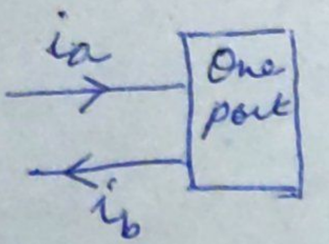
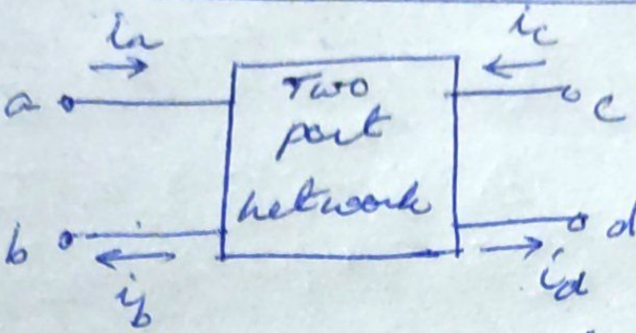
Two port networks





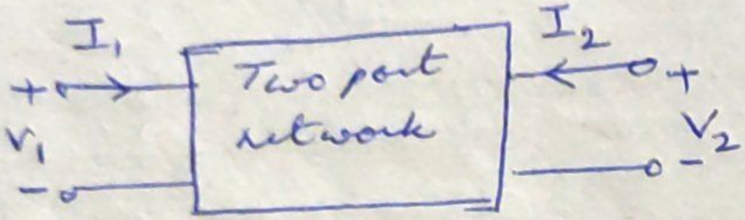
Loop eqns. for a N by N passive network:

$$Z_{11} I_1 + Z_{12} I_2 + \dots + Z_{1N} I_N = V_1$$
$$Z_{21} I_1 + Z_{22} I_2 + \dots + Z_{2N} I_N = V_2$$
$$\vdots$$
$$Z_{N1} I_1 + Z_{N2} I_2 + \dots + Z_{NN} I_N = V_N$$

$$\Rightarrow \begin{bmatrix} Z_{11} & Z_{12} & \dots & Z_{1N} \\ Z_{21} & Z_{22} & \dots & Z_{2N} \\ \vdots & & \ddots & \vdots \\ Z_{N1} & Z_{N2} & \dots & Z_{NN} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ \vdots \\ I_N \end{bmatrix} = \begin{bmatrix} V_1 \\ V_2 \\ \vdots \\ V_N \end{bmatrix}$$

$$\Rightarrow \underline{Z}\,\underline{I} = \underline{V}$$

so $I_1 = \frac{\Delta_{11}}{\Delta_Z} \cdot V_1$; $Z_{in} = \frac{V_1}{I_1} = \frac{\Delta_Z}{\Delta_{11}}$ (minor)



<u>Short circuit admittance parameters</u> y_{ij}

$$I_1 = y_{11} V_1 + y_{12} V_2 \qquad\qquad I_2 = y_{21} V_1 + y_{22} V_2 \quad \Rightarrow \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} \Rightarrow \underline{I} = \underline{Y}\,\underline{V}$$

$$\therefore\ y_{11} = \left.\frac{I_1}{V_1}\right|_{V_2=0} ;\quad y_{12} = \left.\frac{I_1}{V_2}\right|_{V_1=0} ,\quad y_{21} = \left.\frac{I_2}{V_1}\right|_{V_2=0} ,\quad y_{22} = \left.\frac{I_2}{V_2}\right|_{V_1=0}.$$

i/p admittance — $\underbrace{y_{12},\ y_{21}}_{\text{transfer admittances}}$ — o/p admittances.

<u>Open circuit impedance parameters</u> z_{ij}

$$V_1 = z_{11} I_1 + z_{12} I_2 \qquad\qquad V_2 = z_{21} I_1 + z_{22} I_2 \quad \Rightarrow \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} \Rightarrow \underline{V} = \underline{z}\,\underline{I}$$

$$\Rightarrow \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = z^{-1} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} \Rightarrow I_1 = \frac{z_{22}}{\Delta_z} V_1 - \frac{z_{21}}{\Delta_z} V_2 = \frac{\Delta_{11}}{\Delta_z} V_1 - \frac{\Delta_{12}}{\Delta_z} V_2$$

$$I_2 = \frac{-z_{12}}{\Delta_z} V_1 + \frac{z_{22}}{\Delta_z} V_2 = -\frac{\Delta_{21}}{\Delta_z} V_1 + \frac{\Delta_{22}}{\Delta_z} V_2$$

