Ex 6.1 Draw root loci for ch. eqn. of E sys. given as $s(s+4)(s^2+4s+20)+K=0$. Find Gain margin for $K_1=26$, $K_2=2600$.

1. Rewrite $G(s)H(s) = 1 + \dfrac{K}{s(s+4)(s^2+4s+20)} = 0 \quad \therefore G_1H_1(s) = \dfrac{1}{s(s+4)(s^2+4s+20)}$

2. Poles at $s=0, -4, -2 \pm j4 \quad \therefore K=0 \quad P=4$ 4.
Zeros at ∞ $\quad K=\infty \quad Z=0 \quad \therefore$ 4 asymptotes $= N = P-Z$

5. $\angle$ of asymptotes $\pm 45°, \pm 135°$, Centroid $\sigma_A = \dfrac{-4-2-2}{4} = -2$ (06)

6. ch. eqn: $s^4+8s^3+36s^2+80s+K=0$.

R-H array

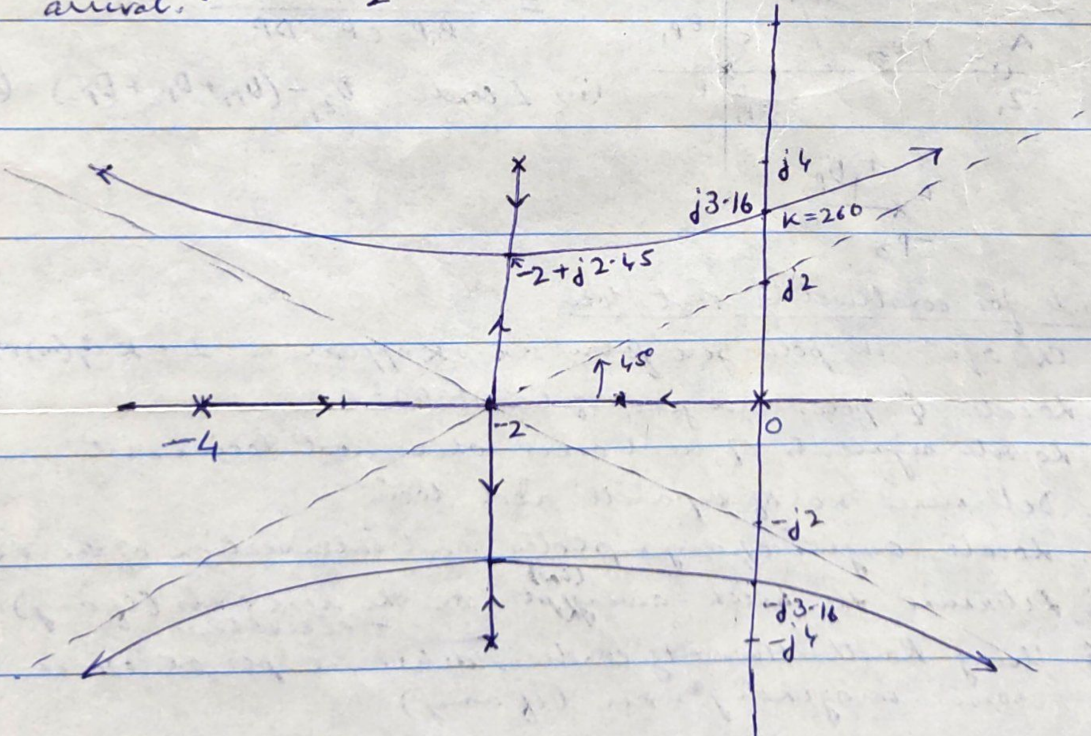
s^4	1	36	K
s^3	$\cancel{8}$ 1	$\cancel{80}$ 10	
s^2	26	K	
s^1	$10-\frac{K}{26}$		
s^0	K		

Aux. eqn: $s^2+10=0$
$\Rightarrow s = \pm j3.16$

$\therefore 0 < K < 260$

7. $K = -(s^4+8s^3+36s^2+80s)$
$\therefore \dfrac{dK}{ds} = 0 = \cancel{4s^3+24s^2+72s+80}$
$\Rightarrow s^3+6s^2+18s+20 = 0 = (s+2)(s+2\pm j2.45) = 0$.

