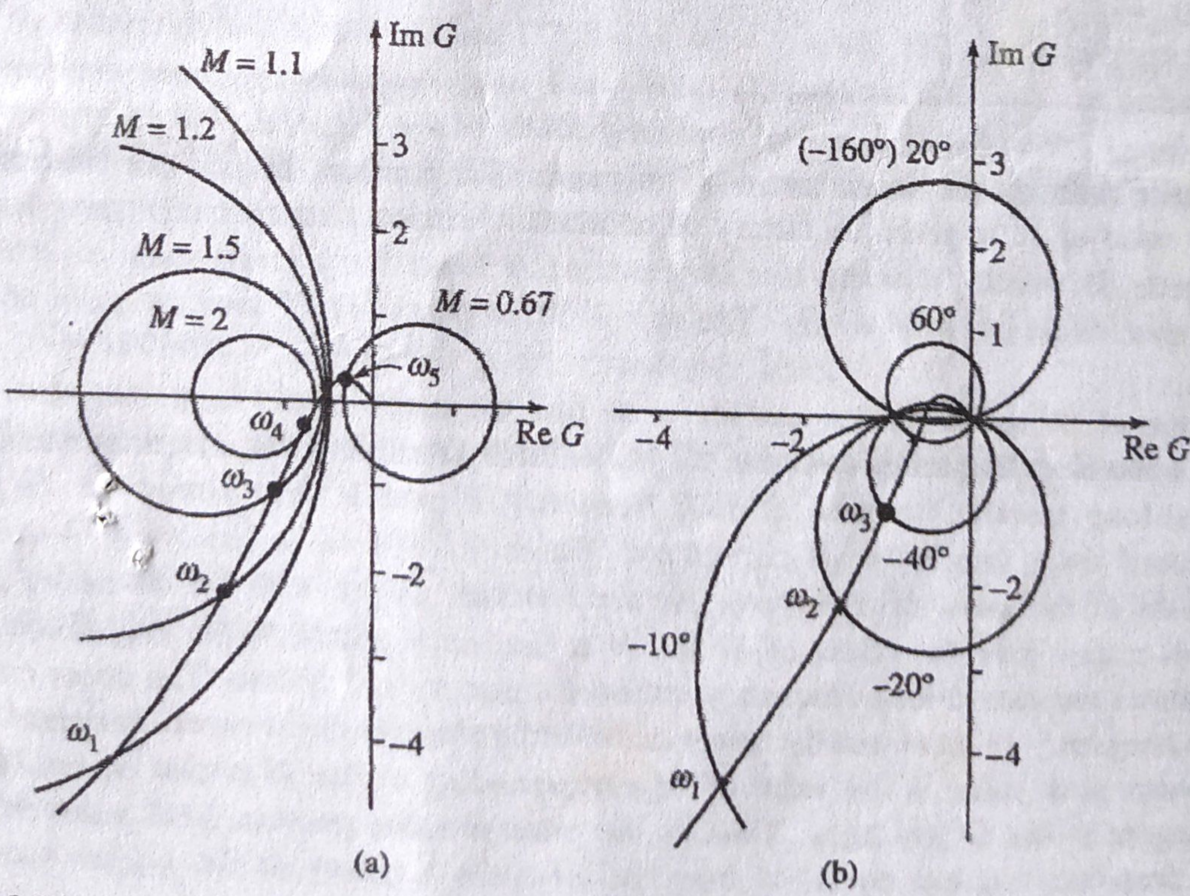


FIGURE 9.53(a) and (b) (a) $G(j\omega)$ locus superimposed on a family of *M*-circles, (b) $G(j$ superimposed on a family of *N*-circles.

