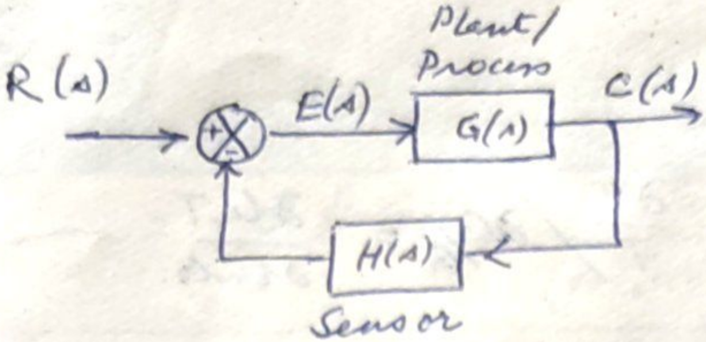
CHARACTERISTICS OF FEEDBACK CONTROL SYSTEMS.

— negative f/b sys. unless otherwise stated



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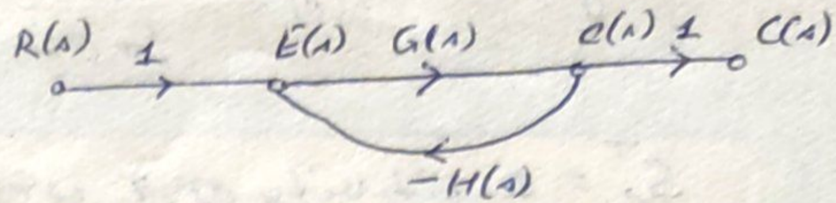


$\therefore \; E(s) = R(s) - H(s)\,C(s)$

$C(s) = G(s)\,E(s) = G(s)\left[R(s) - H(s)\,C(s)\right]$

$$\Rightarrow \; C(s) = \frac{G(s)}{1 + G(s)H(s)}\, R(s)$$

$$E(s) = \frac{1}{1 + G(s)H(s)}\, R(s)$$

$\therefore$ To reduce error, $|G(s)H(s)| \gg 1 \;\Rightarrow\; E(s) < R(s)$. High gain feedback

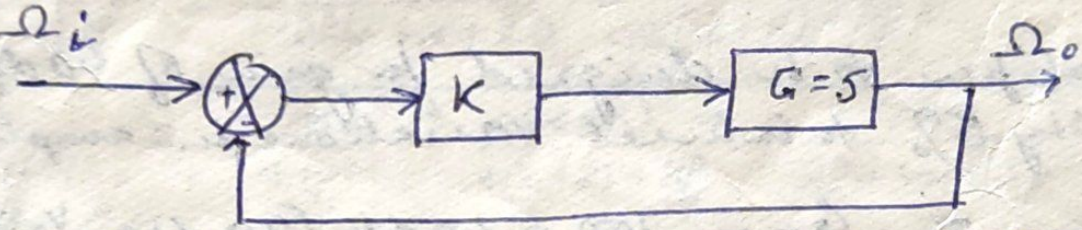
over the desired (reqd.) range of frequencies!

Note:

1. CONTROL SYSTEMS usually low-pass systems $\therefore$ block high freq. So $|G(s)H(s)| \gg 1$ is DIFFICULT to achieve at HF.

2. $G(s)H(s)\big|_{s} \simeq -1 \;\Rightarrow\; E(s)$ very large $\rightarrow$ Oscillators

Example:



Effect of K on s.s. error

$\therefore\ T(s) = \dfrac{KG}{1+KG} = \dfrac{5K}{1+5K}$

OBJECTIVE [to achieve unity gain $\therefore\ \Omega_o = \Omega_i$]

For $K = 1, 5, 10$; $T(s) = \frac{5}{6}(\simeq 0.83)$, $\frac{25}{26}(\simeq 0.96)$, $\frac{50}{51}(\simeq 0.98)$

$\therefore$ % error for K $= 1$ is $\dfrac{1-0.83}{1} \times 100\% = 17\%$

$= 5$ is 4%

$= 10$ is 2%.

$\therefore$ As K increases, s.s. error is reduced.

Say $G \longrightarrow (G + 10\%G)$ due to parameter variations.

$\therefore$ for $K = 1$, $T(s) = \dfrac{1(5.5)}{1+1(5.5)} = 0.85$

$= 5$ $\qquad \dfrac{27.5}{28.5} \simeq 0.96$

$= 10$ $\qquad \dfrac{55}{56} = 0.98$

$\therefore$ As K increases, sensitivity to parameter variations in forward path reduces.

