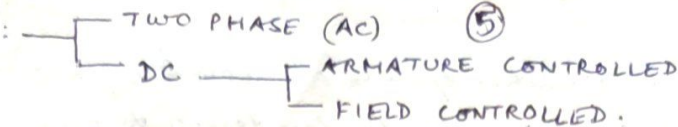


SERVO MOTORS :



- req. max. acceleration (particularly at 0 Torque)
- req. torque to be reduced to 0 when angular velocity = 0.

For max. accel.,  $J$  min. so same  $T$  delivered.

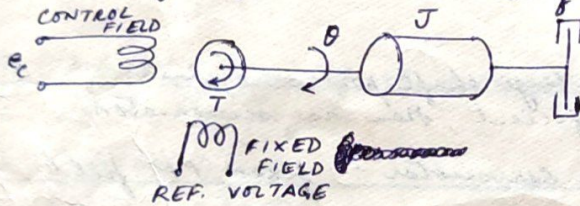
$T_{eq} = J_m + n^2 J_L$       $n < 1 =$  gear ratio bet. motor and load

$f_{eq} = f_m + n^2 f_L$

If  $n^2 J_L$  &  $n^2 f_L$  are small compared to  $J_m, f_m$ , then neglect.

TWO PHASE (AC) SERVOMOTORS: ~ to 2  $\phi$  I.M. with small dia to length ratio  $\therefore J$  small.

- rugged & reliable, used in instrument servomechanisms
- power range frac. of W to few 100 W.



Reliance  
 $T = \frac{J_{eq} \times f(\text{rpm})}{308 \times t} \approx J \ddot{\theta}$   
 $t =$  accelerating time

(usually freq. 50 Hz / 400 Hz / 1000 Hz)  
 ↓                      ↓                      ↓  
 general            military            aerospace

(i) Control signal: suppressed carrier signal  $90^\circ$  phase-shifted w.r.t. fixed phase voltage - see  $e_c(t)$  and  $e_f(t)$  below

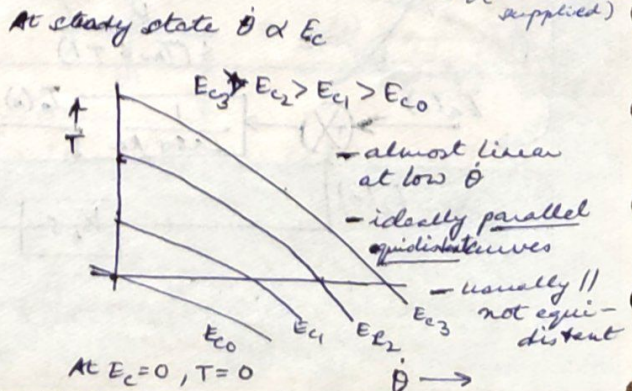
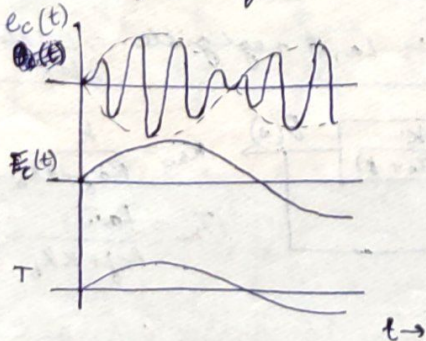
(ii) CONTROL winding & Fixed field windings in space quadrature  
 Both done to maximize Torque generated. (efficiency)

Usually 2  $\phi$  power supply used, else use C in path of <sup>fixed phase</sup> control winding while control winding directly.  
 Polarity of control voltage determines direction of rotation.

$e_c(t) = E_c(t) \sin \omega t \quad \forall E_c(t) > 0$   
 $= |E_c(t)| \sin(\omega t + \pi) \quad \forall E_c(t) < 0.$

Since ref. voltage constant  $\therefore (T, \dot{\theta}) = f(E_c(t)).$

Angular momentum  
 $T = J \ddot{\theta}$  - conservation  
 $\rightarrow \dot{\theta}$  (external energy supplied)

