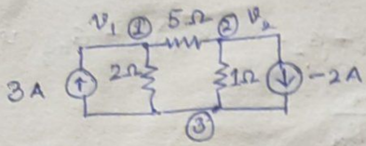
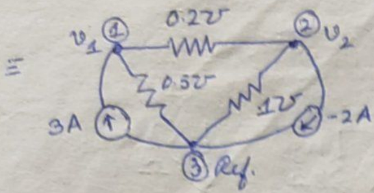
Nodal analysis: N node ckt : (N-1) voltages and (N-1) equations.





Let reference node be ③ → node to which largest no. of branches connected.

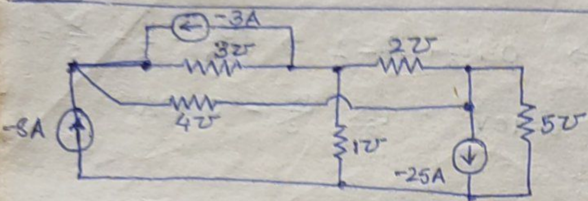
Voltages defined w.r.t. ③.

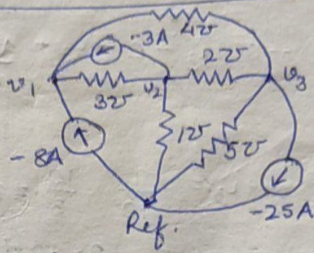
If ref. marked, then signs of v_1, v_2 need not be marked.

KCL at node ①: $-3 + 0.2(v_1 - v_2) + 0.5 v_1 = 0$

node ②: $-2 + 1v_2 + 0.2(v_2 - v_1) = 0$.

$\therefore v_1 = 5V; \quad v_2 = 2.5V$.





KCL at node ①: $+8 + 3 + 3(v_1 - v_2) + 4(v_1 - v_3) = 0$

②: $-3 + 3(v_2 - v_1) + 1v_2 + 2(v_2 - v_3) = 0$.

③: $-25 + 5v_3 + 2(v_3 - v_2) + 4(v_3 - v_1) = 0$.

Current leaving +ve

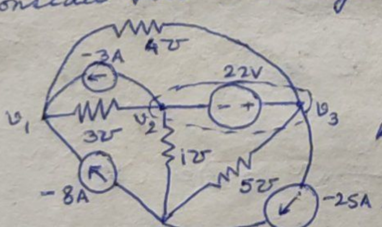
$$\Rightarrow \begin{aligned} 7v_1 - 3v_2 - 4v_3 &= -11 \\ -3v_1 + 6v_2 - 2v_3 &= 3 \\ -4v_1 - 2v_2 + 11v_3 &= 25 \end{aligned}$$

Cramer's rule: $v_1 = \dfrac{\begin{vmatrix} -11 & -3 & -4 \\ 3 & 6 & -2 \\ 25 & -2 & 11 \end{vmatrix}}{\begin{vmatrix} 7 & -3 & -4 \\ -3 & 6 & -2 \\ -4 & -2 & 11 \end{vmatrix}} = 1V$.

$v_2 = 2V, \quad v_3 = 3V$.

Conductance matrix: $G = \begin{bmatrix} 7 & -3 & -4 \\ -3 & 6 & -2 \\ -4 & -2 & 11 \end{bmatrix}$: symmetrical about prin. axis.
all elements on diag: +ve
rest

Consider that a voltage source bet. v_2 and v_3 in place of 2℧ conductance.



If sum of i leaving v_2 or v_3 = 0, then sum of i leaving supernode $(v_2, v_3) = 0$.

KCL at node (v_2, v_3):

$\therefore 3(v_2 - v_1) - 3 + 4(v_3 - v_1) - 25 + 5v_3 + 1v_2 = 0$

~~at node ①~~ $-7v_1 + 4v_2 + 9v_3 = 28$

$7v_1 - 3v_2 - 4v_3 = -11$.