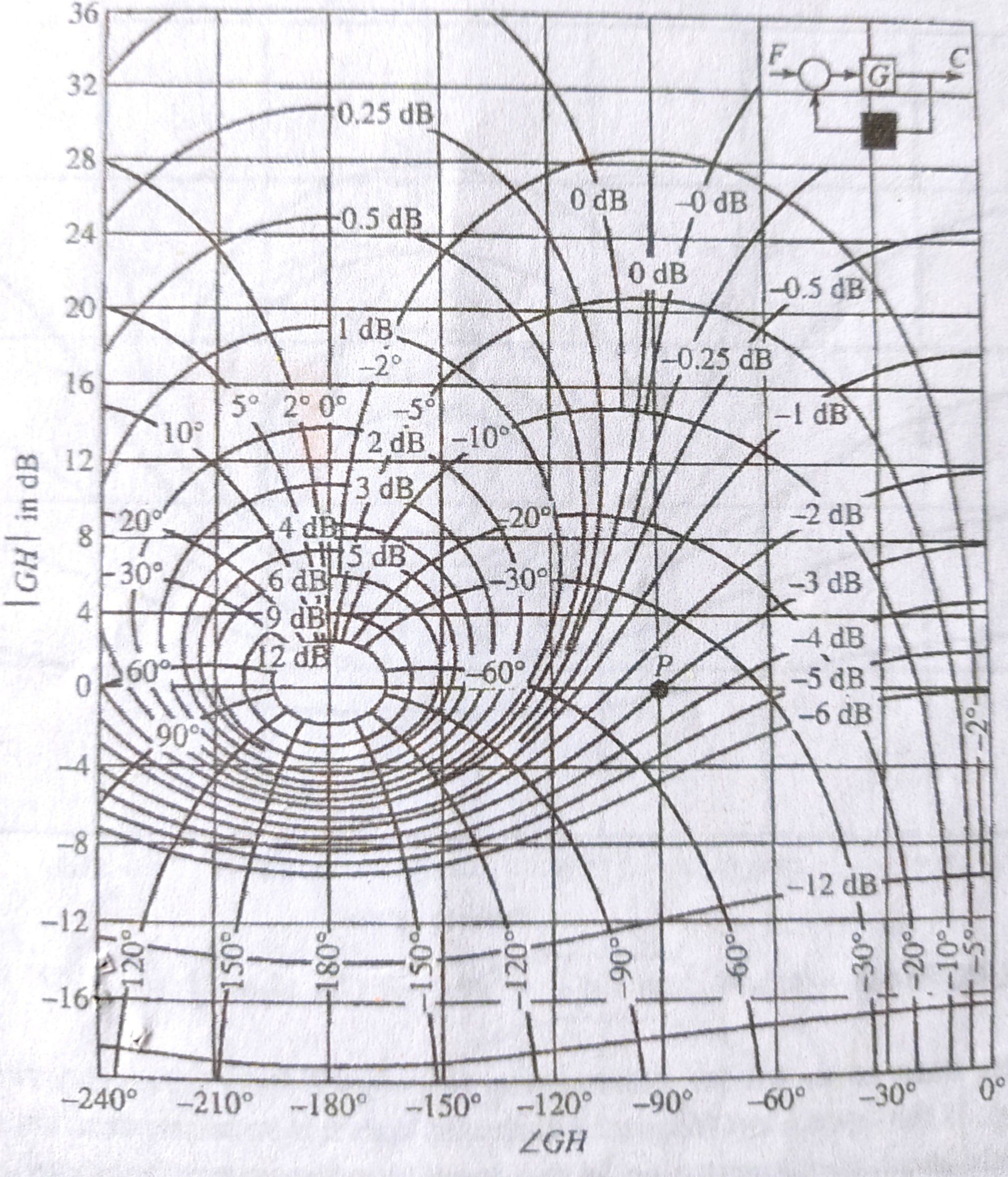


FIGURE 9.56 Nichols chart

Non unity fb system

$$T(s) = \frac{G(s)}{1+GH(s)} = \frac{1}{H(s)} T_0(s)$$

$\therefore$ Plot for $T_0(s) \to$ use M & N circles for $G_0(s)$ to obtain $T_0(s)$

Then $T_0(s) \times \frac{1}{H(s)} \to$ use Bode plot $\to T(s)$

Freq. domain specs for design

2nd order prototype $\frac{\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2}$

$$M_p = \frac{1}{2\zeta\sqrt{1-\zeta^2}} \quad 0<\zeta<0.707 \text{ (given } \zeta)$$

$$= 1 \quad \zeta \geq 0.707$$

$$\Rightarrow \zeta^2 = \frac{1}{2} - \frac{1}{2}\sqrt{\left(1-\frac{1}{M_p^2}\right)} \text{ for } M_p > 1 \text{ (Given } M_p)$$

M_p large $\Rightarrow$ usually large peak OVERSHOOT for STEP RESPONSE

$1.1 < M_p < 1.5 \iff 0.54 > \zeta > 0.36$ for stable system.

