Linear approx. of Physical Systems:

System at rest subjected to excitation $x_1(t) \to$ response $y_1(t)$

	excitation	response
	$x_2(t)$	$y_2(t)$
SUPERPOSITION	$x_1 + x_2$	$y_1 + y_2$
HOMOGENEITY	αx_1	αy_1

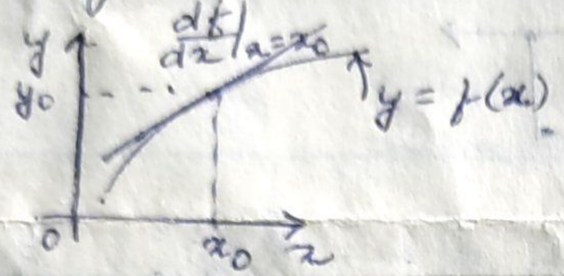
(AFFINE)

$\therefore$ $y = mx + c$ is NOT Linear violates homogeneity

$y = x^{\alpha}$ violates superposition

BUT $y = mx + c$ is linear about an operating point (x_0, y_0) for small changes Δx, Δy

$\therefore \quad y_0 + \Delta y = m x_0 + m \Delta x + c \Rightarrow \Delta y = m \Delta x$



$$y(t) = f(x(t))$$

$$\therefore y = f(x) = f(x_0) + \frac{df}{dx}\Big|_{x=x_0} \cdot \frac{x - x_0}{1!} + \frac{d^2 f}{dx^2}\Big|_{x=x_0} \frac{(x-x_0)^2}{2!}$$

Notions of Jacobian matrix

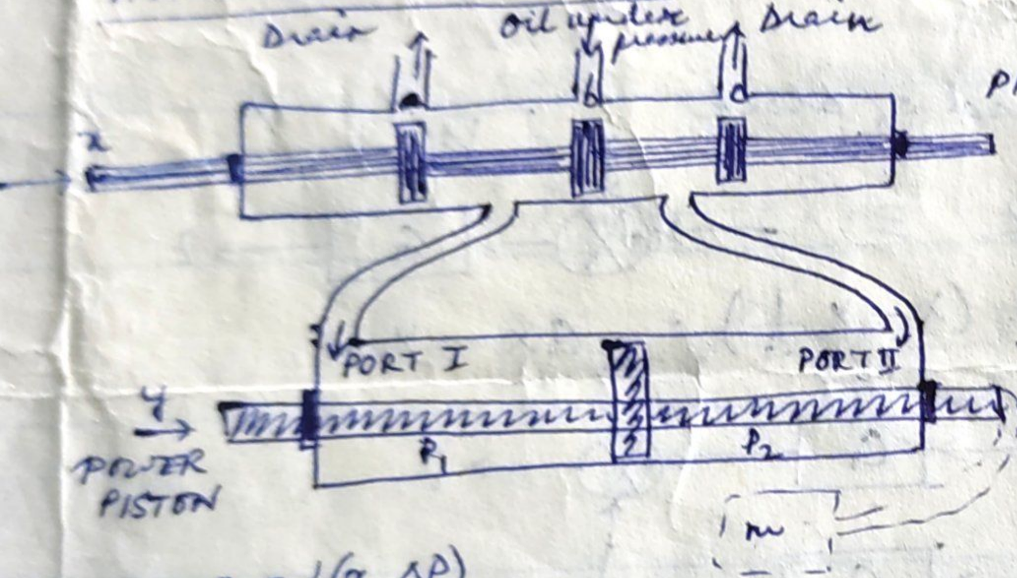
Hessian matrix.

$$y = f(x_1, x_2):$$

$$= f(x_{10}, x_{20}) + \left[\frac{\partial f}{\partial x_1}(x_1 - x_{10}) + \frac{\partial f}{\partial x_2}(x_2 - x_{20})\right]$$

$$+ \frac{1}{2!}\left[\frac{\partial^2 f}{\partial x_1^2}(x_1 - x_{10})^2 + 2\frac{\partial^2 f}{\partial x_1 \partial x_2}(x_1 - x_{10})(x_2 - x_{20}) + \frac{\partial^2 f}{\partial x_2^2}(x_2 - x_{20})^2\right] + \cdots$$

HYDRAULIC ACTUATOR: (FLUIDIC SYS + MECH. SYS)



PILOT VALVE

x shift rt. ⇒ Port I to oil
Port II to drain

∴ y shift rt.

