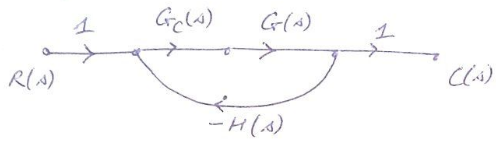
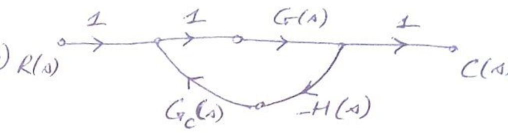
COMPENSATION TECHNIQUES

System performance —
- Time domain – t_p, M_p, t_s : step i/p
  - s.s errors for various inputs
- Freq. domain
  - M_r, ω_r, BW, GM, PM

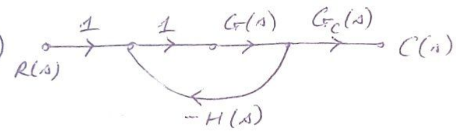
— may need alteration : use compensating networks.

a) 

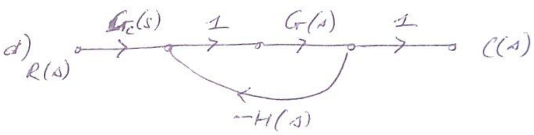
CASCADE COMPENSATION

b) 

FEEDBACK COMPENSATION
(Process control, electronics)

c) 

O/P or LOAD COMPENSATION
(Printer etc.)

d) 

I/P COMPENSATION.
(~~[illegible]~~ Preamplifier)

CASCADE COMPENSATORS : $G_C(s) = K \cdot \dfrac{\prod(s+z_i)}{\prod(s+p_j)}$

– use graphical choice of z_i, p_j by trial and error : see effect on root loci

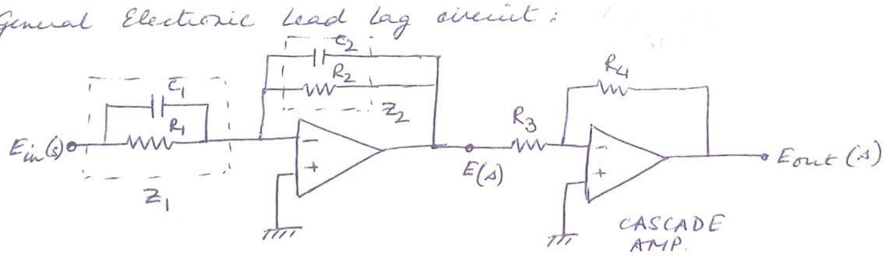
So if $G_C(s) = K \cdot \dfrac{s+z}{s+p} = K' \cdot \dfrac{\left(\frac{1}{z}\right)s+1}{(1/p)s+1}$; $K' = \dfrac{z}{p} \cdot K$

So when $\left(\dfrac{1}{|z|}\right) > \dfrac{1}{|p|} \Rightarrow |p| > |z|$: PHASE LEAD COMPENSATOR : HP : Differentiator (zero tow origin)

$\phi = \tan^{-1}\frac{\omega}{z} - \tan^{-1}\frac{\omega}{p}$

$\dfrac{1}{|z|} < \dfrac{1}{|p|} \Rightarrow |p| < |z|$: PHASE LAG COMPENSATOR : LP : Integrator

General Electronic Lead lag circuit :

A 

$Z_1 = \dfrac{R_1}{sC_1R_1+1}$, $Z_2 = \dfrac{R_2}{sC_2R_2+1}$

$= \dfrac{1}{C_1} \cdot \dfrac{R_1C_1}{sC_1R_1+1}$

$$\frac{E_{out}}{E_{in}}(s) = \left(\frac{-R_4}{R_3}\right) \cdot \left(\frac{-Z_2}{Z_1}\right) = \frac{R_4C_1}{R_3C_2} \frac{s + 1/R_1C_1}{s + 1/R_2C_2}$$

$$= K_c \cdot \frac{s + 1/T}{s + 1/\alpha T}$$ where $K_c = \dfrac{R_4C_1}{R_3C_2}$, $T = R_1C_1$, $\alpha T = R_2C_2$

$\therefore \alpha = \dfrac{R_2C_2}{R_1C_1}$

$$= \underline{\underline{K_c\alpha}} \cdot \frac{Ts+1}{\alpha Ts+1}$$ $\therefore \phi = \tan^{-1}\omega T - \tan^{-1}\alpha\omega T$

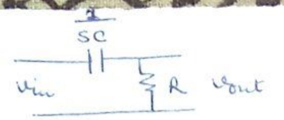
$\therefore$ If $T > \alpha T \Rightarrow R_1C_1 > R_2C_2 \Rightarrow 0 < \alpha < 1$: LEAD

$R_2C_2 > R_1C_1$: LAG.