$\omega_p = \omega_n\sqrt{1-2\zeta^2} \Rightarrow$ system's speed of response

$$BW = \omega_n\left[(1-2\zeta^2)+\sqrt{4\zeta^4-4\zeta^2+2}\right]^{1/2} \approx \omega_n\left[1-2\zeta^2+\frac{1}{2}(4\zeta^4-4\zeta^2+2)\right]^{1/2} \text{ for } \zeta\to 0$$

$$= \omega_n\left[2-4\zeta^2+2\zeta^4\right]^{1/2} = \omega_n(\sqrt{2}-\sqrt{2}\zeta^2)^{2/2}$$

$$= \sqrt{2}\,\omega_n(1-\zeta^2)$$

$$= 2\omega_d$$

larger BW $\Rightarrow$ faster rise time, less noise rejection.

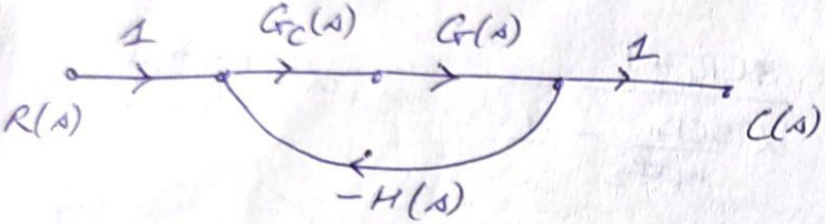
$\therefore$ higher freq. signals passed easily to o/p.

COMPENSATION TECHNIQUES

System performance
- Time domain — t_p, M_p, t_s : step i/p
  - s.s. errors for various inputs
- Freq. domain
  - M_r, ω_r, BW, GM, PM

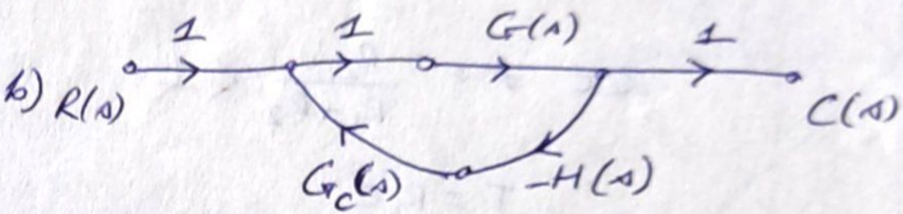
— may need alteration : use compensating networks.

a)



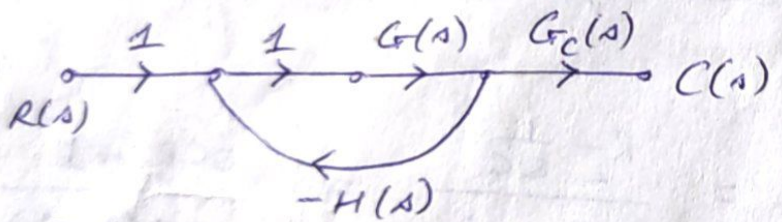
CASCADE COMPENSATION

b)



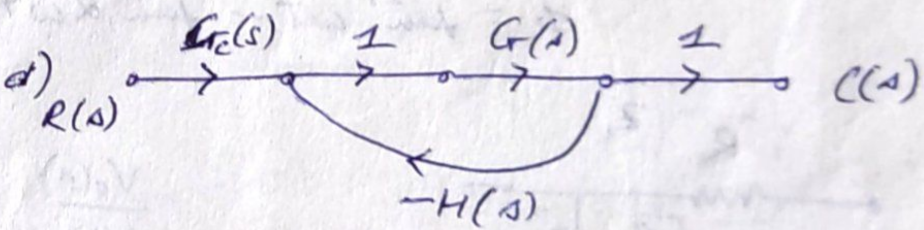
FEED BACK COMPENSATION
(Process control, electronics)

c)



O/P or LOAD COMPENSATION
(Printer etc.)

d)



I/P COMPENSATION.
(~~[illegible]~~ Preamplifier)

