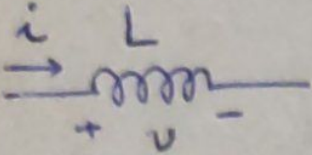
Inductance and Capacitance :

Active element : Capable of furnishing an average power greater than zero to some external device, where the average is taken over an infinite time interval.

Passive element : One that cannot supply an average power that is greater than zero over an infinite time interval.

Practical circuits : Both forcing fns. & responses usually T Varying.

Inductor : Capable of storing and delivering finite amounts of energy — both L&C but not unlimited amount of energy or finite avg. power over infinite time interval



Early 1800s , Oersted — current carrying conductor affected compass needles (produced magnetic field)

Ampere — Mag. field $\propto$ i producing it

Faraday & Henry — changing magnetic field induced voltage in neighbouring circuit.

Math. model :

$$\boxed{v = L \frac{di}{dt}}$$

i produced the mag. field.

Henry (H) — Vs/A

Henry (H) — Vs/A

Physical inductor: winding length of ~~coil~~ wire into coil →

→ effectively increases i causing the mag. field + inc. "no." of neighbouring ckts. in which Faraday's voltage may be induced.

$\boxed{L = \mu N^2 A / l}$ for long helix of v. small pitch.

A: cross sectional area of the HELIX l: axial length of helix

N: no. of complete turns of wire

μ: permeability (constant) μ_0 (vacuum ≈ air) $= 4\pi \times 10^{-7}$ H/m.

① For i = constant $v = 0$ ∴ short ckt. to dc.

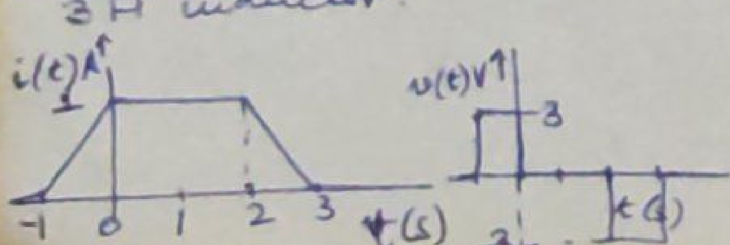
② $i = i_1$ to $i = i_2$ abruptly → $v = \infty$ across L also $p = vi = \infty$.

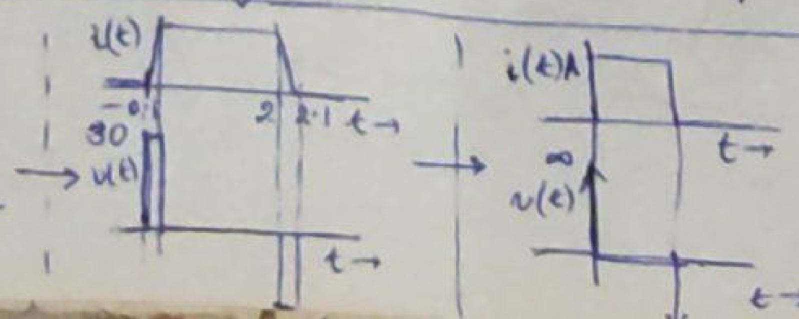
So to avoid ∞ v or p, i is not allowed to jump instantaneously from one value to another.

eg → open ckt. a physical inductor through which i flowing causes "arcing" across switch. — stored energy → ionizes air in path of arc.

used in spark plugs ← of automobile ignition where i through spark coil is interrupted by distributor.

3 H inductor.





$$\boxed{i(t) = \frac{1}{L}\int_{t_0}^{t} v\,dt + i(t_0)} = \frac{1}{L}\int_{-\infty}^{t} v\,dt.$$

So say if $L = 2H$, $v(t) = 6\cos 5t$, then $i(t) = 0.6(\sin 5t - \sin 5t_0) + i(t_0)$.

Say if i is 1A at $t = -\pi/2$ s $\Rightarrow t_0 = -\pi/2$, $i(t_0) = 1$.

$\therefore i(t) = 0.6 \sin 5t + 1.6$.

