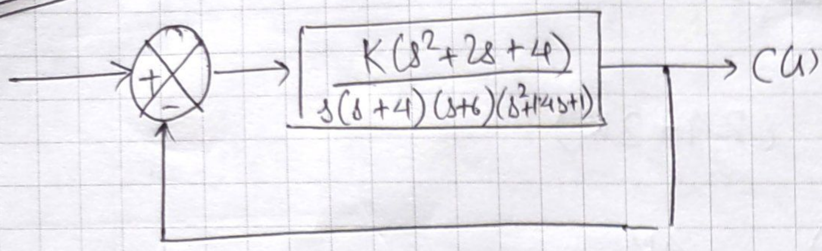
PRANTIKA RAY

2015.

6.13



$$G(s)\,H(s) = \frac{K(s^2+2s+4)}{s(s+4)(s+6)(s^2+1.4s+1)}$$

$$1 + G(s)\,H(s) = 0$$

$$K = -\frac{s(s+4)(s+6)(s^2+1.4s+1)}{(s^2+2s+4)}$$

characteristic equation =

$$s(s+4)(s+6)(s^2+1.4s+1) + K(s^2+2s+4) = 0$$

zeroes of the system $= -1 \pm j\sqrt{3}$

poles of the system $= 0, -4, -6, -0.7 \pm j\,0.7141$

centroid $= \dfrac{\Sigma p - \Sigma z}{n_p - n_z} = -2.8$

asymptotes $= 60°, 180°, 300°$

char. eqn. $= s^5 + 11.4s^4 + 39s^3 + (43.6+K)s^2 + (24+2K)s + 4K = 0$

Routh Hurwitz table

S^5	1	39	$(24+2K)$
S^4	11.4	$(43.6+K)$	$4K$
S^3	$\frac{401-K}{\cancel{11.4}}$	$\frac{273.6+18.8K}{\cancel{11.4}}$	
S^2	$\frac{-K^2+143.08K+14364.56}{401-K}$	$4K$	
S^1	$273.6+18.8K-\frac{4K(401-K)^2}{-K^2+143.08K+14364.56}$		
S^0	$4K$		

Conditions for K

① $4K > 0$

② $-22.8K^3 - 5624.304K^2 - 334003.584K + 3930143.616 > 0$

K for points where root loci cross imaginary axis $K = 163.5567, \ 15.61062, \ 67.5126$

stable regions $\rightarrow 0 < K < 15.61062 \ ; \ 67.5126 < K < 163.5567$

Thus unstable regions $\rightarrow 15.6 < K < 67.5$ ~~$67.5 < K < 163.6$~~ $\infty > K > 163.6$

unstable region $\rightarrow 15.6 < K < 67.5$

$163.6 < K < \infty$

$$\frac{dK}{ds} = \frac{d}{ds}\left(\frac{s^5 + 11.4s^4 + 39s^3 + 43.6s^2 + 24s}{(s^2+2s+4)}\right)$$

$$= \frac{(5s^4 + 45.6s^3 + 117s^2 + 87.2s + 24)(s^2+2s+4) - (2s+2)(s^5+11.4s^4+39s^3+43.6s^2+24s)}{(s^2+2s+4)^2}$$