CASCADE COMPENSATORS : $G_C(s) = K \cdot \dfrac{\prod (s+z_i)}{\prod (s+p_j)}$

– use graphical choice of z_i, p_j by trial and error : see effect on root loci

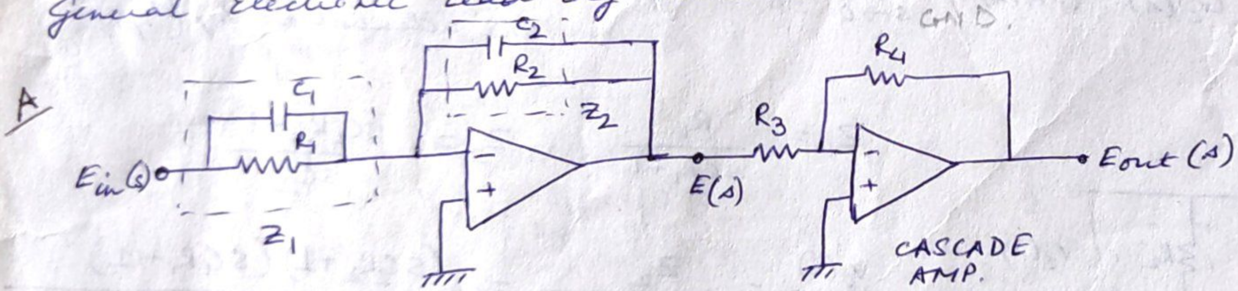
So if $G_C(s) = K \cdot \dfrac{s+z}{s+p} = K' \dfrac{\left(\frac{1}{z}\right)s + 1}{(1/p)s + 1}$; $K' = \dfrac{z}{p} \cdot K$

So when $\left(\dfrac{1}{|z|}\right) > \dfrac{1}{|p|} \Rightarrow |p| > |z|$: PHASE LEAD COMPENSATOR : HP : Differentiator (zero towards origin)

$\phi = \tan^{-1}\dfrac{\omega}{z} - \tan^{-1}\dfrac{\omega}{p}$

$\dfrac{1}{|z|} < \dfrac{1}{|p|} \Rightarrow |p| < |z|$: PHASE LAG COMPENSATOR : LP : Integrator

General Electronic Lead lag circuit : VIRTUAL GND.



$Z_1 = \dfrac{R_1}{sC_1R_1+1}$; $Z_2 = \dfrac{R_2}{sC_2R_2+1}$

$= \dfrac{1}{C_1} \cdot \dfrac{R_1C_1}{sC_1R_1+1}$

$\dfrac{E_{out}}{E_{in}}(s) = \left(\dfrac{-R_4}{R_3}\right) \cdot \left(\dfrac{-Z_2}{Z_1}\right) = \dfrac{R_4 C_1}{R_3 C_2} \; \dfrac{s + 1/R_1C_1}{s + 1/R_2C_2}$

$= K_C \dfrac{s + 1/T}{s + 1/\alpha T}$ where $K_C = \dfrac{R_4 C_1}{R_3 C_2}$, $T = R_1 C_1$, $\alpha T = R_2 C_2$

$\therefore \alpha = \dfrac{R_2 C_2}{R_1 C_1}$

$= \underline{\underline{K_C \alpha}} \cdot \dfrac{Ts + 1}{\alpha Ts + 1}$ $\therefore \phi = \tan^{-1}\omega T - \tan^{-1}\alpha\omega T$

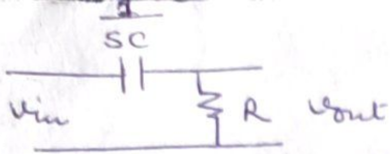
$\therefore$ If $T > \alpha T \Rightarrow R_1C_1 > R_2C_2 \Rightarrow 0 < \alpha < 1$: LEAD

$R_2C_2 > R_1C_1$: LAG.

Note : K_C needed to compensate the attenuation α during LEAD $\therefore$ CASCADE amplifier

or balance gain increase α during LAG

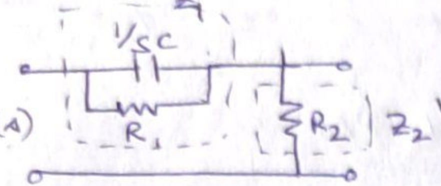
I LEAD



$\frac{R}{R+1/sC} = \frac{sRC}{sRC+1} = \frac{V_o(s)}{V_i(s)}$; $\phi = 90^\circ - \tan^{-1}\omega RC$ = +ve acute

BUT $s=0$ (d.c), gain $=0$ ∴ NO CONTROL ∴ low accuracy ∴ NOT USED IN SERVOS.