Reln. bet. v_2 & v_3: $-v_2 + v_3 = 22$.

$v_1 = \dfrac{\begin{vmatrix} 28 & 4 & 9 \\ -11 & -3 & -4 \\ 22 & -1 & 1 \end{vmatrix}}{\begin{vmatrix} -7 & 4 & 9 \\ 7 & -3 & -4 \\ 0 & -1 & +1 \end{vmatrix}} = -4.5V$

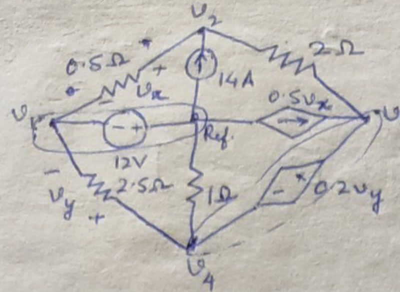
Comments:
1. No conductance matrix
2. No symmetry @ diag.
3. KCL to no. of nodes reduced by 1 for a voltage source.
\+ voltage reln.

$v_1 = -12V$.

KCL at ②: $-14 + \frac{(v_2 - v_1)}{0.5} + \frac{(v_2 - v_3)}{2} = 0$

at supernode ③-④:

$$-0.5v_x + \frac{v_3 - v_2}{2} + \frac{v_4}{1} + \frac{v_4 - v_1}{2.5} = 0$$



$v_3 - v_4 = 0.2v_y = 0.2(v_4 - v_1)$.

Also $v_x = v_2 - v_1$.

$\therefore$ $-14 + \frac{12}{0.5} + 2.5v_2 - 0.5v_3 = 0$

$-2v_1 + 2.5v_2 - 0.5v_3 = 14$

$v_1 = -12$

$0.1v_1 - v_2 + 0.5v_3 + 1.4v_4 = 0$

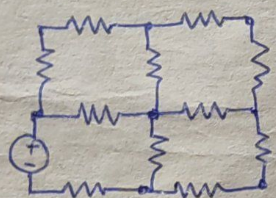
$0.2v_1 + v_3 - 1.2v_4 = 0$.

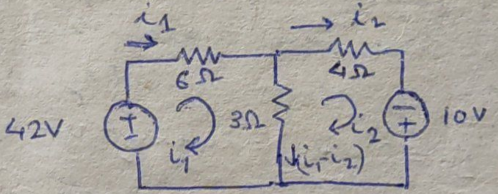
$v_1 = -12V$; $v_3 = 0$, $v_2 = -4V$, $v_4 = -2V$.

<u>Mesh analysis</u>: Applicable <u>only</u> to <u>planar</u> networks.

<u>Planar</u>: If the diag. of a ckt. can be drawn on a plane surface s.t. no branch passes over or under any branch.

<u>Mesh</u>: A loop which does not contain any other loops w/i it.



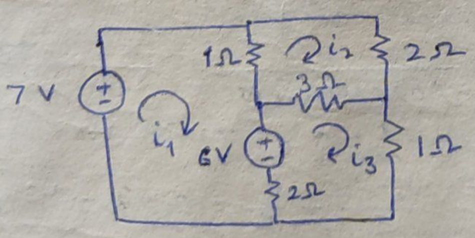


KVL for LH mesh: $-42 + 6i_1 + 3(i_1 - i_2) = 0$

RH : $4i_2 - 10 - 3(i_1 - i_2) = 0$.

$i_1 = 6A$, $i_2 = 4A$, $i_1 - i_2 = 2A$.

$\therefore$ M-meshes: M branch currents & M independent eqns.



$3i_1 - i_2 - 2i_3 = 1$

$-i_1 + 6i_2 - 3i_3 = 0$

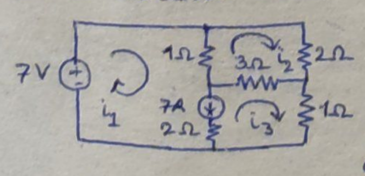
$-2i_1 - 3i_2 + 6i_3 = 6$.