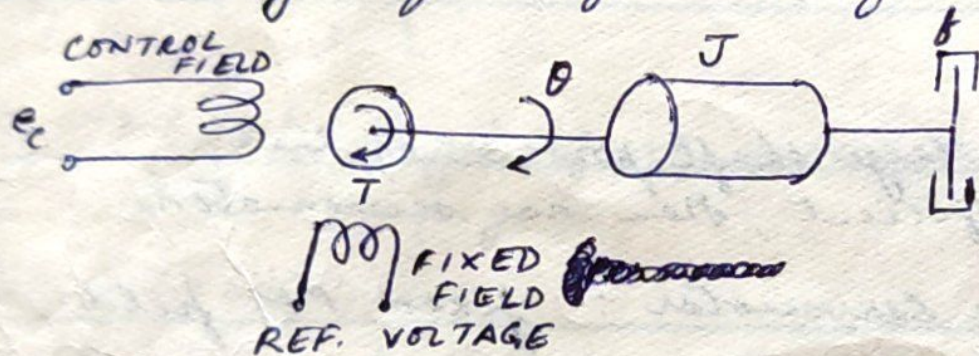


At  $E_c = 0, T = 0$   
 CONSERVATION OF ENERGY AND ANG. MOMENTUM



TWO PHASE (AC) SERVOMOTORS.  $\sim$  to 2  $\phi$  I.M. with small dia to length ratio  $\therefore J$  small.

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$$T = \frac{\text{Reluctance}}{308 \times t} \approx J\ddot{\theta}$$

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(usually freq. 50 Hz / 400 Hz / 1000 Hz)  
 general military aerospace

(i) Control signal: suppressed carrier signal  $90^\circ$  phase-shifted in figure w.r.t. fixed phase voltage - see  $e_c(t)$  and  $E_c(t)$  below

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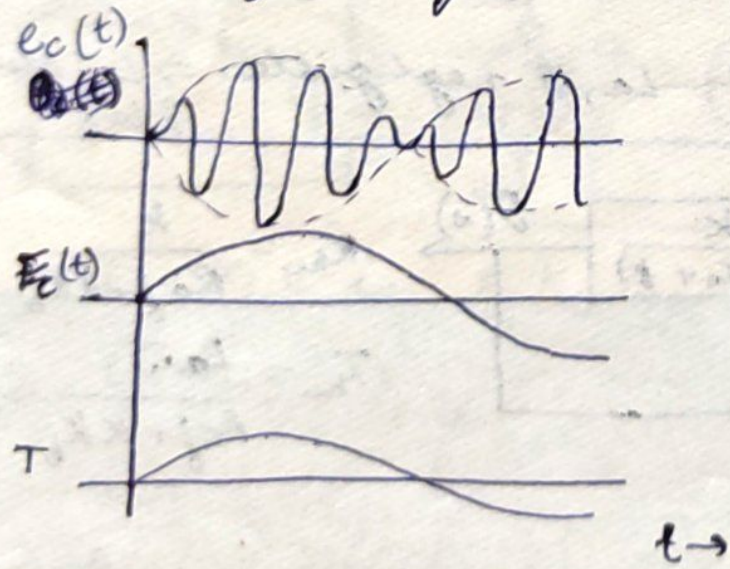
Polarity of control voltage determines direction of rotation.

$$E_c(t) = E_c(t) \sin \omega t \quad \forall E_c(t) > 0$$

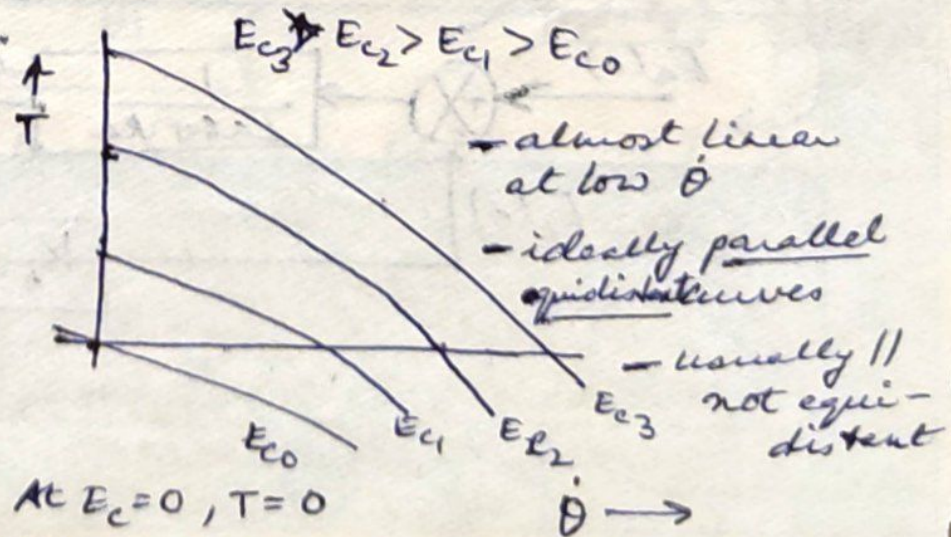
$$= |E_c(t)| \sin(\omega t + \pi) \quad \forall E_c(t) < 0.$$

