Ex $G(s) = \frac{1}{s+1} \rightarrow$ low pass RC $R = 2\Omega$, $C = 0 \cdot 5F$ with i/p $10\cos 4t$ V.

v_0 across C ?

$u(s) = \mathcal{L}[10\cos 4t] = \frac{10s}{s^2+16}$ $\therefore V_0(s) = \frac{10s}{(s+1)(s^2+16)} = \frac{-10/17}{s+1} + \frac{10/17\, s + 160/17}{s^2+16}$

Laplace

$\therefore v_0(t) = -10/17\, e^{-t} + \left(\frac{10}{17}\cos 4t + \frac{40}{17}\sin 4t\right)$

(transient) s.s.

Let $s = j\omega$, $\therefore G(j\omega) = \frac{1}{1+j\omega}$ for $\omega = 4$ $= \frac{1-j4}{17} = \frac{1}{\sqrt{17}} \angle -76^\circ$

Sinusoidal analysis

$\therefore v_0(t) = \frac{10}{\sqrt{17}}\cos(4t - 76^\circ)\, V = \frac{10}{17}\cos 4t + \frac{40}{17}\sin 4t.$

FREQUENCY RESPONSE ANALYSIS:

Sinusoidal signals → steady state behaviour $\sigma = 0$ → finite

& Characteristic eqn. $1 + GH(s) = 0$

$\Rightarrow |GH(s)| = 1$; $\angle GH(s) = -180°$ or $(2k+1)\pi$ for a just stable

system ∴ Indicate Gain & Phase Margins to stability for s.s.

i.e. $s = j\omega$.

Frequency Domain Analysis:

Mag. : $20 \log |GH(s)| = 20 \log 1 = 0\,dB$ $\rightarrow$ gives gain crossover freq. ω_c ~~and~~ PHASE MARGIN.

Phase : $\angle GH(s) = (2k+1)\pi$ $\rightarrow$ gives phase crossover freq.
$= -180^\circ$ GAIN MARGIN

$$\text{Gain Margin} = -20 \log |GH(j\omega_p)|\,dB = +20 \log \left| \frac{1}{GH(j\omega_p)} \right| dB.$$

$$\text{Phase Margin} = \angle GH(j\omega_g) + 180^\circ \Rightarrow \angle GH(j\omega_g) - PM = -180^\circ$$

(Normally lagging systems $\Rightarrow$ add (-) phase

Both +ve $\Rightarrow$ STABLE system.

BODE PLOT : HENDRIK W. BODE b1905

$GH(j\omega) = |GH(\omega)| e^{j\phi(\omega)}$

$\therefore \ln GH(j\omega) = \ln|GH(\omega)| + j\phi(\omega)$ [usually in dB]

log scale ∴ cover large range of freq. ; log ∴ × & / to + and −.

$u = \log_{10}\omega \Rightarrow 10^u = \omega$

$u_2 - u_1 = \log_{10}\omega_2 - \log_{10}\omega_1 = \log_{10}\frac{\omega_2}{\omega_1}$ usually octaves $\omega_2 = 2\omega_1$ or decades $\omega_2 = 10\omega_1$

No. of decades: $\frac{\log_{10}\frac{\omega_2}{\omega_1}}{\log_{10}10} = \log_{10}\frac{\omega_2}{\omega_1} = x \Leftarrow \boxed{\frac{\omega_2}{\omega_1} = 10^x}$ $= 2^y$

$\log_2\frac{\omega_2}{\omega_1} = y$

No. of octaves

~~Let~~ $\cancel{\omega} = 10^u = 2^v \therefore \log_{10}\omega = u$; $\log_2\omega = v$

$\Rightarrow v\log_{10}2 = u \Rightarrow \boxed{v = \frac{u}{\log_{10}2} = \frac{\log_{10}\omega}{\log_{10}2} = \log_2\omega}$.

$\therefore$ So 20 dB/decade $= 20\log_{10}\frac{\omega_2}{\omega_1} = 20 \times (\log_{10}2) \times \left(\log_2\frac{\omega_2}{\omega_1}\right) = 20 \times 0.301 \log_2\frac{\omega_2}{\omega_1}$ $= 6.4$

CHAIN RULE

$= 6$ dB/octave

CONSTRUCTION OF BODE PLOT:

$$GH(s) = \frac{K_S \prod (s^2 + 2\zeta_k \omega_{nk} s + \omega_{nk}^2) \prod (sT_i + 1)}{s^N \prod (s^2 + 2\zeta_m \omega_{nm} s + \omega_{nm}^2) \prod (sT_\ell + 1)}$$

$$GH(j\omega) = \frac{K \prod (j\omega T_i + 1) \prod (1 + j2\zeta_k u_{nk} - u_{nk}^2)}{(j\omega)^N \prod (j\omega T_\ell + 1) \prod (1 + j2\zeta_m u_{nm} - u_{nm}^2)}$$

where $K = \dfrac{K_S \prod \omega_{nk}^2}{\prod \omega_{nm}^2}$ $\quad u_{nk} = \dfrac{\omega}{\omega_{nk}} \quad u_{nm} = \dfrac{\omega}{\omega_{nm}}$

20 log

∴ Mag: $|GH(j\omega)|_{dB} = 20 \log |K| + \sum 20 \log |1 + j\omega T_i|$

$$+ \sum 20 \log |1 + j2\zeta_k u_{nk} - u_{nk}^2|$$

$$- 20N \log |j\omega| - \sum 20 \log (1 + j\omega T_\ell)$$

$$- \sum 20 \log |1 + j2\zeta_m u_{nm} - u_{nm}^2|$$

phase: $\angle GH(j\omega) = 0 + \sum \tan^{-1}(\omega T_i) + \sum \tan^{-1} \dfrac{2\zeta_k u_{nk}}{1 - u_{nk}^2}$

$$- N \cdot \frac{\pi}{2} - \sum \tan^{-1}(\omega T_\ell) - \sum \tan^{-1} \frac{2\zeta_m u_{nm}}{1 - u_{nm}^2}$$

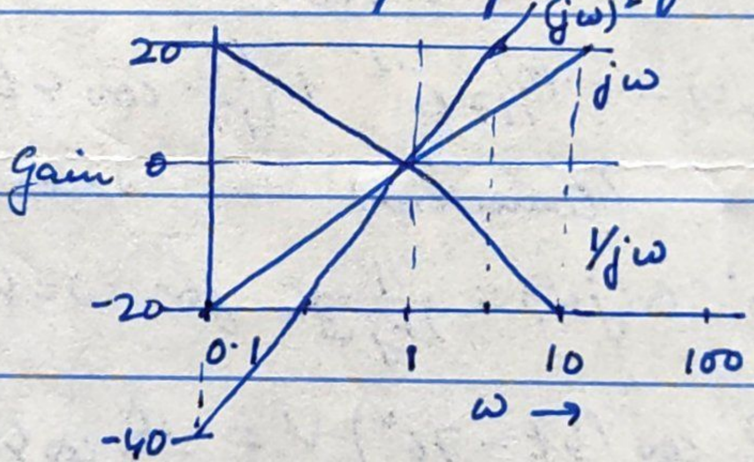
① Constant K $\qquad 20\log|K| = \text{constant}$; phase $= 0$.

