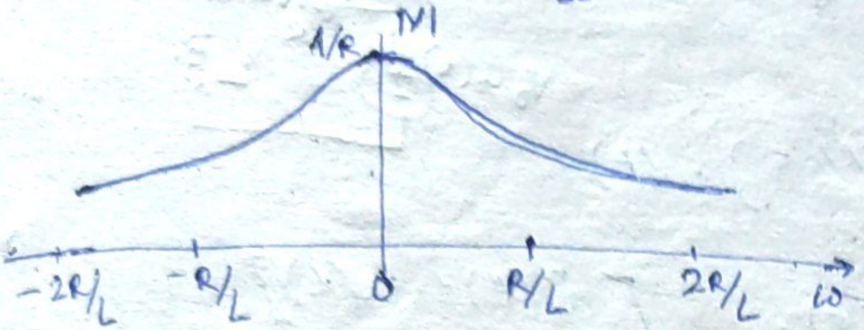
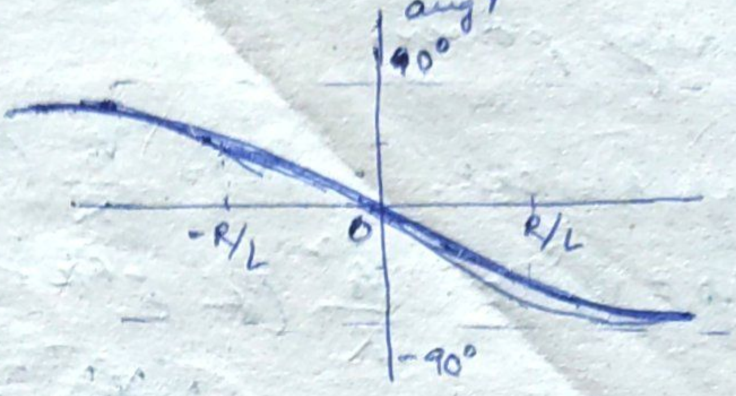
Response as fn. of ω :

1. Consider series RL ckt. $\therefore I = \dfrac{V_s}{R+j\omega L}$ $\therefore Y = \dfrac{1}{R+j\omega L}$

$\therefore |Y| = \dfrac{1}{\sqrt{R^2+\omega^2L^2}}$ $\quad$ $\arg. Y = -\tan^{-1}\dfrac{\omega L}{R}$





$\omega = -100$ rad/s $\Rightarrow v(t) = 50\cos(\omega t + 30°)$

$v(t) = 50\cos(-100t + 30°)$

at $\omega = \pm R/L = \pm 1/\tau \rightarrow |Y| = 0.707|Y_{max}|$, $\arg Y = 45°$.

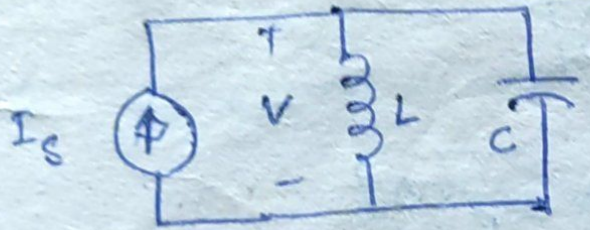
$\downarrow$

avg power = 0.5 max. power.

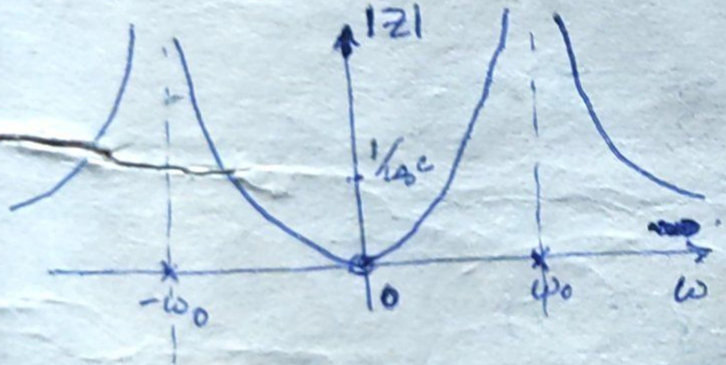
$\therefore$ Half power frequency. $\longrightarrow$ $\therefore$ For same V, $I = \frac{1}{\sqrt{2}} I_{max}$ $\therefore$ avg. power = $(0.707)^2 P_{max} = 0.5 P_{max}$.

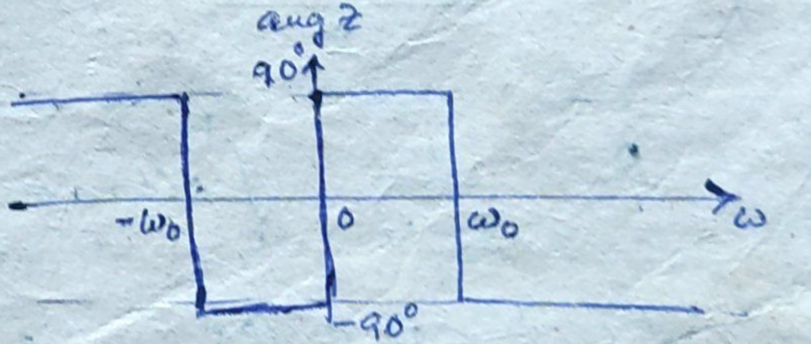
only due to R

2. Parallel LC ckt. driven by sinusoidal source:



$$V = I_S \cdot \frac{(j\omega L)\cdot(1/j\omega C)}{j\omega L - j(1/\omega C)}$$





$$Z = \frac{L/C}{j(\omega L - 1/\omega C)} = -j \cdot \frac{1}{C} \cdot \frac{\omega}{\omega^2 - 1/LC}$$

$$\therefore |Z| = \frac{1}{C} \frac{|\omega|}{|(\omega - \omega_0)(\omega + \omega_0)|}$$