$p = vi = Li\frac{di}{dt}$ W $\rightarrow \begin{cases} \frac{di}{dt} \text{ +ve, } p \text{ +ve.. absorbed} \\ \frac{di}{dt} \text{ -ve, } p \text{ -ve} \rightarrow \text{released} \end{cases}$ $\therefore$ no need for $v = -L\frac{di}{dt}$

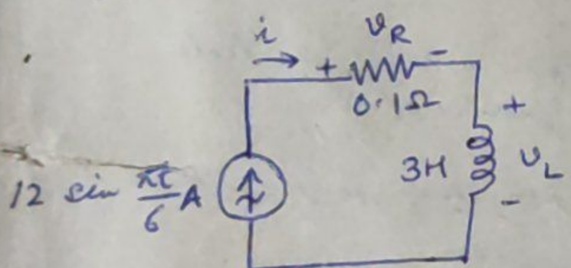
energy $w_L = \int_{t_0}^{t} p\,dt = \int_{t_0}^{t} Li\frac{di}{dt}\,dt \doteq \int_{t_0}^{t} Li\,di = L\frac{i^2}{2}\Big|_{t_0}^{t}$ J.

$$\boxed{w_L = \frac{1}{2}Li^2}$$ Assuming energy $= 0$ at time $t = t_0$. $\Rightarrow i = 0$

$\therefore$ Whenever there is finite i through inductor, energy is stored.

$\therefore$ Power is delivered to & recovered from <u>ideal</u> inductor — all energy may be recovered.

Practical L : associated R $\therefore$ lossless storage / recovery not possible



$v_R = iR = 1.2 \sin\frac{\pi t}{6}$ V.

$v_L = L\frac{di}{dt} = 3 \cdot \frac{d}{dt}\left(12 \sin\frac{\pi t}{6}\right) = 6\pi\cos\frac{\pi}{6}t$ V

$w_L = \frac{1}{2}Li^2 = 216 \sin^2 \pi/6\, t$. J

$\sin^2\theta = \frac{1-\cos 2\theta}{2}$

$\therefore$ period: 2θ.

$\cancel{2\theta}\ 2(\pi/6\, t) = 2\pi$ $\Rightarrow t = 6$s

$\therefore t = 0$, $w_L = 0$; $t = 3$, $w_L = 216$. and so on.

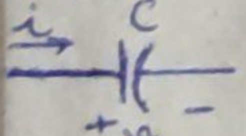
power dissipated in resistor $= p_R = i^2R = 14.4 \sin^2 \pi/6\, t$. W

$\therefore w_R = 14.4 \int_0^6 \sin^2 \pi/6\, t\,dt$ in 6-s

$= \frac{14.4}{2}\int_0^6 (1 - \cos \pi/3\, t)\,dt = 43.2$ J. $= 20\%$ of maxm. stored energy.

For $L = 100\mu H$ typically 2-3% heating losses.

Capacitor:



$$\boxed{i = C\frac{dv}{dt}}$$

farad (F) : C/V or AS/V

Physically: two conducting surfaces on which charge may be stored, separated by thin insulating layer having very large resistance.

Assuming $R=\infty$, no path for equal & opp. charges placed on cap. plates to combine within capacitor

Say i source connected to C so positive i into one plate of capacitor and out through other. — equal currents. In interior of capacitor, +ve charges accumulate on +ve plate as $i = \frac{dq}{dt}$

Maxwell hypothesized a "displacement i" present whenever i or v varies with time s.t. conduction current at leads $= i_{disp} = i = C\frac{dv}{dt}$.

Physical model : $\boxed{C = \epsilon A / d}$ ϵ : permittivity $\epsilon_0 = 8.854\ pF/m$

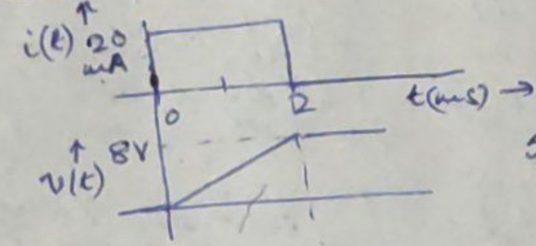
A : area of c.s. d : plate sepn. distance

① open circuit to dc since constant voltage $\Rightarrow$ requires zero current.