Voltage gain $G_V = \frac{V_2}{V_1}$

Current gain $G_I = \frac{I_2}{I_1}$

Power gain $G_p = -G_V G_I = \frac{P_{out}}{P_{in}} = \frac{Re\left[-\frac{1}{2} V_2 I_2^*\right]}{Re\left[-\frac{1}{2} V_1 I_1^*\right]}$

Input impedance $Z_{in} = \frac{V_1}{I_1}$, output impedance Z_{out} = Thevenin impedance

$= \frac{V_2}{I_2} = Z_{22} - \frac{Z_{12} Z_{21}}{Z_{11} + Z_g}$ for generator impedance Z_g.

$V_s = V_1 + I_1 Z_g$

Hybrid parameters h_{ij}

$$V_1 = h_{11} I_1 + h_{12} V_2 \qquad \begin{bmatrix} V_1 \\ I_2 \end{bmatrix} = \underline{h} \begin{bmatrix} I_1 \\ V_2 \end{bmatrix}$$
$$V_2 = h_{21} I_1 + h_{22} V_2$$

$h_{11} = \left.\dfrac{V_1}{I_1}\right|_{V_2=0}$ = s.c. i/p impedance ; $h_{12} = \left.\dfrac{V_1}{V_2}\right|_{I_1=0}$ = o/p ckt. reverse voltage gain

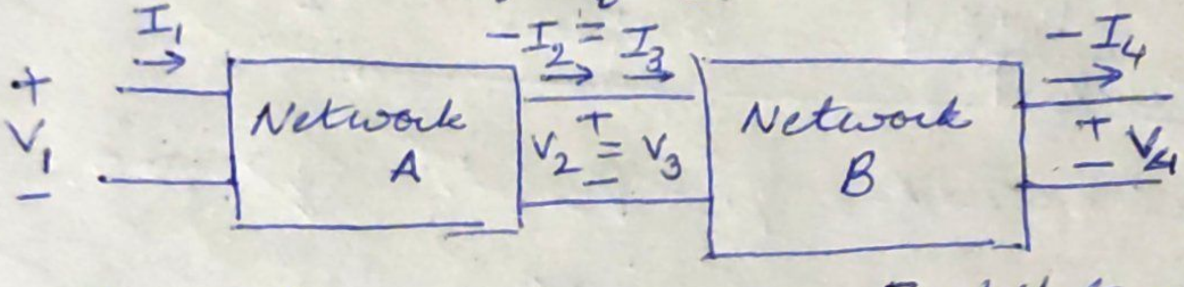
$h_{21} = \left.\dfrac{I_2}{I_1}\right|_{V_2=0}$ = s.c. fwd current gain ; $h_{22} = \left.\dfrac{I_2}{V_2}\right|_{I_1=0}$ = o/p ckt. o/p admittance

For bilateral network, $h_{12} = -h_{21}$

Transmission parameters t_{ij} ABCD parameters.

$$V_1 = t_{11} V_2 - t_{12} I_2$$
$$I_1 = t_{21} V_2 - t_{22} I_2$$
$$\Rightarrow \begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} t_{11} & t_{12} \\ t_{21} & t_{22} \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$$

For reciprocal networks, $\Delta_t = 1$.