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

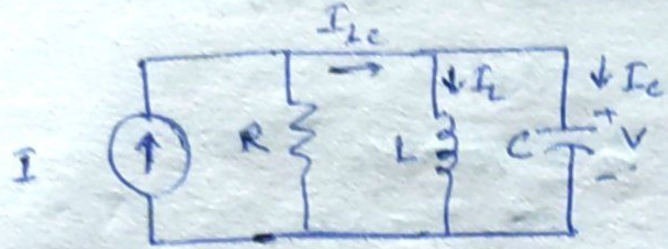
Critical frequencies: at which responses 0 or ∞.

zeros (0), poles (x)

note: $\omega = \pm\infty$ is a zero

Frequency Response: Parallel resonance.

A network is in resonance when the voltage and current at the network input terminals are in phase.



$$\underline{Y}(\underline{s}) = \frac{1}{R} + \frac{1}{\underline{s}L} + \underline{s}C = C\,\frac{\underline{s}^2 + \underline{s}/RC + 1/LC}{\underline{s}}$$

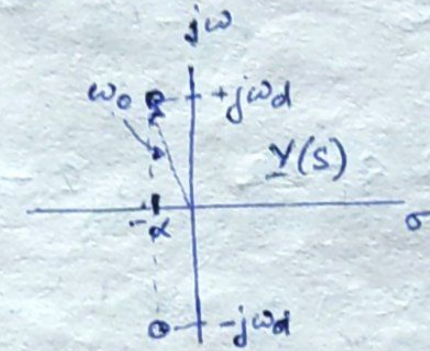
$$= C\,\frac{(\underline{s} + \alpha - j\omega_d)(\underline{s} + \alpha + j\omega_d)}{\underline{s}}$$

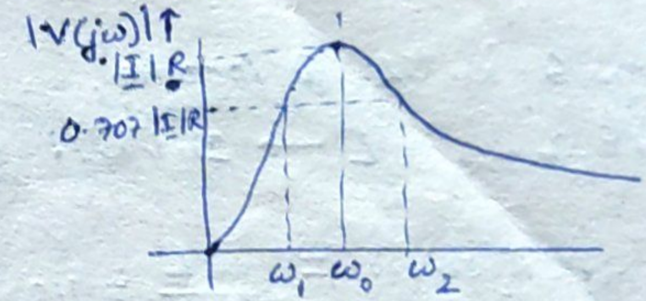
As seen resonance occurs when $\omega C - \frac{1}{\omega L} = 0$

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

$$\underline{Y} = \frac{1}{R} + j\left(\omega C - \frac{1}{\omega L}\right)$$

$\alpha = 1/2RC$, $\omega_d = \sqrt{\omega_0^2 - \alpha^2}$





Ht. of response curve depends only on R for constant amplitude excitation but width & steepness depend on L&C too.

Quality factor $Q = 2\pi \dfrac{\text{maximum energy stored}}{\text{total energy lost per period.}} = 2\pi \dfrac{[w_L(t) + w_C(t)]_{max}}{P_R\, T}$

P_R ↓ avg. power dissipat

Say $i(t) = I_m \cos \omega_0 t$ $\quad v(t) = R\, i(t) = R I_m \cos \omega_0 t$.

$w_C(t) = \frac{1}{2} C v^2 = \frac{I_m^2 R^2 C}{2} \cos^2 \omega_0 t$ $\quad w_L(t) = \frac{1}{2} L i_L^2 = \frac{1}{2} L \left(\frac{1}{L} \int_0^t v\, dt \right)^2$

$= \frac{I_m^2 R^2 C}{2} \sin^2 \omega_0 t$

$\therefore w(t)_{max} = \frac{I_m^2 R^2 C}{2}$. $\quad P_R T = \frac{1}{2} I_m^2 R \cdot T = \frac{1}{2 f_0} I_m^2 R$

$\therefore Q_0 = \omega_0 RC = R \sqrt{\frac{C}{L}} = \frac{R}{X_{C0}} = \frac{R}{X_{L0}}$ (dimensionless)

$\therefore \alpha = \frac{1}{2 (Q_0 / \omega_0 C) C} = \frac{\omega_0}{2 Q_0}$; $\quad \omega_d = \omega_0 \sqrt{1 - \left(\frac{1}{2Q_0}\right)^2}$

$\zeta = \frac{1}{2 Q_0}$

Bandwidth $= \omega_2 - \omega_1 = \mathcal{B}$

* Half power freq. : since for const. I, $V = \frac{1}{\sqrt{2}} V_{max}$

$\therefore$ power dissipated in $R = V^2/R$ is $\frac{1}{2} P_{max}$

$$\underline{Y} = \frac{1}{R} + j\left(\omega C - \frac{1}{\omega L}\right) = \frac{1}{R} + j\frac{1}{R}\left[\frac{\omega \cdot \omega_0 RC}{\omega_0} - \frac{\omega_0 R \cdot C}{\omega \cdot \omega_0 LC}\right]$$

$$= \frac{1}{R}\left[1 + jQ_0\left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right)\right]$$