$$\frac{dK}{ds} = 0$$

on solving

$$3s^6 + 30.8s^5 + 127.4s^4 + 337.6s^3 + 531.2s^2 + 348.8s + 96 = 0$$

We would employ bisection here

at $s=-2$ $f(s) = 67.2$

at $s=-3$ $f(s) = -262.8$

at $s=-2.5$ $f(s) = -29.828$

at $s=-2.4$ $f(s) = -0.743424$

at $s=-2.3$ $f(s) = 23.117$

bisection between $s=-2.3$ and $s=-2.4$

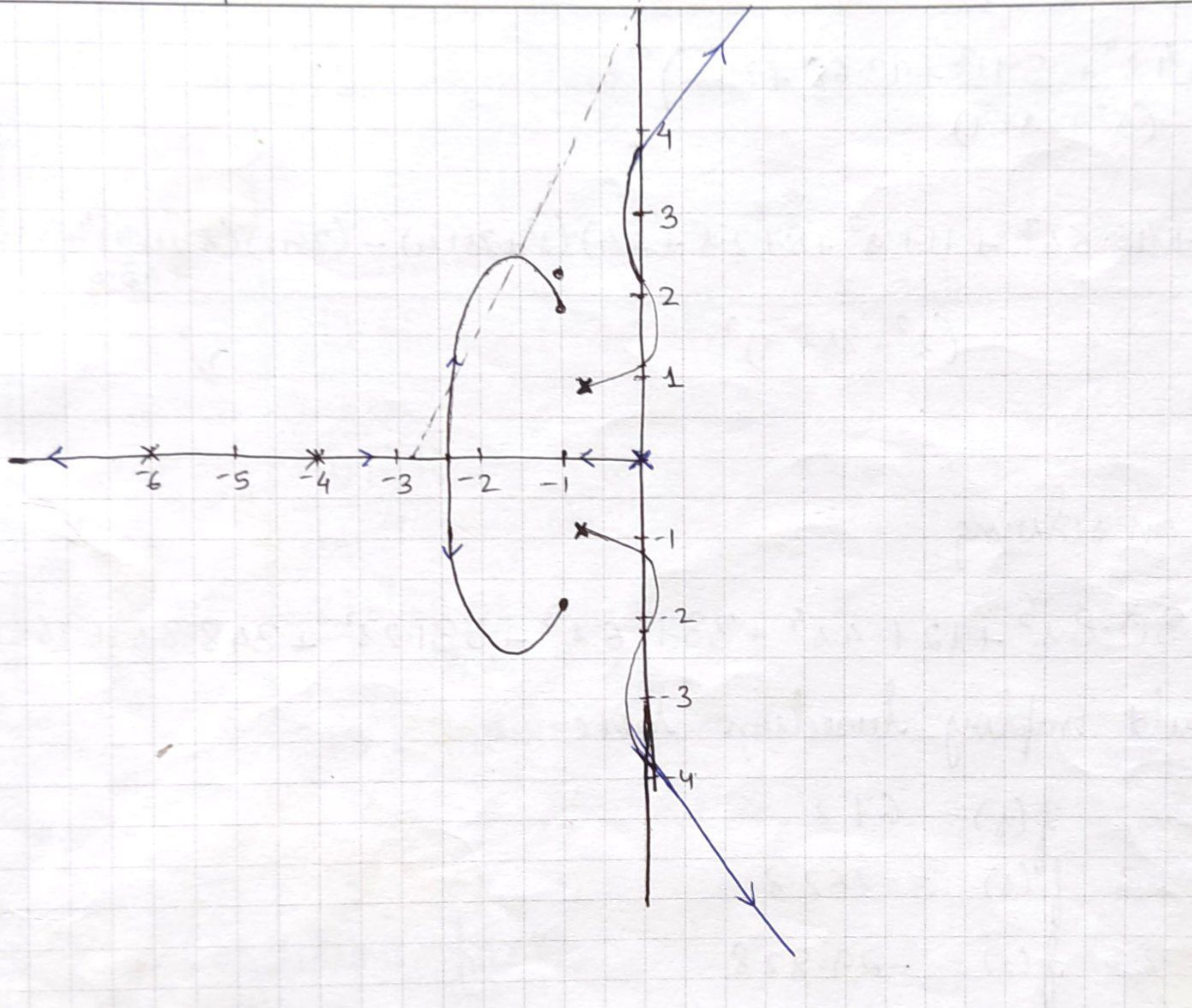
$f(-2.35) = 14.026$

$f(-2.375) = 7.95378$

$f(-2.3875) = 2.517548$

$f(-2.39375) = 0.897$

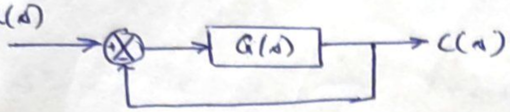
at $(-2.39715) \approx 0$



-6
-5
-4
-3
-2
-1
4
3
2
1
-1
-2
-3
-4

6.13 Root Loci with Compensator (14)

(Pole placement)

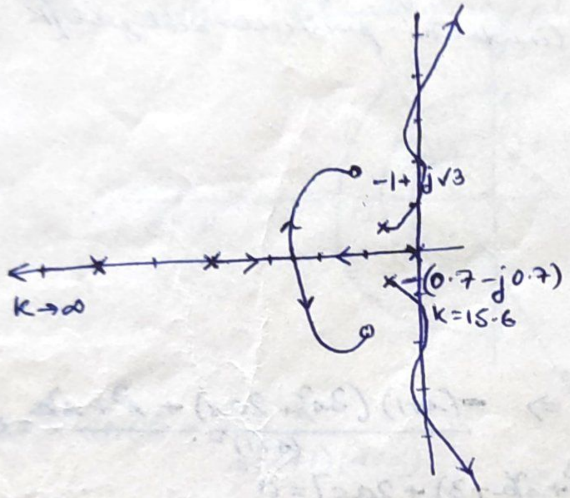


$$-0.7 \pm j0.7$$

$$1+GH(s) = 1 + \frac{K(s^2+2s+4)}{s(s+4)(s+6)(s^2+1.4s+1)} = \frac{s(s+4)(s+6)(s^2+1.4s+1)+K(s^2+2s+4)}{D(s)} = 0.$$

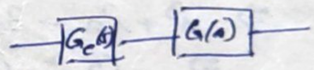
Check: (i) System stable for $0 < K < 15.6$; $67.5 < K < 163.6$

(ii) Unstable for $15.6 < K < 67.5$; $163.6 < K < \infty$.



See worked out sheet

Use compensating network $G_c(s)$ in fwd path with $G(s)$



Say $G_c(s) = \frac{s+3}{s+5}$ ∴ system stable ∀ $0 < K < 373$.

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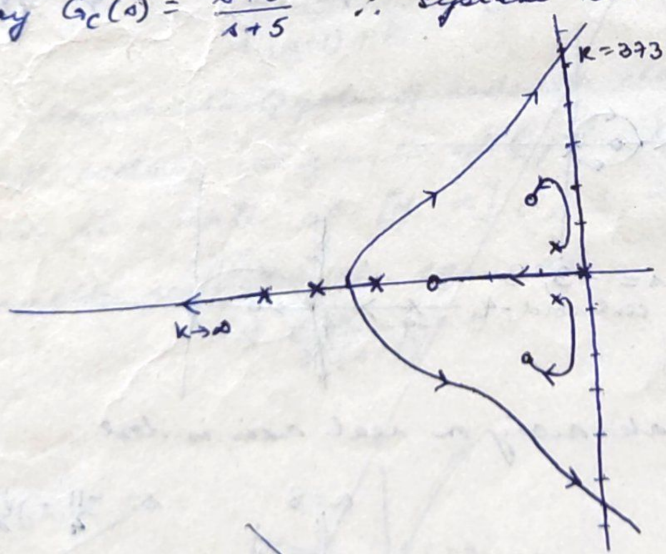
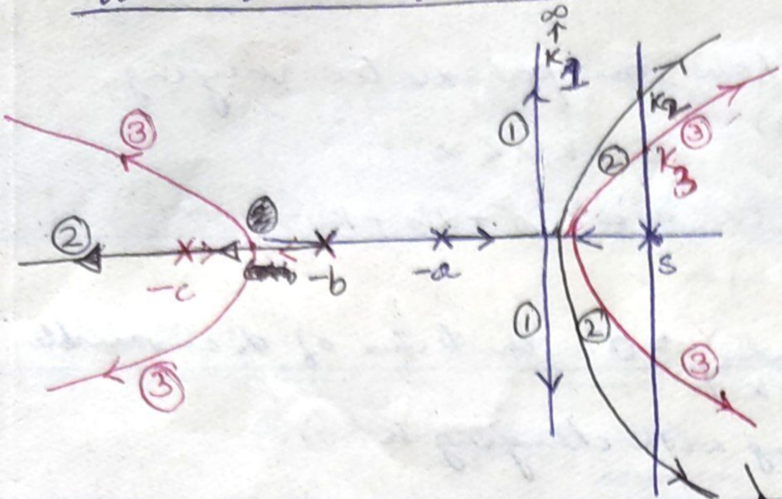


(14a) Root Locus Analysis

Effect of Adding poles.