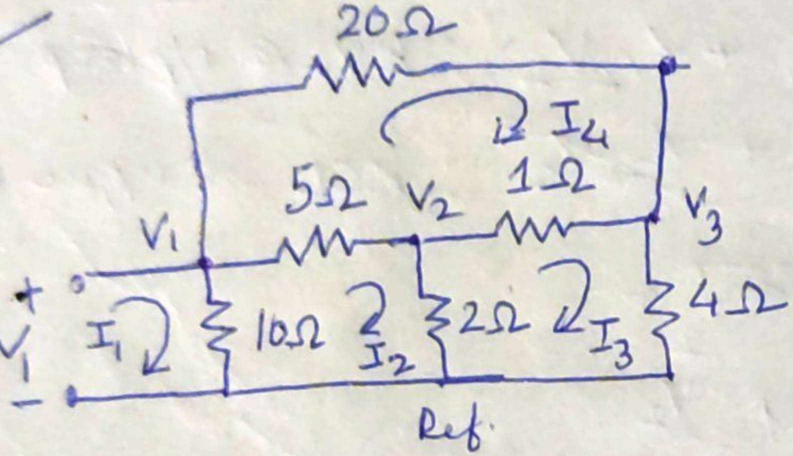


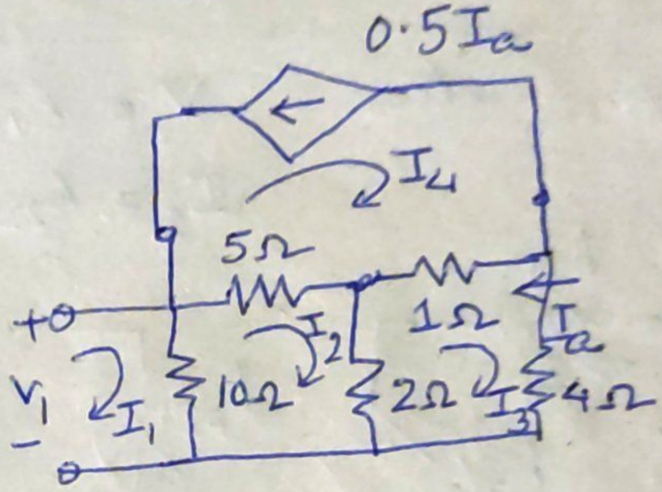
Cascade connections

For bilateral, $\Delta_t = t_{11} t_{22} - t_{12} t_{21} = 1$

Ex 15-1, 15-2, 15-3

Single port network





Mesh equations

$$V_1 = 10I_1 - 10I_2$$
$$0 = -10I_1 + 17I_2 - 2I_3 - 5I_4$$
$$0 = \quad -2I_2 + 7I_3 - I_4$$
$$0 = \quad -5I_2 - I_3 + 26I_4$$

$$V_1 = 10I_1 - 10I_2$$
$$0 = -10I_1 + 17I_2 - 2I_3 - 5I_4$$
$$0 = \quad -2I_2 + 7I_3 - I_4$$
$$I_4 = -0.5I_a = -0.5(I_4 - I_3)$$
$$\Rightarrow 0 = \quad +0.5I_3 - 1.5I_4$$