Note:
1. Some leakage through valves allowed
2. Dither (high freq. low amp. signal) signal superimposed on pilot valve motion — improves sensitivity and linearity

LIQ: $q = f(x, \Delta P)$ $\left[dq = \underbrace{\frac{\partial q}{\partial x}}_{K_1} dx + \underbrace{\frac{\partial q}{\partial P}}_{-K_2} d\Delta P\right]$

$$q - \bar{q} = K_1 (x - \bar{x}) - K_2 (\Delta P - \Delta\bar{P})$$

oil mass flow rate; driving displacement; diff. pressure across power piston

$$q = A\rho \frac{dy}{dt} \quad \therefore \quad K_2 \Delta P = K_1 x - A\rho\frac{dy}{dt} \quad \therefore \quad \Delta F = A\Delta P = \frac{A}{K_2}(K_1 x - A\rho\dot{y})$$

PISTON: $m\ddot{y} + f\dot{y} = \dfrac{A}{K_2}(K_1 x - A\rho\dot{y})$ [+ Ky if spring loaded piston!]

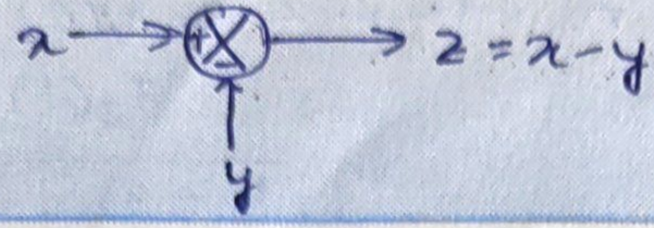
$$\therefore G(s) = \frac{Y(s)}{X(s)} = \frac{1}{s\left[\left(\frac{mK_2}{AK_1}\right)s + \frac{fK_2}{AK_1} + \frac{A\rho}{K_1}\right]} = \frac{K}{s(Ts+1)}$$

where $K = \dfrac{1}{\frac{fK_2}{AK_1} + \frac{A\rho}{K_1}}$; $T = \dfrac{mK_2}{fK_2 + A^2\rho}$

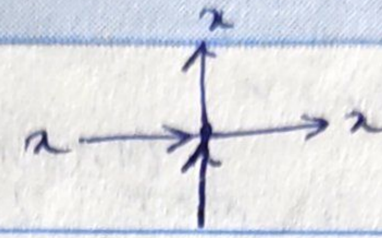
MANAGEMENT REPORT

BLOCK DIAGRAMS :

SUMMING PT.

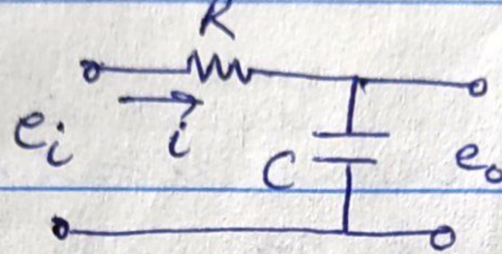


BRANCH PT. / PICK OFF PT.



