Magnetically coupled circuits : TRANSFORMERS

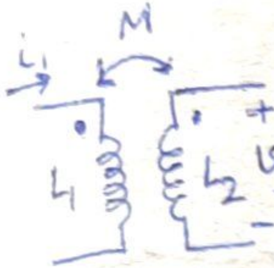
$v = L\frac{di}{dt}$ due to (i) magnetic flux produced by current $(\phi \propto i)$

(ii) prod. of voltage by time varying magnetic field $\left(v \propto \frac{d\phi}{dt}\right)$

Mutual inductance : same concept : current flowing in one coil establishes magnetic flux about that coil and a second coil in its vicinity and thus voltage produced in 2nd coil.

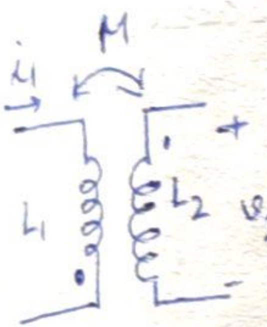
$$v_2 = M_{21}\frac{di_1}{dt} \qquad M_{21} : \text{coeff. of mutual inductance.}$$

units : H (henry).



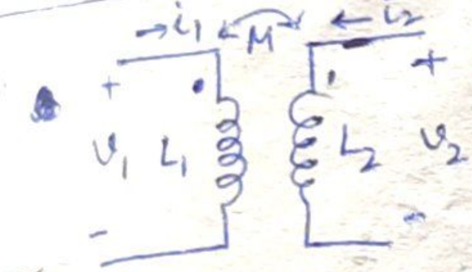
$v_2 = M\frac{di_1}{dt}$

Dot convention : Current entering dotted terminal of one coil produces ~~an~~ an open ckt. voltage bet. the terminals of the second coil which is sensed in the direction indicated by a positive voltage reference at the dotted terminal of this second coil.



$v_2 = -M\frac{di_1}{dt}$

1.



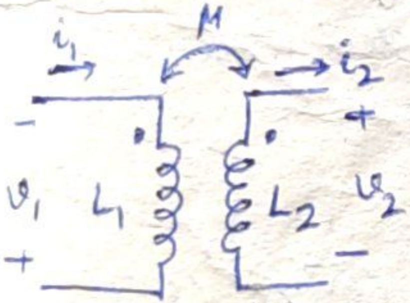
$$v_1 = L_1 \frac{di_1}{dt} + M \frac{di_2}{dt}$$

$$v_2 = L_2 \frac{di_2}{dt} + M \frac{di_1}{dt}.$$

$$\underline{V}_1 = \underline{s} L_1 \underline{I}_1 + \underline{s} M \underline{I}_2$$

$$\underline{V}_2 = \underline{s} L_2 \underline{I}_2 + \underline{s} M \underline{I}_1$$

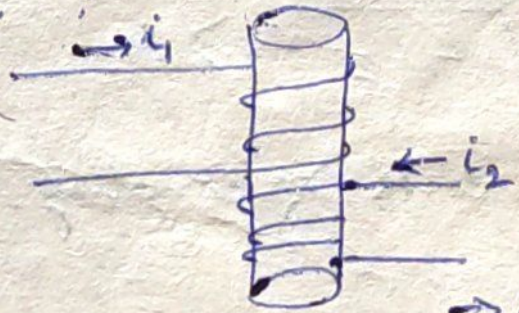
2.