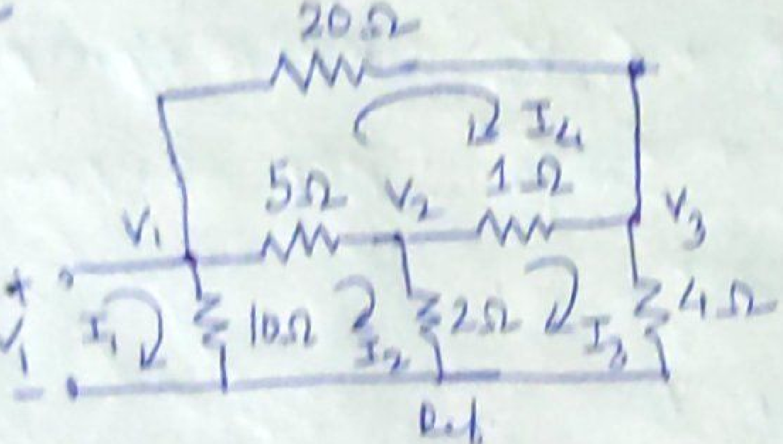
BILATERAL ELEMENTS

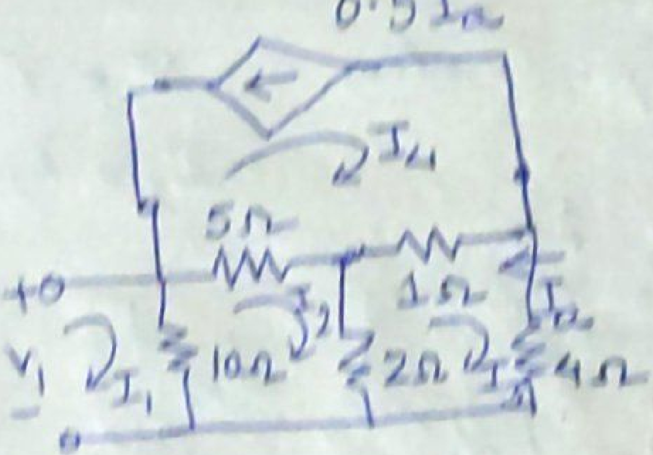
$$\Delta_Z = \begin{bmatrix} 10 & -10 & 0 & 0 \\ -10 & 17 & -2 & -5 \\ 0 & -2 & 7 & -1 \\ 0 & -5 & -1 & 26 \end{bmatrix}$$

$$\Delta_Z = \begin{bmatrix} 10 & -10 & 0 & 0 \\ -10 & 17 & -2 & -5 \\ 0 & -2 & 7 & -1 \\ 0 & 0 & 0.5 & -1.5 \end{bmatrix}$$

Ex 15-1, 15-2,
15-3

Single port network





Nodal for Δy

$$V_1\left(\frac{1}{10}+\frac{1}{5}+\frac{1}{20}\right)-0.2V_2 \quad -0.05V_3=0$$

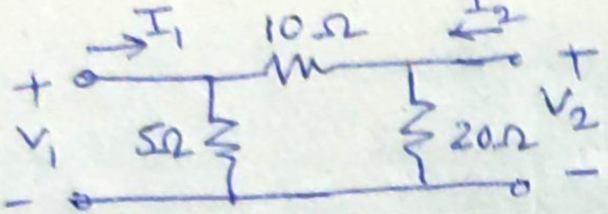
$$-0.2V_1+V_2\left(\frac{1}{5}+\frac{1}{2}+1\right)-V_3=0$$

$$-0.05V_1-V_2 \quad +V_3\left(\frac{1}{20}+1+\frac{1}{4}\right)=0$$

$$\Delta y=\begin{bmatrix}0.35 & -0.2 & -0.05\\ -0.2 & 1.7 & -1\\ -0.05 & -1 & 1.3\end{bmatrix};\quad Y_{in}=\frac{\Delta_Y}{\Delta_{11}}$$

Ex 15.5

Two port network



$$I_1 = V_1\left(\frac{1}{5}+\frac{1}{10}\right) - 0.1V_2$$

$$I_2 = -0.1V_1 + V_2\left(\frac{1}{10}+\frac{1}{20}\right)$$

$$\Rightarrow \quad I_1 = 0.3V_1 - 0.1V_2$$

$$I_2 = -0.1V_1 + 0.15V_2$$

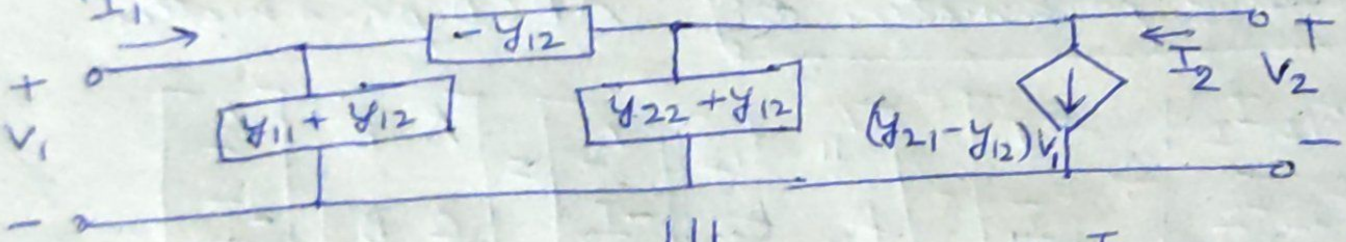
$$\Rightarrow [\underline{Y}] = \begin{bmatrix} 0.3 & -0.1 \\ -0.1 & 0.15 \end{bmatrix}$$

$$I_1 = y_{11}V_1 + y_{12}V_2$$
$$I_2 = y_{21}V_1 + y_{22}V_2$$

$\rightarrow I_2 = y_{12}V_1 + y_{22}V_2 + (y_{21} - y_{12})V_1$

for bilateral network form.