8. Break away at $\pm \dfrac{180°}{2} = 90°$ at each stage to direction of arrival.



Gain margin at $K_1=26$ is $+20$dB : system stable
$\left(-20\log_{10}\dfrac{K}{K_u}\right)$ $\quad K_2=2600 \quad GM = -20\log_{10}\dfrac{2600}{260} = -20\log_{10}10 = -20$dB : UNSTABLE.

Ex 6.2. Draw root loci of unity f/b system with OL T.F.

$$GH(s) = \frac{K(s+2)}{s^2+2s+3}$$

Determine value of K for a) repetitive roots b) range of K for UD sys. c) $\zeta = 0.7$

1. $GH(s) = \dfrac{K(s+2)}{(s+1 \pm j\sqrt{2})}$

2. OL poles : $-1 \pm j\sqrt{2}$ $P = 2$: $K = 0$
   zeros : -2 ; $K = \infty$ $Z = 1$ $\therefore N = P - Z = 1$ infinite zero, 1 asymptote   4.

5. $\angle$ of asymptote : $\theta = \pi$ 3. existence of root loci bet. -2 & $-\infty$

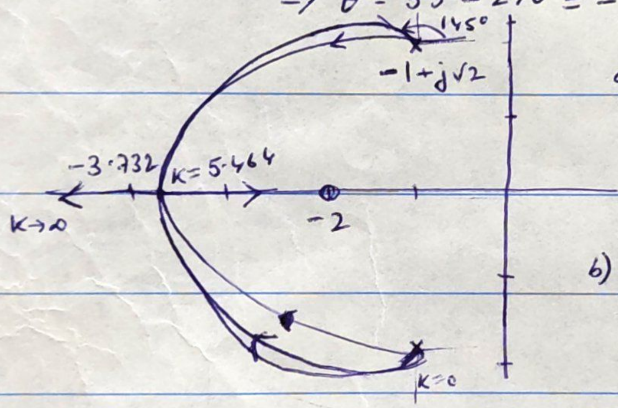
6. No root loci to rt. $\therefore$ R-H array not needed. – no intersection with $j\omega$ axis

7. Break-in pt. $K = \dfrac{-(s^2+2s+3)}{s+2}$ ✓ $\therefore \dfrac{dK}{ds} = \dfrac{-(2s+2)(s+2)+(s^2+2s+3)}{(s+2)^2} = 0$

$\Rightarrow s^2+4s+1=0 \Rightarrow s = -3.732, -0.268 \rightarrow$ NOT VALID PT. ON ROOT LOCI

8. $\angle$ of departure : $\theta_z - (\theta_p + \theta_p^*) = 180° = \tan^{-1}\dfrac{\sqrt{2}}{1} - (\theta + 90°) = 55° - \theta - 90°$

$\Rightarrow \theta = 55° - \overset{360}{270°} = -215° = 145°$



a) repetitive roots at break in pt.

$\therefore s_1 = -3.732$.   Note : $\zeta = 1$

$\therefore K = \left.\dfrac{-(s^2+2s+3)}{s+2}\right|_{s=s_1} = 5.464$

b) range of K $0 < K < 5.464$ for U.D. OL system.

c) $\zeta = 0.7 \Rightarrow \cos\theta = 0.7 \Rightarrow \theta = 45.57° \Rightarrow \boxed{\tan\theta = 1.0202 = \dfrac{\omega}{\sigma}}$ for $s = \sigma + j\omega$