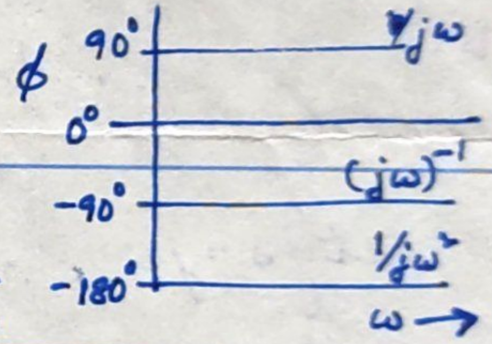
② Zeros/poles at origin $(j\omega)^{\pm N}$ $\qquad$ ⓓ 20dB/dec = 6dB/octave

$$\pm 20N\log(j\omega) = \pm 20N\log|\omega| \text{ dB}$$

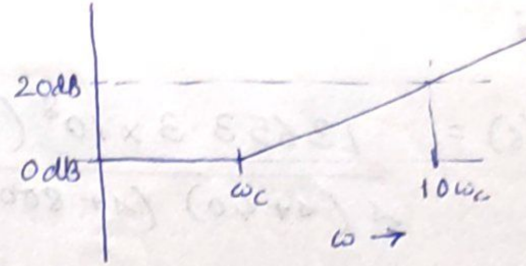
– straight line with $\pm 20N$ slope passing through 0dB at $\omega=1$.

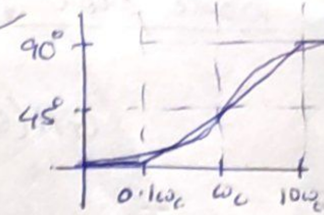
angle of $\pm N(\pi/2)$





Simple zero and pole $\quad (1+j\omega T)^{\pm 1}$





Let $GH(j\omega) = 1 + j\omega T \qquad$ Let $T = 1/\omega_c$

$= 1 + j\left(\frac{\omega}{\omega_c}\right)$

Mag: $20 \log \sqrt{1+\omega^2 T^2} = 20 \log \sqrt{1+\left(\frac{\omega}{\omega_c}\right)^2}$ dB

Phase $\tan^{-1}\omega T = \tan^{-1}\frac{\omega}{\omega_c}$

$\omega T = \frac{\omega}{\omega_c} \ll 1 \qquad 20 \log 1 = 0$ dB $\qquad \tan^{-1} 0 = 0^\circ$

$\omega T = 1 \qquad \cancel{1}0 \log 2 = 3.01$ dB $\qquad \tan^{-1} 1 = 45^\circ$

$\omega T \gg 1 \qquad \cancel{2}0 \log\left(\frac{\omega}{\omega_c}\right) = 20 \log \omega + 20 \log T \qquad \tan^{-1}\frac{\omega}{\omega_c} \rightarrow \tan^{-1}\infty = 90^\circ$

For $\frac{\omega}{\omega_c} = K$, $20 \log K$ $\therefore K = 10 \Rightarrow 20$dB.

$\therefore$ 20 dB/decade slope, origin at $\omega = \omega_c = \frac{1}{T}$

Approx: 0° upto $0.1\,\omega_c$

90° beyond $10\omega_c$

st. line from 0.1 to $10\,\omega_c$

with 45° at ω_c.

Error: $20 \log \sqrt{1+\left(\frac{\omega}{\omega_c}\right)^2}$ for $\omega < \omega_c$

$20 \log \sqrt{1+\left(\frac{\omega}{\omega_c}\right)^2} - 20 \log \frac{\omega}{\omega_c}$ for $\omega > \omega_c$

$y = 45\left[\log \frac{\omega}{\omega_c}\right] + (45^\circ)$ — at $\omega = \omega_c$ $\because \log 1 = 0$. origin

$= 45\left[\log \frac{\omega}{\omega_c} + 1\right]$

Let $\frac{\omega}{\omega_c} = K$ for $\omega > \omega_c$ $\therefore$ Error $= 20 \log \sqrt{\frac{1+K^2}{K^2}}$

For $\frac{\omega}{\omega_c} = \frac{1}{K}$ — do —

Phase error

$\tan^{-1}\omega T - y$.

$\therefore K = 0.1$ or $10 \qquad +0.043$ dB $\qquad \pm 5.71^\circ$

$K = 0.5$ or $2 \qquad +0.97 \approx 1$ dB $\qquad \mp 4.89^\circ$

$K = 1 \qquad +3.01$ dB $\qquad 0$

Phase error 0 at approx $K = 0.1586 = 1/K$

Quadratic pole or zero:

$$GH(j\omega) = \frac{1}{1 + j2\zeta u - u^2}$$

For O.D. ~ to 2 real poles

U.D. $\zeta < 1$ $\quad \omega/\omega_n = u \ll 1$, $\quad$ Mag = 0 dB

$u \gg 1 \quad -20 \log \sqrt{u^4} = -40 \log u$ through corner freq.

Resonant freq. when $(1-u^2)^2 + (2\zeta u)^2 = g(\omega)$ is minimum

$G(j\omega_n) = -j\,\frac{1}{2\zeta}$

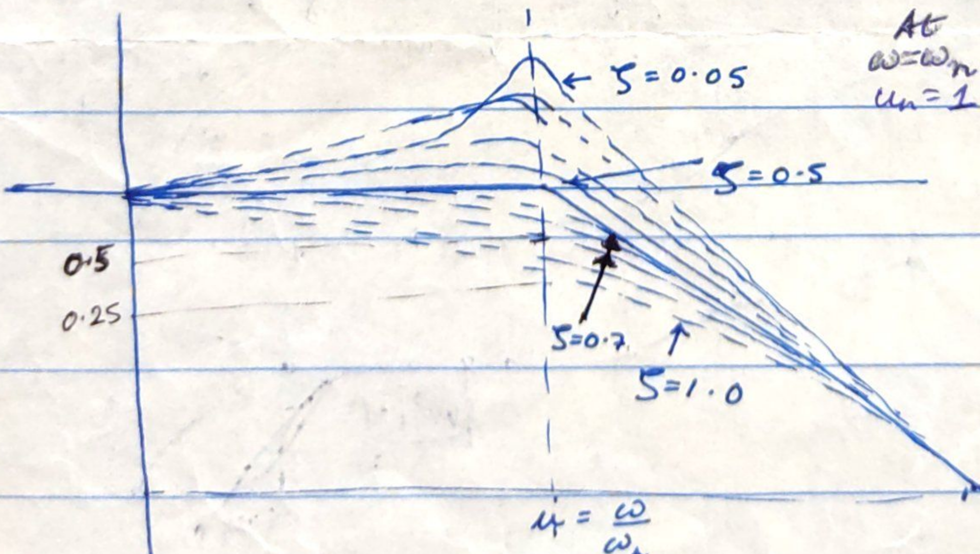
$$\therefore g(\omega) = 1 + \left(\frac{\omega^2}{\omega_n^2}\right)^2 - 2\left(\frac{\omega^2}{\omega_n^2}\right) + 4\,\frac{\omega^2}{\omega_n^2}\,\zeta^2$$

