I

Laplace transform for Transfer function:

Transfer fn. $G(s) = \dfrac{\mathcal{L}(o/p)}{\mathcal{L}(i/p)} = \dfrac{P(s)}{Q(s)}$

Values of s for which $P(s) = 0 \longrightarrow$ ZEROS $\circ$ $G(s) = 0$

$Q(s) = 0 \longrightarrow$ POLES $\times$ $G(s) = \infty$

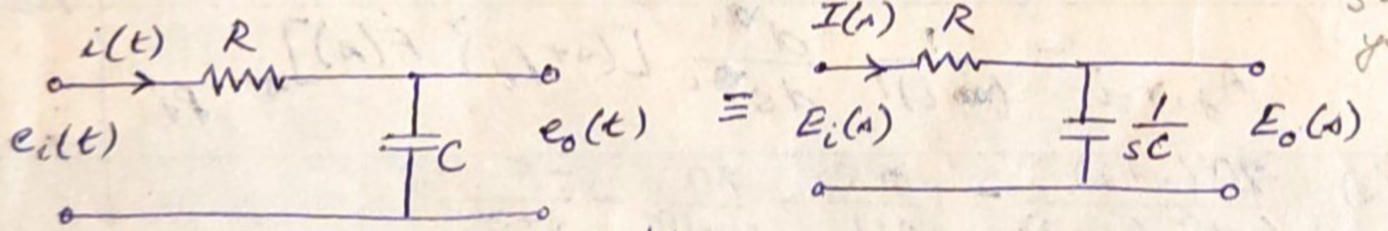
(EIGENVALUES)

$\downarrow$ characteristic equation of system — characteristic roots

$G(s)$ is a RATIONAL fn. $\therefore$ $\eta(\text{zeros}) = \eta(\text{poles})$ including those at ∞.

FINITE & INFINITE ZEROS

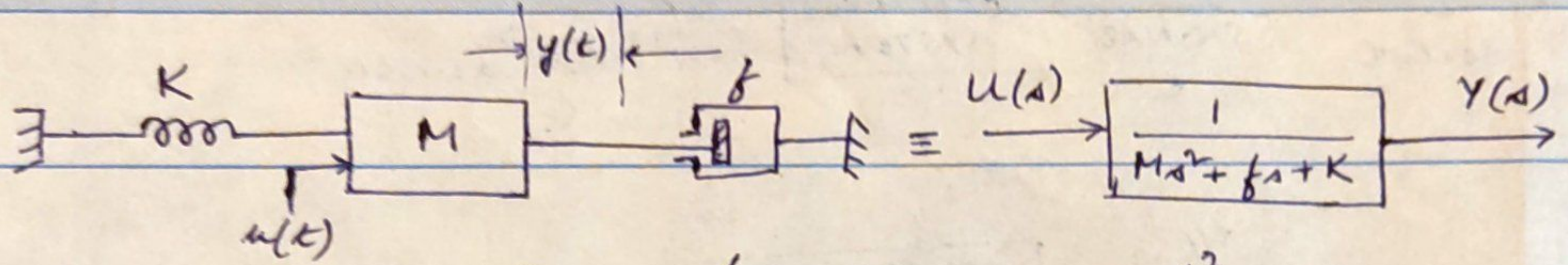
$S = \infty$ in expression yields $G(s) = 0$.



$$\therefore G(s) = \frac{E_0(s)}{E_i(s)} = \frac{1/sC}{R + \frac{1}{sC}} = \frac{1}{1 + RCs}$$

$\longrightarrow$ STATE VARIABLE

(II)



$F_s = -Ky$; $F_f = -f\frac{dy}{dt}$; $F_M = M\frac{d^2y}{dt^2}$

(−) because spring recoils
opp. to applied force $u(t)$

$$\therefore M\frac{d^2y}{dt^2} = u(t) - Ky - f\frac{dy}{dt}$$

$$\Rightarrow u(t) = M\frac{d^2y}{dt^2} + f\frac{dy}{dt} + Ky \quad \therefore G(s) = \frac{Y(s)}{U(s)} = \frac{1}{Ms^2+fs+K}$$

STATE VARIABLE FORMULATION :

$$M \frac{d^2y}{dt^2} + f \frac{dy}{dt} + Ky = u(t)$$

STATE VECTOR $X(t) = [x_1(t) \; x_2(t) \; \ldots \; x_n(t)]^T$

$x_1 \ldots x_n$ are STATE VARIABLES (minimal set) reqd. to describe state of system (dynamic) (describes fn. al behaviour of system for any time $t > t_0$)

IV

Let $y = x_1$ $\qquad \therefore \; M\dfrac{dx_2}{dt} + f x_2 + K x_1 = u(t)$.