Now, $|\underline{Y}| = \sqrt{2}/R$ only when $\left|Q_0\left(\frac{\omega_2}{\omega_0} - \frac{\omega_0}{\omega_2}\right)\right| = 1$

$$\therefore \omega_{1,2} = \omega_0\left[\sqrt{1 + \left(\frac{1}{2Q_0}\right)^2} \mp \frac{1}{2Q_0}\right]$$

$\therefore \mathcal{B} = \frac{\omega_0}{Q_0}$, $\omega_0 = \sqrt{\omega_1 \omega_2}$

$\therefore \alpha = \frac{1}{2}\mathcal{B}$

Similarly analyze series resonance ckt. $\omega_0 = 1/\sqrt{LC}$ $\quad Q_0 = \omega_0 L/R$

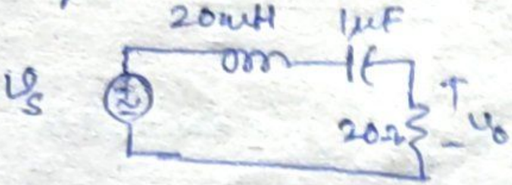
$$\omega_{1,2} = \omega_0\left[\sqrt{1 + \left(\frac{1}{2Q_0}\right)^2} \mp \frac{1}{2Q_0}\right] \cong \omega_0 \mp \frac{1}{2}\mathcal{B} \qquad \mathcal{B} = \frac{\omega_0}{Q_0}$$

Ex. Dr. Noe gave Pat $R = 20\Omega$, $L = 20mH$, $C = 1\mu F$ (nominal)

connect variable freq. voltage source to series combination

& meas. resultant v_R as fn. of freq. then calculate ω_0, Q_0, B.

& predict them



Theo.

$f_0 = \dfrac{1}{2\pi\sqrt{LC}} = 1125 Hz$

$Q_0 = \dfrac{\omega_0 L}{R} = 7.07$

$B = \dfrac{f_0}{Q_0} = 159 Hz$

Prac.

$f_0 = 1000 Hz$

$Q_0 = 0.625$

$B = 1600 Hz$.

Check on Q-meter @ 1000 Hz:

$R = 18\Omega$ $\quad L \rightarrow 21.4 mH$ with $Q = 1.2$ $\quad$, $C \rightarrow 1.41\mu F$ with $1/Q = 0.123$

dissipation factor.

$\therefore$ L with R_L series $= \dfrac{\omega L}{Q} = 112\Omega$

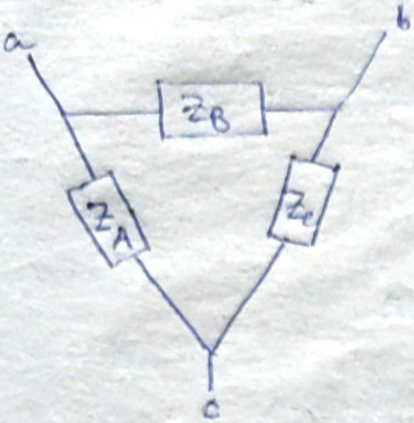
C with series $R = \dfrac{1}{\omega C Q} = 13.9\Omega$

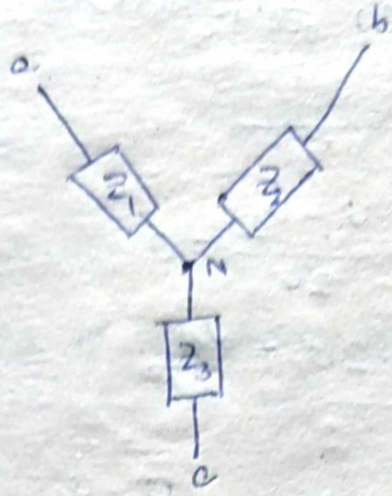
$\therefore$ $f_0 = 916 Hz$. $\quad Q_0 = \dfrac{2\pi \times 916 \times 21.4 \times 10^{-3}}{143.9} = 0.856$ $\quad B = \dfrac{916}{0.856} = 1070 Hz$.

Further, o/p impedance of voltage source 50Ω.

$\therefore$ $Q_0 = 0.635$, $\quad B = 1442 Hz$.

Δ-Y transformation





Between a & b:

$$\bar{Z}_B \parallel (\bar{Z}_A + \bar{Z}_c) = \frac{\bar{Z}_B(\bar{Z}_A + \bar{Z}_c)}{\bar{Z}_A + \bar{Z}_B + \bar{Z}_c} = \bar{Z}_1 + \bar{Z}_2$$

$$\therefore \bar{Z}_1 = \frac{\bar{Z}_A \bar{Z}_B}{\bar{Z}_A + \bar{Z}_B + \bar{Z}_c} \; ; \; \bar{Z}_2 = \frac{\bar{Z}_B \bar{Z}_c}{\Sigma Z_i} \; ; \; \bar{Z}_3 = \frac{\bar{Z}_A \bar{Z}_c}{\Sigma Z_i}$$

~~Bet~~. Short a and b.