At $\zeta = \frac{1}{\sqrt{2}} = 0.707$

$$= \left[\frac{\omega^2 - \omega_n^2(1-2\zeta^2)}{\omega_n^2}\right]^2 + 4\zeta^2(1-\zeta^2)$$

$g(\omega_r) = \frac{4}{2}\left(1 - \frac{1}{2}\right) = 1$

$\therefore \omega_r = \omega_n\sqrt{1-2\zeta^2} \qquad 0 \le \zeta \le 0.707. \qquad \therefore |GH(j\omega_r)| = \frac{1}{2\zeta\sqrt{1-\zeta^2}}$



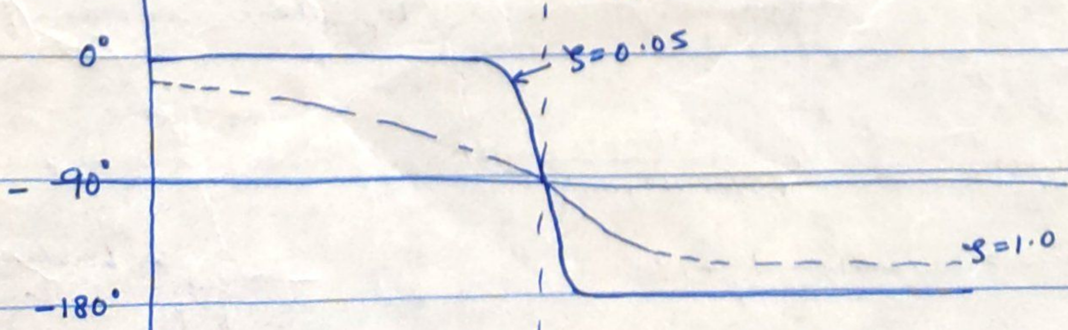
At $\omega = \omega_n$, $u_n = 1$, $\quad -\tan^{-1}\left(\frac{2\zeta u_n}{1-u_n^2}\right) = -\tan^{-1}(\infty)$

$\Rightarrow \theta = -90°$

$\omega \ge \omega_n \quad -\tan^{-1}\left(\frac{2\zeta u}{1-u^2}\right)$

$\rightarrow -\tan^{-1}(-0)$

$\Rightarrow \theta = -180°$.



Quadratic zero and pole : $Mag \propto \frac{1}{g(\omega)}$; At $\omega = \omega_n$, $G(j\omega_n) = -j \cdot \frac{1}{2\xi}$

$$g(\omega) = \left[\frac{\omega^2 - \omega_n^2(1-2\xi^2)}{\omega_n^2}\right]^2 + 4\xi^2(1-\xi^2)$$

$\omega_r = \omega_n\sqrt{1-2\xi^2}$ for resonant ω

		$\omega \ll \omega_n$	$\omega = \omega_n$	$G(j\omega_n)$	$\omega \gg \omega_n$	$\omega = \omega_r$	Value of ω_r
For $\xi = 1$	$g(\omega) = \left[\frac{\omega^2 + \omega_n^2}{\omega_n^2}\right]^2 > 1$	$\rightarrow 1$	4	$\frac{-j}{2}$	$\rightarrow \infty$	NA	Not valid
$\xi = \frac{1}{\sqrt{2}} = 0.707$	$\left[\frac{\omega^2}{\omega_n^2}\right]^2 + \frac{4}{2} \cdot \frac{1}{2} = 1 + \left(\frac{\omega^2}{\omega_n^2}\right)^2$	$\rightarrow 1$	2	$-j/\sqrt{2}$	$\rightarrow \infty$	1	0
$\xi = \frac{1}{2} = 0.5$	$\left[\frac{2\omega^2 - \omega_n^2}{2\omega_n^2}\right]^2 + \frac{4}{4} \cdot \frac{3}{4}$	$\rightarrow 1$	1.0	$-j$	$\rightarrow \infty$	0.75	$\frac{\omega_n}{\sqrt{2}}$
$\xi \ll 1$			.		$\rightarrow \infty$	0	ω_n
$\xi = 0.05$	$\left[\frac{\omega^2 - \omega_n^2(0.99975)}{\omega_n^2}\right]^2 + 0.0005$	$\rightarrow 1$	0.000500062	$-10j$	$\rightarrow \infty$	0.0005	$0.999875\,\omega_n$

3a 1. Identify critical frequencies, magnitude & phase at these frequencies

2. Plot bode components while stating characteristics

3. Total plot — additive : semi log plot for magnitude & phase

4. Identify ω_g, ω_p, GM, PM from plot. Verify with tabular value
   Comment – % error, why & expectations.

5. Infer & stability.

f	M_{n}	$\angle_{n}$

$$\frac{K(s+150)}{s(s+20)(s^2+800s+64\times10^4)}$$

Bode plot :

$$GH(s) = \frac{13653.3 \times 10^8 (s+150)}{s(s+40)(s+800)(s^2+2000s+4\times10^6)}$$

$$\therefore GH(j\omega) = \frac{1600\left(1+\frac{j\omega}{150}\right)}{j\omega\left(1+\frac{j\omega}{40}\right)\left(1+\frac{j\omega}{800}\right)\left[1-\left(\frac{\omega}{2000}\right)^2+\frac{j\omega}{2000}\right]}$$

Four factors:

1) Constant $K = 1600 \triangleq 64\,dB = (20\log 1600)$; shift of origin, 0° phase

2) Pole at origin : $(j\omega)^{-1}$: Mag : $-20\,dB/dec$ through $\omega = 1$, phase $= -90^\circ$

3) Simple poles and zeros :

	Mag	Phase
a) $\left(1+j\frac{\omega}{40}\right)^{-1}$	0 dB till 40, then $-20\,dB/dec$	0 till 4, -45° at 40, -90° from 400
b) $\left(1+j\frac{\omega}{150}\right)$	150, $+20\,dB/dec$	till 15, 45° at 150, 90° from 1500
c) $\left(1+j\frac{\omega}{800}\right)^{-1}$	800, $-20\,dB/dec$	till 80, -45° at 800, -90° from 8000