$\dot{y} = \dot{x}_1 = x_2$

So $\dot{x}_1 = x_2$

$\dot{x}_2 = -\dfrac{K}{M}x_1 - \dfrac{f}{M}x_2 + \dfrac{1}{M}u(t)$

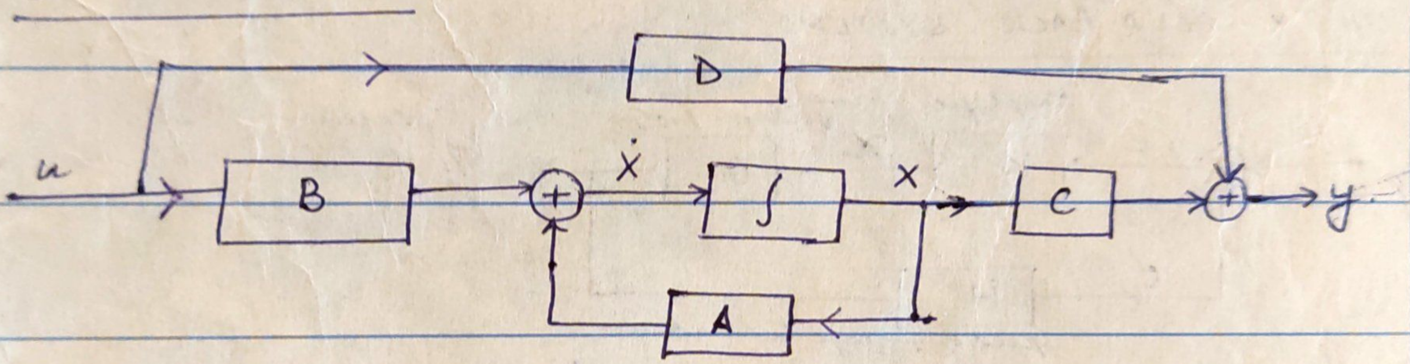
$\therefore \; \dot{X} = AX + Bu$

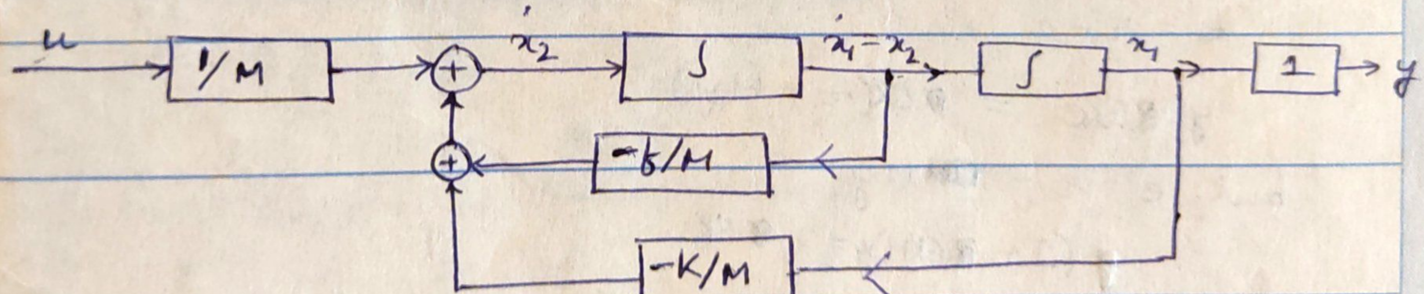
$y = CX + Du$

$A = \begin{bmatrix} 0 & 1 \\ -K/M & -f/M \end{bmatrix} \quad B = \begin{bmatrix} 0 \\ 1/M \end{bmatrix} \quad C = [1 \;\; 0] \quad D = [0]$

$G(s)$

BLOCK DIAGRAM:





Transfer fn. derivation:

$sX(s) - X(0) = AX(s) + BU(s)$

$Y(s) = CX(s) + DU(s)$

$\therefore [sI - A]X(s) = BU(s)$ Assuming $X(0) = 0$.