Now, from $\angle$ condn.,

$$\angle s+2 - (\angle s+1+j\sqrt{2} + \angle s+1-j\sqrt{2}) = \pm(2k+1)\pi$$

let $s = \sigma + j\omega$, $\tan^{-1}\left(\dfrac{\omega}{\sigma+2}\right) \pm (2k+1)\pi = \tan^{-1}\left(\dfrac{\omega+\sqrt{2}}{\sigma+1}\right) + \tan^{-1}\left(\dfrac{\omega-\sqrt{2}}{\sigma+1}\right)$

$$\Rightarrow \frac{\frac{\omega}{\sigma+2} \pm 0}{1 \mp \frac{\omega}{\sigma+2}\cdot 0} = \frac{\frac{\omega+\sqrt{2}}{\sigma+1} + \frac{\omega-\sqrt{2}}{\sigma+1}}{1 - \frac{(\omega+\sqrt{2})(\omega-\sqrt{2})}{(\sigma+1)^2}}$$

$\boxed{\tan(x \pm y) = \dfrac{\tan x \pm \tan y}{1 \mp \tan x \tan y}}$

$\Rightarrow \omega[(\sigma+2)^2 + \omega^2 - 3] = 0 \Rightarrow \omega = 0, (\sigma+2)^2 + \omega^2 = (\sqrt{3})^2 \rightarrow$ Eqn. of circle radius $\sqrt{3}$, centre $(-2, 0)$

$\therefore \tan\theta = \dfrac{\omega}{\sigma} \Rightarrow \omega = 1.0202\sigma \Rightarrow |\sigma| = |-1.6659| = \zeta\omega_n = 0.7\omega_n \Rightarrow \omega_n = 2.38$

$\omega = \omega_n\sqrt{1-\zeta^2} = 1.6995 \therefore s_2 = -1.6659 + j1.6995$

$\therefore K = \left.-\dfrac{(s^2+2s+3)}{s+2}\right|_{s=s_2} = 1.3318$

Ex 6.8, 6.9, 6.10 (to correct), 6.11 / 6.12, 6.13 – Nyquist, compensator

*Note : Pole-zero cancellation DOES NOT reflect in root locus plot $\therefore$ Any varn. in pole – not accounted – large instability may occur.

Assignment problems :

Ex 6·3 $G(s)H(s) = \dfrac{K}{s(s+4)(s+4\pm j4)}$

Ex 6·5. $G(s)H(s) = \dfrac{K}{s(s^2+6s+25)}$; G.M. at $K_1 = 15$, $K_2 = 1500$

(11)

Ex 6·6 $G(s)H(s) = \dfrac{K}{s(s+2)(s^2+6s+25)}$

Ex 6·7 $G(s)H(s) = \dfrac{K(s+3)}{s(s+2)}$

Ex 6·8 T.F. $= \dfrac{20(1+ks)}{s(s+1)(s+4)}$; find k s.t. $\zeta = 0.4$. [Let $20k = K$]

$1+GH = s^3+5s^2+4s+20+20ks \rightarrow GH(s) = \dfrac{Ks}{(s+5)(s\pm j2)} = \dfrac{Ks}{(s+5)(s^2+4)}$

for $\zeta = 0.4 = \cos\theta \Rightarrow \theta = \pm 66.42° \rightarrow s = -1.05+j2.4$ at P (from root locus) → rel. bet. σ, ω into $G, H, (K)$ ∠ relation

$K = 8.98 = 20k \therefore k = 0.449$. better response.

$s = -2.15+j4.95$ at Q $\therefore K = 28.26 \Rightarrow k = 1.413$.

For $k = 0.449$, CE poles $s = -1.05\pm j2.4, -2.902$.

Asymptote centroid $= \dfrac{-5-0}{3-1} = -2.5$

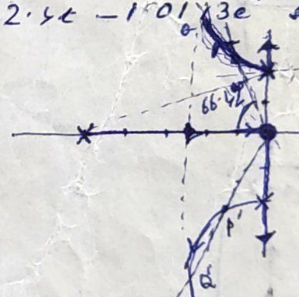
∠ of dep: $180° = \cancel{90°}-\cancel{90°} - \tan^{-1}12/5 - \theta$; 21.77

$\Rightarrow \theta = -180° - 21.77° = -201.77°$

$\therefore \dfrac{C(s)}{R(s)} = \dfrac{20(1+9.04s)}{s^3+5s^2+12.98s+20}$

VERIFY $\longrightarrow c(t) = 1 - 0.747e^{-2.902t} - 0.253e^{-1.05t}\cos 2.4t - 1.0113e^{-1.05t}\sin 2.4t$.