4) Quadratic pole : $1-\left(\frac{\omega}{2000}\right)^2+\frac{j\omega}{2000}$ $\therefore \xi = 0.5$

Mag: Asymptote : 0 till 2000, $-40\,dB/dec$ then

$G(j2000) = \frac{-j}{1} = -j \therefore$ Mag 1

$\omega_r = \omega_n\sqrt{1-2\xi^2} = \frac{2000}{\sqrt{2}} = 1414.$

$|G(j\omega_r)| = \frac{1}{\sqrt{\frac{10}{16}}} = \frac{4}{\sqrt{10}} = 1.265 = 2.042\,dB$

Phase : 0° till 200, -90° at 2000, -180° from 20000

$\therefore$ Critical Freq.	Mag. (Th.)	Ph. (Th.)
1		
4		
15		
40		
80		
150		
200		
400		
800		
1500		
2000		
8000		
20000		
Theo.	GM +12 dB	-180° ω_p (1150 Hz)
	1 ω_g (420Hz)	PM (44°)

Comparison of Theo. and Practical (ω_p, GM, ω_g, PM.

Gain Margin : $20\log\frac{1}{|GH(j\omega_p)|}$ dB at ω_p : If stable, then +ve GM ($\because \frac{1}{|GH(\omega_p)|} > 1$)

Phase Margin : $\angle GH(j\omega_g) - (-180^\circ)$: If stable, then +ve PM (amount of lag allowable)

Infer & stability.

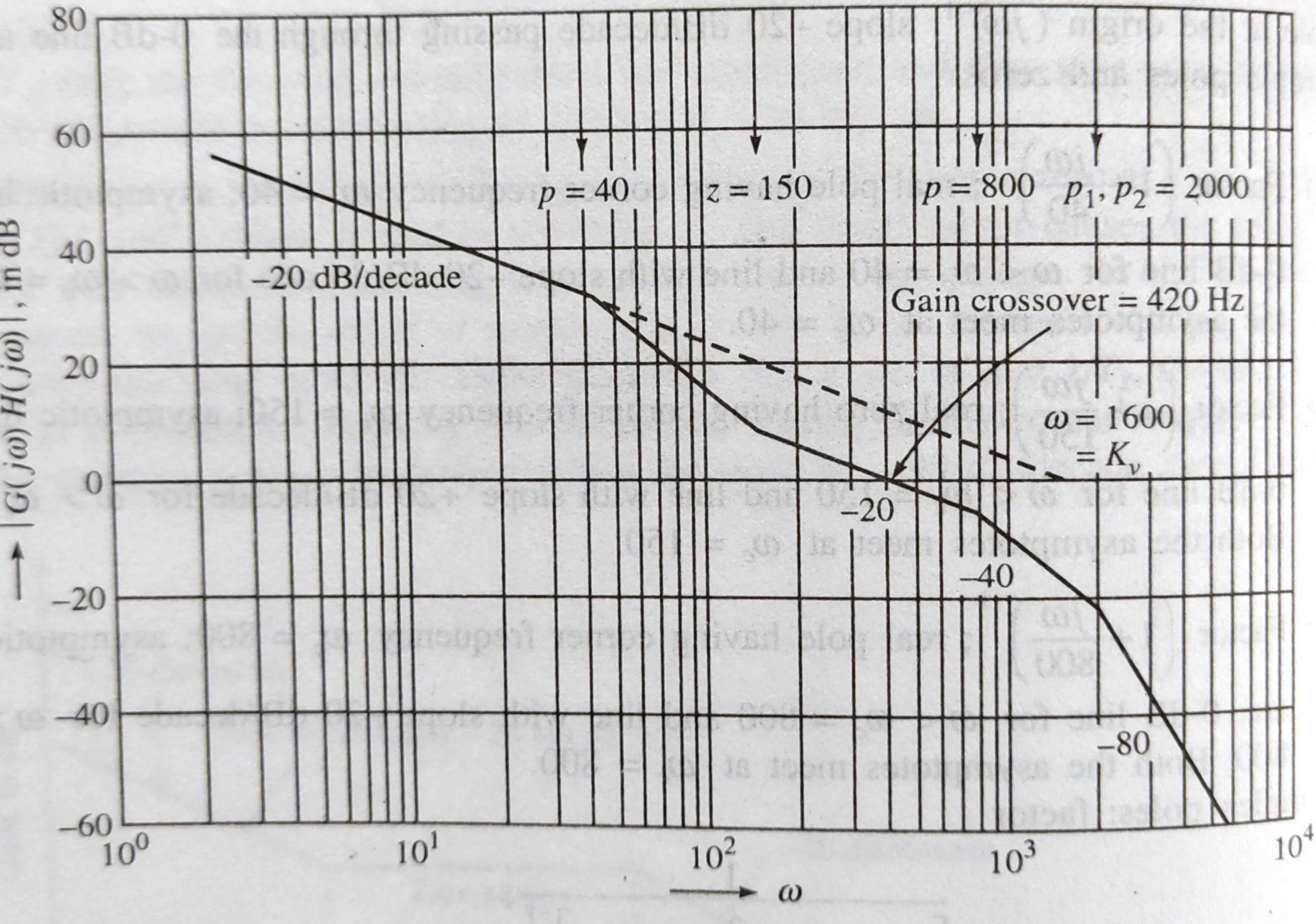


FIGURE 8.10 Example 8.3: construction of magnitude asymptotes.