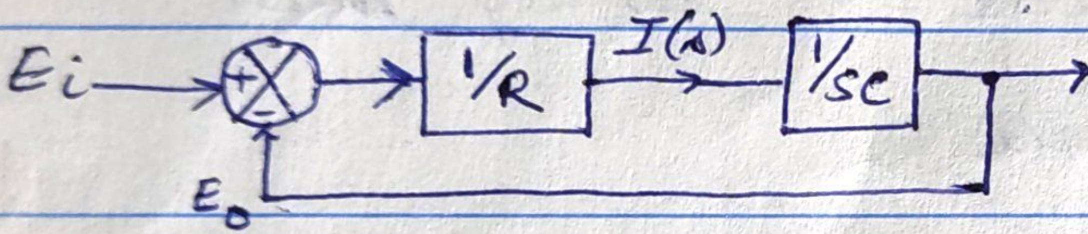
$$i = \frac{e_i - e_o}{R}$$

$$e_o = \frac{1}{C}\int i\,dt.$$



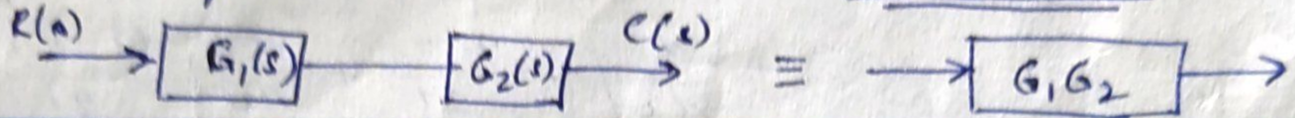
$$I(s) = \frac{E_i - E_o}{R}$$

$$E_o(s) = \frac{I(s)}{sC}.$$

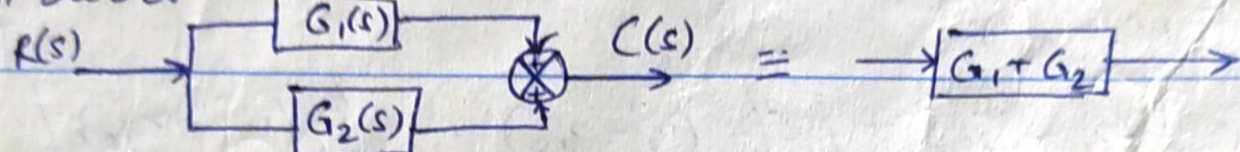


Rules:

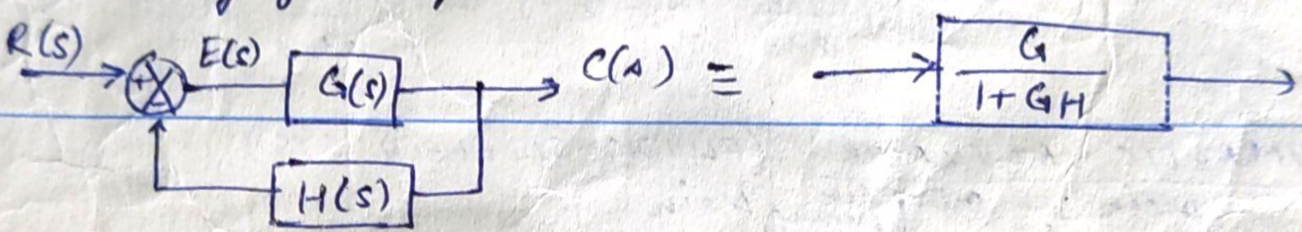
1. Combining blocks in cascade: NO LOADING



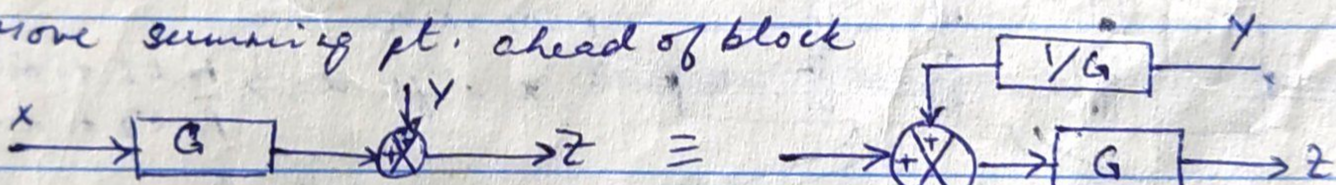
2. ∥ blocks



3. Eliminating fb. loop



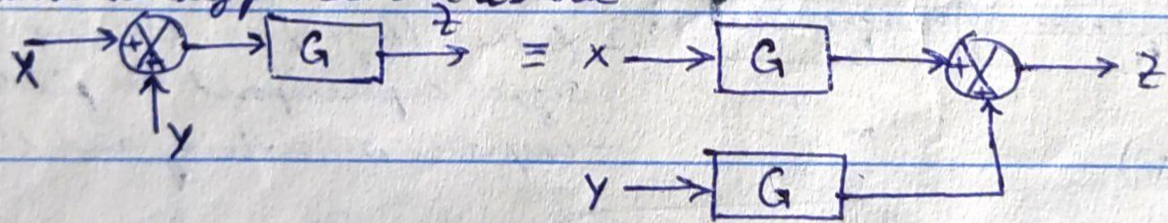
4. Move summing pt. ahead of block



$$Z = G\left(X + Y \cdot \frac{1}{G}\right) = GX + Y$$

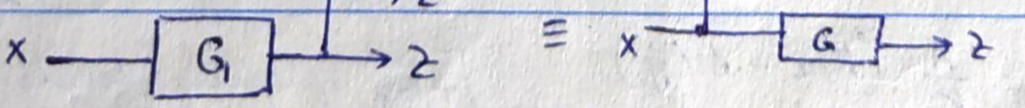
move summing pt. behind a block

5.