$$-v_1 = L_1 \frac{di_1}{dt} - M \frac{di_2}{dt} \Rightarrow v_1 = -L_1 \frac{di_1}{dt} + M \frac{di_2}{dt}$$

$$v_2 = -L_2 \frac{di_2}{dt} + M \frac{di_1}{dt}.$$

Physically: Rt. hand rule

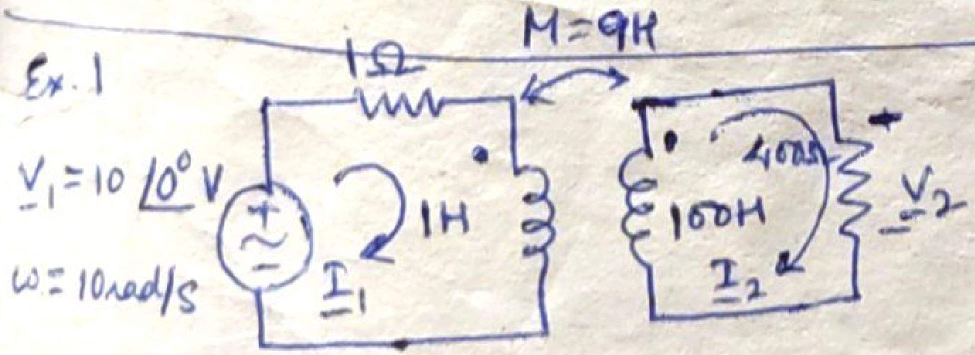


Assume i_1, i_2 +ve and ↑ with time.
downward flux for both.

∴ Additive fluxes.

$\Rightarrow i_1, i_2 \xrightarrow{\text{entering}}$ dot marked terminals

Ex. 1



$$\underline{I}_1(1+j10) - j90\,\underline{I}_2 = 10$$

$$\underline{I}_2(400+j1000) - j90\,\underline{I}_1 = 0$$

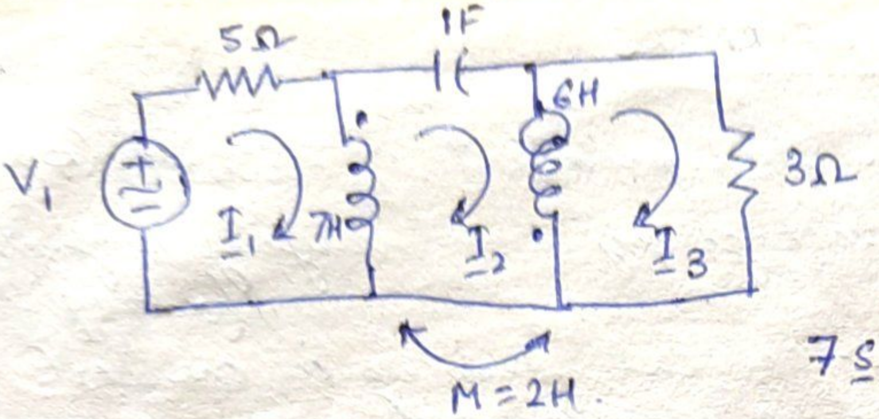
$$\therefore\ \underline{I}_2 = 0.1724\angle -16.7^\circ$$

$$\therefore\ \frac{\underline{V}_2}{\underline{V}_1} = \frac{400\,(0.1724\angle -16.7^\circ)}{10} = 6.90\angle -16.7^\circ$$

General expression for $\underline{I}_2(\underline{s}) = \dfrac{9\underline{s}\,\underline{V}_1}{19\underline{s}^2 + 500\underline{s} + 400}$

$$\therefore\ \frac{\underline{V}_2}{\underline{V}_1} = 189.5\,\frac{\underline{s}}{(\underline{s}+0.826)(\underline{s}+25.5)}$$

Ex 2



$\cancel{(5+7s)\underline{I}_1}$.

$$5\underline{I}_1 + 7\underline{s}(\underline{I}_1 - \underline{I}_2) + 2\underline{s}(\underline{I}_3 - \underline{I}_2) = \underline{V}_1$$

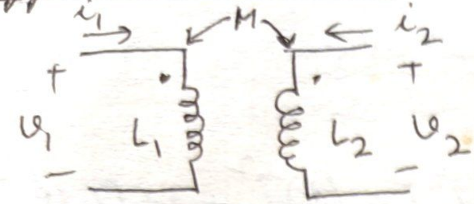
$$7\underline{s}(\underline{I}_2 - \underline{I}_1) + \frac{1}{\underline{s}}\underline{I}_2 + 6\underline{s}(\underline{I}_2 - \underline{I}_3)$$

$$+ 2\underline{s}(\underline{I}_2 - \underline{I}_3) + 2\underline{s}(\underline{I}_2 - \underline{I}_1) = 0.$$