Admittance bet. a(b) & c:

$$\frac{1}{\bar{Z}_A} + \frac{1}{\bar{Z}_c} = \frac{1}{(\bar{Z}_1 \parallel \bar{Z}_2) + \bar{Z}_3} = \frac{\bar{Z}_1 + \bar{Z}_2}{\bar{Z}_1\bar{Z}_2 + \bar{Z}_2\bar{Z}_3 + \bar{Z}_3\bar{Z}_1}$$

$$\therefore \bar{Z}_A = \frac{\sum_{i,j=1}^{3} \bar{Z}_i \bar{Z}_j}{\bar{Z}_2} \; ; \; \bar{Z}_B = \frac{\Sigma \bar{Z}_i \bar{Z}_j}{\bar{Z}_3} \; ; \; \bar{Z}_c = \frac{\Sigma \bar{Z}_i \bar{Z}_j}{\bar{Z}_1}$$

and thus the voltage $\mathbf{V}_{ab}$ may be found, with an eye on the subscripts:

$$\begin{aligned}\mathbf{V}_{ab} = \mathbf{V}_{an} + \mathbf{V}_{nb} &= \mathbf{V}_{an} - \mathbf{V}_{bn} \\ &= 100\underline{/0^\circ} - 100\underline{/-120^\circ} \\ &= 100 - (-50 - j86.6) \\ &= 173.2\underline{/30^\circ}\end{aligned}$$

The three given voltages and the construction of the phasor $\mathbf{V}_{ab}$ are shown on the phasor diagram of Fig. 11-3.

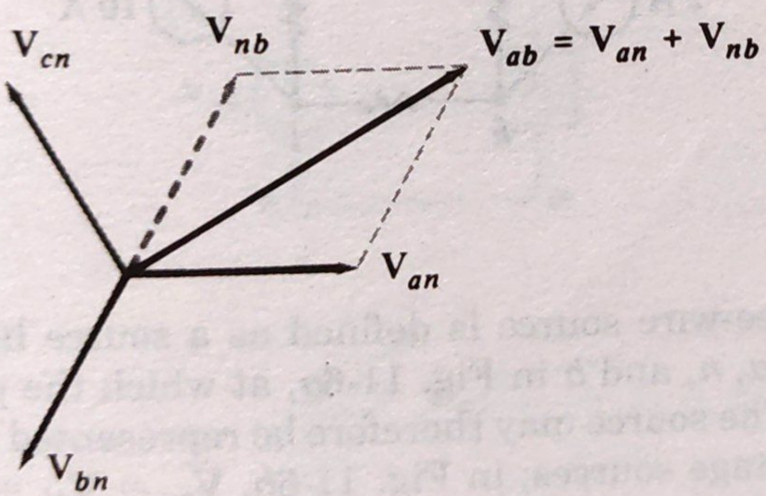


Figure 11-3

This phasor diagram illustrates the graphical use of the double-subscript voltage convention to obtain $\mathbf{V}_{ab}$ for the network of Fig. 11-2.

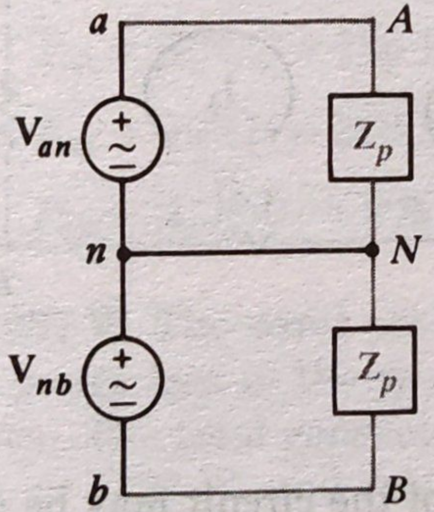


Figure 11-7

A simple single-phase three-wire system. The two loads are identical, and the neutral current is zero.

Since

$$\mathbf{V}_{an} = \mathbf{V}_{nb}$$

then,

$$\mathbf{I}_{aA} = \frac{\mathbf{V}_{an}}{\mathbf{Z}_p} = \mathbf{I}_{Bb} = \frac{\mathbf{V}_{nb}}{\mathbf{Z}_p}$$

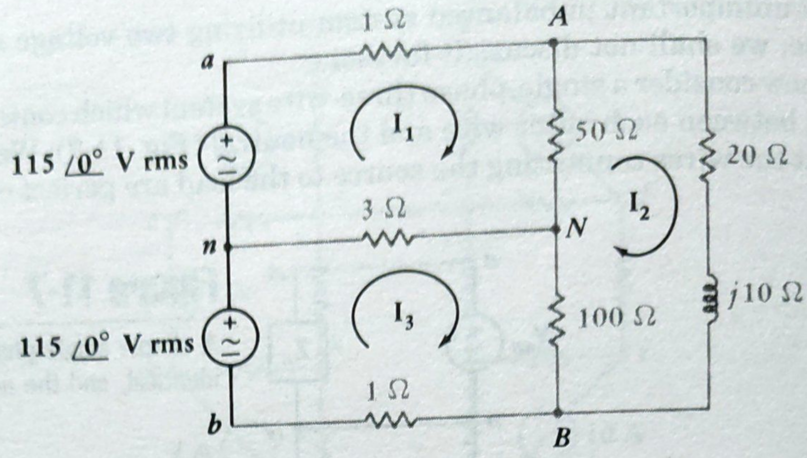
and therefore

$$\mathbf{I}_{nN} = \mathbf{I}_{Bb} + \mathbf{I}_{Aa} = \mathbf{I}_{Bb} - \mathbf{I}_{aA} = 0$$

Th

Figure 11-8

A typical single-phase three-wire system.