$i_3 = 3A$, $i_1 = 3A$, $i_2 = 2A$.

Resistance matrix $R = \begin{bmatrix} 3 & -1 & -2 \\ -1 & 6 & -3 \\ -2 & -3 & 6 \end{bmatrix}$

Consider current source in place of voltage source of 6V.

— Supermesh whose interior is that of meshes 1 & 3.



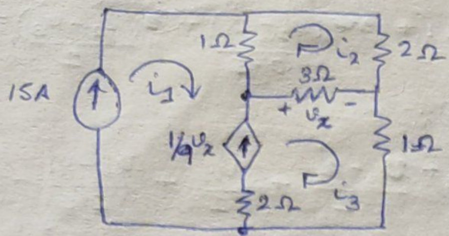
KVL of S.mesh : $-7 + 1(i_1 - i_2) + 3(i_3 - i_2) + 1i_3 = 0$

KVL@1 : $1(i_2 - i_1) + 2i_2 + 3(i_2 - i_3) = 0$.

Current source relu : $i_1 - i_3 = 7$

$i_3 = 2A$, $i_1 = 9A$, $i_2 = 2.5A$.

For both dependent and independent source:



Only one KVL for mesh 2 : $1(i_2-i_1)+2i_2+3(i_2-i_3)=0$

$i_1 = 15A$

$i_3 - i_1 = \frac{1}{9}v_x = \frac{1}{9}[3(i_3-i_2)]$

$i_1 = 15$, $i_2 = 11$, $i_3 = 17A$.

Linearity and Superposition :

Linear element : Passive element that has a linear voltage-current relationship

Linear : satisfies principle of superposition — v. useful practically.

HOMOGENEITY ① $a \cdot Kb = K(ab)$ | $f(a_1x_1 + a_2x_2) = a_1f(x_1) + a_2f(x_2)$.

SUPERPOSITION ② $\alpha(a+b) = \alpha a + \alpha b$. $y = mx + c$ violates (2) | $y = ax^1 + b.x^0$, $2y = a.2x^1 + b.2x^0$

or linear current-voltage reln. → multiplication of ~~time varying~~ it i through element by constant K results in multiplication of v across element by same constant K.

$[v = iR \therefore (Ki).R = K(iR) = K.v]$

Principle of superposition: the response at any pt. in the linear ckt. having more than one independent source can be obtained as a sum of the responses caused by each independent source acting alone. $[v = R(i_1+i_2) = Ri_1 + Ri_2 = v_1 + v_2]$

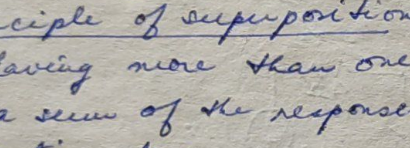
Resistor, inductor, capacitor, mutually coupled inductor — all linear.

Linear dependent source: Dependent current or voltage source whose o/p i or v $\propto$ only to first power of some i or v variable in the circuit or to the sum of such quantities.

$\therefore v_s = 0.6\,i_1 - 14v_2$ linear NOT $v_s = 0.6i_1^2$ or $v_s = 0.6\,i_1v_2$.

$\Rightarrow \alpha v_s = 0.6(\alpha i_1) - 14.(\alpha v_2)$ $\quad \alpha v_s \neq 0.6(\alpha i_1)^2 \quad \alpha v_s \neq 0.6(\alpha i_1).(\alpha v_2)$.

Linear circuit : ckt. composed entirely of independent sources, linear dependent sources and linear elements.



Sources ⟶ FORCING FNS.

Voltages produced ⟶ RESPONSE FNS.