* (current leaving dot terminals)

$$6\underline{s}(\underline{I}_3 - \underline{I}_2) + 3\underline{I}_3 + 2\underline{s}(\underline{I}_1 - \underline{I}_2) = 0.$$

Energy considerations:



1. Let all i & v be zero.
$\therefore$ 0 initial energy storage

2. o.c. rt. terminal pair
$i_1 \to 0$ to I_1 in time $t = t_1$
$\therefore$ power entering network from left

instantaneously : $v_1 i_1 = L_1 \frac{di_1}{dt} \cdot i_1$

from rt. : $v_2 i_2 = 0$. $\therefore i_2 = 0$.

Energy stored w/i the network when $i_1 = I_1$

$$\int_0^{t_1} v_1 i_1 \, dt = \int_0^{I_1} L_1 i_1 \, di_1 = \frac{1}{2} L_1 I_1^2$$

3. i_1 cons. at I_1, change $i_2 \to 0$ to I_2 in time $t_1 \to t_2$.

$\therefore$ Energy delivered from rt. source is

$$\int_{t_1}^{t_2} v_2 i_2 \, dt = \int_0^{I_2} L_2 i_2 \, di_2 = \frac{1}{2} L_2 I_2^2$$

Energy delivered by left source

$$\int_{t_1}^{t_2} v_1 i_1 \, dt = \int_{t_1}^{t_2} M_{12} \frac{di_2}{dt} \cdot i_1 \, dt = M_{12} I_1 I_2$$

$\therefore$ Total energy stored when i_1 & i_2 reach constant values

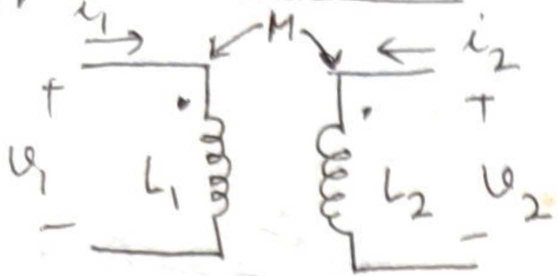
$$W_{total} = \frac{1}{2} L_1 I_1^2 + \frac{1}{2} L_2 I_2^2 + M_{12} I_1 I_2$$

~ly starting from rt. $= \frac{1}{2} L_1 I_1^2 + \frac{1}{2} L_2 I_2^2 + M_{21} I_1 I_2$.

i.c & final condns. same $\therefore$ total energy same $\therefore M_{12} = M_{21} = M$.

$$\therefore W = \frac{1}{2} L_1 I_1^2 + \frac{1}{2} L_2 I_2^2 + M I_1 I_2.$$

Energy considerations:



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$$\int_{t_1}^{t_2} v_2 i_2 \, dt = \int_0^{I_2} L_2 i_2 \, di_2 = \frac{1}{2} L_2 I_2^2$$

Energy delivered by left source

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~ ly starting from rt. $= \frac{1}{2} L_1 I_1^2 + \frac{1}{2} L_2 I_2^2 + M_{21} I_1 I_2$.

i.e. final condns. same $\therefore$ total energy same $\therefore$ $M_{12} = M_{21} = M$.

$$\therefore \quad W = \frac{1}{2} L_1 I_1^2 + \frac{1}{2} L_2 I_2^2 + M I_1 I_2 .$$

Say i_1 enters • terminal, i_2 leaves • terminal, then

$$W = \frac{1}{2}L_1 I_1^2 + \frac{1}{2}L_2 I_2^2 - M I_1 I_2$$

$\therefore$ Instantaneously: $w(t) = \frac{1}{2} L_1 [i_1(t)]^2 + \frac{1}{2} L_2 [i_2(t)]^2 \pm M [i_1(t)] [i_2(t)]$