Solution: The analysis of the circuit may be achieved by assigning mesh currents and writing the appropriate equations. The three mesh currents are

$$\mathbf{I}_1 = \frac{\begin{vmatrix} 115 & -50 & -3 \\ 0 & 170 + j10 & -100 \\ 115 & -100 & 104 \end{vmatrix}}{\begin{vmatrix} 54 & -50 & -3 \\ -50 & 170 + j10 & -100 \\ -3 & -100 & 104 \end{vmatrix}} = 11.24\underline{/-19.83^\circ} \text{ A rms}$$

$$\mathbf{I}_2 = \frac{\begin{vmatrix} 54 & 115 & -3 \\ -50 & 0 & -100 \\ -3 & 115 & 104 \end{vmatrix}}{\text{denom}} = 9.39\underline{/-24.47^\circ}$$

$$\mathbf{I}_3 = \frac{\begin{vmatrix} 54 & -50 & 115 \\ -50 & 170 + j10 & 0 \\ -3 & -100 & 115 \end{vmatrix}}{\text{denom}} = 10.37\underline{/-21.80^\circ}$$

The currents in the outer lines are thus

$$\mathbf{I}_{aA} = \mathbf{I}_1 = 11.24\underline{/-19.83^\circ} \text{ A rms}$$

$$\mathbf{I}_{bB} = -\mathbf{I}_3 = 10.37\underline{/158.20^\circ}$$

and the smaller neutral current is

$$\mathbf{I}_{nN} = \mathbf{I}_3 - \mathbf{I}_1 = 0.946\underline{/-177.7^\circ} \text{ A rms}$$

The power drawn by each load may be determined:

$$P_{50} = |\mathbf{I}_1 - \mathbf{I}_2|^2(50) = 206 \text{ W}$$

which could represent two 100-W lamps in parallel. Also,

$$P_{100} = |\mathbf{I}_3 - \mathbf{I}_2|^2(100) = 117 \text{ W}$$

which might represent one 100-W lamp. Finally,

$$P_{20+j10} = |\mathbf{I}_2|^2(20) = 1763 \text{ W}$$

which we may think of as a 2-hp induction motor. The total load power is 2086 W. The loss in each of the wires is next found:

$$P_{aA} = |\mathbf{I}_1|^2(1) = 126 \text{ W}$$

$$P_{bB} = |\mathbf{I}_3|^2(1) = 108 \text{ W}$$

$$P_{nN} = |\mathbf{I}_{nN}|^2(3) = 3 \text{ W}$$

giving a total line loss of 237 W. The wires are evidently quite long; otherwise, the relatively high power loss in the two outer lines would cause a dangerous temperature rise. The total generated power must therefore be 206 + 117 + 1763 + 237, or 2323 W, and this may be checked by finding the power delivered by each voltage source:

$$P_{an} = 115(11.24) \cos 19.83° = 1216 \text{ W}$$

$$P_{bn} = 115(10.37) \cos 21.80° = 1107 \text{ W}$$

or a total of 2323 W. The transmission efficiency for this system is

$$\text{Eff.} = \frac{2086}{2086 + 237} = 89.8\%$$

A phasor diagram showing the two source voltages, the currents in the outer lines, and the current in the neutral is constructed in Fig. 11-9. The fact that $\mathbf{I}_{aA} + \mathbf{I}_{bB} + \mathbf{I}_{nN} = 0$ is indicated on the diagram. ■

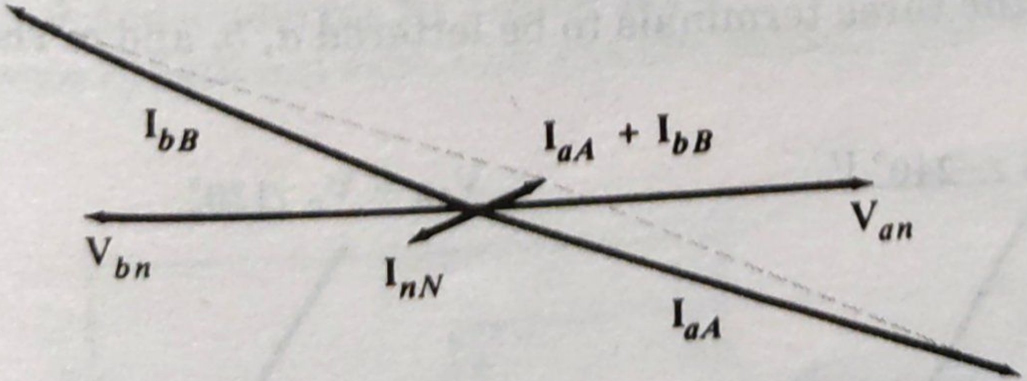


Figure 11-9

The source voltages and three of the currents in the circuit of Fig. 11-8 are shown on a phasor diagram. Note that $\mathbf{I}_{aA} + \mathbf{I}_{bB} + \mathbf{I}_{nN} = 0$.