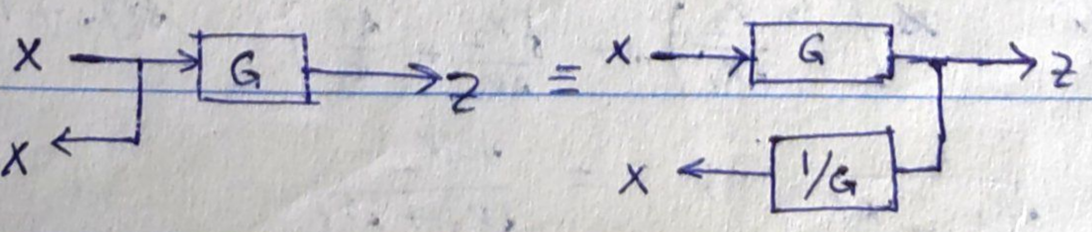


$$Z = (X+Y)G \qquad = XG + YG$$

6. Move a pick-off pt. ahead of block

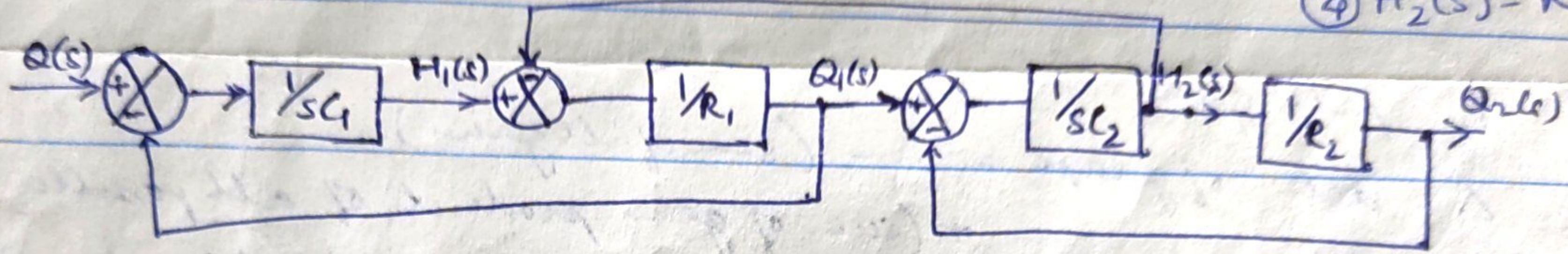


7. Move pick-off pt. behind a block



① $sC_1 H_1(s) = Q(s) - Q_1(s)$ ② $H_1(s) - H_2(s) = R_1 Q_1(s)$ ③ $sC_2 H_2(s) = Q_1(s) -$

④ $H_2(s) = R_2 Q_2(s)$.



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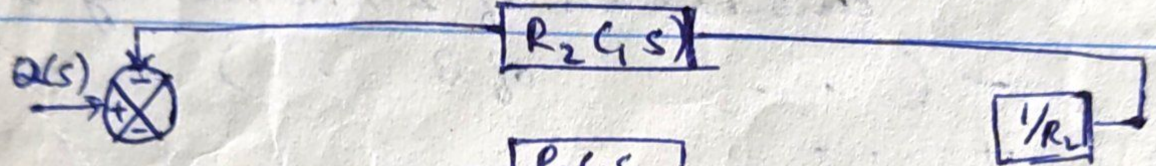