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Angular momentum  
 $T\omega$  - conservation  
 $\rightarrow$   $\dot{\theta}$  (external energy supplied)



At steady state  $\dot{\theta} \propto E_c$



At  $E_c = 0, T = 0$   
 CONSERVATION OF ~~ENERGY~~ ANG. MOMENTUM



1978 AUTOMATIC DATA PROCESSING INC  
— Usually servo mechs. operate at low  $\dot{\theta}$  when curves // & equidistant

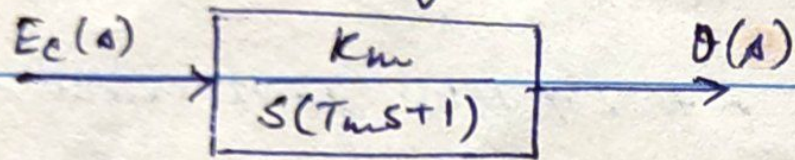
—  $\partial E_c(t) \Rightarrow \partial T$   $\therefore$  motor develops torque to stop rotation.

$$T = -K_R \dot{\theta} + K_C E_c = J \ddot{\theta} + f \dot{\theta}$$

$$\Rightarrow J \ddot{\theta} + (f + K_R) \dot{\theta} = K_C E_c$$

$$\therefore \frac{\theta(s)}{E_c(s)} = \frac{K_C}{J s^2 + (f + K_R) s} = \frac{K_M}{s(T_M s + 1)}$$

where  $K_M = \frac{K_C}{f + K_R}$  ;  $T_M = \frac{J}{f + K_R} = \text{motor time constant}$   
= motor gain constant



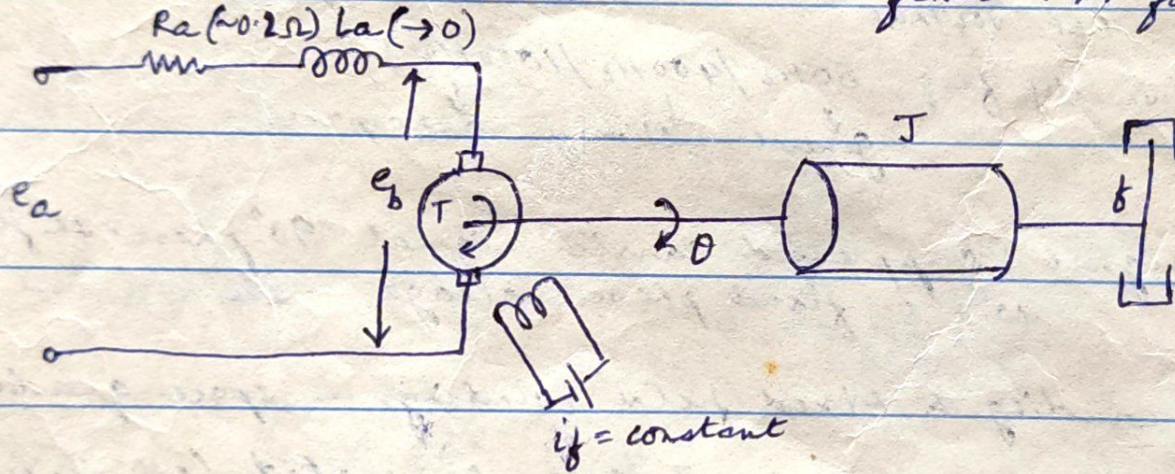
— For steep  $K_R$  (negative of slope of T-speed curves), damping of motor is more. Note: unity f.b. yields typical 2<sup>nd</sup> order system.

— If  $J$  low, then  $|T_M s| \ll 1$ , then servomotor as INTEGRATOR.