For O case $T(s) = G(s)$

C $T(s) = \dfrac{G}{1+GH}$

S_G^T = sensitivity of T wrt G $= \dfrac{\partial T}{T} \Big/ \dfrac{\partial G}{G} = \dfrac{\partial \ln T}{\partial \ln G}$.

∴ For O $= 1$.

$$C = \frac{\partial T}{\partial G} \cdot \frac{G}{T} = \frac{1}{(1+GH)^2} \cdot \frac{G}{G/1+GH} = \frac{1}{1+GH}$$

∴ Improvement (Reduced sensitivity) of factor $\dfrac{1}{1+GH} \approx \dfrac{1}{GH}$ if $|GH| \gg 1$

∴ If $GH \gg 1$ ⇒ [10% change of G appears as changes of $\left(\dfrac{10}{GH}\right)\%$ in $T(s)$]

$$G'(s) = 1.1\, G(s).$$

$$\therefore T'(s) = \frac{G'(s)}{1+G'(s)H} = \frac{1.1G}{1+1.1GH}$$

$$\Delta T(s) = T'(s) - T(s) = \frac{0.1G}{(1+1.1GH)(1+GH)}.$$

$$\therefore \frac{\Delta T}{T} \times 100\% = \left(\frac{10}{1+1.1GH}\right)\% \approx \left(\frac{10}{GH}\right)\% \text{ if } GH \gg 1.$$

~ly for $G'(s) = 0.9\, G(s)$.

Ex Fb amplifier to be designed for gain of 40 dB and sensitivity 5% to internal variations in amplifier gain

$$T = \frac{G}{1+GH} \approx \frac{G}{GH} = 100 = 40\,dB \left[20 \log_{10} \frac{V_2}{V_1}\right].$$

Tutorial Q1

$$S_G^T = \frac{1}{1+GH} = 0.05 \simeq \frac{1}{GH}$$

$\therefore$ $GH = 20$, $\quad G = 20/H = 2000 = 66\,dB$.

$\therefore$ Reduced gain (G) to achieve reduced sensitivity. (eg. OP AMP $>1000 \to 10$)

EFFECT ON SENSITIVITY OF PARAMETER VARN. IN FB ELEMENT.

Say $H(s) = \beta$

$$\therefore S_\beta^T = \frac{\partial T}{\partial \beta} \cdot \frac{\beta}{T} = \left(\frac{\beta}{H}\frac{dH}{d\beta}\right)\left(\frac{H}{T}\cdot\frac{\partial T}{\partial H}\right) \quad \text{CHAIN RULE}$$

$$= S_\beta^H \cdot \left(\frac{H}{T} \cdot \frac{-G(G)}{(1+GH)^2}\right) = S_\beta^H \cdot \frac{H(1+GH)}{G} \cdot \frac{-G^2}{(1+GH)^2}$$

$$= 1 \cdot \frac{-GH}{1+GH}$$

$\therefore$ $S_\beta^T = 1 = S_\beta^H$ for $GH \gg 1$ $\therefore$ No effect

$\therefore$ Require FB elements to be more precise (No parameter variations desired)

Note: $S_H^T = \dfrac{-GH}{1+GH} \simeq -1.$

If α is a parameter of $G(s)$, then

$$S_\alpha^G = \frac{d\ln G}{d\ln\alpha} = \frac{\alpha}{G}\cdot\frac{dG}{d\alpha}$$

$$S_\alpha^T = \frac{\alpha}{T}\cdot\frac{dT}{d\alpha} = \left(\frac{\alpha}{G}\cdot\frac{dG}{d\alpha}\right)\left(\frac{G}{T}\cdot\frac{dT}{dG}\right)$$

$$= \frac{\alpha}{G}(1+GH)\cdot\frac{1}{(1+GH)^2}\cdot\frac{dG}{d\alpha} = \frac{\alpha}{G(1+GH)}\cdot\frac{dG}{d\alpha}$$

$$\therefore S_\alpha^T = \frac{1}{1+GH}\cdot S_\alpha^G = S_G^T\cdot S_\alpha^G \text{ (chain rule)}$$

Since $T(s) = \frac{N(s)}{D(s)}$, $\therefore S_\alpha^T = S_\alpha^N - S_\alpha^D = \frac{\partial\ln T}{\partial\ln\alpha}$