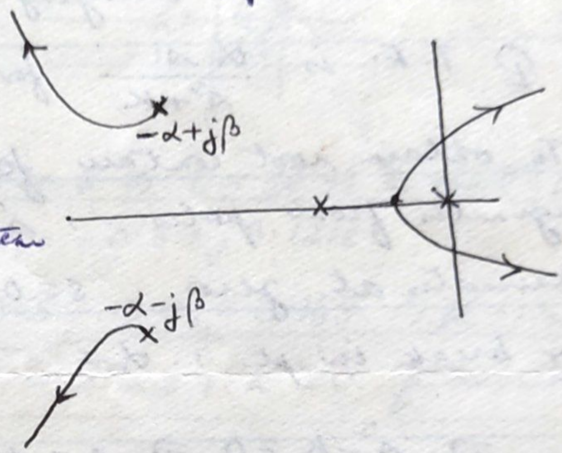
① $GH_1 = \dfrac{K}{s(s+a)}$ $a > 0$

② $GH_2 = \dfrac{K}{s(s+a)(s+b)}$ $|b| > |a|$

③ $GH_3 = \dfrac{K}{s(s+a)(s+b)(s+c)}$ $|c| > |b| > |a|$

$\therefore K_1 = \infty > K_2 > K_3$

$GH_4 = \dfrac{K}{s(s+a)(s+\alpha \pm j\beta)}$



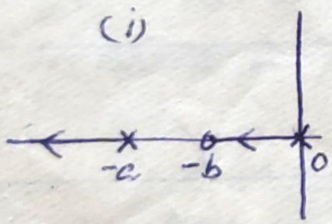
Addn. of pole ≡ inc. ORDER of system
— decreases stability

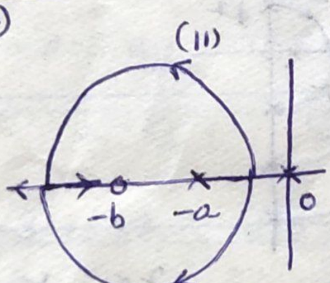
Effect of Adding Zeros :

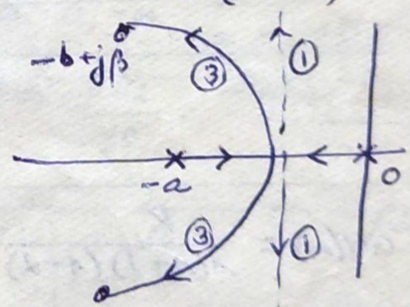
$GH_1 = \dfrac{K}{s(s+a)}$ $a > 0$

$GH_2 = \dfrac{K(s+b)}{s(s+a)}$

$GH_3 = \dfrac{K(s+b \pm j\beta)}{s(s+a)}$







Addn. of zero : stabilizes system

Q. — effect on transient ? (loss of causality)?
complex conjugates $\therefore$ UD - speed of response — ω_d?

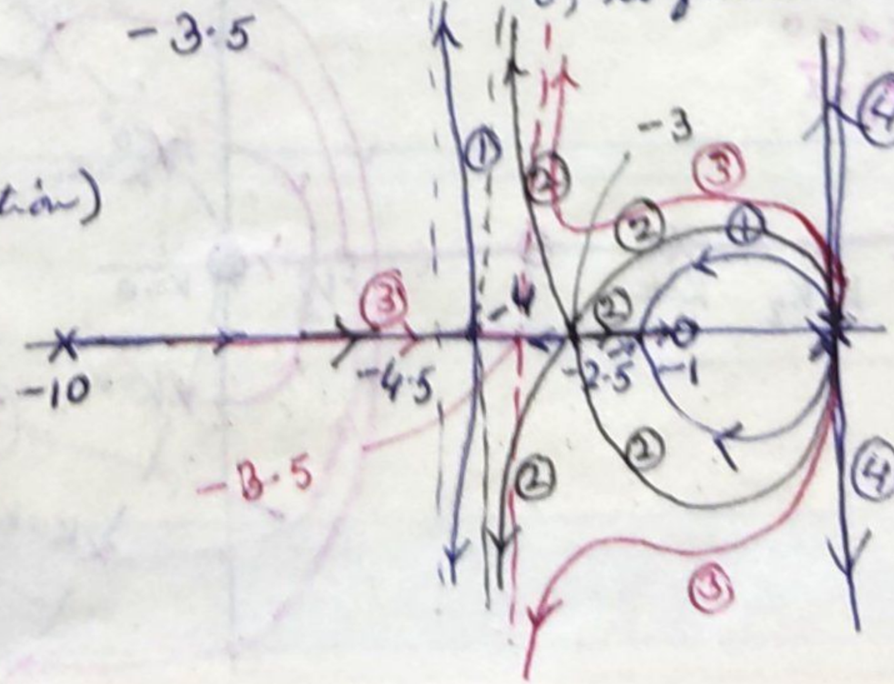
Effect of varying pole position:

$$GH = \frac{K(s+1)}{s^2(s+a)} \quad \therefore\ K = -\frac{s^2(s+a)}{s+1} \text{ for root loci.}$$

$$\therefore \frac{dK}{ds} = \frac{-(s+1)(3s^2+2as)+s^2(s+a)}{(s+1)^2} = 0 \Rightarrow s(2s^2+(a+3)s+2a)=0$$