For $k = 1.413$, $s = -2.15\pm j4.95, -0.6823$.
∴ sluggish



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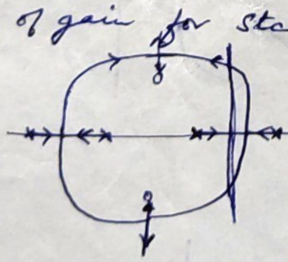
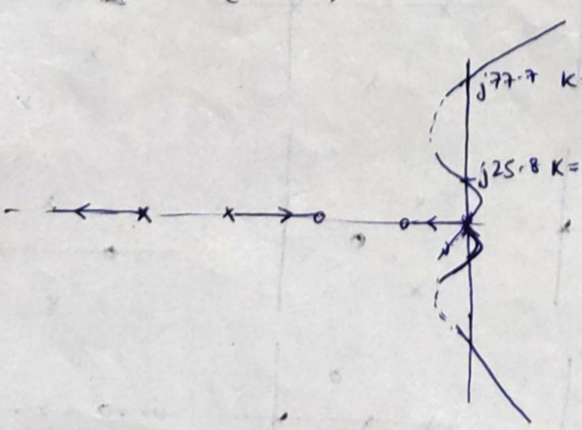
Ex 6·9 $GH(s) = \dfrac{K(s^2+1.5s+1.5625)}{(s-0.75)(s+0.25)(s+1.25)(s+2.25)}$

Values of gain for stable CE eqn.



Ex 6·12 $GH(s) = \dfrac{100K(s+5)(s+40)}{s^3(s+100)(s+200)}$



(3a)

Ex 6.8

$$T.F. = \frac{20(1+ks)}{s(s+1)(s+4)} \quad \text{find } k \text{ s.t. } \zeta = 0.4 \quad [\text{Let } 20k = K]$$

$$1 + GH(s) = s^3 + 5s^2 + 4s + 20 + \underbrace{20k}_{K}s \quad \therefore GH(s) = \frac{Ks}{(s+5)(s^2+4)} = \frac{Ks}{(s+5)(s \pm j2)} \quad \cdots ①$$

For $\zeta = 0.4 = \cos\theta$, $\theta = \pm 66.42^\circ \Rightarrow \tan\theta = 2.29 = \frac{\omega_d}{\sigma}$

$$\therefore 1 + GH(s) = s^3 + 5s^2 + (4+K)s + 20 = \cancel{(\sigma+j\omega)^3 + 5(\sigma+j\omega)^2 + (4+K)(\sigma+j\omega) + 20 = 0}$$

$\angle$ condn. in ①, put $s = \sigma + j\omega$.

$$\therefore \angle GH(s) = \frac{\angle \sigma + j\omega}{(\sigma+5+j\omega)(\sigma+j(\omega+2))(\sigma+j(\omega-2))} = \tan^{-1}\frac{\omega}{\sigma} - \tan^{-1}\frac{\omega}{\sigma+5} - \tan^{-1}\frac{\omega+2}{\sigma} - \tan^{-1}\frac{\omega-2}{\sigma}$$

$$= (2k+1)\pi$$

$$\left[\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}\right]$$

$$\Rightarrow \tan^{-1}\left(\frac{\frac{\omega}{\sigma} - \frac{\omega}{\sigma+5}}{1 + \frac{\omega^2}{\sigma^2+5\sigma}}\right) - \tan^{-1}\left(\frac{\frac{2\omega}{\sigma}}{1 - \frac{\omega^2-4}{\sigma^2}}\right) = (2k+1)\pi$$