$\Rightarrow I_2 - (y_{21} - y_{12})V_1 = y_{12}V_1 + y_{22}V_2$



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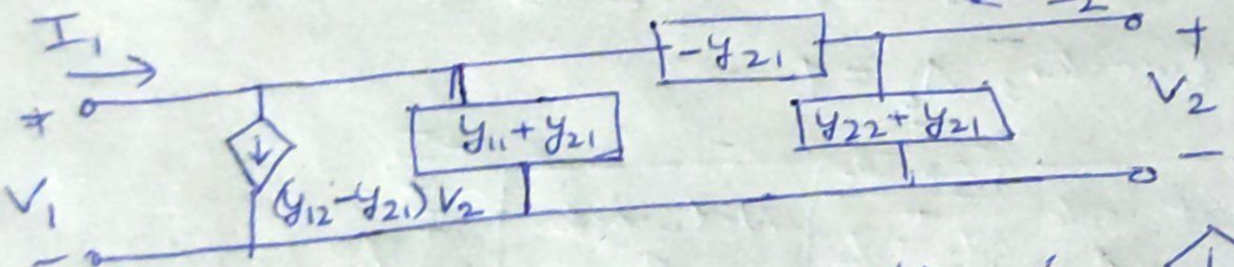
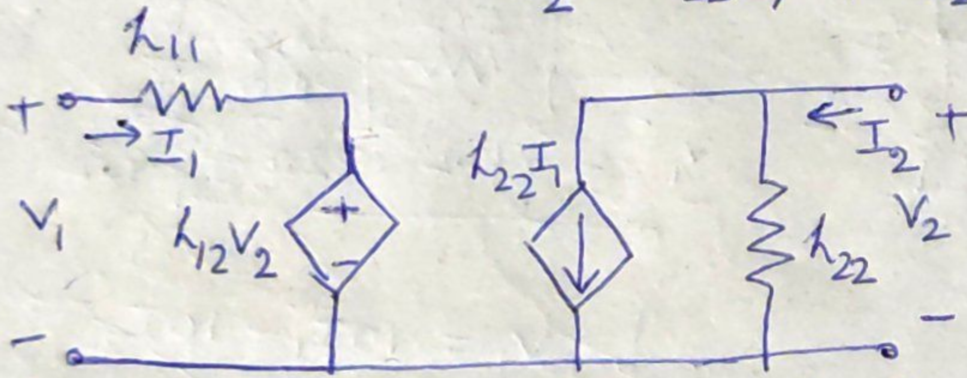


For bilateral network, $y_{12} = y_{21}$ $\therefore$  $\rightarrow 0$.

Also $\nabla \rightarrow Y$ conversion possible.

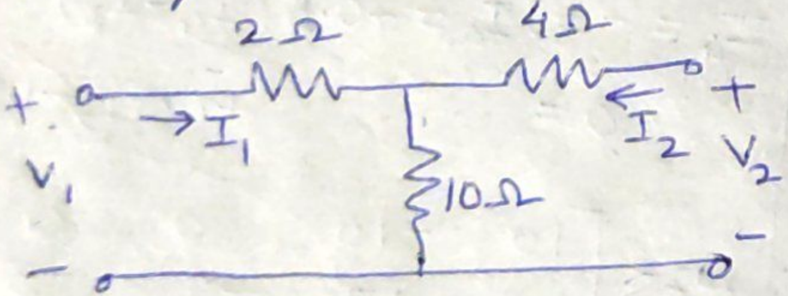
For h-parameters $V_1 = h_{11} I_1 + h_{12} V_2$: KVL @ i/p loop

$I_2 = h_{22} I_1 + h_{22} V_2$; KCL at upper o/p node



Transistor equivalent circuit

For t-parameters



$V_1 = t_{11} V_2 - t_{12} I_2$

$I_1 = t_{21} V_{12} - t_{22} I_2$

$V_1 = 12 I_1 + 10 I_2$

$V_2 = 10 I_1 + 14 I_2 \rightarrow I_1 = \underbrace{0.1}_{t_{21}} V_1 - \underbrace{1.4}_{t_{22}} I_2$

$V_1 = \underbrace{1.2}_{t_{11}} V_1 - \underbrace{6.8}_{t_{12}} I_2$