For upper lt. on M, assume i_1 & i_2 both +ve or −ve

$\therefore$ only case when $w(t)$ can be −ve is

$$w(t) = \frac{1}{2} L_1 i_1^2 + \frac{1}{2} L_2 i_2^2 - M i_1 i_2$$

$$= \underbrace{\frac{1}{2}\left(\sqrt{L_1}\, i_1 - \sqrt{L_2}\, i_2\right)^2}_{\geq 0} + \sqrt{L_1 L_2}\, i_1 i_2 - M i_1 i_2$$

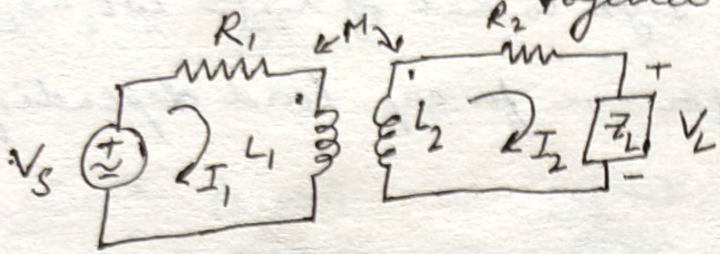
$\therefore$ since $w(t) \geq 0$

$$\therefore \sqrt{L_1 L_2} - M \geq 0 \quad \therefore M \leq \sqrt{L_1 L_2}.$$

coeff. of coupling $k = \dfrac{M}{\sqrt{L_1 L_2}}$, $0 \leq k \leq 1$.

The linear transformer: ϕ vs. i linear

network containing two or more coils deliberately coupled together.



Core: usually iron alloy.

Primary, Secondary (has load connected)

:)

R_1, R_2: winding resistance & other losses.

Usually secondary: resonant (tuned)
also primary tuned – ideal voltage source replaced by current source + // large R & C.

Mesh eqns.

$$V_S = I_1(R_1 + sL_1) - I_2 sM \quad \ldots (11)$$

$$0 = -I_1 sM + I_2(R_2 + sL_2 + Z_L) \quad \ldots (12)$$

Let. $Z_{11} = R_1 + sL_1 \qquad Z_{22} = R_2 + sL_2 + Z_L$

$$\therefore V_S = I_1 Z_{11} - I_2 sM \quad \ldots (13)$$

$$0 = -I_1 sM + I_2 Z_{22} \quad \ldots (14)$$

Solve (14) for I_2 & replace in (13)

$$\therefore Z_{in} = \frac{V_S}{I_1} = Z_{11} - \frac{s^2M^2}{Z_{22}} \quad \ldots (15)$$

Note: 1. independent of dot placement

2. reduce coupling (M) to 0, then $\underline{Z}_{in} = \underline{Z}_{11}$

as M inc., $\underline{Z}_{in}$ changes from $\underline{Z}_{11}$ by $\left(-\dfrac{\underline{s}^2 M^2}{\underline{Z}_{22}}\right) \rightarrow$ reflected impedance.

Under s.s. sinusoidal oper. $\underline{s} = j\omega$

$$\therefore \underline{Z}_{in}(j\omega) = \underline{Z}_{11}(j\omega) + \frac{\omega^2 M^2}{R_{22} + jX_{22}}$$

$$= \underline{Z}_{11} + \frac{\omega^2 M^2 R_{22}}{R_{22}^2 + X_{22}^2} + \frac{-j\omega^2 M^2 X_{22}}{R_{22}^2 + X_{22}^2}$$

3. Presence of sec. inc. losses in pri. ckt. $\because \dfrac{\omega^2 M^2 R_{22}}{R_{22}^2 + X_{22}^2}$ is +ve.

4. Reactance reflected into pri. opp. sign to that of $X_{22} (= \omega L_2 + X_L)$ & X_{22} is (+ve) for inductive load & +ve or −ve for cap. load depending on mag. of X_L.