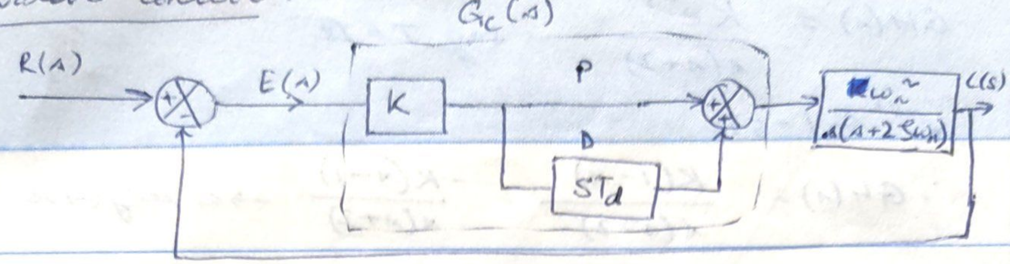
$$\Rightarrow s=0 \text{ or } \frac{-(a+3)}{4} \pm \frac{\sqrt{a^2-10a+9}}{4}$$ are possible break-in pts. or breakaway

	a	∠asymptotes	intersection of asymptotes σ_A	Break-in/away pts.
1	10	± 90°	−4.5	0, −2.5, −4
2	9	± 90°	−4	0, −3
3	8	± 90°	−3.5	0, no finite non-zero pt.
** 4	1	(Pole zero cancellation) NOT reflected in root locus.		



To 14
then To 15

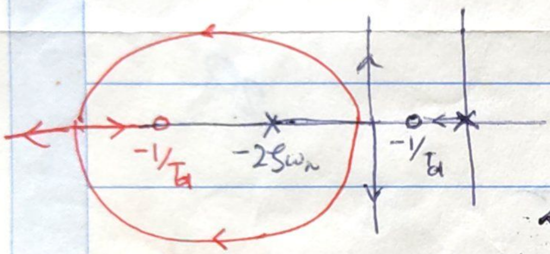
Derivative Control:



$\therefore$ Q w/o derivative control: $\dfrac{K\omega_n^2}{s(s+2\zeta\omega_n)}$

with derivative $\dfrac{K\omega_n^2(1+sT_d)}{s(s+2\zeta\omega_n)}$

$\rightarrow$ addn. of zero.



$|1/T_d| > |2\zeta\omega_n| \rightarrow$ then U.D. response

Else O.D. totally

If U.D. ($\zeta<1$), then overshoot but speed of response more.

$**$**NOTE**: Derivative control of error voltage (low voltage signal) in f/w path may req. power amplifier
$\Rightarrow$ K high $\Rightarrow$ system O.D. $\therefore$ slow.

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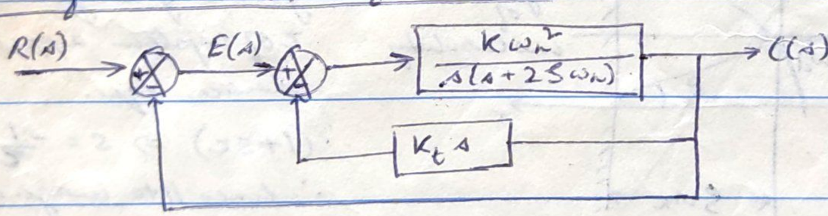
Rate feedback (Tacho f/b) Control:



$$\frac{C(s)}{R(s)} = \frac{K\omega_n^2}{s^2+(2\zeta\omega_n+KK_t\omega_n^2)s+K\omega_n^2}$$ $\therefore$ Charac. eqn. same.

just replace T_d by K_t $\therefore$ Same change of damping ratio

*But rise time in derivative control faster due to added zero although same damping ratio. $\left[t_r = \frac{\cancel{\pi}}{\cancel{\omega_d}} \tan^{-1}\left(\frac{\omega_d}{-\sigma}\right)\right]$

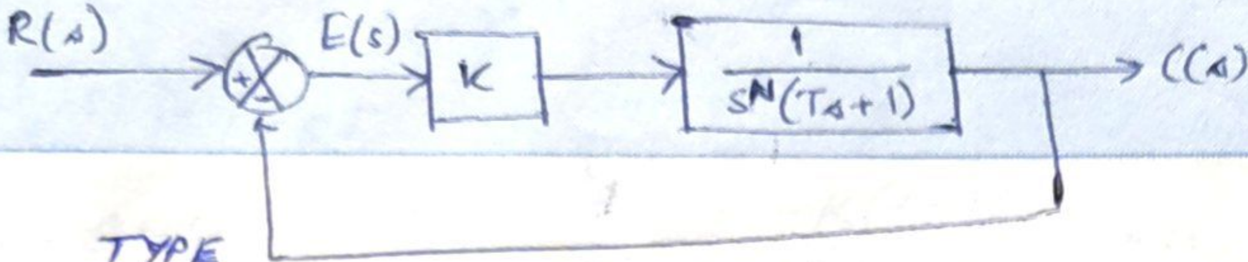
wt. changes: $\dfrac{C(s)}{R(s)} = \dfrac{K(1+T_d s)\omega_n^2}{s^2+(2\zeta\omega_n+KT_d\omega_n^2)s+K\omega_n^2}$ | DERIVATIVE CONTROL

But Rate f/b on o/p $\therefore$ higher voltage level
Derivative control on f/w path $\therefore$ error signal low
$\therefore$ use high pass RC (else derivative of noise too!)

ex. o/p of potentiometer is discontinuous
$\therefore$ the jumps differentiated more.
+ power amplifier $\therefore K\uparrow \therefore$ O.D.

Integral controller

To reduce offset in response to step i/ps.