$$\mathbf{V}_{an} = V_p\underline{/0^\circ}$$

where we shall consistently use V_p to represent the rms *amplitude* of any of the phase voltages, then the definition of the three-phase source indicates that either

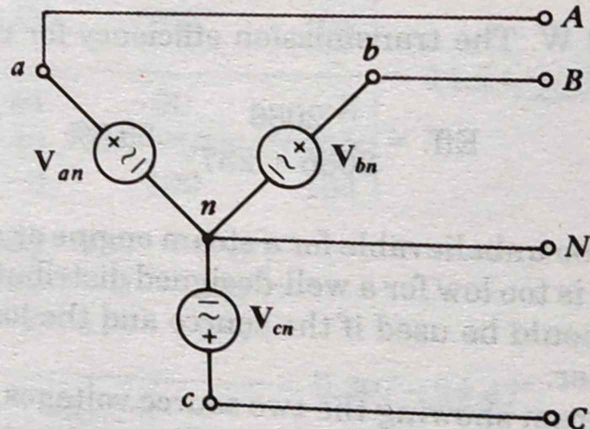
$$\mathbf{V}_{bn} = V_p\underline{/-120^\circ} \qquad \mathbf{V}_{cn} = V_p\underline{/-240^\circ}$$

or

$$\mathbf{V}_{bn} = V_p\underline{/120^\circ} \qquad \mathbf{V}_{cn} = V_p\underline{/240^\circ}$$

Figure 11-10

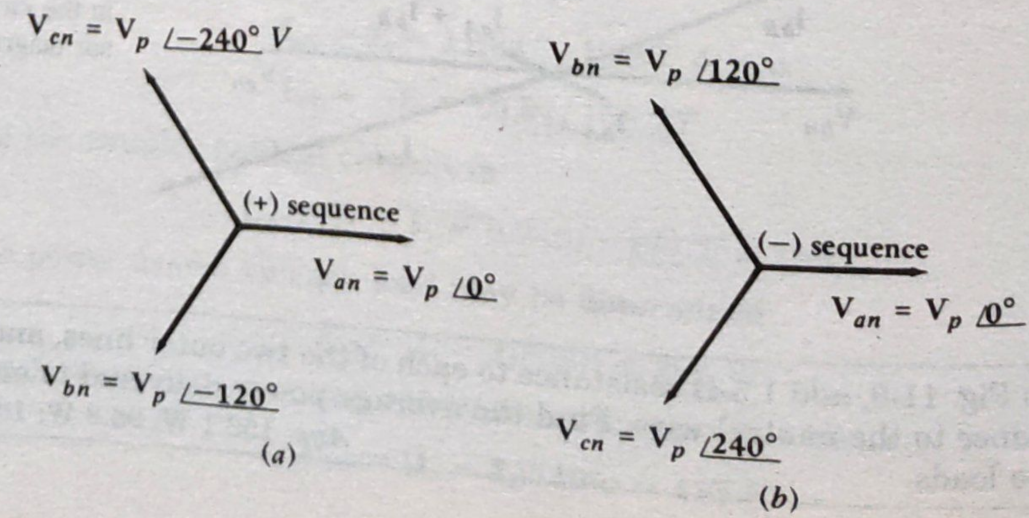
A Y-connected three-phase four-wire source.



The former is called *positive phase sequence,* or *abc* phase sequence, and is shown in Fig. 11-11*a*; the latter is termed *negative phase sequence,* or *cba* phase sequence, and is indicated by the phasor diagram of Fig. 11-11*b*. It is apparent that the phase sequence of a physical three-phase source depends on the arbitrary choice of the three terminals to be lettered *a*, *b*, and *c*. They may always

Figure 11-11

(*a*) Positive, or *abc*, phase sequence.
(*b*) Negative, or *cba*, phase sequence.



$$\mathbf{V}_{ab} = \sqrt{3}V_p \angle 30^\circ$$

$$\mathbf{V}_{bc} = \sqrt{3}V_p \angle -90^\circ$$

$$\mathbf{V}_{ca} = \sqrt{3}V_p \angle -210^\circ.$$

Kirchhoff's voltage law requires the sum of these three voltages to be zero, and it is zero.