Sp. case : 1. pri. has C in series added.

2. pri. & sec. are identical series resonant ckts.

$R_1 = R_2 = R$, $L_1 = L_2 = L$. $\underline{Z}_L = C$.

$\therefore \omega_0 = 1/\sqrt{LC}$ $\therefore$ $\underline{Z}_{11} = R_{11}$, $\underline{Z}_{22} = R_{22}$

$\underline{Z}_{in}$ = purely resistive, no reflected reactance.

$\therefore$ Resonant condn.

3. Say $\omega \doteq \omega_0 + \delta$ $\therefore$ X_{11} & X_{22} both inductive

$\therefore$ reflected reactance is capacitive (associated -ve sign)

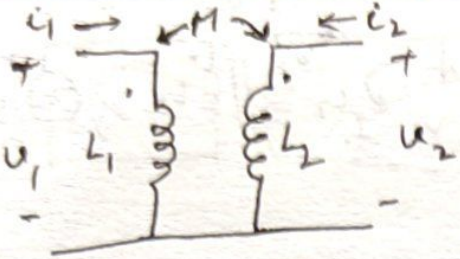
if k is large, then $\underline{Z}_{in}$ may again be resistive $\therefore$ 2nd resonant condn.

vice versa for $\omega = \omega_0 - \delta$.

Useful in communications sys. $\longrightarrow$ AM & FM radio, TV, radar, telemetry etc

$\therefore$ allow large sec. i allowed over a band of frequencies from slightly below to slightly above ω_0. $\therefore$ Max. response over wider range of freq.

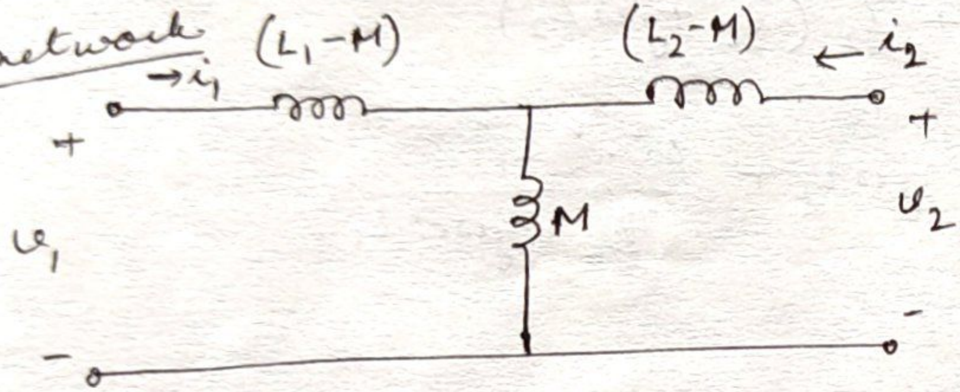
...uivalent T & π networks: Say ckt.



$$\therefore \quad v_1 = L_1 \frac{di_1}{dt} + M \frac{di_2}{dt} \quad \ldots (16)$$

$$v_2 = M \frac{di_1}{dt} + L \frac{di_2}{dt}. \quad \ldots (17)$$

T network



If dot changed, then replace M by (−M) all through

Note: Equi. ckt. may have (−ve) inductance.
– useful in network synthesis where such is reqd. by using appropriate transformer.

π network req. nodal eqn.