$$\Rightarrow \tan^{-1}\left(\frac{5\omega}{\sigma^2+5\sigma+\omega^2}\right) - \tan^{-1}\left(\frac{2\sigma\omega}{\sigma^2-\omega^2+4}\right) = \tan^{-1}\cancel{\frac{5\omega(\sigma^2-\omega^2+4) - 2\sigma\omega(\sigma^2+5\sigma+\omega^2)}{10\sigma\omega^2 + (\sigma^2+5\sigma+\omega^2)(\sigma^2-\omega^2+4)}} = (2k+1)\pi$$

$$\cancel{\angle GH(s) = \frac{K(2\sigma + K s)}{\angle s(s+1)(s+4)}}$$

$$1 + GH = \frac{Ks}{s^3 + 5s^2 + 4s + 20} = \frac{K \cdot s}{(s+5)(s^2+4)} \qquad s \to \sigma + j\omega$$

$\sigma + j\omega$; $s^2 \to \sigma^2 - \omega^2 + 2\sigma\omega j$

$$\Rightarrow \tan^{-1}\frac{5\omega}{\sigma^2 + 5\sigma + \omega^2} = (2k+1)\pi + \tan^{-1}\frac{2\sigma\omega}{\sigma^2 - \omega^2 + 4} =$$

$$\Rightarrow \frac{5\omega}{\sigma^2 + 5\sigma + \omega^2} = \frac{2\sigma\omega}{\sigma^2 - \omega^2 + 4} \Rightarrow 5\sigma^2 - 5\omega^2 + 20 = 2\sigma^3 + 10\sigma^2 + 2\sigma\omega^2$$

$$\Rightarrow 20 = 2\sigma^3 + 5\sigma^2 + \underbrace{2\sigma\omega^2}_{10.5\sigma^3} + \underbrace{5\omega^2}_{26.22\sigma^2}$$

$$\Rightarrow \omega = 2.29\sigma \left[\because \tan\theta = \frac{\omega_d}{\sigma} = 2.29\right] \Rightarrow 12.5\sigma^3 + 31.22\sigma^2 - 20 = 0$$

$$12.5\sigma^3 + 31.22\sigma^2 - 20 = 0$$

0.1 0.7 → −0.5

+0.7

$$(\sigma - 0.708)(12.5\sigma^2 + 40.07\sigma + 28.37)$$

$$= \sqrt{187.1} = 13.68$$

$$28.37 \to \sigma = \frac{-40.07 \pm \sqrt{40.07^2 - 4(28.37)(12.5)}}{25} = \underbrace{-1.0556 ; -2.15}_{\omega = 2.417 ; 4.92}$$

$$\left|\frac{Ks}{(s+5)(s^2+4)}\right| = 1 \Rightarrow K = \left|\frac{(s+5)(s^2+4)}{s}\right| = \left|\frac{\sqrt{(\sigma+5)^2+\omega^2}\cdot\sqrt{[(\sigma^2-\omega^2+4)^2+(2\sigma\omega)^2]}}{\sqrt{\sigma^2+\omega^2}}\right|$$

$s_1 = -1.0556 + j\,2.417$

$s_2 = -2.15 + j\,4.92$

$$① = \frac{(4.63)(5.15)}{2.64} = 9.04 = K$$

$\Rightarrow k = 0.452$

$$s^2+4 = (\sigma+j\omega)^2+4 = \underbrace{\sigma^2-\omega^2+4}+j\,\underbrace{2\sigma\omega}$$

$(s+j2) = \sigma + j(\omega+2)$

$= \sqrt{\sigma^2+(\omega+2)^2}$

$$② = \frac{(5.685)(26.28)}{\cancel{28.88}\ 5.37} = 27.825 = K$$

$k = 1.39$

$$Ⓣ = \frac{G}{1+G} = \frac{20+Ks}{s^3+5s^2+(4+K)s+20}$$

$$① = \frac{20+9.04s}{s^3+5s^2+13.04s+20} \longrightarrow s = -1.0556 \pm j2.417,\ -2.9$$