Note : K_c needed to compensate the attenuation α : CASCADE amplifier during LEAD

or balance gain increase α during LAG

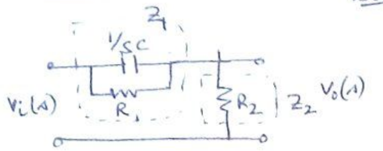
I LEAD



$$\frac{R}{R+1/sC} = \frac{sRC}{sRC+1} = \frac{V_o(s)}{V_i(s)} \quad ; \quad \phi = 90^\circ - \tan^{-1}\omega RC = +\text{ve acute}$$

BUT $s=0$ (d.c), gain $=0$ ∴ NO CONTROL ∴ low accuracy ∴ NOT USED IN SERVOS.

II LEAD



$$\frac{V_o(s)}{V_i(s)} = \frac{Z_2}{Z_1+Z_2} = \frac{R_2}{\frac{R_1/sC}{R_1+1/sC}+R_2} = \frac{R_2}{\frac{R_1}{sCR_1+1}+R_2}$$

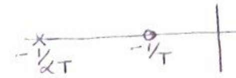
$$= \frac{sCR_1R_2+R_2}{R_1+sCR_1R_2+R_2} = \frac{R_2(sCR_1+1)}{(R_1+R_2)\left(\frac{sC\,R_1R_2}{R_1+R_2}+1\right)}$$

Let $\alpha = \frac{R_2}{R_1+R_2} < 1$

$T = R_1C$

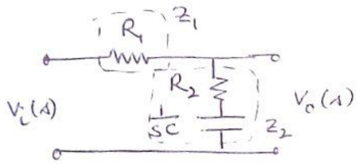
$$= \alpha\,\frac{sCR_1+1}{\alpha\,sCR_1+1} = \alpha\,\frac{Ts+1}{\alpha Ts+1}$$

$$= \frac{s+1/T}{s+1/\alpha T}$$



$\phi = \tan^{-1}\omega T - \tan^{-1}\alpha\omega T = +$ve acute angle for $0<\alpha<1$ HP filter

III ~~LAG~~



$$\frac{V_o(s)}{V_i(s)} = \frac{Z_2}{Z_1+Z_2} = \frac{R_2+\frac{1}{sC}}{R_1+R_2+\frac{1}{sC}} = \frac{sCR_2+1}{sC(R_1+R_2)+1}$$

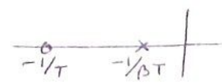
Let $\beta = \frac{R_1+R_2}{R_2} > 1$

$R_2C = T$

$$= \frac{Ts+1}{\beta Ts+1} = \frac{1}{\beta}\left(\frac{s+1/T}{s+1/\beta T}\right)$$

$\phi = \tan^{-1}\omega T - \tan^{-1}\beta\omega T$ ∴ LP filter

pole at $s = \frac{-1}{\beta T}$, zero at $s = \frac{-1}{T}$



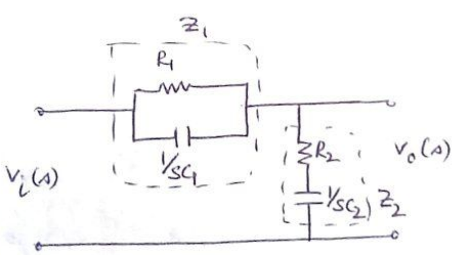
~~LEAD-LAG~~

For compensator design: $\phi < 5^\circ$ ∴ pole and zero close and near origin ∴ NOT change overall root loci appreciably.

Also, to keep $\frac{V_o(s)}{V_i(s)} \approx \frac{1}{\beta}$ → HANDRULE: $1<\beta<15$, typically $\beta = 10$

Note: $\hat{K}_v = \lim_{s\to 0} s\,G_c(s)\cdot GH(s) = \lim_{s\to 0} G_c(s)\cdot K_v$ ∴ req. addl. gain K_c to compensate the attenuation by $(1/\beta)$.