② sudden jump in voltage req. infinite i

$$\boxed{v(t) = \frac{1}{C}\int_{t_0}^{t} i\,dt + v(t_0) = \frac{1}{C}\int_{-\infty}^{t} i\,dt}$$

$\therefore$ $\boxed{q = Cv}$

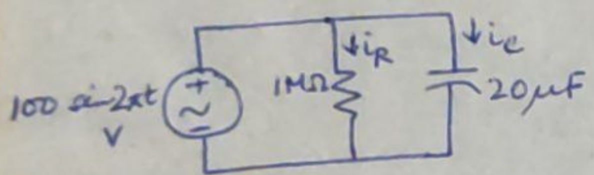


5μF capacitor

$$\therefore v(t) = \begin{cases} 0 & t \leq 0 \\ 4000t & 0 \leq t \leq 2\,ms \\ 8 & t \geq 2\,ms. \end{cases}$$

$$p = vi = Cv\frac{dv}{dt}$$

$$w_C = \frac{1}{2}Cv^2$$



// resistor : resis. of insulator/dielectric bet. plates of cap.

$$i_R = \frac{v}{R} = 10^{-4}\sin 2\pi t \ \ A$$

$$i_C = C\frac{dv}{dt} = 4\pi \times 10^{-3}\cos 2\pi t \ \ A$$

$$w_C = \frac{1}{2}Cv^2 = 0.1\sin^2 2\pi t \ \ J$$

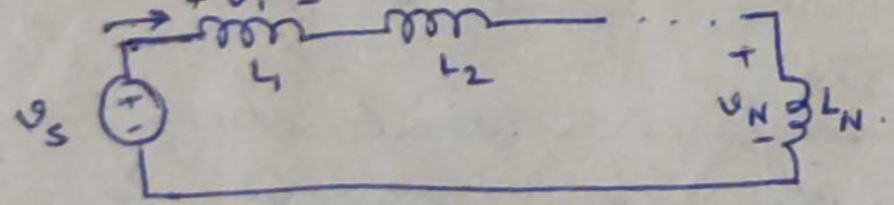
$t = 0$ $w_C = 0$

$t = 1/4\,s$ $w_C = 0.1\,J$

$$w_R = \int_0^{0.5} p_R\,dt = 2.5\,mJ.$$

$\rightarrow$ 2.5% of max an. stored energy of C. is lost in the process of storing & recovering the energy.

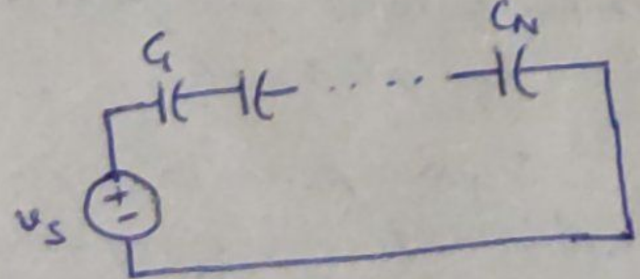
Inductance and capacitance combinations:



$$v_S = v_1 + \dots + v_N$$

$$= L_1 \frac{di_1}{dt} + \dots + L_N \frac{di}{dt}$$

$$= \sum_n L_i \frac{di}{dt} = L_{eq} \frac{di}{dt}$$

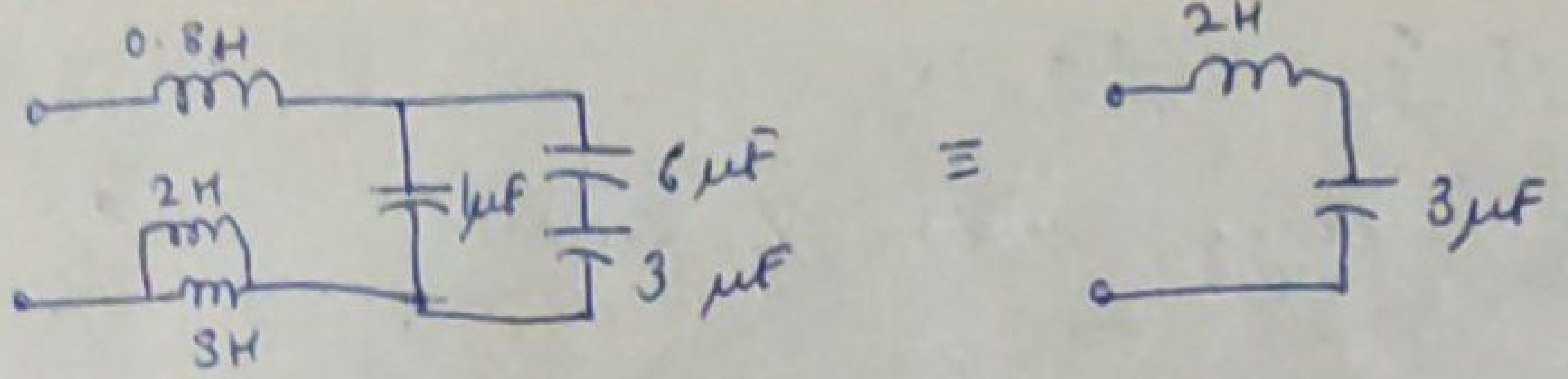


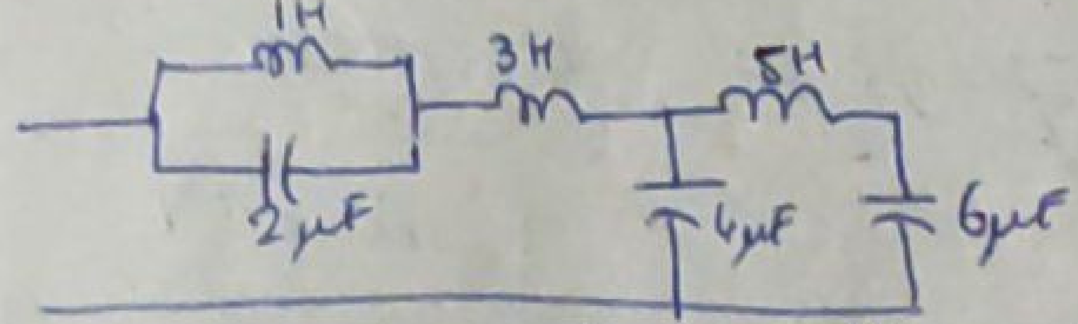
$$v_s = v_1 + \dots + v_N$$

$$= \sum_n \left[\frac{1}{C_i} \int_{t_0}^{t} i\,dt + v_n(t_0) \right]$$

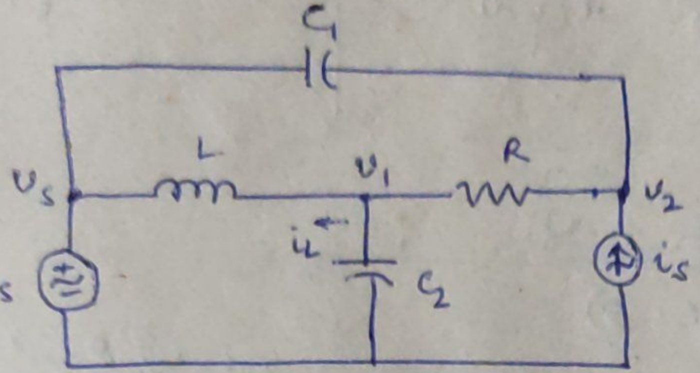
$$= \sum_n \frac{1}{C_n} \int_{t_0}^{t} i\,dt + v_s(t_0) \qquad \therefore \frac{1}{C_{eq}} = \sum_i \left(\frac{1}{C_i} \right)$$

~ly for ∥ ckts.





no series / || combinations possible



KCL at ①,

$$\frac{1}{L}\int_{t_0}^{t}(v_1-v_s)\,dt + i_L(t_0) + \frac{v_1-v_2}{R} + C_2\frac{dv_1}{dt} = 0.$$