KCL at ① : $i_a = \frac{v_1 - v_2}{5} + \frac{v_1}{2} = 0.7v_1 - 0.2v_2$

② : $i_b = -0.2v_1 + 1.2v_2$

changing sources to $i_{ax}, i_{bx} \longrightarrow v_{1x}, v_{2x}$ and then $i_{ay}, i_{by} \longrightarrow v_{1y}, v_{2y}$ then add or superpose :

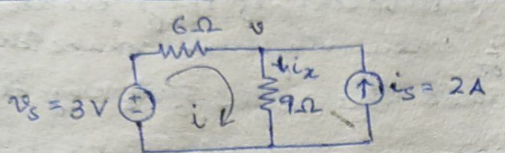
$i_{ax} + i_{ay} = 0.7(v_{1x}+v_{1y}) - 0.2(v_{2x}+v_{2y})$

$i_{bx} + i_{by} = -0.2(v_{1x}+v_{1y}) + 1.2(v_{2x}+v_{2y})$.

$\therefore$ Responses can be taken independently and simply added.

Superposition Theorem :

In any linear resistive network containing several sources, the voltage across or the current through any resistor or source may be calculated by adding algebraically all the individual voltages or currents caused by each independent source acting alone, with all the other independent voltage sources replaced by short ckts. & all other independent current sources replaced by open circuits.



Reqd. : expression for i_x.

1. Current source = 0 ∴ open ckt.

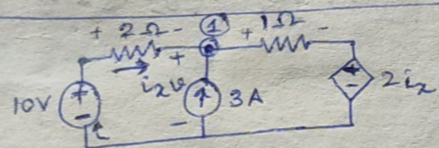
$\therefore i_x = \frac{3V}{15\Omega} = \frac{1}{5} A = 0.2 A.$

2. Voltage source = 0, short ckt.

$\therefore i_x = \frac{6}{15} \times 2 A = \frac{12}{15} A = \frac{4}{5} A = 0.8 A$ ∴ Total $i_x = 1 A$

① $\frac{v-3}{6} + \frac{v}{9} = 2 \Rightarrow \frac{5v}{18} = \frac{5}{2} \Rightarrow v = 9V$ ∴ $i_x = \frac{9}{9} = 1A$.

② $-3 + 6i + 9(i+2) = 0 \Rightarrow 15i + 15 = 0 \Rightarrow i = -1$ ∴ $i_x = i + 2 = 1A$.



Seek i_x

1. Open ckt. 3A source.

$\therefore -10 + 5 i_x' = 0$ ∴ $i_x' = 2 A$.

2. Short ckt. 10V source. KCL at node ①: $\frac{v''}{2} + \frac{v'' - 2i_x''}{1} = 3.$

short ckted left branch: $v'' = -2i_x''$

$\therefore i_x'' = -0.6 A$

$\therefore i_x = 2 - 0.6 = 1.4 A.$

Limitation of Superposition: ① not much time saved if dependent sources present since then at least two sources to be considered each time.

② applicable only to linear responses ∴ power/energy not subject to superposition.

Source transformations:

Practical source ≡ ideal source only as long as relatively small currents / power drawn.

Automobile start with headlights on: starter current of 100 A ∴ headlights perceptibly dimmed.

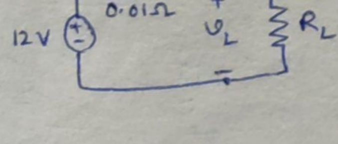
Say, for no current flow, battery terminal voltage = 12V

100 A , — do — = 11V.

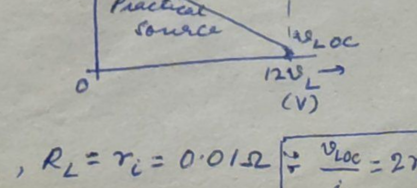
∴ Practical voltage source : 12 V in series with 0.01Ω

∴ Practical voltage source : 12V in series with 0.01Ω when 100 A flows.