IV LEAD-LAG



$$Z_1 = \frac{R_1}{sC_1R_1+1} \quad ; \quad Z_2 = \frac{sC_2R_2+1}{sC_2}$$

$$\therefore \frac{V_o(s)}{V_i(s)} = \frac{Z_2}{Z_1+Z_2} = \frac{(sC_2R_2+1)(sC_1R_1+1)}{sC_2R_1+(sC_2R_2+1)(sC_1R_1+1)}$$

Let $T_1 = R_1C_1$, $T_2 = R_2C_2$;

$$\frac{T_1}{\beta} + \beta T_2 = (T_1+T_2+R_1C_2)$$

$$= \boxed{\frac{(T_1s+1)(T_2s+1)}{\left(\frac{T_1}{\beta}s+1\right)(\beta T_2 s+1)}} = \frac{s+1/T_1}{s+\beta/T_1}\cdot\frac{s+1/T_2}{s+1/\beta T_2}$$

LEAD LAG

So $\omega_1 = \frac{1}{\sqrt{T_1T_2}}$ ∴ $0<\omega<\omega_1$ LAG — Mag 0dB at low freq.

$\omega_1<\omega<\infty$ LEAD — 0dB at high freq.

$$\phi = \tan^{-1}\omega T_1 + \tan^{-1}\omega T_2 - \left[\tan^{-1}\frac{\omega T_1}{\beta} + \tan^{-1}\omega\beta T_2\right]$$

$$\phi = \tan^{-1}\frac{\omega(T_1+T_2)}{1-\omega^2T_1T_2} - \tan^{-1}\frac{\omega\left(\frac{T_1}{\beta}+\beta T_2\right)}{1-\omega^2T_1T_2}$$

$$= \tan^{-1}\frac{-\omega R_1C_2(1-\omega^2T_1T_2)}{(1-\omega^2T_1T_2)^2+\omega^2(T_1+T_2)^2+\omega^2(T_1+T_2)R_1C_2}$$

$$\left(\frac{1-\beta}{\beta}\right)T_1 + (\beta-1)T_2 = R_1C_2$$

$$\beta T_2 + T_1 = \beta(T_1+T_2)+\beta R_1C_2$$

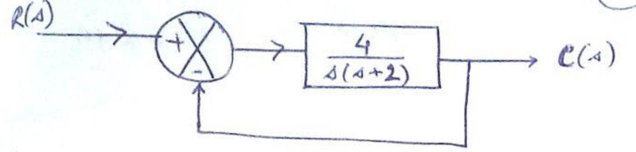
$$\beta^2R_2C_2+R_1C_1 = \beta[R_1C_1+R_2C_2+R_1C_2]$$

$$(\beta-1)\left[\beta T_2 - \frac{T_1}{\beta}\right] = \beta R_1C_2$$

$$-\tan^{-1}\frac{\omega(T_1+T_2+R_1C_2)}{1-\omega^2T_1T_2} = \tan^{-1}\frac{\omega(T_1+T_2)-\omega(T_1/\beta+\beta T_2)}{\ldots}$$

$$= \tan^{-1}\frac{-\omega R_1C_2(1-\omega^2T_1T_2)}{1-2\omega^2T_1T_2+\ldots}$$

Ex1.



Desired $\omega_{nT} = 4$ rad/s , ξ same

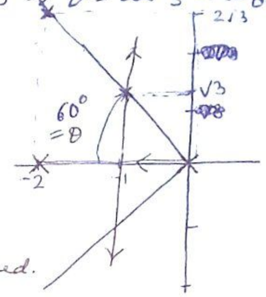
ch. eqn : $1 + GH(s) = s^2 + 2s + 4 = 0 \Rightarrow \omega_n = 2$ rad/s $\quad \xi = 0.5 \Rightarrow \theta = \cos^{-1}\xi = \pm 60^\circ$

ξ pole : $-1 + j\sqrt{3} = s$

For $\omega_{nd} = 4$, $\xi = 0.5$, Desired ch. eqn. : $s^2 + 4s + 16 = 0$

$\therefore$ Desired ξ poles : $s_d = -2 \pm j2\sqrt{3}$ (on same ξ line)

$$\angle GH(s)\Big|_{s_d = -2+j2\sqrt{3}} = \angle \frac{4}{s(s+2)}\Big|_{s_d = -2+j2\sqrt{3}} = -210^\circ = -180^\circ - 30^\circ$$



$\therefore$ Change of $(+30^\circ)$ lead (from -180° for pt. on root loci) required.