6.9 $$\frac{K(s^2+1.5s+1.5625)}{(s-0.75)(s+0.25)(s+1.25)(s+2.25)} = GH(s).$$

$$1+GH(s) = (s^2+1.5s-1.6875)(s^2+1.5s+0.3125) + K(s^2+1.5s+1.5625)$$

$$= s^4+3s^3+(0.875+K)s^2+(1.5K-2.0625)s+(1.5625K-0.527) = 0.$$

s^4	1	$(0.875+K)$	$(1.5625K-0.527)$
s^3	3	$(1.5K-2.0625)$	
s^2	$(1.5625+0.5K)$	$1.5625K-0.527$	
s^1	$\frac{(1.5K-2.0625)(1.5625+0.5K)-3(1.5625K-0.527)}{(1.5625+0.5K)}$		
s^0	$1.5625K-0.527.$		

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$$s^0>0 \Rightarrow K \geq \frac{0.527}{1.5625} = 0.34$$

s^1 row $=0 \Rightarrow$ roots on imaginary axis

$$\Rightarrow 0.75K^2-3.375K-1.642=0 \qquad \therefore K = \frac{+3.375}{1.5} \pm \frac{\sqrt{16.32}}{1.5} = 4.04$$

$$= 2.25 \pm 2.69 = 4.94, \cancel{-0.44}$$

$\therefore K = 4.94. \Rightarrow s^1$ row is 0.

$$\therefore A(s) = (1.5625+0.5K)s^2+(1.5625K-0.527)\Big|_{K=4.94} = 0$$

$$= 4.0325s^2+7.192 \Rightarrow s^2 = -\frac{7.192}{4.0325} = -1.7834 \Rightarrow s = \pm j1.335.$$

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$1 + GH = s^3 + 5s^2 + 4s + 20 + 20ks \rightarrow GH(s) = \dfrac{Ks}{(s+5)(s \pm j2)} = \dfrac{Ks}{(s+5)(s^2+4)}$

for $\zeta = 0.4 = \cos\theta \Rightarrow \theta = \pm 66.42^\circ \rightarrow s = -1.05 + j2.4$ at P from root locus → sub. σ, ω into $G, H_1(s)$ [?] retain

$K = 8.98 = 20k \therefore k = 0.449$. better response

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For $k = 0.449$, CL poles $s = -1.05 \pm j2.4, -2.902$.

$\therefore \dfrac{C(s)}{R(s)} = \dfrac{20(1+0.449s)}{s^3 + 5s^2 + 12.98s + 20}$

VERIFY → $c(t) = 1 - 0.747e^{-2.902t} - 0.253e^{-1.05t}\cos 2.4t - 1.013e^{-1.05t}\sin 2.4t$.

For $k = 1.413$, $s = -2.15 \pm j4.95, -0.6823$.

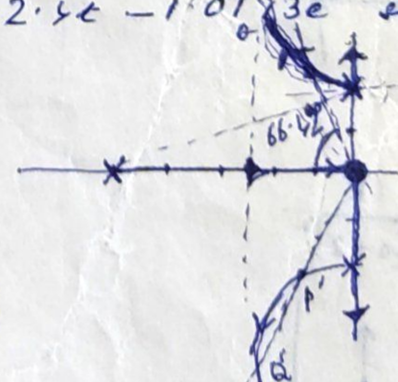
$\therefore$ sluggish

Asymptote centroid $= \dfrac{-5-0}{3-1} = -2.5$

$\angle$ of dep: $180^\circ = 90^\circ - 90^\circ - \tan^{-1}2/5 - \theta$

$\tan^{-1}2/5 = 21.77$

$\Rightarrow \theta = -180^\circ - 21.77^\circ = -201.77^\circ$



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Ex 6.9 $GH(s) = \dfrac{K(s^2 + 1.5s + 1.5625)}{(s-0.75)(s+0.25)(s+1.25)(s+2.25)}$

Values of gain for stable CL sys.