∴ drop of 1V across 0.01Ω when 100 A flows.



v_L : practical voltage



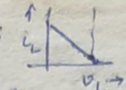
$v_{LOC} = r_i \cdot i_L + v_L$

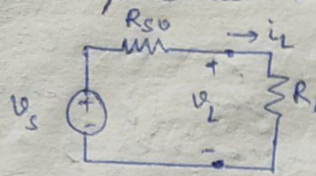
$12 = 0.01 i_L + v_L$

$\therefore i_L = 1200 - 100 v_L$

So, each pt. on line → diff. R_L, when $v_L = \frac{1}{2} v_{LOC}$, $R_L = r_i = 0.01\Omega$ ∵ $\frac{v_{LOC}}{i_L} = 2 r_i$

General practical voltage source :

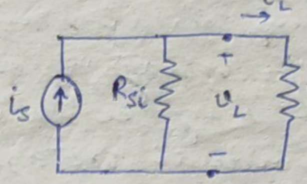




$v_L = v_s - R_{sv}\, i_L$

$v_{Loc} = v_s$

$i_{Lsc} = \dfrac{v_s}{R_{sv}}$



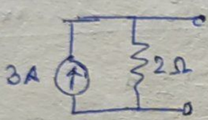
$i_L = i_s - \dfrac{v_L}{R_{si}}$

$i_{Lsc} = i_s \quad , \quad v_{Loc} = R_{si}\, i_s$

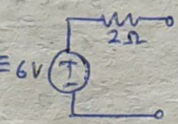
Equivalent sources : Two sources are equivalent if they produce identical values of v_L and i_L when they are connected to identical values of R_L, no matter what the value of R_L may be.

$\therefore$ $R_L = \infty$, $R_L = 0$ $\longrightarrow$ Equivalent sources provide same open ckt. voltage and short circuit current.

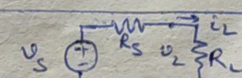
$\therefore$ $v_{Loc} = v_s = R_{si} i_s \quad ; \quad i_{Lsc} = \dfrac{v_s}{R_{sv}} = i_s \qquad \therefore R_{si} = R_{sv} = R_s$ & $v_s = i_s R_s$



$\equiv$



Maximum Power Theorem :



For a practical voltage source, power delivered to the load is

$$p_L = i_L^2 R_L = \left(\frac{v_s}{R_s + R_L}\right)^2 . R_L \qquad \frac{v\,du - u\,dv}{v^2} = \frac{d}{dx}\left(\frac{u}{v}\right)$$

To find value of R_L that absorbs maxm. power from given practical source, differentiate p_L wrt R_L and equate to zero

$$\frac{dp_L}{dR_L} = \frac{(R_s+R_L)^2 . v_s^2 - v_s^2 R_L . 2(R_s+R_L)}{(R_s+R_L)^4} = 0$$

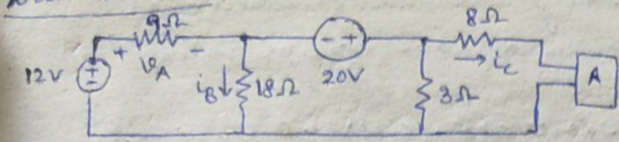
$\Rightarrow (R_s + R_L) - 2R_L = 0 \Rightarrow R_s = R_L$.

$\boxed{\text{Diff. again : } \dfrac{d^2 p_L}{dR_L^2} \longrightarrow \text{-ve for } R_s = R_L \text{ Check.}}$

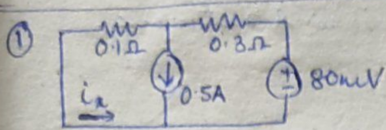
$R_L = 0$ or $R_L = \infty$ $\rightarrow$ both give minm. $p_L = 0$ $\therefore$ proved.

Maxm. power theorem : An independent voltage source in series with a resistance R_s or an independent current source with a resistance R_s in parallel delivers maximum power to that load resistance R_L for which $R_L = R_s$.