II LEAD



$$\frac{V_o(s)}{V_i(s)} = \frac{Z_2}{Z_1+Z_2} = \frac{R_2}{\frac{R_1/sC}{R_1+1/sC}+R_2} = \frac{R_2}{\frac{R_1}{sCR_1+1}+R_2}$$

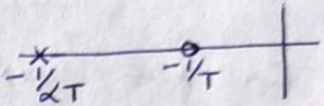
$$= \frac{sCR_1R_2+R_2}{R_1+sCR_1R_2+R_2} = \frac{R_2(sCR_1+1)}{(R_1+R_2)\left(\frac{sC\,R_1R_2}{R_1+R_2}+1\right)}$$

Let $\alpha = \frac{R_2}{R_1+R_2} < 1$

$$= \alpha \cdot \frac{sCR_1+1}{\alpha\frac{sCR_1}{\cancel{R_1+R_2}}+1} = \alpha\frac{Ts+1}{\alpha Ts+1}$$

$T = R_1C$

$$= \frac{s+1/T}{s+1/\alpha T}$$

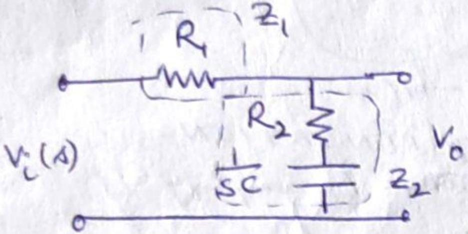


HP filter

$\phi = \tan^{-1}\omega T - \tan^{-1}\alpha\omega T$ = +ve acute angle for $0<\alpha<1$

III

LAG



$$\frac{V_o(s)}{V_i(s)} = \frac{Z_2}{Z_1+Z_2} = \frac{R_2+\frac{1}{sC}}{R_1+R_2+\frac{1}{sC}} = \frac{sCR_2+1}{sC(R_1+R_2)+1}$$

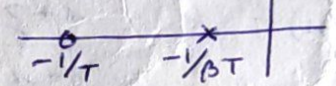
Let $\beta = \frac{R_1+R_2}{R_2} > 1$ $\qquad = \frac{Ts+1}{\beta Ts+1} = \frac{1}{\beta}\left(\frac{s+1/T}{s+1/\beta T}\right)$

$R_2 C = T$

$\therefore$ LP filter

$\phi = \tan^{-1}\omega T - \tan^{-1}\beta\omega T$

pole at $s = \frac{-1}{\beta T}$, zero at $s = \frac{-1}{T}$



IV

~~LEAD-LAG~~

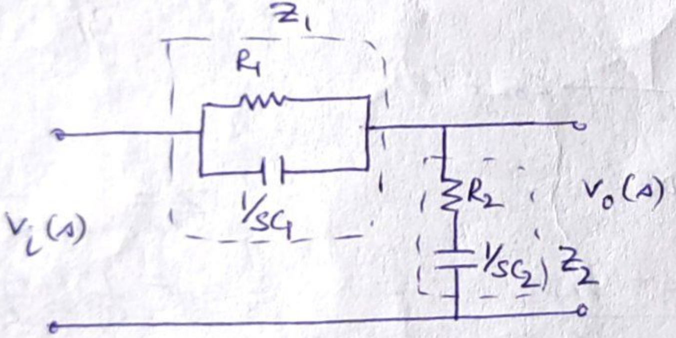
For compensator design : $\phi < 5^\circ$ $\therefore$ pole and zero : close and near origin

$\therefore$ NOT change overall root loci appreciably.

Also, to keep $\frac{V_o(s)}{V_i(s)} \approx \frac{1}{\beta} \longrightarrow$ HANDRULE: $1 < \beta < 15$, typically $\beta = 10$

Note : $\hat{K}_v = \lim_{s\to 0} s\,G_c(s)\cdot GH(s) = \lim_{s\to 0} G_c(s)\cdot K_v$ $\quad \therefore$ req. addl. gain K_c to compensate the attenuation by $(1/\beta)$.

IV LEAD-LAG



$Z_1 = \dfrac{R_1}{sC_1R_1+1}$; $Z_2 = \dfrac{sC_2R_2+1}{sC_2}$

$$\frac{V_o(s)}{V_i(s)} = \frac{Z_2}{Z_1+Z_2} = \frac{(sC_2R_2+1)(sC_1R_1+1)}{sC_2R_1 + (sC_2R_2+1)(sC_1R_1+1)}$$

Let $T_1 = R_1C_1$, $T_2 = R_2C_2$;

$\dfrac{T_1}{\beta} + \beta T_2 = (T_1 + T_2 + R_1C_2)$

$$= \frac{(T_1 s+1)(T_2 s+1)}{\left(\frac{T_1}{\beta}s+1\right)(\beta T_2 s+1)} = \frac{s+1/T_1}{s+\beta/T_1} \cdot \frac{s+1/T_2}{s+1/\beta T_2}$$

LEAD LAG

So $\omega_1 = \dfrac{1}{\sqrt{T_1T_2}}$

$0 < \omega < \omega_1$ LAG — Mag 0dB at low freq.