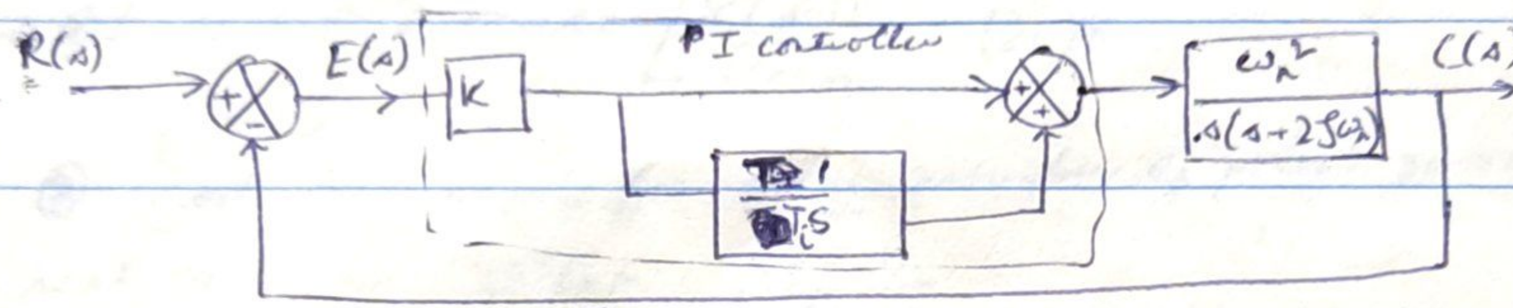


TYPE

For $N=0$; $K_p = \lim_{s\to 0} GH(s) = K$ $\therefore e_{ss} = \frac{1}{1+K}$ = finite.

$N \neq 0$; $K_p = \infty$ $\therefore e_{ss} = 0$

∴



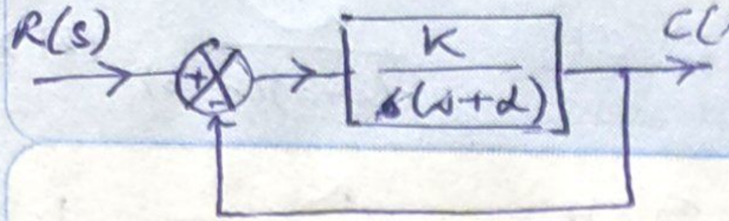
$$G(s) = \frac{K\omega_n^2\left(1+\frac{1}{T_i s}\right)}{s(s+2\zeta\omega_n)} = \frac{K\omega_n^2(T_i s+1)}{T_i s^2(s+2\zeta\omega_n)} \rightarrow \frac{s+1/T_i}{s}$$

∴ Eq. to adding zero at $-1/T_i$ and a pole at 0.

∴ inc. system order by one ∴ ss error improved

∴ For PID controller : $G_c(s) = K\left(1 + T_d s + \frac{1}{T_i s}\right)$.

Root Contour :: More than one parameter varying.



$0 < K, \alpha < \infty$

ch. eqn. $s^2 + \alpha s + K = 0$.

Rewrite as $1 + \alpha\left(\frac{s}{s^2+K}\right) = 0$. in terms of α as variable.

with the root loci changing with changing K.

$\therefore$ Q T.F. is $\frac{\alpha s}{s^2+K}$ for any particular K.

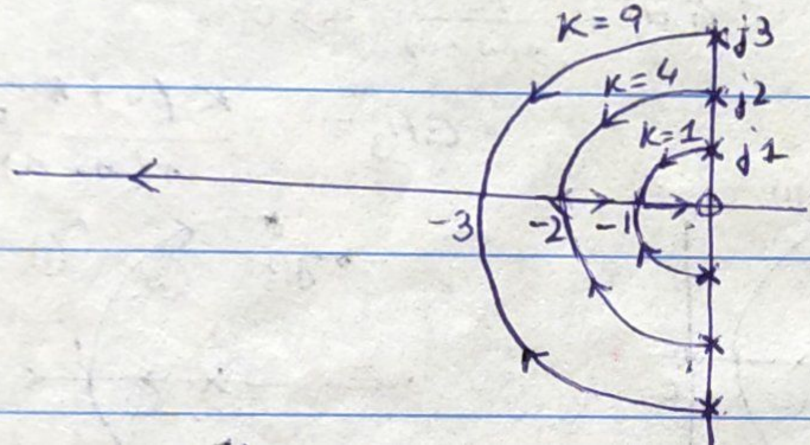
$\therefore$ To obtain root contour for various K $\longrightarrow$

originates from poles $\therefore s^2 + K = 0 \Rightarrow s = \pm j\sqrt{K}$

terminates at zeros $s = 0, -\infty$.

For break in pt. : $\alpha = -\frac{(s^2+K)}{s}$ $\therefore \frac{d\alpha}{ds} = \frac{-2s^2 + s^2 + K}{s^2} = 0$

$\Rightarrow s^2 - K = 0 \Rightarrow s = \pm\sqrt{K}$ possible $\therefore$ valid $s = -\sqrt{K}$.



Ex 6.15 $\quad GH(s) = \dfrac{K}{s(s+1)(s+\alpha)}$ $\quad \therefore$ ch. equ. $s^2(s+1) + \alpha s(s+1) + K = 0$

$\therefore \quad 1 + \dfrac{\alpha\, s(s+1)}{s^3+s^2+K} = 0 \quad \therefore$ Q T.F. $\dfrac{s(s+1)}{s^3+s^2+K}$ II

Root contours originate ($\alpha = 0$) at Q poles of reduced ch. equ.

$s^3+s^2+K=0$. Rewrite as $1 + \dfrac{K}{s^2(s+1)} = 0$

I $\quad \therefore$ Root locus of $\dfrac{K}{s^2(s+1)}$ provide start of root contour

$\sigma_A = -1/3$, $N=3$, $\angle s = \pi, \pm\pi/3$

Root contours terminate at $s = 0, -1, -\alpha$.