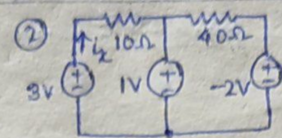
Drill Problems:



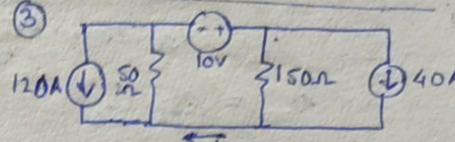
Find v_A if A is
① 4V voltage source 4V   23.7V
② 9Ω 23.6V
③ 600mA   23.9V



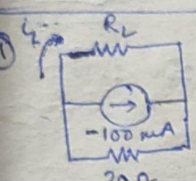
−175 mA



200 mA

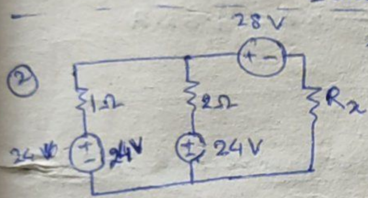


i_x = 50 mA.



a) $R_L = 80\Omega$, $i_L = ?$ b) transform to prac. voltage source,
$R_L = 80\Omega$, $i_L = ?$ c) Power supplied by ideal source in each case?   d) value of R_L for maxm. power, maxm power = ?

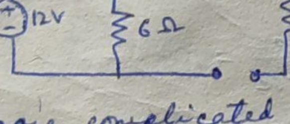
20mA; 20mA; 160mW, 40mW; 20Ω, 50mW.



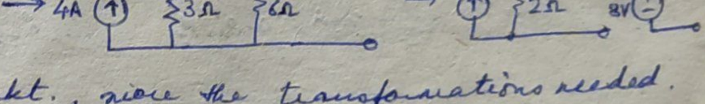
transform both 24V sources to prac. current sources combine resistors & ideal current sources, transform prac. current source back to prac. voltage source, combine ideal voltage sources.

Power delivered to $R_x = ?$
a) if $R_x = 2\Omega$, power delivered to any $R_x = ?$
b) maxm. power
c) what two values of $R_x \rightarrow$ exactly 5W delivered to them?
4.5W; 6W; 0.280 & 1.587Ω.

Thevenin's Theorem:



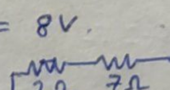
Source transformations



Prob.: ① more complicated the ckt., more the transformations needed.
② dependent sources → usually inapplicable.

THEVENIN'S THEOREM: Given ~~that~~ any linear ckt., rearrange it in the form of two networks A and B that are connected together by two resistanceless conductors. If either network contains a dependent source, its control variable must be in the same network. Define a voltage v_{oc} as the open ckt. voltage which would appear across the terminals of A if B were disconnected so that no current is drawn from A. Then all the currents and voltages in B will remain unchanged if A is killed (all inde. voltages → shorts, i sources → open) and an independent voltage source v_{oc} is connected, with proper polarity in series with the dead (inactive) A network.

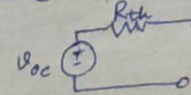
Disconnect R_L ∴ $v_{oc} = v_{6\Omega} = \frac{6}{9} \times 12V = 8V$.
Kill A ; Replace 12V by short. ∴



= Equivalent resistance R_{th}

∴ Result.

∴



Only restriction → ① linear ckt. — original ckt. composed of A & B.
② dependent sources in A have control variables in A.

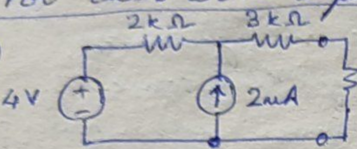
Dual : Norton's theorem : Define short ckt. current i_{sc} ... if B were short ckt.ed so that no voltage is provided by A.

∴ i_{sc} connected in parallel with dead A network.

$v_{oc} = R_{th}\, i_{sc}$

For ckts with dependent sources, evaluate v_{oc}, i_{sc}, then R_{th}.