DC Servomotors: when large shaft power required.  
 - more efficient than ac servomotors

Armature controlled dc servomotor: fixed PM field



Airgap flux  $\Psi = k_f i_f$  and  $T = k_t i_a \Psi = k_f k_t i_f i_a = K i_a$ .

$$e_b = k_b \frac{d\theta}{dt} \quad \text{--- (1)}$$

Speed of motor is controlled by  $e_a$  supplied by amplifier.

$$\therefore L_a i_a + R_a i_a + e_b = e_a \quad \text{--- (2)}$$

$$I_a(s) = \frac{E_a(s)}{sL_a + R_a}$$

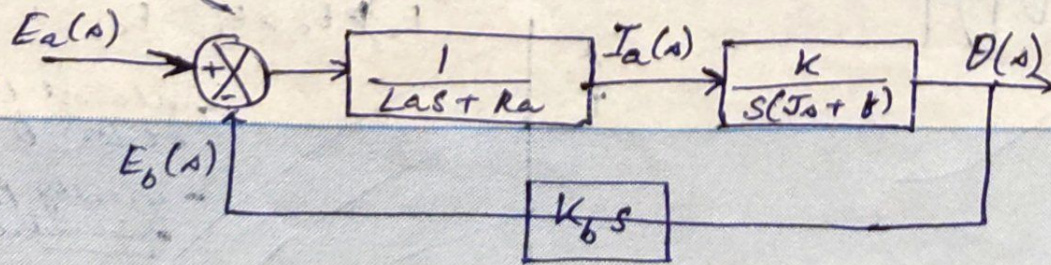
$$k_T I_a(s) = \frac{K}{s(sJ + f)} \theta(s)$$

$$J\ddot{\theta} + f\dot{\theta} = T = K i_a \quad \text{--- (3)}$$

$$\therefore \frac{\theta(s)}{E_a(s)} = \frac{K}{s[L_a J s^2 + (L_a f + R_a J)s + R_a f + K k_b]}$$

$$\frac{K_m v}{s(T_m s + 1)}$$

$\therefore L_a, \text{ negligible. } \Rightarrow L_a J \text{ also negligible}$   
 $\& L_a f$



$$K_m = \frac{K}{R_a f + K k_b}$$

$$T_m = \frac{R_a J}{R_a f + K k_b}$$

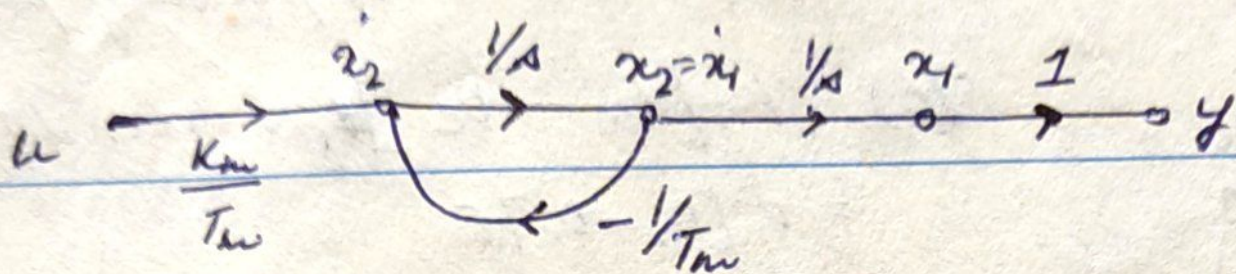


$$\ddot{\theta} + \frac{1}{T_m} \dot{\theta} = \frac{K_m}{T_m} e_a$$

Let  $x_1 = \theta$        $x_2 = \dot{\theta}$

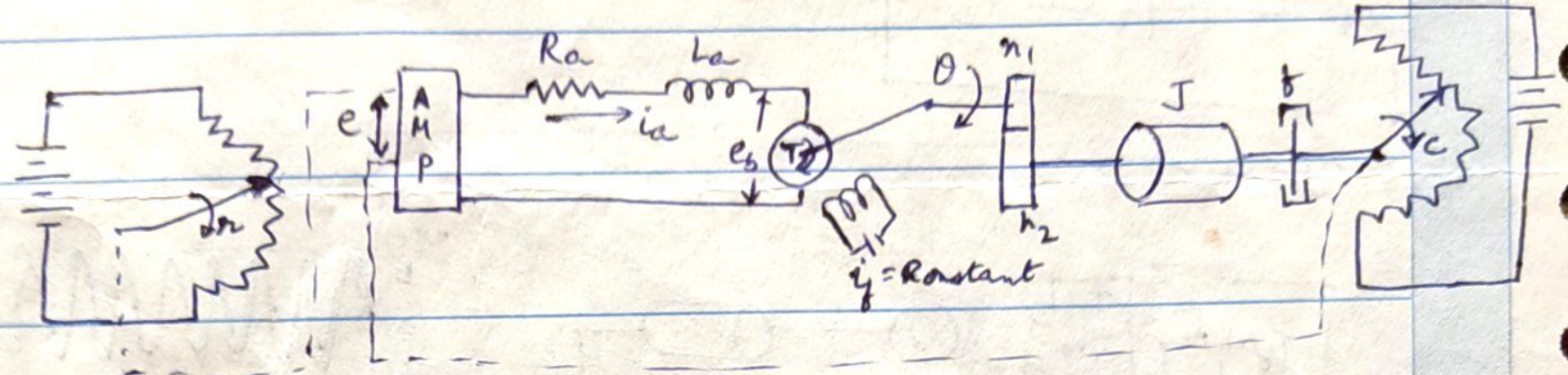
$$\therefore \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & -\frac{1}{T_m} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{K_m}{T_m} \end{bmatrix} u$$