KCL at ②,

$$C_1\frac{d(v_2-v_s)}{dt} + \frac{v_2-v_1}{R} - i_s = 0.$$

$$\therefore \quad \frac{v_1}{R} + C_2\frac{dv_1}{dt} + \frac{1}{L}\int_{t_0}^{t} v_1\,dt - \frac{v_2}{R} = \frac{1}{L}\int_{t_0}^{t} v_s\,dt - i_L(t_0).$$

$$-\frac{v_1}{R} + \frac{v_2}{R} + C_1\frac{dv_2}{dt} = C_1\frac{dv_s}{dt} + i_s$$

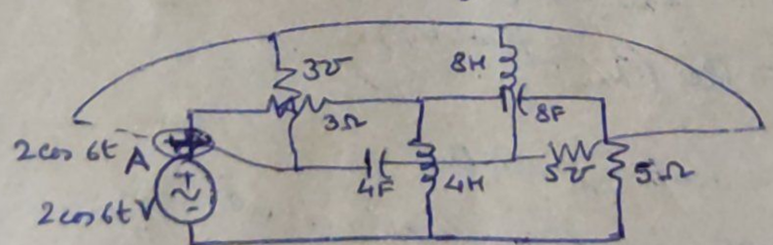
→ INTEGRO DIFFERENTIAL EQNS.

① v_S - source voltage enters as integral as well as derivative (but not as v_S. → ; R not connected to it)

② $i_L(t_0)$ → acts as a (constant) source current at node ①.

③ For sinusoidal forcing fns. v_s & i_s, will be possible to define impedance / admittance for each of the 3 passive elements.

Dual :



Ⓐ

$$3i_1 + 4\frac{di_1}{dt} - 4\frac{di_2}{dt} = 2\cos 6t$$

$$-4\frac{di_1}{dt} + 4\frac{di_2}{dt} + \frac{1}{8}\int_0^t i_2\,dt + 5i_2 = -10.$$

STAR DELTA CONVERSION

13 Refer to the network shown in Fig. 3-30 and find (*a*) R_{eq} if each element is a 10-Ω resistor; (*b*) L_{eq} if each element is a 10-H inductor; and (*c*) C_{eq} if each element is a 10-F capacitor.

14 In Fig. 3-31, let elements A, B, C, and D be (*a*) 1-H, 2-H, 3-H, and 4-H inductors, respectively, and find the input inductance with x-x' first open-circuited and then short-circuited; (*b*) 1-F, 2-F, 3-F, and 4-F capacitors, respectively, and find the input capacitance with x-x' first open-circuited and then short-circuited.