$j\omega$ axis crossings : R-H array ; $s^3 + s^2(\alpha+1) + \alpha s + K = 0$

$$\begin{array}{c|cc} s^3 & 1 & \alpha \\ s^2 & \alpha+1 & K \\ s^1 & \dfrac{\alpha^2+\alpha-K}{\alpha+1} & \\ s^0 & K & \end{array}$$

$\therefore \quad K > 0$

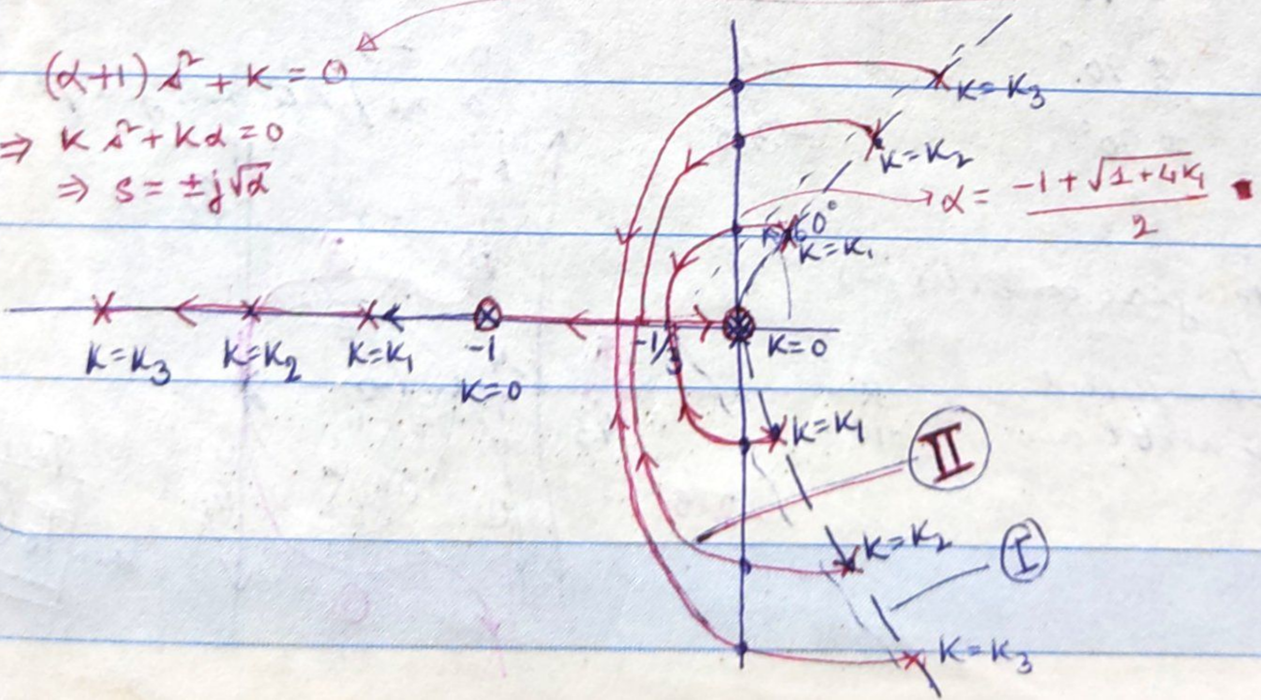
$\alpha(\alpha+1) - K = 0$ are cross over pts. $\Rightarrow (\alpha+1) = \dfrac{K}{\alpha}$

$\Rightarrow \alpha = \dfrac{-1 \pm \sqrt{1+4K}}{2} \quad (\because \alpha > 0)$.

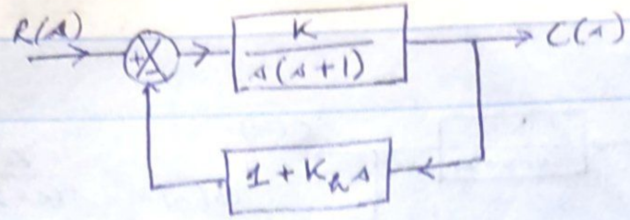
$(\alpha+1)s^2 + K = 0$

$\Rightarrow K s^2 + K\alpha = 0$

$\Rightarrow s = \pm j\sqrt{\alpha}$



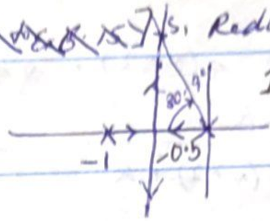
Ex 6.16



(i) Let $K_h = 0$ So $GH(s) = \dfrac{K(1+K_h s)}{s(s+1)}$ ⇒ ch. eqn. $s(s+1) + KK_h s + K = 0$.

becomes $s^2 + s + K = 0$ [~~...~~] ∴ Reduced ch. eqn. $1 + KK_h \cdot \dfrac{s}{s^2+s+K} = 0$

∴ $G_1(s) = \dfrac{K}{s(s+1)}$ with root locus plot



(ii) value of K for $\zeta = 0.158 = \cos\theta$ ⇒ $\theta = 80.9°$ ⇒ $\tan\theta = 6.243 = \frac{p}{b}$

∴ $b = -0.5$ (break away decided) ⇒ $p = 3.1216$ ⇒ $s_1 = -0.5 \pm j\,3.1216$

⇒ $\zeta\omega_n = 0.5$ ⇒ $\omega_n = 3.165$ ⇒ $K = \omega_n^2 = 10$

$\boxed{\because\ s^2 + (2\zeta\omega_n)s + \omega_n^2 = s^2 + s + K = 0 \text{ from reduced ch. eqn}}$

(iii) For $K = 10$, draw root loci with K_h as variable.