$$y = [1 \ 0] \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + 0 \cdot u \quad \therefore y = \theta, \quad u = e_a$$





Ex



Tutorial

$K_1 = 7.64 \text{ V/rad} = \text{gain of pot. error detector}$

$K_p = 10 \text{ V/V} = \text{amp. gain}$

$K_b = 0.055 \text{ V-s/rad.}$

$R_a = 0.2 \Omega, L_a \rightarrow 0.$

$k = \text{motor torque constant} = 6 \times 10^{-5} \text{ N-m/V}$

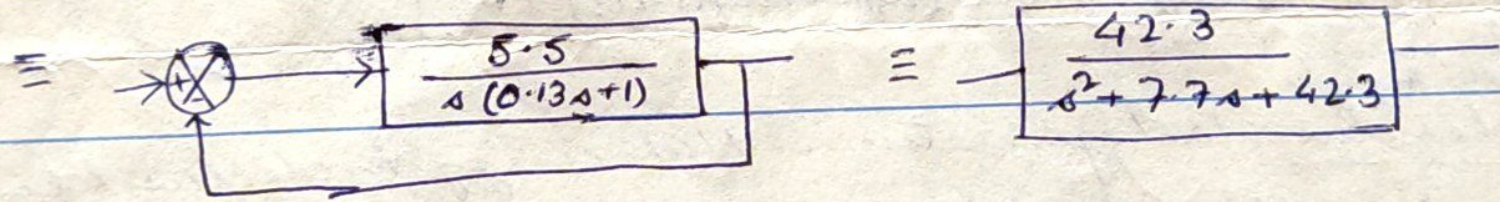
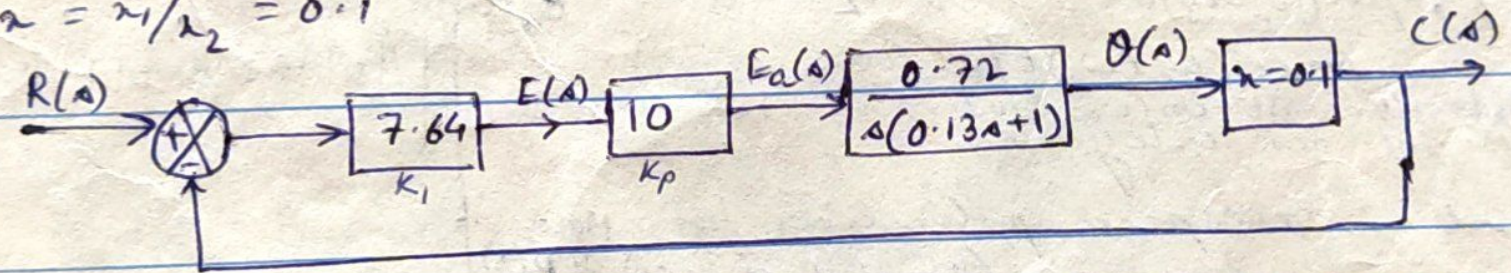
$J_m = 1 \times 10^{-5} \text{ kg m}^2$

$\omega_m \rightarrow 0 \quad J_{eq} = J_m + n^2 J_L; \omega_{eq} = \omega_m + n^2 \omega_L$