$\omega_1 < \omega < \infty$ LEAD — 0dB at high freq.

$\phi = \tan^{-1}\omega T_1 + \tan^{-1}\omega T_2 - \left[\tan^{-1}\omega\frac{T_1}{\beta} + \tan^{-1}\omega\beta T_2\right]$

$\therefore \phi = \tan^{-1}\dfrac{\omega(T_1+T_2)}{1-\omega^2T_1T_2} - \tan^{-1}\dfrac{\omega\left(\frac{T_1}{\beta}+\beta T_2\right)}{1-\omega^2T_1T_2}$

$-\tan^{-1}\omega(T_1+T_2+R_1C_2)$

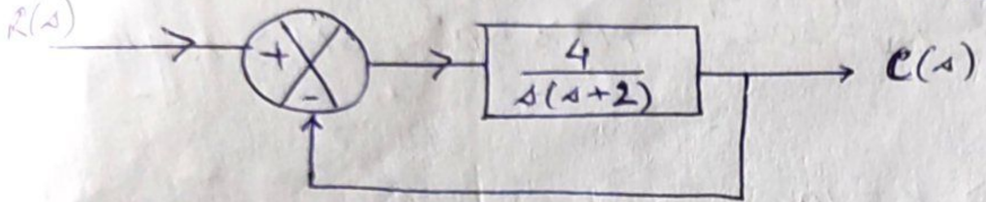
$= \tan^{-1}\dfrac{-\omega R_1C_2(1-\omega^2T_1T_2)}{(1-\omega^2T_1T_2)^2+\omega^2(T_1+T_2)^2+\omega^2(T_1+T_2)R_1C_2}$

$\left(\dfrac{1-\beta}{\beta}\right)\underset{R_1C_1}{T_1} + (\beta-1)\underset{R_2C_2}{T_2} = R_1C_2$

$\cancel{\beta T_2 + T_1 = \beta(T_1+T_2)+\beta R_1C_2}$

$\cancel{\beta^2R_2C_2 + R_1C_1 = \beta[R_1C_1+R_2C_2+R_1C_2]}$

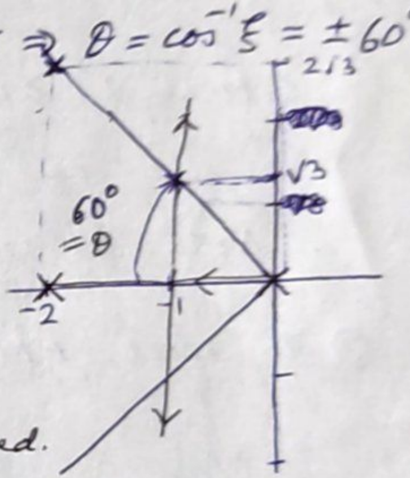
$(\beta-1)[\beta T_2 - T_1] \ldots$



Desired $\omega_{nd} = 4$ rad/s, ξ same

ch. eqn : $1 + GH(s) = s^2 + 2s + 4 = 0 \Rightarrow \omega_n = 2$ rad/s $\quad \xi = 0.5 \Rightarrow \theta = \cos^{-1}\xi = \pm 60^\circ$

& pole : $-1 + j\sqrt{3} = s$



For $\omega_{nd} = 4$, $\xi = 0.5$, Desired ch. eqn. : $s^2 + 4s + 16 = 0$

$\therefore$ Desired & poles : $s_d = -2 \pm j2\sqrt{3}$ (on same ξ line)

$$\angle GH(s)\Big|_{s_d = -2+j2\sqrt{3}} = \angle \frac{4}{s(s+2)}\Big|_{s_d = -2+j2\sqrt{3}} = -210^\circ = -180^\circ - 30^\circ$$

$\therefore$ Change of $(+30^\circ)$ lead (from -180° for pt. on root loci) required.