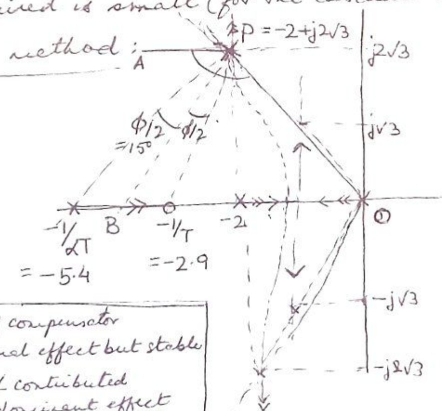
$$= \angle \frac{4}{s(s+2)}\Big|_{s = -1+j\sqrt{3}} = \text{orig. } \xi \text{ poles}$$

Ex2 Design of the LEAD compensator. a) Introduce pole zero pair so root loci passes through s_d.

b) Several possible T $\therefore$ Choose T with largest α so that compensator gain K_c required is small (for the cascade amplifier).

Graphical method :

$\phi_d = 30^\circ$



(i) mark P and -180° line on P=PA

(ii) bisect $\angle P$ till it intersects σ axis at B.

(iii) plot $\frac{-1}{T}$ and $\frac{-1}{\alpha T}$ as zero and pole resp. at $\pm \frac{\phi_d}{2}$

Note $G_c(s) = K_c \cdot \alpha \cdot \frac{Ts+1}{\alpha Ts + 1}$, $0 < \alpha < 1$

Note: zeros & poles of compensator
A) along PA : minimal effect but stable
B) along PO : min. $\angle$ contributed but dominant effect
$\therefore$ BISECT

(iv) $\therefore$ Desired ϕ T.F. $= G_c(s) \cdot G(s)$

$$= \frac{4K_c(s+2.9)}{(s+5.4) \cdot s(s+2)}$$

(v) To satisfy magnitude condition,

$$\left| \frac{4K_c\,(s+2.9)}{s\,(s+2)(s+5.4)} \right|_{s_d = -2+j2\sqrt{3}} = 1 \qquad \therefore K_c = 4.68 \quad \text{while } K = 4K_c = 18.7$$

Note: 1. $K_v = \lim_{s \to 0} sG_c(s)\,GH(s) = 5.02\,s^{-1}$

2. 3^{rd} ξ pole : $s(s+2)(s+5.4) + 18.7(s+2.9) = (s+2\pm j2\sqrt{3})(s+3.4)$

So, since (-3.4) is close to the added ~~zero~~ at (-2.9) $\therefore$ effect of pole on transient response is less. (effect at $\lim_{s\to\infty}$)

$\therefore$ POSSIBLE DESIGN choices :

II : $\frac{V_0}{V_i} = \frac{\cancel{s\,RC}}{\cancel{sRC+1}}\ \alpha\ \frac{Ts+1}{\alpha Ts+1}$: $T = R_1C$, $\alpha = \frac{R_2}{R_1+R_2}$ so $R_1 = 345\,k\Omega$, $R_2 = 400\,k\Omega$, $C = 1\,\mu F$ and separate K_c

IA 1. $R_1 = 345\,k\Omega$ $\quad R_2 = 400\,k\Omega$, $\quad C_1 = 1\,\mu F$, $\quad C_2 = 0.47\,\mu F$ $\quad R_3 = 4.7\,k\Omega$, $\quad R_4 = 10\,k\Omega$

2. $34.5\,k\Omega$ $\quad 40\,k\Omega$ $\quad C_1 = C_2 = 10\,\mu F$ $\quad R_3 = 10\,k\Omega$ $\quad R_4 = 46.8\,k\Omega$

— Balanced resistors and capacitors.

Ex3 Design a compensator for Ex 1 s.t. $K_v = 20\,s^{-1}$, $PM \geq 50°$, $GM \geq 10\,dB$.

① To achieve $K_{v_d} = 20\,s^{-1}$

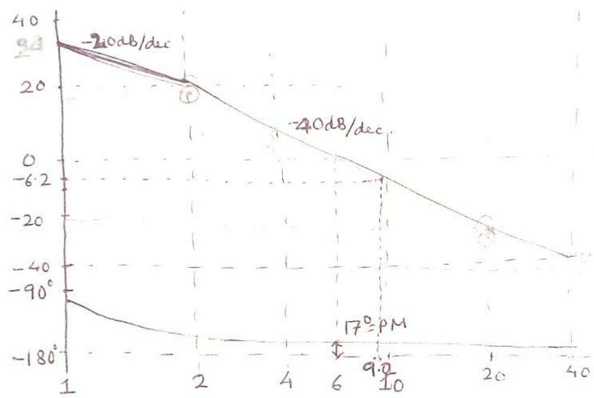
$$K_{v_d} = \lim_{s\to 0} sGH(s)K = \lim_{s\to 0} \frac{4K}{s+2} = 20 \Rightarrow K = 10.$$

② GM reqd. $\geq 10\,dB$ → satisfied since 2^{nd} Order Q eqs $GM = \infty$ ∵ $\phi \cancel{\neq} (-180°)$

else use cascade amplifier / attenuator

Note: at least one of GM / K_v constraints have to be inequality constraints.