$\therefore$ Solve (17) for $\frac{di_2}{dt}$ & substitute in (16)

$$\therefore \; v_1 = L_1 \frac{di_1}{dt} + M\left(\frac{1}{L_2}\right)\left[v_2 - M\frac{di_1}{dt}\right]$$

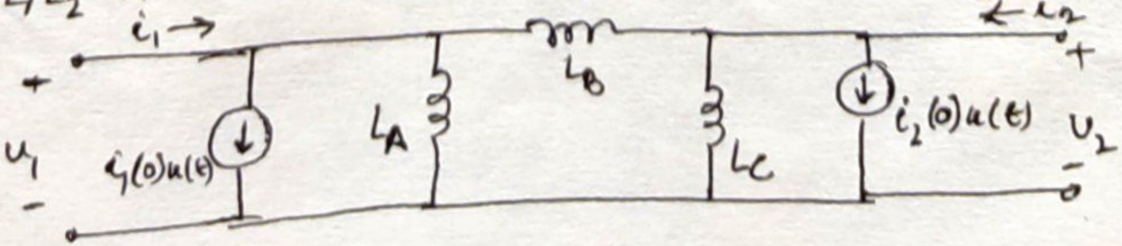
$$\text{or} \quad \frac{di_1}{dt} = \frac{L_2}{L_1L_2 - M^2}\, v_1 - \frac{M}{L_1L_2 - M^2}\, v_2$$

Integrate from 0 to t current source.

$$\therefore \; i_1 - i_1(0)\,u(t) = \frac{L_2}{L_1L_2 - M^2}\int_0^t v_1\,dt - \frac{M}{L_1L_2 - M^2}\int_0^t v_2\,dt$$

$$\text{||ly} \quad i_2 - i_2(0)\,u(t) = \frac{-M}{L_1L_2 - M^2}\int_0^t v_1\,dt + \frac{L_1}{L_1L_2 - M^2}\int_0^t v_2\,dt$$

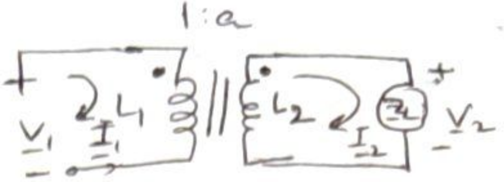
$\therefore \; 1/L_B = \frac{M}{L_1L_2 - M^2}$, coeff. of $\int_0^t v_1\,dt = \frac{1}{L_A} + \frac{1}{L_B}$ $\therefore \; \frac{1}{L_A} = \frac{L_2 - M}{L_1L_2 - M^2}$



||ly $\frac{1}{L_C} = \frac{L_1 - M}{L_1L_2 - M}$

Ideal transformer: $k = 1$

$$\therefore \quad \frac{L_2}{L_1} = \frac{N_2^2}{N_1^2} = a^2 \qquad \text{where } \frac{N_2}{N_1} = a.$$



sinusoidal s.s.

$$\underline{V}_1 = \underline{I}_1 j\omega L_1 - \underline{I}_2 j\omega M$$

$$0 = -\underline{I}_1 j\omega M + \underline{I}_2 (\underline{Z}_L + j\omega L_2)$$

$$\therefore \quad \underline{Z}_{in} = j\omega L_1 + \frac{\omega^2 M^2}{\underline{Z}_L + j\omega L_2} = j\omega L_1 + \frac{\omega^2 a^2 L_1^2}{\underline{Z}_L + j\omega a^2 L_1}$$

$$= \frac{j\omega L_1 \underline{Z}_L}{\underline{Z}_L + j\omega a^2 L_1}$$

If $L_1 \to \infty$, then $\underline{Z}_{in} = \frac{\underline{Z}_L}{a^2}$ for finite $\underline{Z}_L$

Note Z_{22} = Reflected impedance is $\propto$ $[a \cdot L_1]$

$\therefore$ $\underline{Z}_{in}$ show diff. from cancellation of $\underline{Z}_{11}$ & reflected $\underline{Z}_{22}$.

$$N_1 \underline{I}_1 = N_2 \underline{I}_2$$

$$\frac{V_2}{V_1} = a^2 \frac{I_2}{I_1} = a = \frac{N_2}{N_1} \quad \& \quad V_2 I_2 = V_1 I_1 \longleftrightarrow$$ A2L for condns. when L_1 & L_2 large compared to $\underline{Z}_L$

Can be represented as Th. / Norton's equivalent on pri. / sec. side as req.