Ex. $\underline{Q}$ position control system with armature controlled dc servomotor

$G(s) = \frac{K}{s(s+\alpha)}$ $\quad H(s) = \beta \quad K = 20, \alpha = 4, \beta = 1$

$\therefore G(s) = \frac{20}{s(s+4)} \quad H(s) = 1. \quad T = \frac{20}{s^2+4s+20}$

$\underline{G}$ sensitivities

$S_K^G = \frac{K}{G}\cdot\frac{\partial G}{\partial K} = s(s+\alpha)\cdot\frac{1}{s(s+\alpha)} = 1.$

$S_\alpha^G = \frac{\alpha}{G}\cdot\frac{\partial G}{\partial \alpha} = \frac{\alpha}{K/s(s+\alpha)}\cdot\frac{-K}{s(s+\alpha)^2} = \frac{-\alpha}{s+\alpha} = \frac{-4}{s+4}.$

$S_\beta^H = 1$

$\underline{T}$ sensitivities

$S_K^T = \frac{S_K^G}{1+GH} = \frac{s^2+4s}{s^2+4s+20} = \frac{s(s+\alpha)}{s(s+\alpha)+K\beta}$

$S_\alpha^T = \frac{S_\alpha^G}{1+GH} = \frac{-\alpha}{s+\alpha}\cdot\frac{s(s+\alpha)}{s(s+\alpha)+K\beta} = \frac{-4s}{s^2+4s+20}$

$S_\beta^T = -S_\beta^H\cdot\frac{GH}{1+GH} = \frac{-K\beta}{s^2+\alpha s+K\beta} = \frac{-20}{s^2+4s+20}$

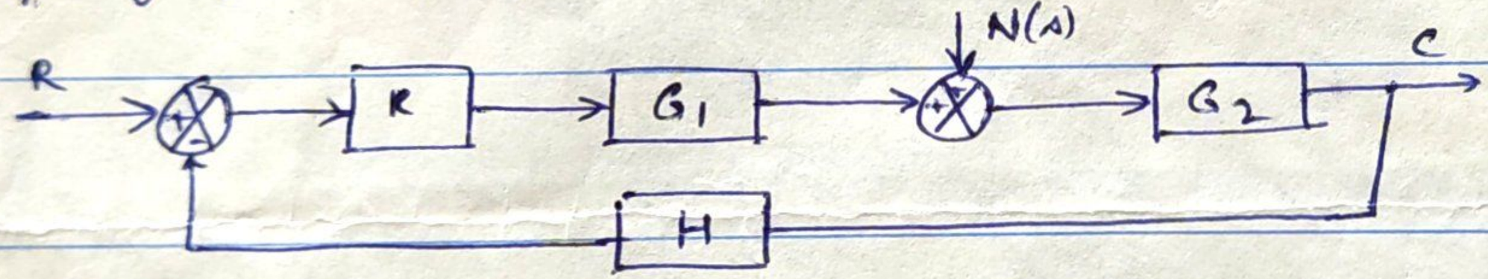
at low freq. & dc, S_K^T & S_α^T negligible or small wrt parameter variation

$S_\beta^T = -1$ at $s=0$

No reduced sensitivity for β element

EFFECT OF DISTURBANCE SIGNALS (DISTURBANCE REJECTION)

*1 only if in F/W path.

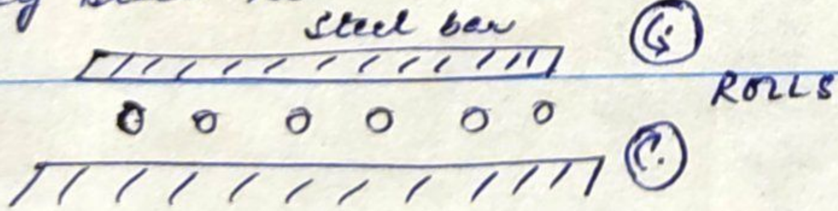


$$C(s) = \frac{KG_1G_2}{1+KG_1G_2H}R(s) + \frac{G_2}{1+KG_1G_2H}N(s)$$

∴ Noise reduced by factor of $\frac{1}{KG_1H}$ over freq. range of interest

More noise rejection if noise source near o/p.