①

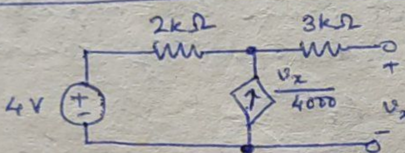


$\xrightarrow{\text{Th. equi}}$ $v_{oc} = v_{oc}\big|_{4V} + v_{oc}\big|_{2mA} = 4 + 2\times10^{-3}\times2\times10^{3}$
$= 8V$

$R_{th} = 5k\Omega$

$\xrightarrow{\text{Norton equi}}$ $i_{sc}\big|_{4V} + i_{sc}\big|_{2mA} = \frac{4}{5}\,mA + 2\cdot\frac{2}{5}\,mA = 1.6\,mA$

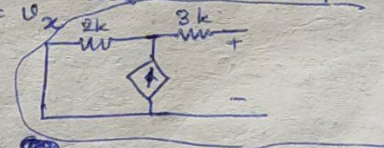
②



Find both v_{oc} & i_{sc}

and i_x must pass through 2kΩ

$v_{oc} = v_x$



∴ Summing voltage @ outer loop:

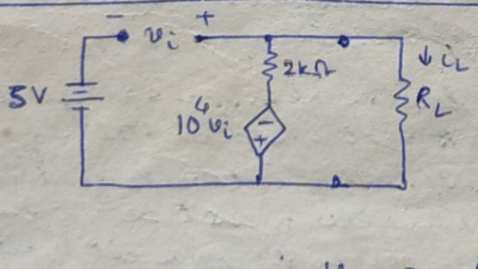
$-4 + 2\times10^{3}\times\left(\frac{-v_x}{4000}\right) + 3\times10^{3}\times0 + v_x = 0$

∴ $v_x = 8 = v_{oc}$

i_{sc}, short o/p ∴ $v_x = 0$. ∴ dependent $i_x = 0$.

∴ $i_{sc} = \frac{4}{5}\,mA = 0.8\,mA$ ∴ $R_{th} = \frac{v_{oc}}{i_{sc}} = 10k\Omega$

③



v_{Loc} , $R_L = \infty$.

∴ no current through 2kΩ

∴ $v_{Loc} = -10^4 v_i$

$v_i = v_{Loc} - 5$.

∴ $v_{Loc} = \frac{10^4}{10001}\times5 \simeq 5V$.

i_{Lsc} : R_L by short. KVL for right mesh.

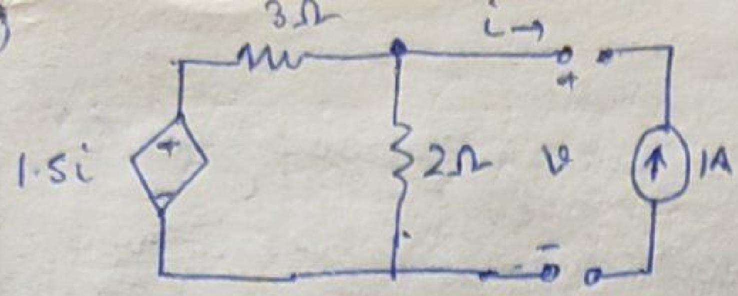
$10^4 v_i + 2000\, i_{sc} = 0$ ∴

KVL @ perimeter : $-5 - v_i = 0$

∴ $10^4\cdot(-5) + 2000\, i_{Lsc} = 0$ ∴ $i_{Lsc} = 25A$.

∴ $R_{th} = \frac{v_{Loc}}{i_{Lsc}} = 0.2\,\Omega$.

(4)



$v_{oc} = 0$ ∵ no inde. source.

∴ Apply 1-A source externally, measure resultant voltage, then set $R_{th} = v/1$.

∴ $i = -1$, $\dfrac{v - 1.5i}{3} + \dfrac{v}{2} = 1$. (KCL at node)

∴ $v = 0.6\,V$.

$R_{th} = 0.6\,\Omega$. ∴ Th. equi. 0.6Ω