$J_L = 4.4 \times 10^{-3} \text{ kg m}^2$

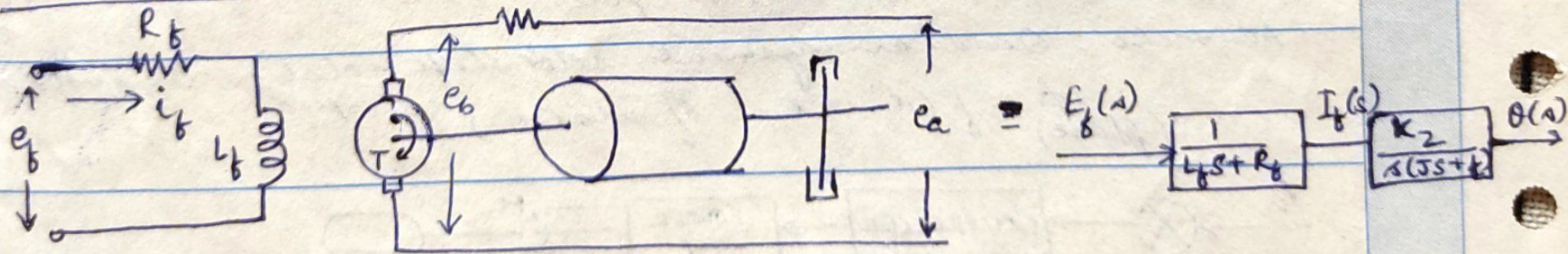
$f_L = 4 \times 10^{-2} \text{ Nm/rad/s}$

$n = n_1/n_2 = 0.1$





## Field controlled dc servomotor:



$$T_a = K_1 \psi i_a = K_2 i_f = J\ddot{\theta} + f\dot{\theta} \quad [\text{Assuming } e_b \text{ constant}]$$

$$L_f \frac{di_f}{dt} + R_f i_f = e_f$$

$$\frac{\theta(s)}{E_f(s)} = \frac{K_2}{s(L_f s + R_f)(Js + f)} = \frac{K_m}{s(T_f s + 1)(T_m s + 1)}$$

$$K_m = K_2 / R_f$$

$$T_f = L_f / R_f$$

$$T_m = J / f$$

MANAGEMENT REPORT



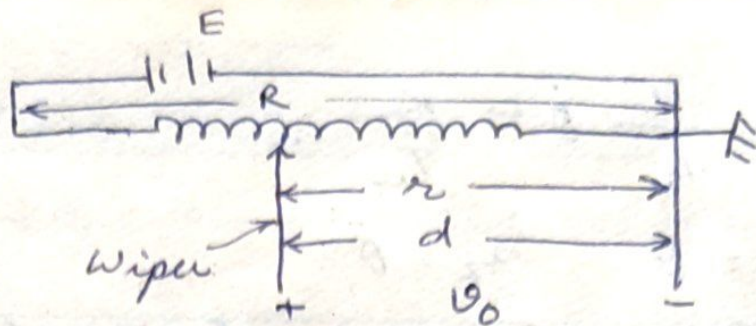
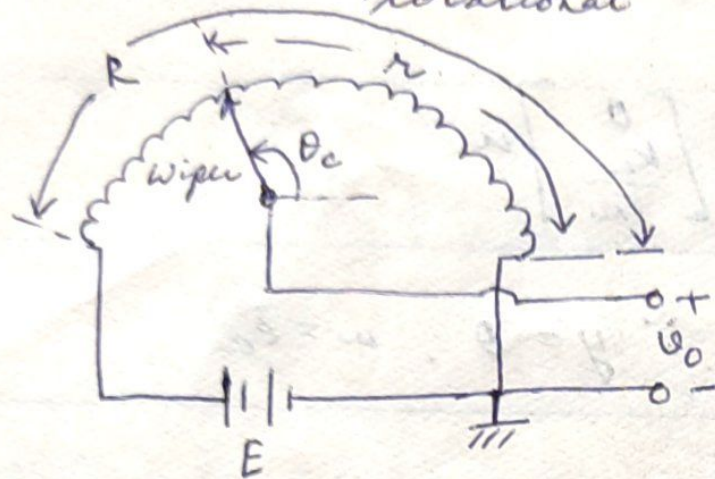
①  $L_f$  NOT negligible.

② not  $i_a$  constant needed but  $e_b$  cons.