③ To meet $PM \geq 50°$



Presently $PM = 17°$ at $\omega_g = 6\,rad/s$

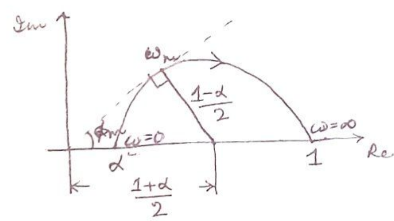
∴ Req. addl. at least $50° - 17° = 33°$ phase lead + Tolerance margin of $5° \rightarrow 38°$ phase lead to be designed.

∴ $\phi_m = 38°$

$$\boxed{\begin{array}{l} 25\,dB \text{ at } \omega = 1 \\ 17\,dB \text{ at } \omega = 2 \\ 0.46\,dB \text{ at } \omega = 6 \\ -2.1\,dB \text{ at } \omega = 7 \\ -6.34\,dB \text{ at } \omega = 9 \end{array}}$$

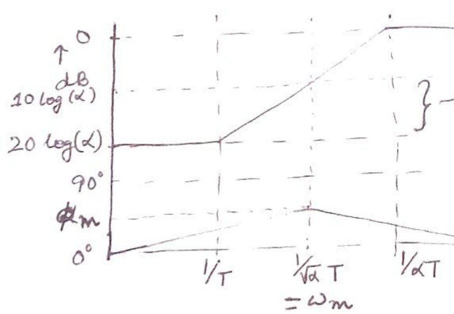
General: Phase plot for compensator $G_c(j\omega) = \alpha \dfrac{j\omega T + 1}{j\alpha\omega T + 1}$, $\alpha < 1$ $= \dfrac{j\omega + 1/T}{j\omega + 1/\alpha T}$

$\cancel{\phi = \tan^{-1}\omega T - \tan^{-1}\alpha\omega T = \tan^{-1}\frac{\omega T - \alpha\omega T}{1 + \omega^2\alpha T^2}}$



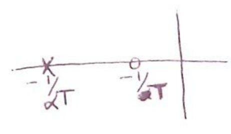
∴ $\sin\phi_m = \dfrac{1-\alpha}{1+\alpha}$ where ϕ_m = maxm. phase lead angle

BODE plot of lead compensator $\alpha < 1$; corner freq: $1/T$, $1/\alpha T$



$$\log \omega_m = \frac{1}{2}\left[\log\frac{1}{T} + \log\frac{1}{\alpha T}\right]$$

∴ $\omega_m = \dfrac{1}{\sqrt{\alpha}\,T}$ Note: log scale for freq.



So $\sin\phi_m = \sin 38° = \dfrac{1-\alpha}{1+\alpha}$ ∴ $\alpha = 0.24$

But adding compensator changes magnitude curve at ω_m → $\left|\dfrac{1+j\omega T}{1+j\alpha\omega T}\right|_{\omega = \frac{1}{\sqrt{\alpha}T}} = \dfrac{1}{\sqrt{\alpha}} = \dfrac{1}{\sqrt{0.24}} = 6.2\,dB$

∴ Magnitude curve will shift up by 6.2 dB at ω_m w.r.t $\omega \leq 1/T$.

∴ select ω_m to be the new ω_g. Original $-6.2\,dB$ at $\omega_{g_{new}} = 9\,rad/s$ (also adjust for low PM because of earlier tolerance margin of $5°$)
= new 0dB

$$\therefore \omega_{g_{new}} = 9\,rad/s = \frac{1}{\sqrt{\alpha}\,T_{new}} \quad \therefore \frac{1}{T_{new}} = 4.41, \quad \frac{1}{\alpha T_{new}} = 18.4$$

$$\therefore \text{Lead network} = K_c \cdot \frac{s+4.41}{s+18.4} = K_c \cdot (0.24)\frac{0.227s+1}{0.054s+1} \quad \therefore K_c = \frac{1}{0.24} = 4.17$$ [to keep overall compensator gain = 1]

$$\therefore \text{Overall new T.F.} = 4.17\,\frac{s+4.41}{s+18.4} \cdot 10 \cdot \frac{4}{s(s+2)} = \underbrace{\boxed{41.7\,\frac{s+4.41}{s+18.4}}}_{G_c(s)} \underbrace{\frac{4}{s(s+2)}}_{G(s)}.$$