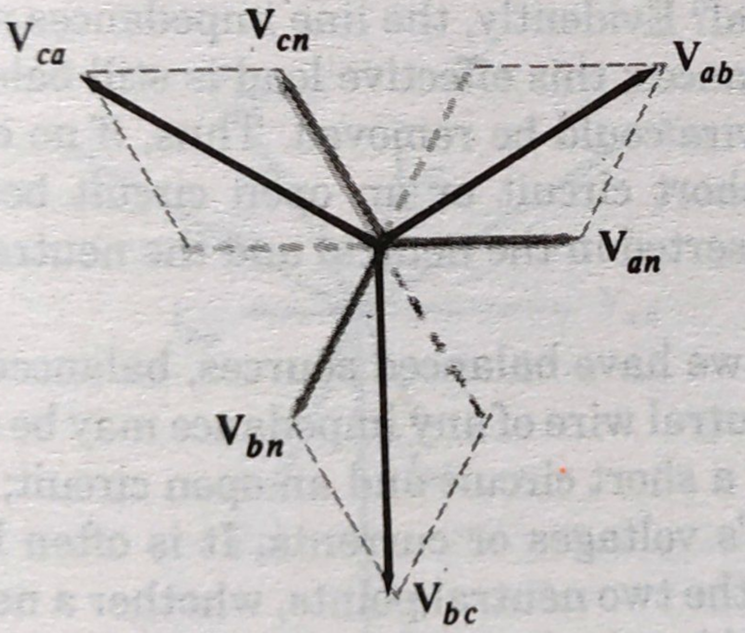
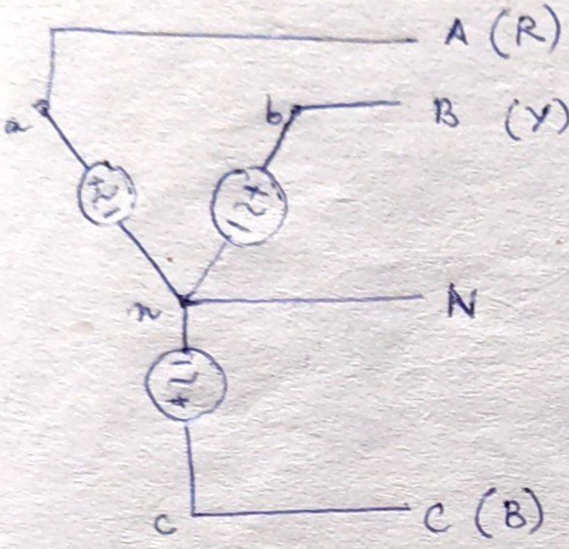


Figure 11-12

A phasor diagram which is used to determine the line voltages from the given phase voltages.

Three phase Y-Y connection:

|All in rms|



3-phase 4 wire system with 4th wire from common or neutral pt.

In balanced 3-phase systems,

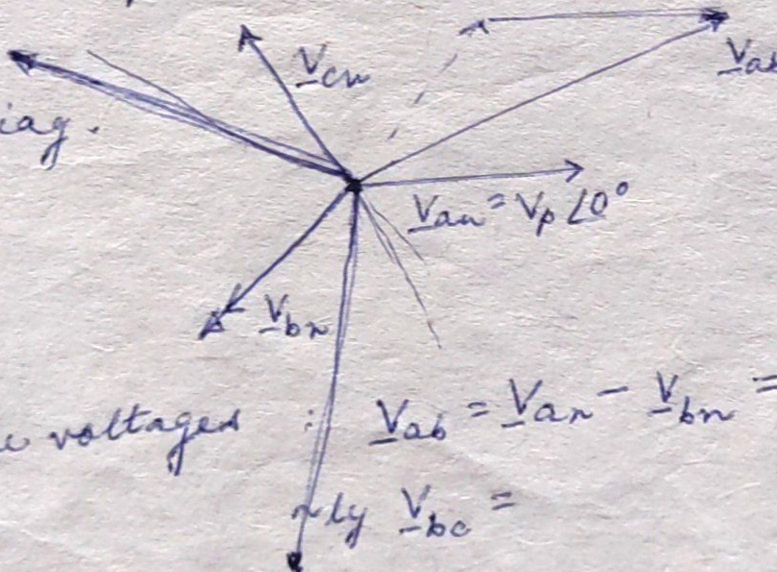
$$|\underline{V}_{an}| = |\underline{V}_{bn}| = |\underline{V}_{cn}| \qquad [\underline{V}_{an} = \underline{V}_a - \underline{V}_n]$$

and $\underline{V}_{an} + \underline{V}_{bn} + \underline{V}_{cn} = 0$.

$\longrightarrow$ PHASE VOLTAGES

Let $\underline{V}_{an} = V_p \angle 0^\circ$, then $\underline{V}_{bn} = V_p \angle -120^\circ$, $\underline{V}_{cn} = V_p \angle -240^\circ$ — POSITIVE PHASE SEQUENCE

Phasor diag.