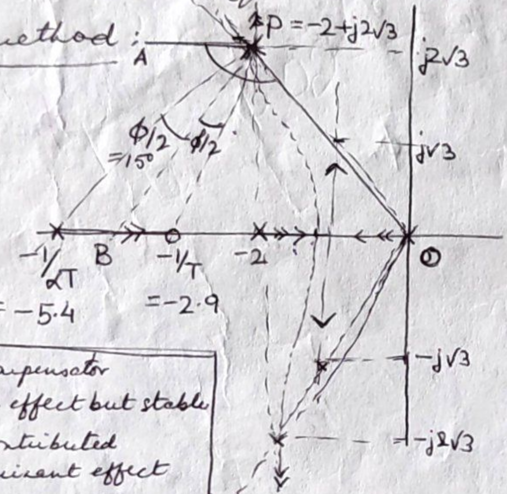
$$= \angle \frac{4}{s(s+2)}\Big|_{s=-1+j\sqrt{3}}$$ = orig. & poles

Ex2 <u>Design of the LEAD compensator</u> : a) Introduce pole zero pair so root loci passes through s_d.

b) Several possible T ∴ Choose T with largest α so that compensator gain K_c required is small (for the cascade amplifier).

<u>Graphical method</u> :

$\phi_d = 30^\circ$



(i) mark P and -180° line on P=PA

(ii) bisect $\angle P$ till it intersects σ axis at B.

(iii) plot $-\frac{1}{T}$ and $-\frac{1}{\alpha T}$ as zero and pole resp. at $\pm \frac{\phi_d}{2}$

Note $G_c(s) = K_c \cdot \alpha \cdot \frac{Ts+1}{\alpha Ts+1}$, $0<\alpha<1$

Note: zeros & poles of compensator
A) along PA : minimal effect but stable
B) along PO : min. $\angle$ contributed but dominant effect
∴ BISECT

(iv) ∴ Desired CL T.F. $= G_c(s) \cdot G(s)$

$$= \frac{4K_c(s+2.9)}{(s+5.4)\cdot s(s+2)}$$

(v) To satisfy magnitude condition,

$$\left| \frac{4K_c\,(s+2.9)}{s\,(s+2)(s+5.4)} \right|_{s_d=-2+j2\sqrt{3}} = 1 \quad \therefore\ K_c = 4.68 \quad \text{while } K = 4K_c = 18.7$$

Note: 1. $K_v = \lim_{s\to 0} sG_c(s)\,GH(s) = 5.02\,s^{-1}$

2. 3rd CL pole : $s(s+2)(s+5.4) + 18.7(s+2.9) = (s+2\pm j2\sqrt{3})(s+3.4)$

So, since (-3.4) is close to the added zero at (-2.9) ∴ effect of pole on transient response is less. (effect at $\lim_{s\to\infty}$)

∴ POSSIBLE DESIGN choices :

II: $\frac{V_o}{V_i} = \cancel{\frac{sRC}{sRC+1}}\ \alpha \frac{Ts+1}{\alpha Ts+1}$; $T = R_1C$, $\alpha = \frac{R_2}{R_1+R_2}$ so $R_1 = 345k\Omega$, $R_2 = 400k\Omega$, $C = 1\mu F$ and separate K_c

A
1. $R_1 = 345k\Omega$ $R_2 = 400k\Omega$, $C_1 = 1\mu F$, $C_2 = 0.47\mu F$ $R_3 = 4.7k\Omega$, $R_4 = 10k\Omega$

2. $34.5k\Omega$ $40k\Omega$ $C_1 = C_2 = 10\mu F$ $R_3 = 10k\Omega$ $R_4 = 46.8k\Omega$

— Balanced resistors and capacitors.

Ex3 Design a compensator for Ex 1 s.t. $K_v = 20s^{-1}$, $PM \geq 50°$, $GM \geq 10dB$.

① To achieve $K_{v_d} = 20s^{-1}$

$$K_{v_d} = \lim_{s\to 0} s\,GH(s)\,K = \lim_{s\to 0} \frac{4K}{s+2} = 20 \Rightarrow K = 10.$$

② GM reqd. $\geq 10dB$ → satisfied since 2^{nd} order & eqs $GM = \infty$ ∵ $\phi \not> (-180°)$

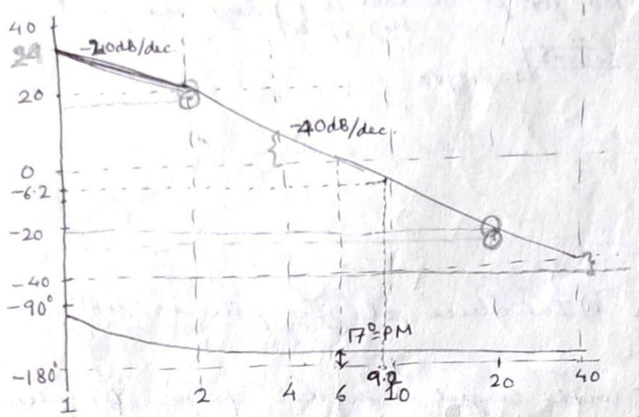
else use cascade amplifier / attenuator

Note: at least one of GM / K_v constraints have to be inequality constraints.