# Sensors

1. Potentiometers Linear  
rotational

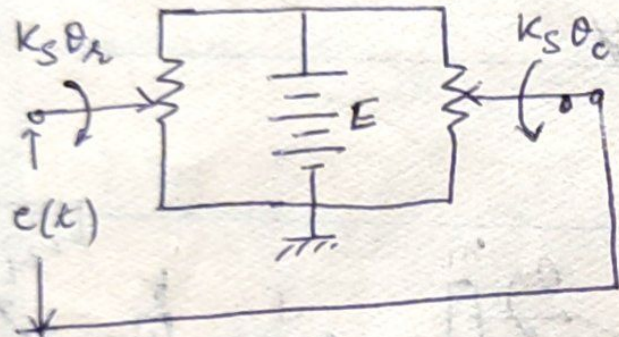


$$V_o = K_s \theta_c(t)$$

$$\text{or } = K \theta_c \cdot E$$

where  $K_s = \frac{E}{2\pi N}$  V/rad;  $K = \frac{1}{2\pi N}$  rad<sup>-1</sup>

when it is a  $N$ -turn rotary pot.

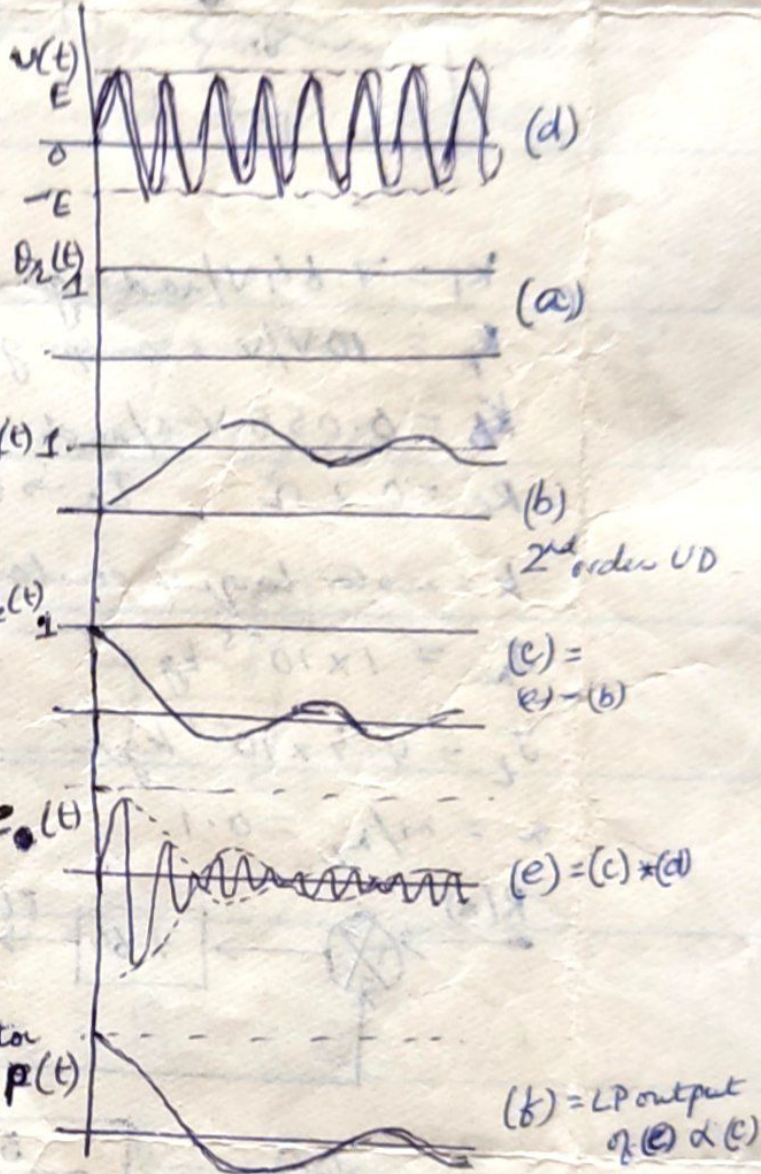
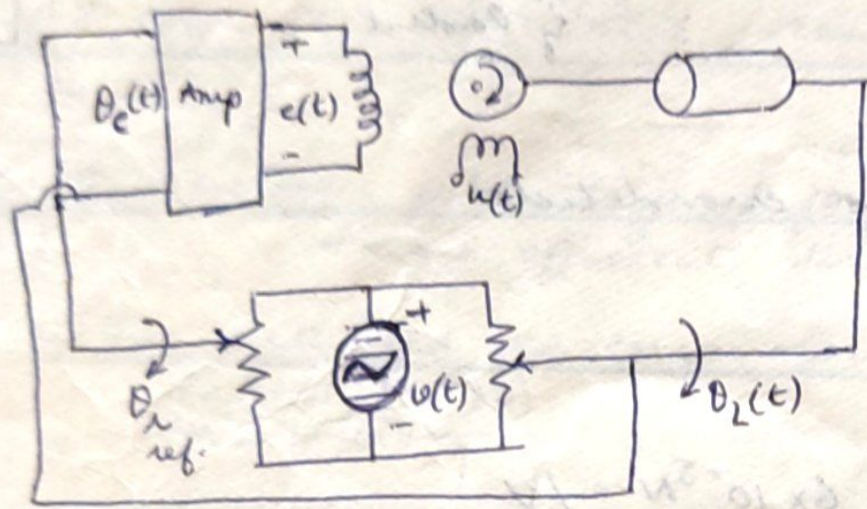