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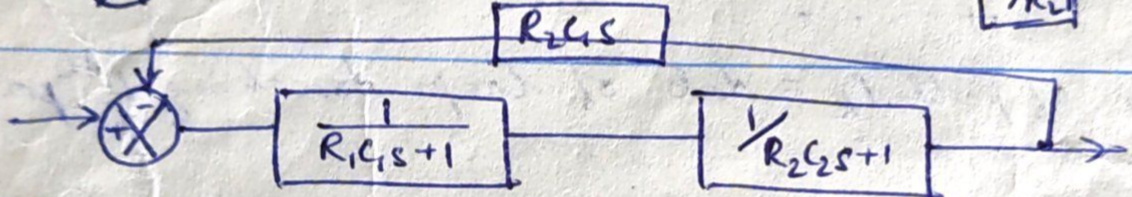
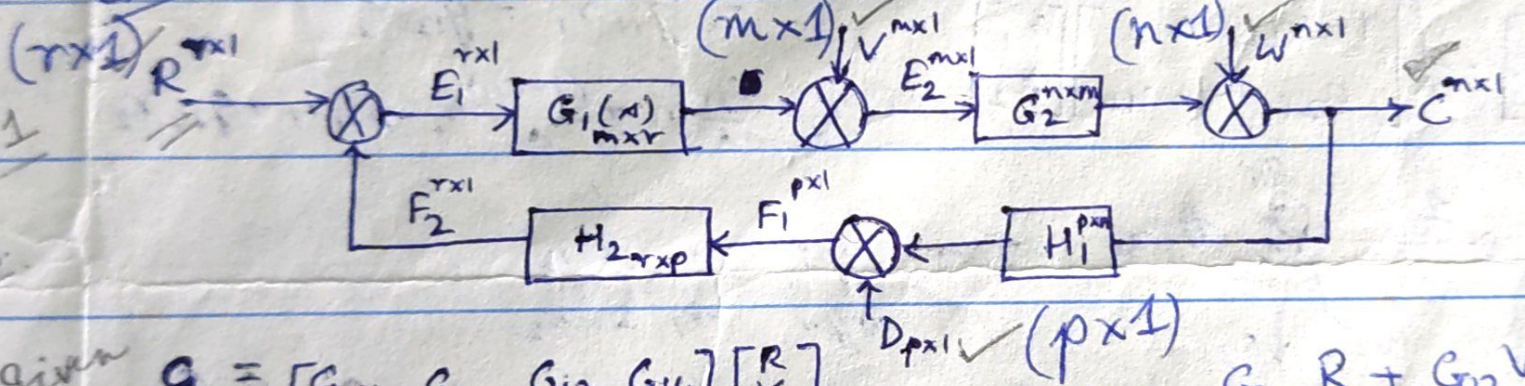


≡

$$Q(s) \longrightarrow \boxed{\frac{1}{R_1C_1R_2C_2s^2 + (R_1C_1 + R_2C_2 + R_2C_1)s + 1}} \longrightarrow Q_2(s).$$

TRANSFER MATRIX OF MULTIVARIABLE SYSTEMS:

torial 1



given $C = [G_{11}\ \ G_{12}\ \ G_{13}\ \ G_{14}]\begin{bmatrix} R \\ V \\ W \\ D \end{bmatrix}$ $\Rightarrow$ $C = G_{11}R + G_{12}V + G_{13}W + G_{14}D$

Find $G_{11} = \left.\frac{C}{R}\right|_{V,W,D=0} = \frac{G_1G_2}{1+G_1G_2H_1H_2}$ $\quad G_{12} = \left.\frac{C}{V}\right|_{R,W,D=0} = \frac{G_2}{1+G_1G_2H_1H_2}$

$G_{13} = \left.\frac{C}{W}\right|_{R,V,D=0} = \frac{\cancel{G_2}\ 1}{1+G_1G_2H_1H_2}$ $\quad G_{14} = \left.\frac{C}{D}\right|_{R,V,W=0} = \frac{H_2G_1G_2}{1+G_1G_2H_1H_2}$

Dimensions of all matrices and intermediate variables.

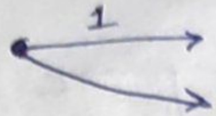
SIGNAL FLOW GRAPH:

Node
Branch
path — no pt. crossed while traversing from pt. 1 to 2
Loop

Loopgain : ~~transmittance~~ gain product in a loop.

Forward path
Feedback path.

Input node
SOURCE



O/p node
SINK

Mixed node

NON TOUCHING LOOPS

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MASON'S GAIN FORMULA

$$T(s) = \frac{\sum_k P_k \Delta_k}{\Delta}$$

P_k : path gain

Δ = det. of graph = 1 − (sum of all gains)
+ (sum of gain products of all possible combinations of 2 nontouching loops)
− (. . . of 3 . . .)
+ . . .