∴ $s^2 + 10K_h s + s + 10 = 0$ ⇒ $1 + K_h \cdot \dfrac{10s}{s^2+s+10} = 0$

∴ Poles at $-0.5 \pm j\,3.1216$, zeros at $0, -\infty$

For Break in pt.; $K_h = -\dfrac{(s^2+s+10)}{10s}$ ⇒ $\dfrac{dK_h}{ds} = \dfrac{-10s(2s+1) + 10(s^2+s+10)}{100s^2} = 0$

$= \dfrac{-Ks(2s+1) + K(s^2+s+K)}{(Ks)^2}$

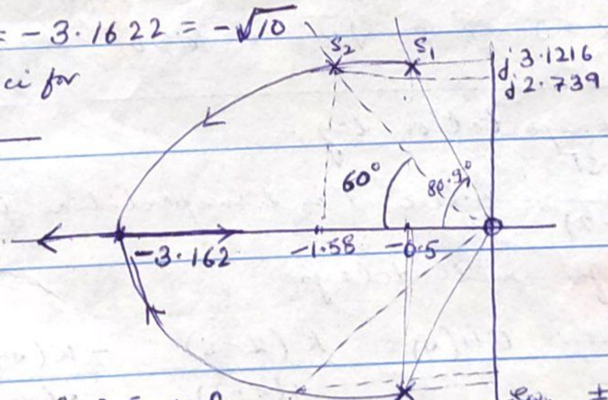
⇒ $s = -3.1622 = -\sqrt{10}$

$= \dfrac{-10(s^2 - 10)}{100s^2} = 0$

⇒ $-K(s^2 - K) = 0$

⇒ $s = \pm\sqrt{K}$ ⇒ $-\sqrt{K}$ possible

Root loci for K=10



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(iv) K_h so that $\zeta = 0.5 = \cos\theta$ $= -\zeta\omega_n \pm j\omega_n\sqrt{1-\zeta^2}$

⇒ $\theta = 60°$ also $\omega_n = \sqrt{10} = 3.1622$ ∴ $s_2 = -1.581 \pm j\,2.739$

∴ $2\zeta\omega_n = \cancel{2(1.581)} = 3.1622 = 1 + 10K_h$ ⇒ $K_h = 0.216$

$= 2(0.5)\omega_n$

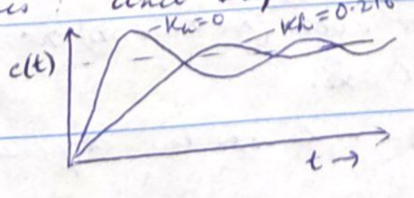
ch. eqn. $= s^2 + (1 + KK_h)s + K = 0$

∴ $K = 10$

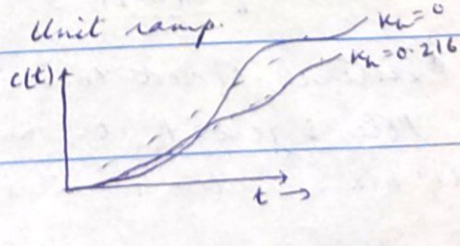
and $2\zeta\omega_n = 2(0.5)\sqrt{10} = 1 + 10K_h$

∴ $K_h = 0.216$.

Responses: Unit step

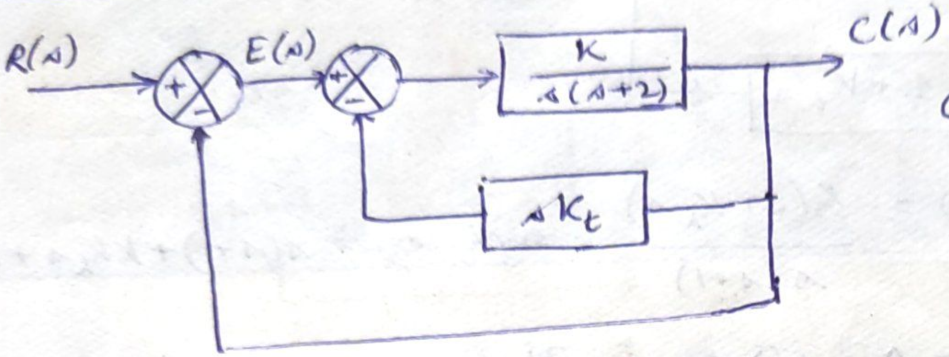


Unit ramp.



Note: $\zeta\omega_n \triangleq \frac{1}{\tau}$ for 1st order
for 2nd order

Multiple loop system:



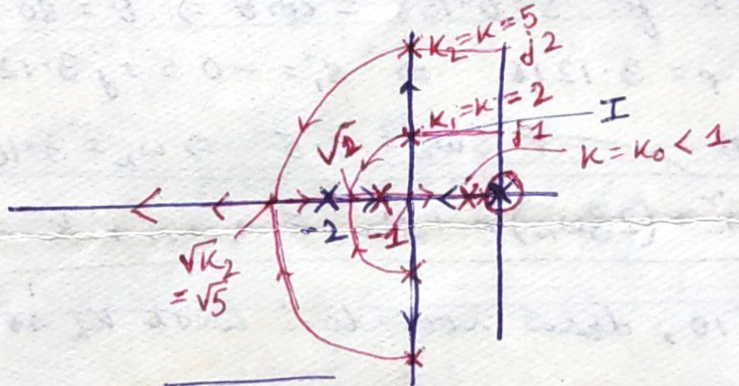
$$G(s) = \frac{\frac{K}{s(s+2)}}{1 + \frac{sKK_t}{s(s+2)}} = \frac{K}{s(s+2) + sKK_t}$$

$\therefore$ ch. eqn. : $s(s+2) + sKK_t + K = 0$.

$$\therefore\ 1 + \frac{sK \cdot K_t}{s(s+2)+K} = 1 + \frac{\alpha s}{s^2+2s+K} = 0 \quad \text{where } KK_t = \alpha. \qquad \text{II}$$

Reduced ch. eqn. : $s(s+2) + K = 0 \Rightarrow 1 + \dfrac{K}{s(s+2)} = 0$ $\quad$ I

$\therefore$



Root contour

Poles : $s = \dfrac{-2 \pm \sqrt{4-4K}}{2} = -1 \pm \sqrt{1-K}$

Zeros : $s = 0, -\infty$.

For Break in pt. : $\alpha = -\dfrac{(s^2+2s+K)}{s}$ $\therefore \dfrac{d\alpha}{ds} = \dfrac{-s(2s+2)+s^2+2s+K}{s^2} = 0$

$\Rightarrow s^2 - K = 0 \Rightarrow s = \sqrt{K} \quad \because K > 0$.