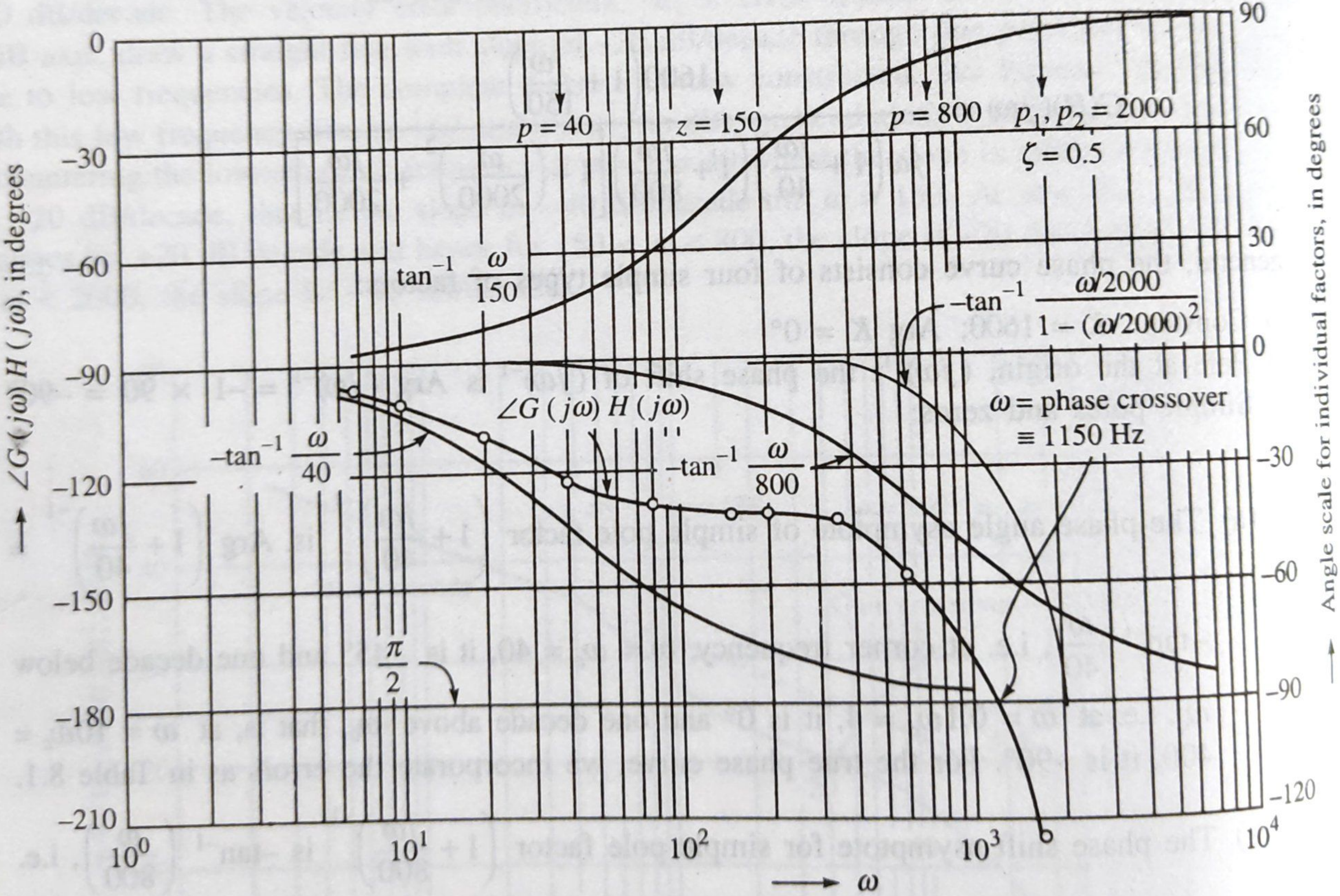


FIGURE 8.11 Example 8.4: phase angle curve.

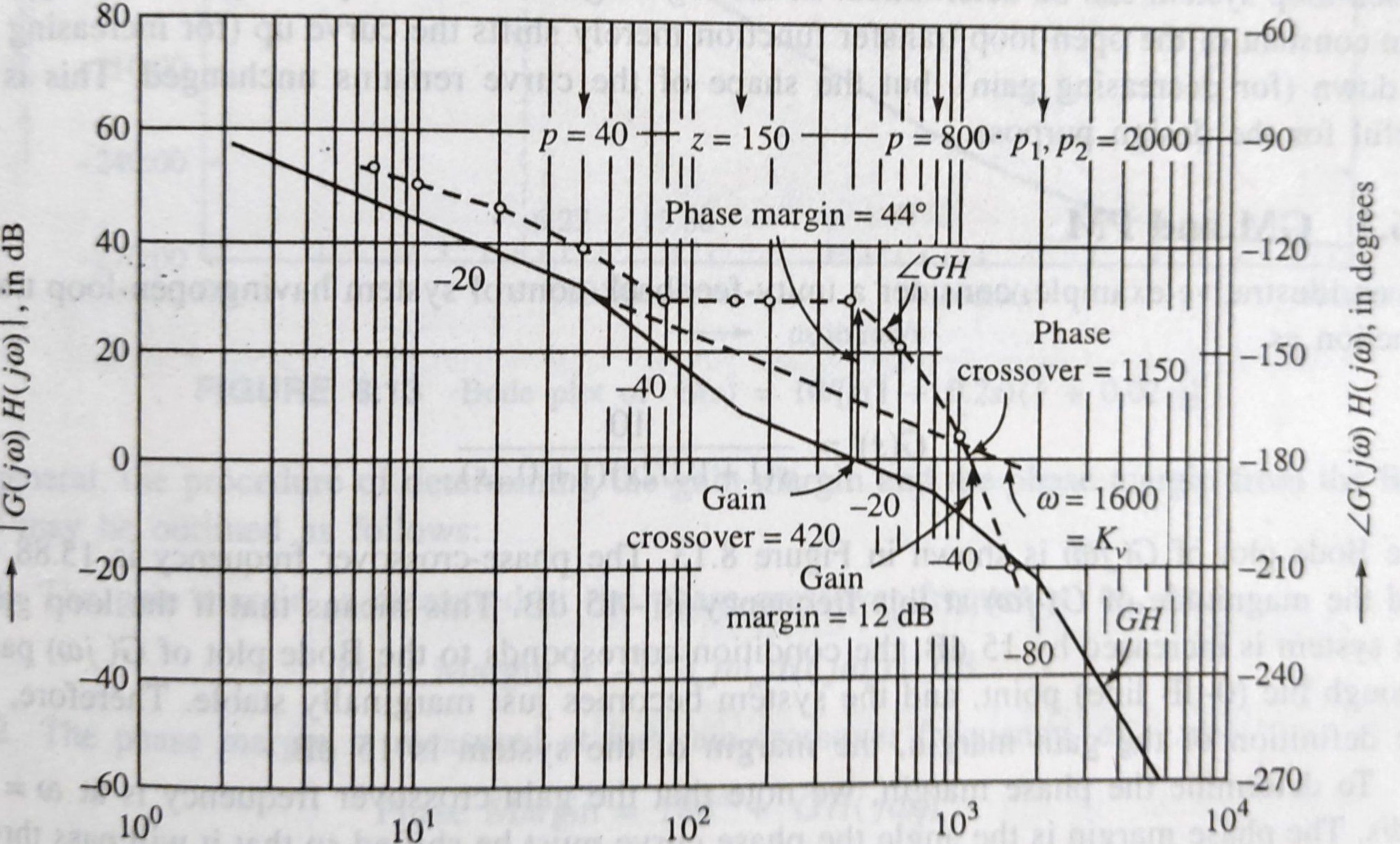
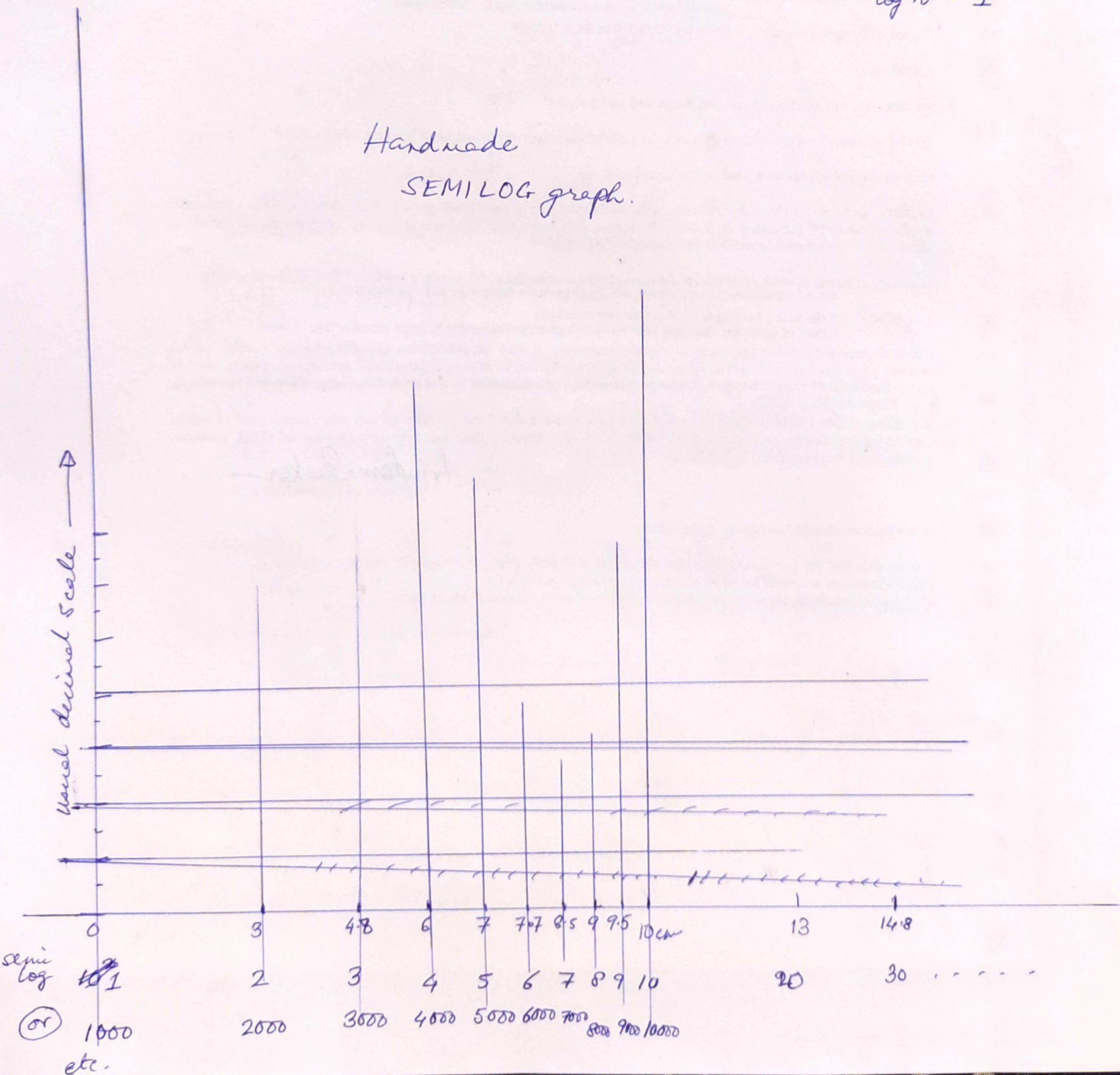