Δ_k = cofactor of path P_k = det. of loops touching k^{th} path.
remove all loops touching path P_k from Δ to obtain Δ_k

Say

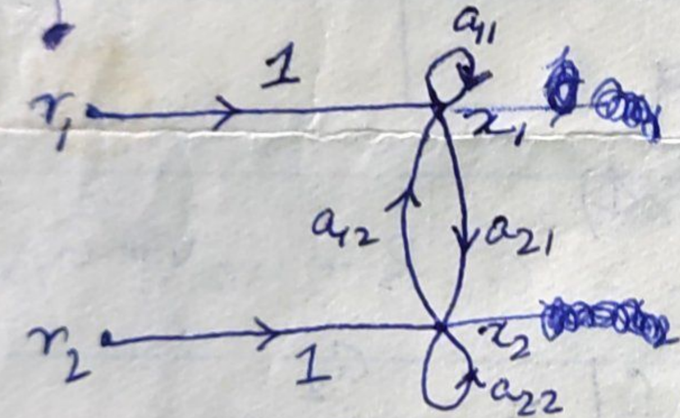
$$\begin{cases} a_{11}x_1 + a_{12}x_2 + r_1 = x_1 \\ a_{21}x_1 + a_{22}x_2 + r_2 = x_2 \end{cases}$$

$$\Rightarrow \begin{bmatrix} 1-a_{11} & -a_{12} \\ -a_{21} & 1-a_{22} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} r_1 \\ r_2 \end{bmatrix} \Rightarrow x_1 = \frac{(1-a_{22})r_1 + a_{12}r_2}{\Delta}$$

$$\Rightarrow \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \frac{1}{\Delta} \begin{bmatrix} 1-a_{22} & a_{12} \\ a_{21} & 1-a_{11} \end{bmatrix} \begin{bmatrix} r_1 \\ x_2 \end{bmatrix} \qquad = g_{11}r_1 + g_{12}r_2$$

$$x_2 = \frac{a_{21}}{\Delta}r_1 + \frac{(1-a_{11})}{\Delta}r_2 = g_{21}r_1 + g_{22}r_2$$

$\therefore$



Two i/ps r_1 & r_2

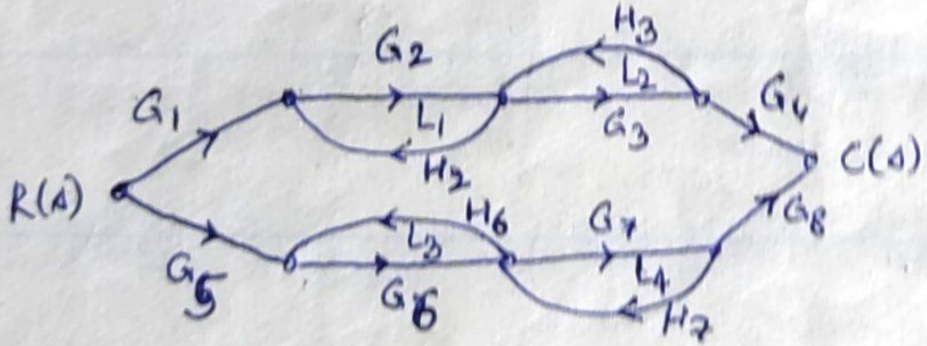
o/ps x_1 & x_2

Individual loop gains a_{11}, a_{22}, $a_{12}a_{21}$

Gain product of two NON TOUCHING LOOPS $a_{11}a_{22}$

$$\therefore \Delta = 1 - (a_{11} + a_{22} + a_{12}a_{21}) + a_{11}a_{22}$$

Ex 1



a) Forward paths : $P_1 = G_1 G_2 G_3 G_4$ $\quad P_2 = G_5 G_6 G_7 G_8$

b) Four selfloops (individual)

$P_{11} = G_2 H_2 \quad P_{21} = H_3 G_3 \quad P_{31} = G_6 H_6$

$P_{41} = G_7 H_7$

c) $P_{12} = P_{11} P_{31} = G_2 G_6 H_2 H_6$; $P_{22} = P_{11} P_{41} = G_2 G_7 H_2 H_7$

$P_{32} = P_{21} P_{31} = G_3 G_6 H_3 H_6 \quad P_{42} = P_{21} P_{41} = G_3 G_7 H_3 H_3$

} Combinations of two non-touching loops.

d) No 3 NON-TOUCHING LOOPS.

e) Cofactor of Δ / P_1 → remove loops touching P_1

$$T(s) = \frac{P_1 \Delta_1 + P_2 \Delta_2}{\Delta} = \frac{G_1 G_2 G_3 G_4 (1 - G_6 H_6 - G_7 H_7) + G_5 G_6 G_7 G_8 (1 - G_2 H_2 - G_3 H_3)}{1 - P_{11} - P_{21} - P_{31} - P_{41} + P_{12} + P_{22} + P_{32} + P_{42}}$$