System with Transportation lag:

$$GH(s) = \frac{Ke^{-sT}}{s(s+2)}$$ where T is transportation lag/delay

$e^{-sT} \approx 1 - sT$ for small delays.

Let $T = 1s$, then $GH(s) = \frac{K(1-s)}{s(s+2)} = \frac{-K(s-1)}{s(s+2)}$

$\therefore \sim$ to positive fb: $1 - G_1H_1(s) = 0$. $\Rightarrow G_1H_1(s) = \frac{K(s-1)}{s(s+2)}$

(i) $\angle$ condn. $\angle \frac{K(s-1)}{s(s+2)} = 2k\pi$

So

(ii) Existence of root loci: for even (NOT ODD) no. of Poles & zeros to rt. on real axis.

All other rules same; magnitude condn. dependent.

R-H

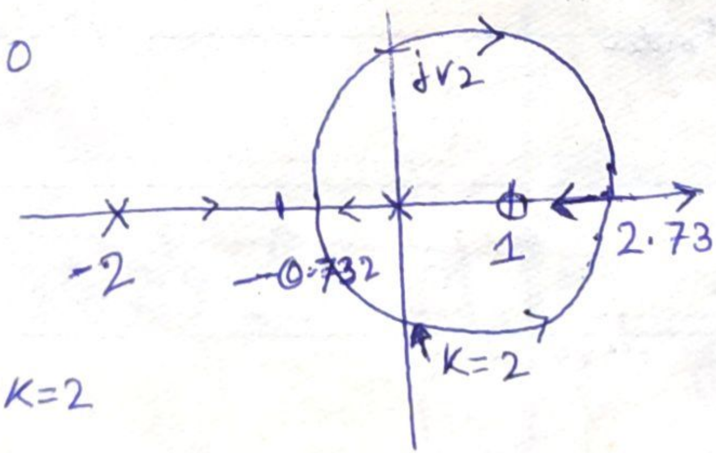
$s^2 + (2-K)s + K = 0$

s^2	1	K
s^1	2-K	
s^0	K	

~~K=0~~ $2-K=0 \Rightarrow K=2$

$\Rightarrow s^2 + K = 0$

$\Rightarrow s = \pm j\sqrt{K} = \pm j\sqrt{2}$.



Breakaway/in

$K = -\frac{s^2+2s}{1-s}$; $\frac{dK}{ds} = 0$

$\Rightarrow +(s-1)(2s+2) - (s^2+2s)$

$= +2(s^2-1) - s^2 - 2s$

$= +s^2 - 2s - 2 = 0$

$\Rightarrow s = \frac{+2 \pm \sqrt{4+8}}{+2} = 1 \pm \sqrt{3}$

$= -0.732, 2.732$

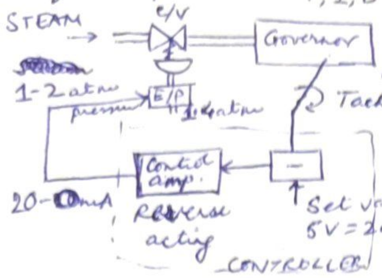
Process Control System

(1770)
James Watt : steam engine governor — speed ↑, flyweights out due to centrifugal force ∴ steam C/V closes ∴ continuous control (any posn. - Full open to full close)

CONTROL — Electrical / Pneumatic

- ON-OFF: fast response
- CONTINUOUS: slow, P, I, D

200 rpm → 5V
10 mA → 0.6 kg/cm² Pg → 50% open C/V



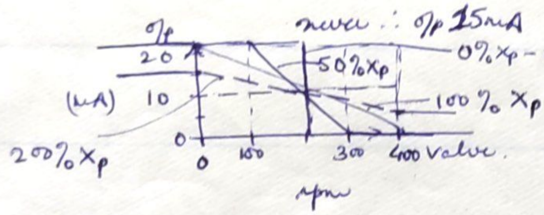
DIRECT VS REVERSE ACTING C/V & CONTROLLERS.

Reverse acting C/V : 400 rpm : Full close
0 rpm : Full open

Reverse acting controller : i/p ↑, o/p ↓

Proportional Band : X_p : indicative of range of operation $= \frac{100}{K}\%$.
∴ inverse to gain.

Say operating range	Full close	Full open	
	400 rpm	0 rpm	— 100% X_p
	300 rpm	100 rpm	— smaller X_p, steeper K. 50% X_p
	never ∴ o/p 15mA	never ∴ o/p 5mA	— larger X_p, small K. — 200% X_p.



I action - to reduce P offset
D action - to improve response time (faster).

Tuning rules for PID controllers:

$$u(t) = K_p e + K_i \int e\,dt + K_d \frac{de}{dt}$$

P, I & D gains : K_p, K_i, K_d ∴ $K_p = \frac{1}{X_p}$; $X_p = PB$.

$$G_c(s) = \frac{Y(s)}{E(s)} = K_p\left(1 + \frac{1}{T_i s} + T_d s\right)$$

$$\therefore y = \frac{1}{X_p} e + \frac{1}{X_p T_i}\int e\,dt + \frac{T_d}{X_p}\frac{de}{dt}$$

Note : $T_i = \infty$ → no I action : large time constant.
$T_d = 0$ → no D action

Z-N I	K_p	T_i	T_d
P	$0.5K_c$	∞	0
PI	$0.45K_c$	$0.83T$	0
PID	$0.6K_c$	$0.5T$	$0.125T$

K_c : critical gain - sys oscillates
T : period of oscillation

$$\hookrightarrow G_c(s) = 0.6K_c\left(1 + \frac{1}{0.5Ts} + 0.125Ts\right) = 0.075K_cT\,\frac{(s + 4/T)^2}{s}$$

Z-N II	K_p	T_i	T_d
P	T/L	∞	0
PI	$0.9(T/L)$	$L/0.3$	0
PID	$1.2(T/L)$	$2L$	$0.5L$