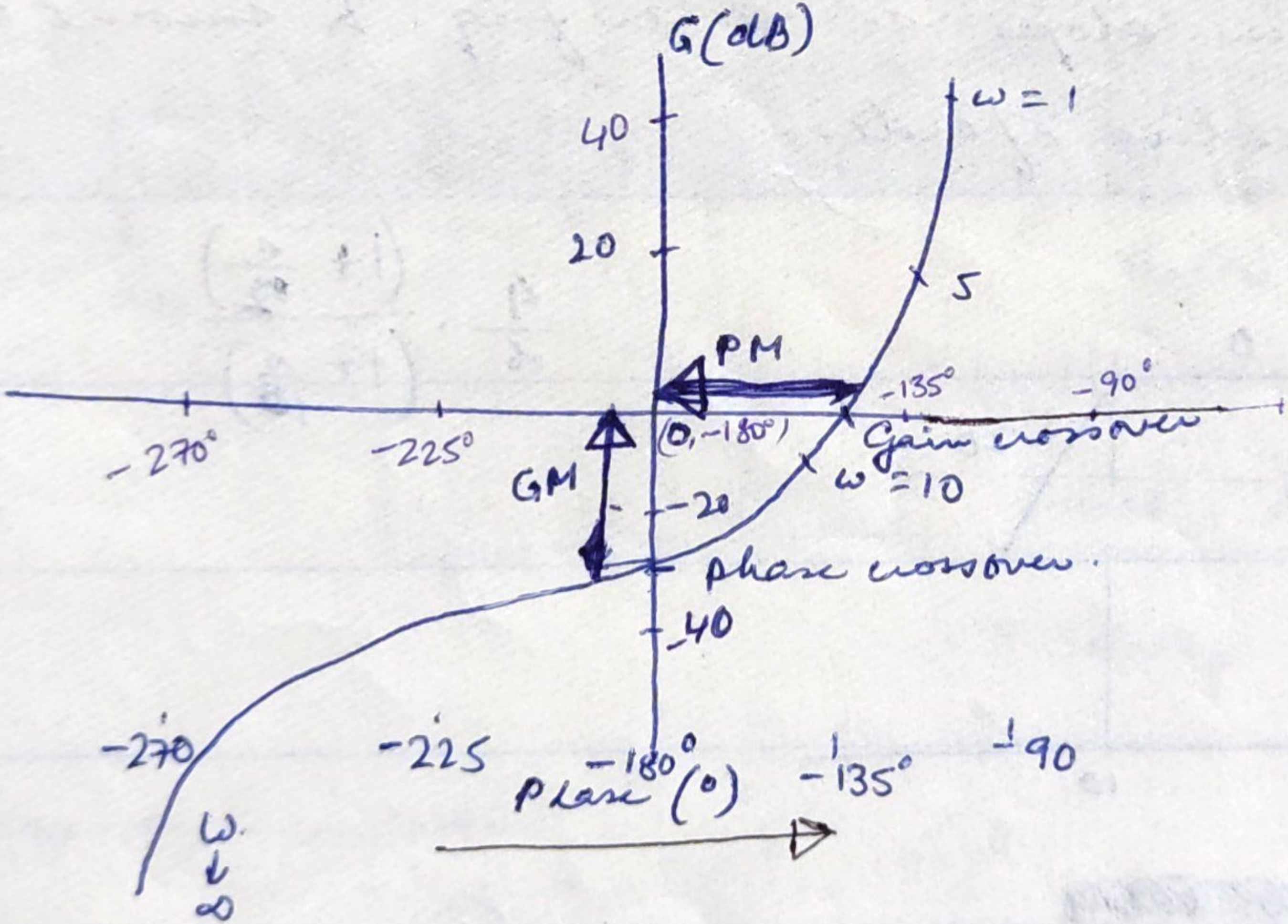
FIGURE 8.12 Example 8.5: magnitude and phase curves.