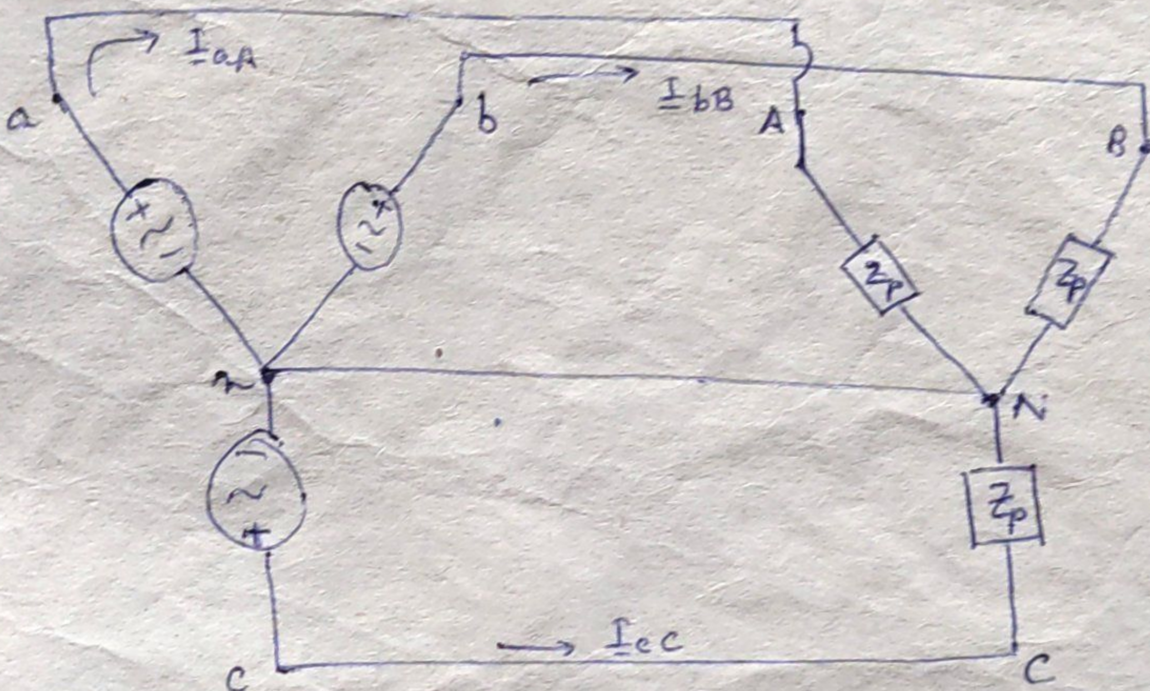


For line voltages : $\underline{V}_{ab} = \underline{V}_{an} - \underline{V}_{bn} = \sqrt{3}\, V_p \angle 30^\circ = V_L \angle 30^\circ$

||ly $\underline{V}_{bc} = \sqrt{3}\, V_p \angle -90^\circ = V_L \angle -90^\circ$

$\underline{V}_{ca} = \sqrt{3}\, V_p \angle -210^\circ = V_L \angle -210^\circ$

$\therefore V_L = \sqrt{3}\, V_p$



Balanced Y-connected load to Y-connected source.

$\therefore \; \underline{I}_{aA} = \dfrac{\underline{V}_{an}}{\underline{Z}_p} = I_L \angle 0^\circ ; \quad \underline{I}_{bB} = \dfrac{\underline{V}_{bn}}{\underline{Z}_p} = \underline{I}_{aA} \angle -120^\circ = I_L \angle -120^\circ$

$\underline{I}_{cC} = \dfrac{\underline{V}_{cn}}{\underline{Z}_p} = \underline{I}_{aA} \angle -240^\circ = I_L \angle -240^\circ$

$\therefore \; \underline{I}_{Nn} = \underline{I}_{aA} + \underline{I}_{bB} + \underline{I}_{cC} = 0$ $\therefore$ no current in neutral for balanced load.

$\therefore$ For Balanced $\begin{cases} \text{source} \\ \text{load} \\ \text{line impedances} \end{cases}$, a neutral wire of ANY impedance may be replaced by any other including short / open ckt.

— system remains unaffected.

For nN open, $I_L = I_p$ [$\because I_{nN} = 0$]

let us assume known line voltages

$$V_L = |\mathbf{V}_{ab}| = |\mathbf{V}_{bc}| = |\mathbf{V}_{ca}|$$

or known phase voltages

$$V_p = |\mathbf{V}_{an}| = |\mathbf{V}_{bn}| = |\mathbf{V}_{cn}|$$

where

$$V_L = \sqrt{3}V_p \quad \text{and} \quad \mathbf{V}_{ab} = \sqrt{3}V_p \underline{/30^\circ}$$

and so forth, as before. Since the voltage across each branch of the Δ is known, the *phase currents* are found:

$$\mathbf{I}_{AB} = \frac{\mathbf{V}_{ab}}{\mathbf{Z}_p} \qquad \mathbf{I}_{BC} = \frac{\mathbf{V}_{bc}}{\mathbf{Z}_p} \qquad \mathbf{I}_{CA} = \frac{\mathbf{V}_{ca}}{\mathbf{Z}_p}$$

and their differences provide us with the line currents, such as

$$\mathbf{I}_{aA} = \mathbf{I}_{AB} - \mathbf{I}_{CA}$$

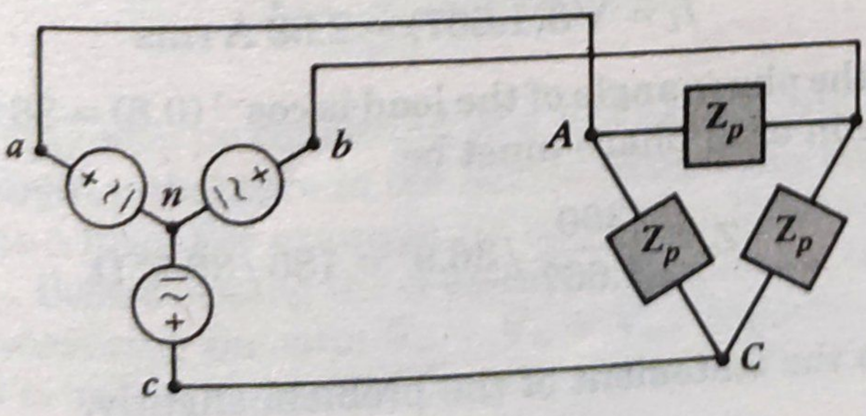


Figure 11-17

A balanced Δ-connected load is present on a three-wire three-phase system. The source happens to be Y-connected.