Say steel roll mill



Rolls loaded when steel bars enter ⇒ step change in disturbance D ∴ $N(s) = D/s$

Say $KG_1 = 10/(s+1)$, $G_2 = \frac{1}{5s+1}$

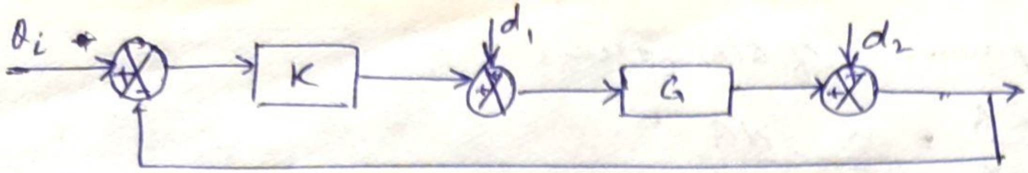
$$\therefore C(s) = \frac{2}{s^2+1.2s+2.2}R(s) + \frac{0.2(s+1)}{s^2+1.2s+2.2}N(s)$$

∴ $C/R \approx 10\frac{C}{N}$ improvement in SNR in o/p $= 10 = \frac{C/R}{C/N}$

*2. D(s) near o/p, if near i/p then no significant ~~[illegible]~~ SNR improvement.

*3. in fb element $C/N \approx 1/H$ ∴ $S^T_H(\cancel{s}) \approx 1$ reqd.

Ex



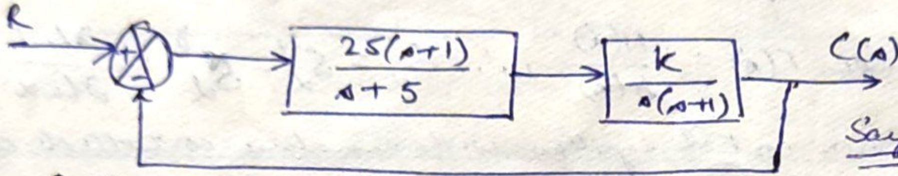
Tutorial 2

$$\therefore \theta_0 = \frac{KG}{1+KG}\theta_i + \frac{G}{1+KG}d_1 + \frac{1}{1+KG}d_2$$ DISTURBANCE REJECTION.

DESIGN USING SENSITIVITY ANALYSIS

Say $G(s) = \frac{K}{s(s+1)}; K=1$, Reqd. $T(s) = \frac{25}{s^2+5s+25}$ at $\omega = 5$ rad/s

$\therefore$ ①



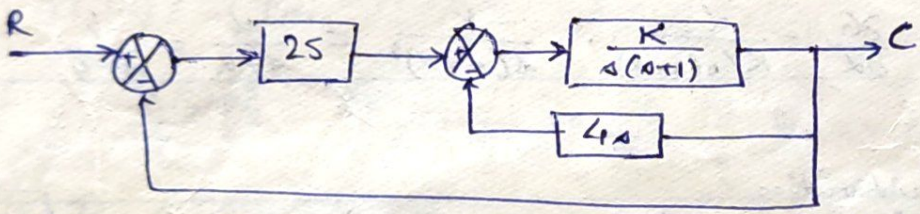
POLE-ZERO CANCELLATION - not adviseble!

Say K = 1

$$T(s) = \frac{25K}{s^2+5s+25K} \qquad \therefore S_K^T = \frac{\partial T}{\partial K}\cdot\frac{K}{T} = \frac{s(s+5)}{s^2+5s+25K} = \frac{s(s+5)}{s^2+5s+25}$$

$$\therefore |S_K^T|_{\omega=5} = 1.41 = \left|\frac{-\omega^2+j5\omega}{(25-\omega^2)+j5\omega}\right| = \left|\frac{-25+j25}{j25}\right| = \left|\frac{-1+j}{j}\right| = |1+j|$$

②



Tutorial 2/3

$$T(s) = \frac{25K}{s^2+(1+4K)s+25K} \qquad G(s) = 25\cdot\frac{K}{s(s+1)+4Ks}$$

$$S_K^T = \frac{s(s+1)}{s(s+1)+K(4s+25)} = \frac{s(s+1)}{s^2+5s+25} \text{ for } K=1. \quad \frac{-\omega^2+j\omega}{(25-\omega^2)+j5\omega}$$

$$\therefore |S_K^T|_{\omega=5} = \sqrt{\frac{26}{25}} \approx 1. \qquad |S_K^T|_{\omega=5} = \left|\frac{-25+j5}{+j25}\right| = \left|\frac{-5+j1}{+j5}\right|$$

$\therefore$ Use 2 loop system for reduced sensitivity.