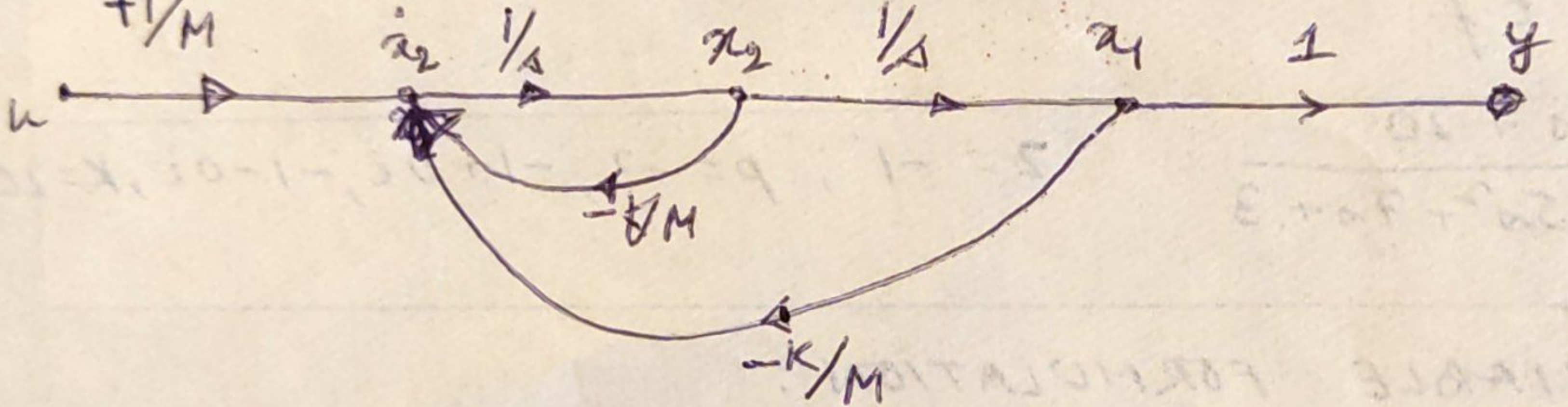
$\therefore X(s) = (sI - A)^{-1}B\,U(s)$

$Y(s) = CX(s) + DU(s)$

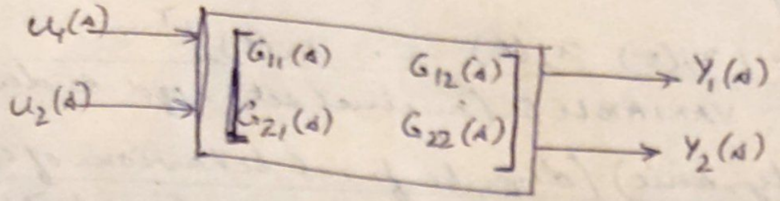
$= [C(sI - A)^{-1}B + D]\,U(s)$

$\therefore G(s) = \dfrac{Y(s)}{U(s)} = C(sI - A)^{-1}B + D$

SIGNAL FLOW GRAPH

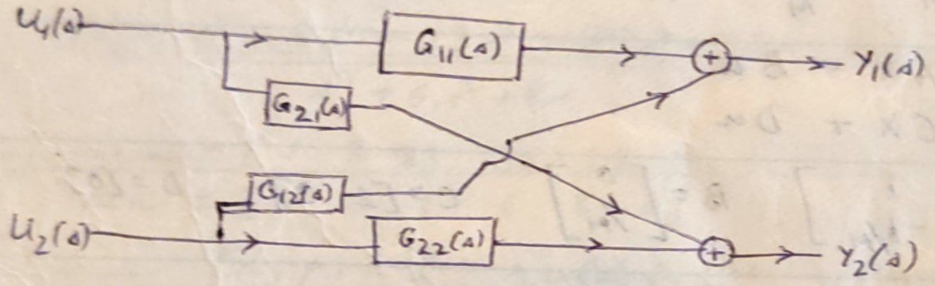


VIII MULTIVARIABLE SYSTEM:

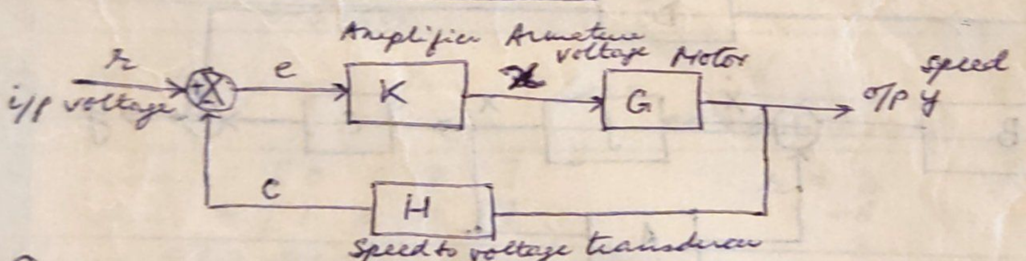


$$Y_1(s) = G_{11}(s)\,U_1(s) + G_{12}(s)\,U_2(s)$$
$$Y_2(s) = G_{21}(s)\,U_1(s) + G_{22}(s)\,U_2(s)$$

$$\therefore\ G_{ij}(s) = \left.\frac{Y_i(s)}{U_j(s)}\right|_{U_k(s)=0\ \ k\neq j}$$ → defines Relative Gain Array RGA



VII UNITY FEED BACK SYSTEMS:



① $\therefore\ Gx = y \qquad c = Hy \qquad x = Ke$

$e = r - c$

$y = GKe = GK(r - Hy)$

but, $e = r - Hy$

$\therefore\ (1 + GKH)y = GKr$

$\therefore\ \dfrac{y}{r} = \dfrac{GK}{1 + GKH} = \dfrac{GK}{1 + GKH}$

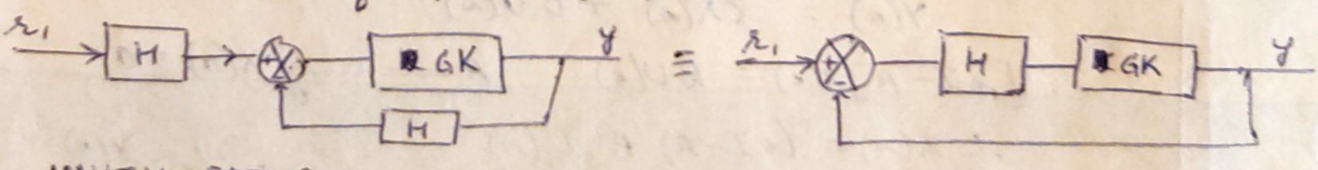
$\dfrac{y}{e} = GK$ F/w path gain

$\dfrac{y}{r} = \dfrac{GK}{1+GKH}$ closed loop gain

(Q) Loop gain: $\dfrac{y}{r}$ (open c) $= GKH$

② More convenient if r_1 as ref. speed s.t. $Hr_1 = r \Rightarrow (1+GKH)y = GKHr_1$

$\therefore$



UNITY FEEDBACK

$$\therefore\ \frac{y}{r_1} = \frac{GKH}{1 + GKH} = \frac{G'}{1 + G'}$$

MATLAB : Partial fraction expansion :

① $F(s) = \frac{P(s)}{Q(s)} = \frac{b_m s^m + b_{m-1} s^{m-1} + \ldots + b_1 s + b_0}{s^n + a_{n-1} s^{n-1} + \ldots + a_1 s + a_0}$

$= k(s) + \frac{r_1}{s-p_1} + \frac{r_2}{s-p_2} + \ldots + \frac{r_n}{s-p_n}$

$k(s)$: direct term ; r_i : residues ; p_i : poles.

$F(s) = \frac{s^5 + 8s^4 + 23s^3 + 35s^2 + 28s + 3}{s^3 + 6s^2 + 8s} = (s^2+2s+3) + \frac{0.375}{s+4} + \frac{0.25}{s+2} + \frac{0.3}{s}$

Say num = [1 8 23 35 28 3]
den = [0 0 1 6 8 0]

Command : [r, p, k] = residue (num, den)

r = 0.3750 p = −4 k = 1 2 3
0.2500 −2
0.3750 0

② Strictly proper fn. $F(s) = \frac{6}{s^3+6s^2+11s+6} = \frac{3}{s+3} - \frac{6}{s+2} + \frac{3}{s+1}$

num = [0 0 0 6] r = 3 p = −3 k = [ ]
den = [1 6 11 6] −6 −2
[r, p, k] = residue (num, den) 3 −1

printsys (num, den, 's') $num/den = \frac{-4e^{-15}s^2 - 1.4e^{-14}s + 6}{s^3 + 6s^2 + 11s + 6}$

③ Multiple poles

$F(s) = \frac{A_{j,1}}{s+p_j} + \frac{A_{j,2}}{(s+p_j)^2} + \ldots + \frac{A_{j,r}}{(s+p_j)^r}$

where $A_{j,r} = [(s+p_j)^r F(s)]_{s=-p_j}$

$A_{j,r-1} = \frac{d}{ds} [(s+p_j)^r F(s)]_{s=-p_j}$

$A_{j,r-i} = \frac{1}{(i)!} \frac{d^i}{ds^i} [(s+p_j)^r F(s)]_{s=-p_j}$

$F(s) = \frac{20(s+2)}{(s+1)^2 (s+3)} = \frac{5}{s+1} + \frac{10}{(s+1)^2} + \frac{-5}{s+3}$

den 1 = conv([1 1],[1 1])
den 2 = conv(den 1, [1 3])
num = [0 0 20 40]
[r, p, k] = residue(num, de

r = −5
5
10
p = −3
−1
−1
k = [ ]
den = $s^3 + 5s^2 + 7s + 3$
= [1 5 7 3]

Zeros & Poles:

[z, p, k] = tf2zp (num, den)

k = gain of t.f.

$$F(s) = \frac{20s + 20}{s^3 + 5s^2 + 7s + 3}$$

z = -1 , p = -3, -1+0i, -1-0i, k = 20.