log 1 = 0
2 = 0.3
3 = 0.477 ≃ 0.5
4 = 0.6
5 = 0.7
6 = 0.77 ≃ 0.8
7 ≃ 0.85
8 = 0.9
9 ≃ 0.95
log 10 = 1



Log Magnitude vs. Phase Plot

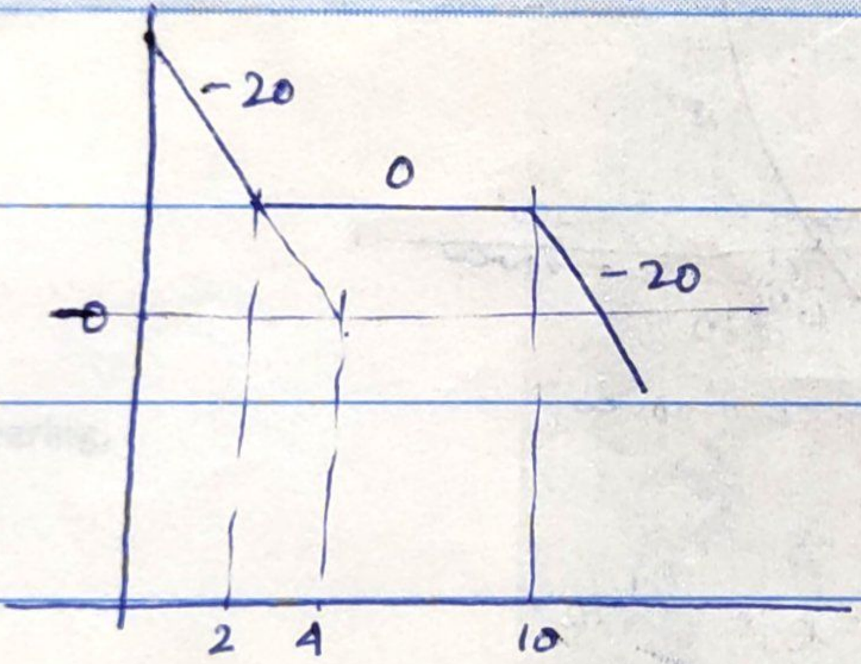
From Bode plot ⟶ crossover of mag. axis & crossover of $-180°$



Note: phase : x axis
log mag. : y axis

Estimating TF from Bode plot:

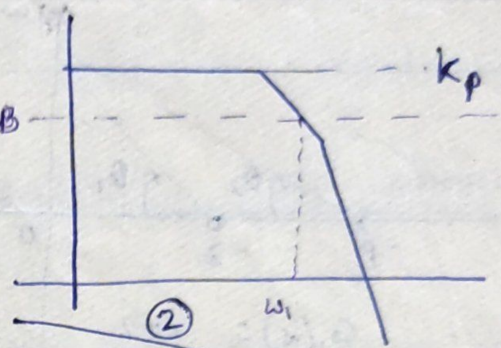
check changes in slope as corner freq. & amount of change as indicative of order.



$$\frac{4}{s} \cdot \frac{\left(1+\frac{s}{2}\right)}{\left(1+\frac{s}{10}\right)}$$

Application in finding error coeff.

For type 0, $GH(s) = \dfrac{K}{(s+1)(s+10)}$ $\therefore$



$K_p = \lim_{s \to 0} GH(s) = K/10$

Type 1 $\quad GH(s) = \dfrac{K}{s}$ $\quad \therefore K_v = \lim_{s\to 0} sGH(s) = K$.

② $\therefore$ at $s = j\omega_1 = K$, $|GH(j\omega)|_{\omega=\omega_1} = \left|\dfrac{K_v}{j\omega}\right|_{\omega=\omega_1} = 20\log\left(\dfrac{K_v}{\omega_1}\right) = 0\text{dB}$.

$\therefore$ Intersection with 0dB gives $\omega_1 = K_v$

① $\therefore$ Initial -20dB/dec. $\Rightarrow$ type 1 and so on.

$\therefore \boxed{\text{at } \omega = 1,\ |GH(j1)| = 20\log K_v}$.

Type 2 : $\quad GH(s) = \dfrac{K_a}{s^2}$ $\therefore 20\log\left|\dfrac{K_a}{(j\omega)^2}\right| = 20\log K_a$ at $\omega = 1$.

$\therefore$ Intersection with 0dB line gives $\sqrt{K_a} = \omega_a$

$\therefore 20\log\left|\dfrac{K_a}{(j\omega)^2}\right|_{\omega=\omega_a} = 20\log 1 = 0\,\text{dB}$ when $\omega_a = \sqrt{K_a}$.

Estimating ξ : For $G(s) = \dfrac{K}{s(Js+F)} = \dfrac{K/F}{s\left(\frac{J}{F}s+1\right)}$

$\therefore \omega_c = \dfrac{1}{T} = \dfrac{F}{J}$; $\quad K_v = K/F = \omega_1$ ✓ (-20dB/dec slope)

$\omega_2 = F/J$ for 1st order part), $\omega_3 = \sqrt{K/J}$ when expressed as

$$\frac{K/J}{s(s+F/J)} \approx \frac{\omega_n^2}{s(s+2\zeta\omega_n)} \approx \frac{\omega_n^2}{s^2} \text{ for } 2\xi\omega_n \ll s$$

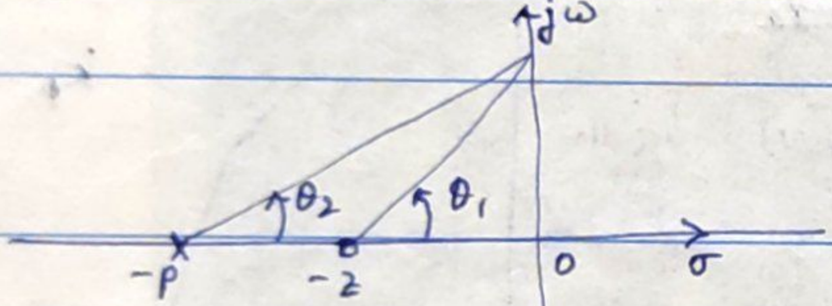
$\therefore$ (-40dB/dec slope

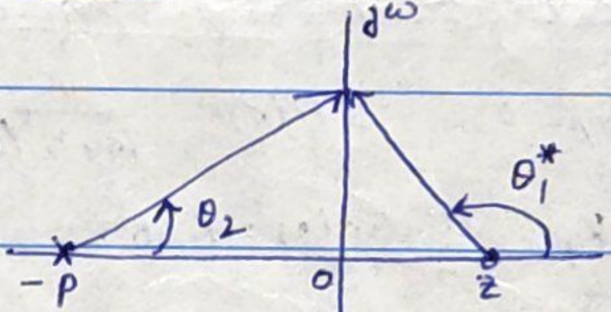
$\therefore$ ~~$\zeta = F/2\sqrt{KJ} = \dfrac{\omega_2}{2\omega_3}$~~