$$e(t) = K_s [\theta_r - \theta_c] \text{ for error detector}$$

$$= K [\theta_r - \theta_c] E$$



AC position servo:



CARRIER  $v = E \sin \omega_c t$

AMP. MODULATION

$e = k_s \theta_e v$      let  $\theta_e = \sin \omega_s t$       $\omega_s \ll \omega_c$

$= \frac{1}{2} k_s E [\cos(\omega_c - \omega_s)t + \cos(\omega_c + \omega_s)t]$      (where  $k = \frac{1}{2kN}$ )

$= \frac{1}{2} k_s [\cos(\omega_c - \omega_s)t - \cos(\omega_c + \omega_s)t]$   
 Motor acts as integrator

ac — used in aerospace applications for noise suppression (signal drifts - LF!) of  $\theta(t)$

- 50Hz Normal
- 400Hz Military
- 1000Hz Aerospace

(f) = LP output of (e) & (c)

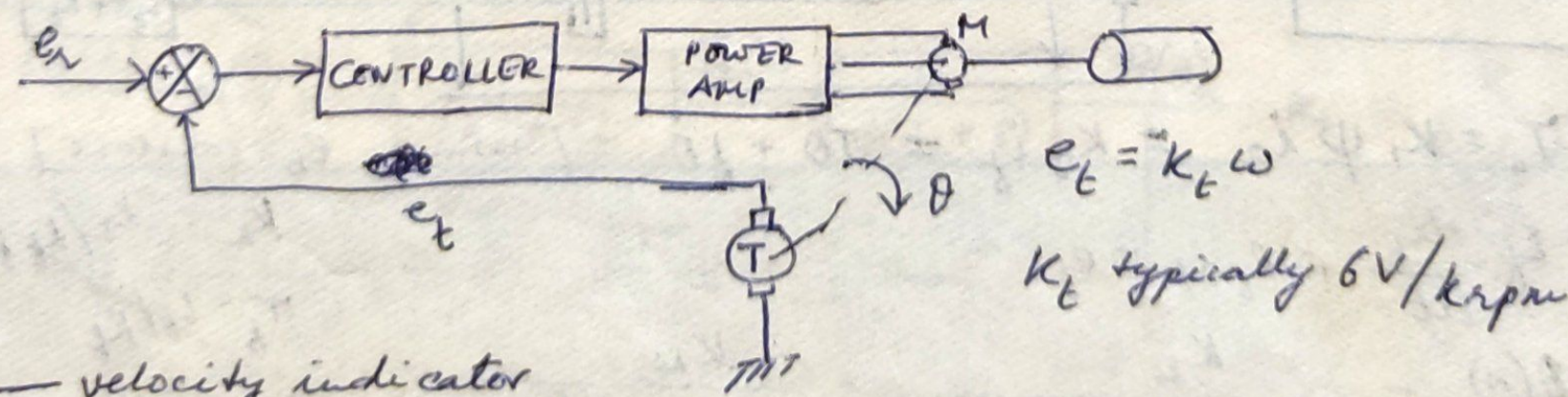


2. Tachometers: Tachogenerators — Mech energy to electrical energy

DC tachometer: a) iron core rotor, field from PM, no external supply voltage  
 b) moving coil tacho: iron less rotor + armature cantilevered bet. PM poles

AC tacho: Quad. arrangement, rotor shaft rotation  $\propto$   $\omega$  of voltage

$\phi$  (phase)  $= f$  (direction of rotation)  $\propto$  Speed



- velocity indicator
- velocity fb. for speed control [need accuracy as in fb loop]
- velocity fb for improving stability & damping — inner fb loop

∴ NON CRITICAL.