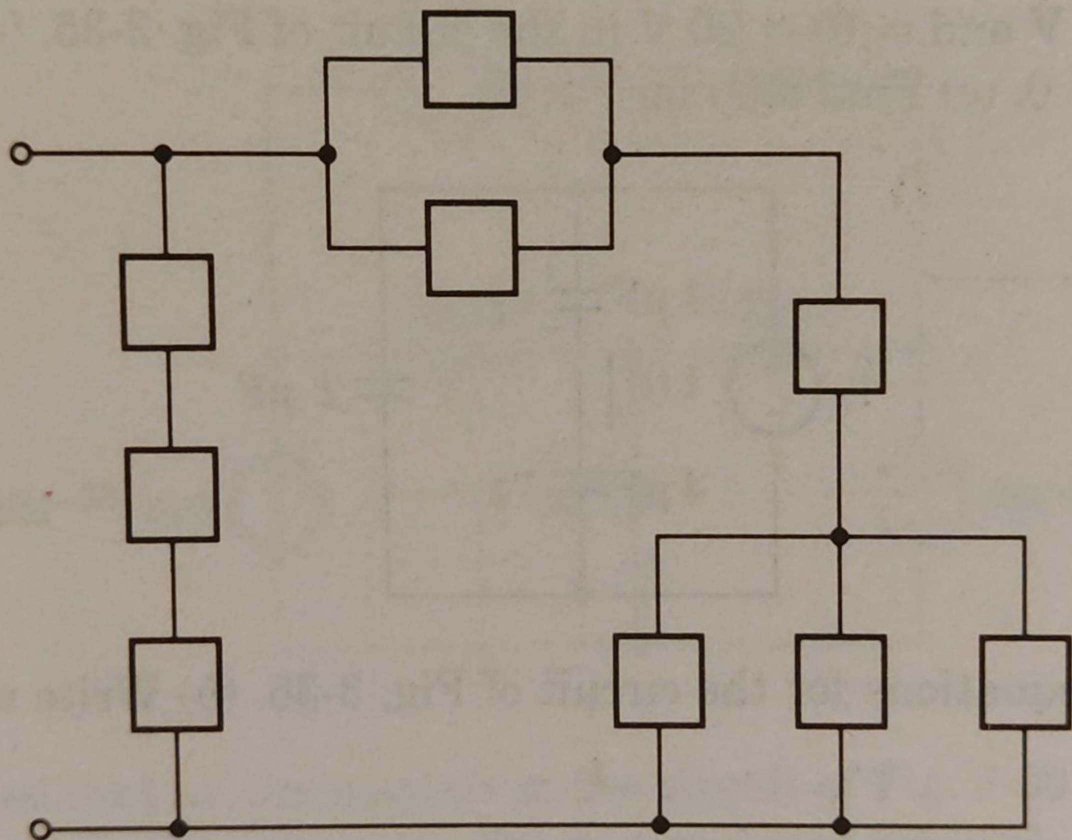


Figure 3-30

See Prob. 13.

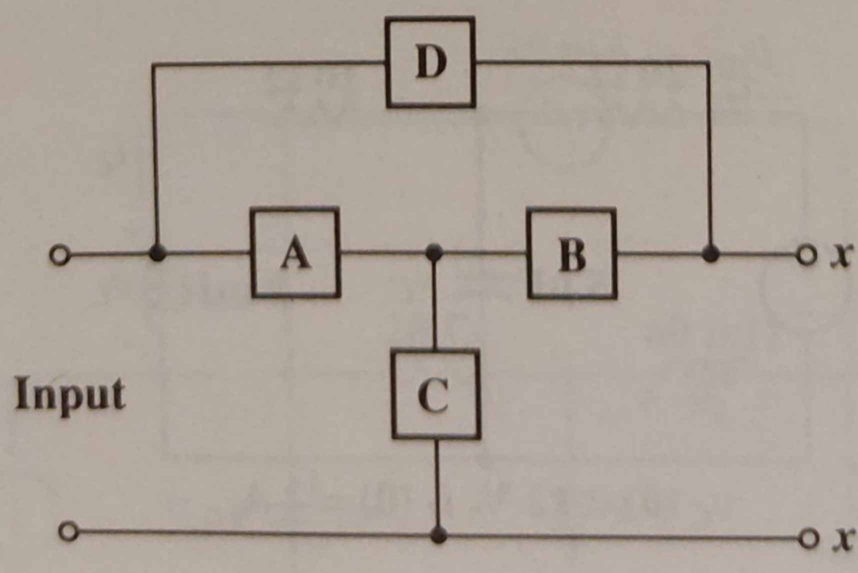


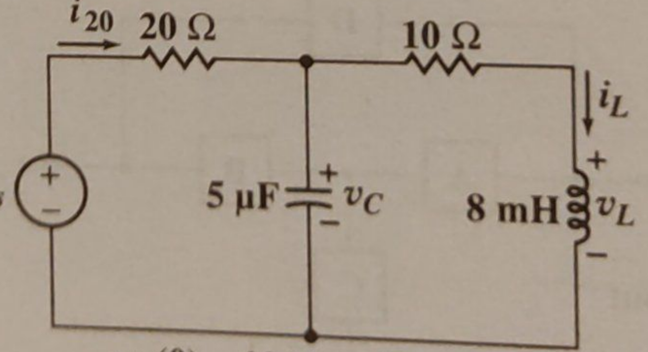
Figure 3-31

See Prob. 14.

20 (*a*) Write nodal equations for the circuit of Fig. 3-36. (*b*) Write mesh equations for the same circuit.

Figure 3-36

See Probs. 20 and 21.



$v_C(0) = 12$ V, $i_L(0) = 2$ A

21 (*a*) Draw the exact dual of the circuit shown in Fig. 3-36. Specify the dual variables and the dual initial conditions. (*b*) Write nodal equations for the dual circuit. (*c*) Write mesh equations for the dual circuit.