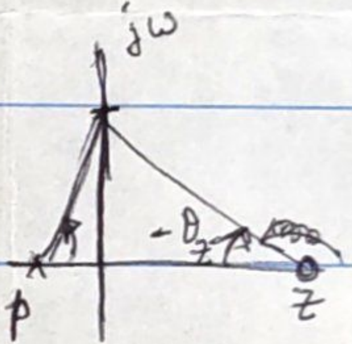
$\omega_3^2 = K/J \quad \therefore \omega_2 = \dfrac{\omega_3^2}{\omega_1}$

and $\xi = \dfrac{F}{2\sqrt{KJ}} = \dfrac{\omega_2}{2\omega_3} = \dfrac{\omega_3}{2\omega_1}$

Minimum & Non Minimum Phase T.F. (4)







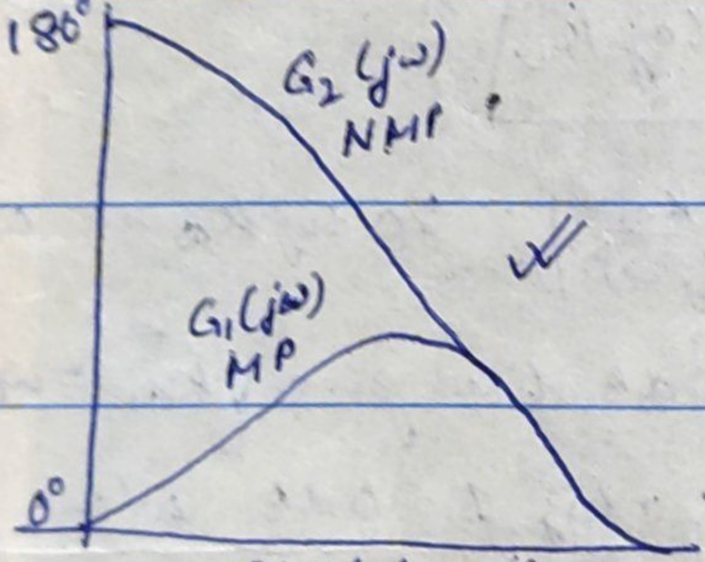
$$G_1(s) = \frac{s+z}{s+p}$$

$$G_2(s) = \frac{s-z}{s+p}.$$

$$\angle G_1(j\omega_1) = \theta_1 - \theta_2 = \angle \tan^{-1}\frac{\omega_1}{z} - \angle \tan^{-1}\frac{\omega_1}{p}$$

$\frac{\omega_1}{z} > \frac{\omega_1}{p}$

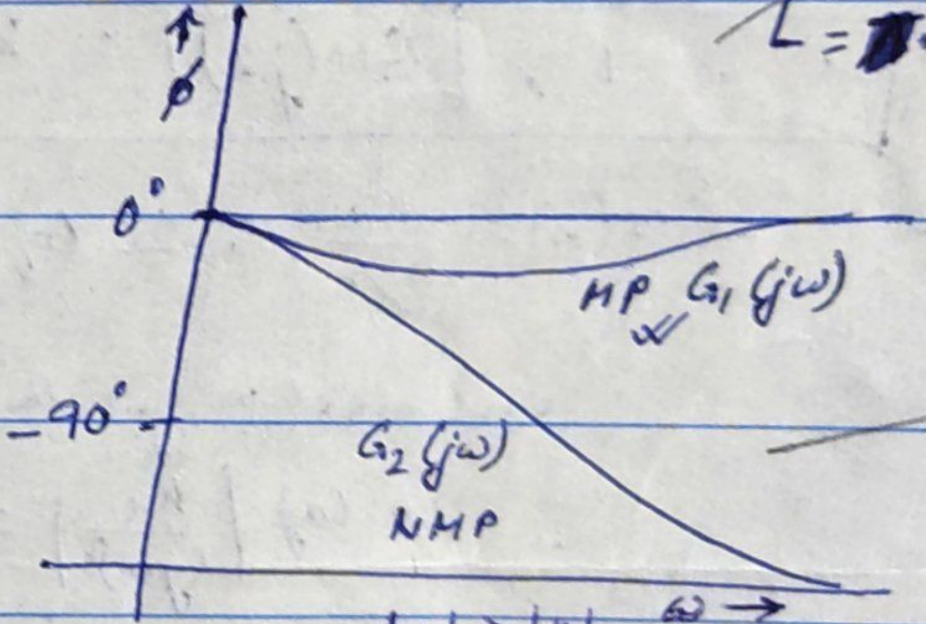
$$\angle G_2(j\omega_1) = \theta_1^{*} - \theta_2 = \left(\pi - \angle \tan^{-1}\frac{\omega_1}{z}\right) - \angle \tan^{-1}\frac{\omega_1}{p}$$



$|z| < |p|$

$1/|z| > 1/|p|$

phase



$|z| > |p|$

$\frac{1}{|z|} < \frac{1}{|p|}$

$\angle G_2(j\omega_1) = \theta_1^* - \theta_2$

For $|z| < |p|$ may be interpreted as $\left(\pi - \angle \tan^{-1}\frac{\omega_1}{z}\right) - \angle \tan^{-1}\frac{\omega_1}{p}$

$$= \pi - \tan^{-1}\frac{\frac{\omega_1}{z} + \frac{\omega_1}{p}}{1 - \frac{\omega_1^2}{zp}} \rightarrow \pi \text{ as } \omega_1 \rightarrow 0.$$

$$\rightarrow \pi - \frac{\pi}{2} - \frac{\pi}{2} = 0 \quad \text{as } \omega_1 \rightarrow \infty$$

For $|z| > |p|$ may be interpreted as $-\tan^{-1}\frac{\omega_1}{p} + \tan^{-1}\left(-\frac{\omega_1}{z}\right)$

$$= -\tan^{-1}\frac{\frac{\omega_1}{z} + \frac{\omega_1}{p}}{1 - \frac{\omega_1^2}{zp}} \rightarrow 0 \text{ as } \omega_1 \rightarrow 0$$

$$\rightarrow -\pi \text{ as } \omega_1 \rightarrow \infty.$$