③ To meet $PM \geq 50°$

Presently $PM = 17°$ at $\omega_g = 6$ rad/s

∴ Req. addl. at least $50° - 17° = 33°$ phase lead + Tolerance margin of $5° \to 38°$ phase lead to be designed.



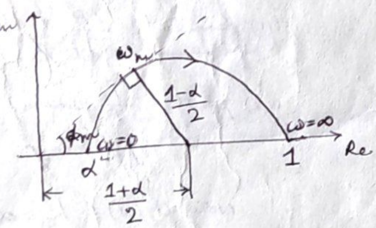
∴ $\phi_m = 38°$

25dB at $\omega = 1$
17dB at $\omega = 2$
0.46dB at $\omega = 6$
-2.1 dB at $\omega = 7$
-6.34 dB at $\omega = 9$

General: Phase plot for compensator $G_c(j\omega) = \alpha \dfrac{j\omega T + 1}{j\alpha\omega T + 1}$, $\alpha < 1$

$= \dfrac{j\omega + 1/T}{j\omega + 1/\alpha T}$

~~$\phi = \tan^{-1}\omega T - \tan^{-1}\alpha\omega T = \tan^{-1}\dfrac{\omega T - \alpha\omega T}{1 + \omega^2\alpha T^2}$~~



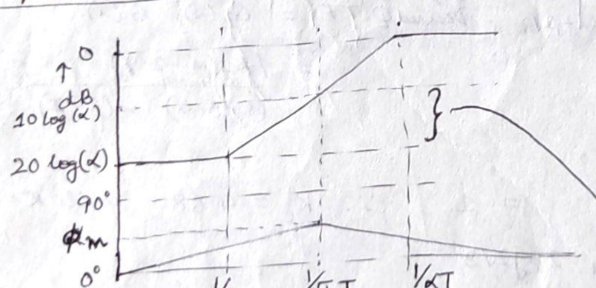
∴ $\sin\phi_m = \dfrac{1-\alpha}{1+\alpha}$ where ϕ_m = maxm. phase lead angle

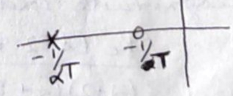
BODE plot of lead compensator $\alpha < 1$; corner freq: $1/T$, $1/\alpha T$

$$\log \omega_m = \frac{1}{2}\left[\log\frac{1}{T} + \log\frac{1}{\alpha T}\right]$$

$$\therefore \omega_m = \frac{1}{\sqrt{\alpha}\,T}$$

Note: log scale for freq.





So $\sin\phi_m = \sin 38° = \dfrac{1-\alpha}{1+\alpha}$ ∴ $\alpha = 0.24$

magnitude curve at $\omega_m \to \left|\dfrac{1+j\omega T}{1+j\alpha\omega T}\right|_{\omega = \frac{1}{\sqrt{\alpha}T}} = \dfrac{1}{\sqrt{\alpha}} = \dfrac{1}{\sqrt{0.24}} = 6.2dB$

But adding compensator changes magnitude curve at ω_m w.r.t $\omega \leq 1/T$.

∴ Magnitude curve will shift up by 6.2dB at ω_m. Original -6.2dB at $\underline{\omega_{g_{new}} = 9 \text{ rad/s}}$ (also adjust for low PM because of earlier tolerance margin of 5°)

∴ select ω_m to be the new ω_g = new 0dB

∴ $\omega_{g_{new}} = 9$ rad/s $= \dfrac{1}{\sqrt{\alpha}\,T_{new}}$ ∴ $\dfrac{1}{T_{new}} = 4.41$, $\dfrac{1}{\alpha T_{new}} = 18.4$

∴ Lead network $= K_c \cdot \dfrac{s+4.41}{s+18.4} = K_c \cdot (0.24) \dfrac{0.227s+1}{0.054s+1}$ ∴ $K_c = \dfrac{1}{0.24} = 4.17$ [to keep overall compensator gain = 1]

∴ Overall new T.F. $= 4.17 \dfrac{s+4.41}{s+18.4} \cdot 10 \cdot \dfrac{4}{s(s+2)} = \underbrace{\boxed{41.7 \dfrac{s+4.41}{s+18.4}}}_{G_c(s)} \cdot \underbrace{\dfrac{4}{s(s+2)}}_{G(s)}$.