Electrical circuit : an interconnection of simple electrical devices in which there is at least <u>one</u> closed path in which current may flow.

Analysis : a (mathematical) study of a complex entity and the interrelationship of its parts.

All matter ⟶ fundamental building blocks - ATOMS - $\bar{e}$, protons and neutrons ⟶ charged particles.

Ref. Millman Halkias (tran. at low freq.)

Coulomb (C) : Fund. unit

a) max., avg (dc) & rms by uppercase

Q or q : symbol — (Difference?) — TI vs. instantaneous

b) lowercase

~ly. i_c, i_C, I_c, I_C ⟶ different quantities. c) Avg (dc) & inst. <u>total</u> - uppercase subscript

d) varying comp. from some quiescent

Current : $\frac{dq}{dt} = i$ I or i Amperes (A)

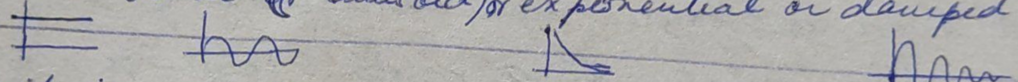
$i_1(t)$

Charge transferred bet. t_0 and t : $q \Big|_{t_0}^{t} = \int_{t_0}^{t} i\,dt.$

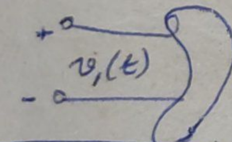
Total charge transferred over all time $\int_{t_0}^{t} i\,dt + q(t_0) = q$

$\xrightarrow{3A} \equiv \xleftarrow{-3A}$

Types : dc or ac (sinusoid) or exponential or damped sinusoid



Any circuit is a system composed of several elements <u>through</u> which <u>i</u> <u>flows</u> when <u>v</u> is applied <u>across</u> it.



Voltage v or V : Volts (V) : work reqd. to move charge through element.

$\therefore E = qv.$

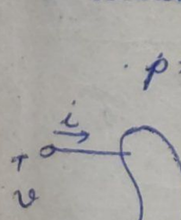
Let $v_1(t) = 100 \cos 250t$ Find a) $v_1(1ms)$ b) $v_1(8ms)$

(c) energy reqd. to move 4C from <u>lower</u> to <u>upper</u> terminal at $t = 4ms$

96.89 V, -41.6 V, -216 J

Power absorbed by any ckt. element in terms of voltage across it and current through it : P or p Watts (W)

$\therefore p = vi$



Passive sign convention : energy or power absorbed +ve

Physical device ≠ mathematical model of device → CKT. ELEMENT

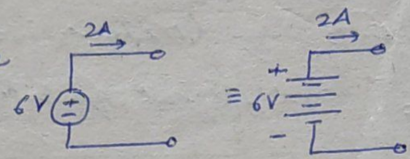
<u>General ckt. element</u> composed of one or more than one <u>simple ckt. element</u> (cannot be further subdivided).

CLASSIFICATION acc. to reln. bet. i & v.

a) Independent sources: v totally independent of i or vice versa.

b) Dependent (controlled) sources: Source v or i depends on i or v elsewhere in ckt.

Independent voltage source (Ideal)



d.c. battery.

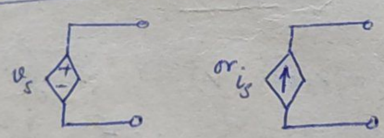
For currents less than 20A or so, ord. household electrical outlet approx. an independent voltage source $v_s = 220\sqrt{2}\cos 2\pi 50t$ V.

Independent current source

: used in electronic ckts.

Independent dc current source represents very closely the proton beam of a <u>cyclotron</u> operating at a constant beam current of @ 1μA

Dependent sources



Sources: ACTIVE ELEMENTS — capable of delivering power to some external device.

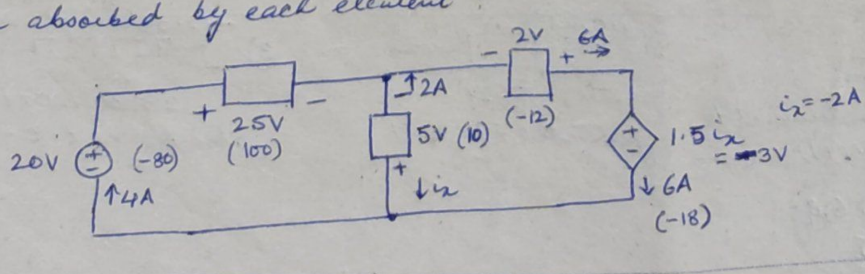
All others: PASSIVE ELEMENTS: receive power. (can store too ∴ deliver).

Electric NETWORK : Interconnection of two/more simple ckt. elements.

CKT. : if at least <u>one</u> closed path so i can flow through.

PASSIVE ckt. elements: Resistor, inductor, capacitor and pair of mutually coupled inductors.

Power absorbed by each element:

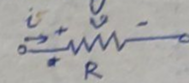


Notation summarized

Inst. total value	v_B	i_B	Source: Millman Halkias
Quiescent	V_B	I_B	Transistor at low frequencies.
Inst. value of varying component	v_b	$i_b = i_B - I_B$	
Effective value of varying component	V_b	I_b	
Supply voltage	V_{BB}		

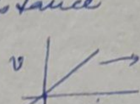
Resistive circuits

Ohm's law : LINEAR resistor : voltage across many types of conducting materials is directly prop. to the current flowing through the material.



$$v = Ri$$

R: constant of proportionality — Resistance
unit: Ω (ohms) = 1 V/A.



→ st. line through origin

provided temp. & other environmental factors are held constant (for certain ranges of i, v, power)

normally +ve quantity.

*(−ve resistances simulated with sp. ckt.)

Actual resistors: nonlinear. — eg. ex. zener diodes, tunnel diodes, fuses.

Absorbed power dissipated totally as heat (always +ve) — no power delivering or storing capacity. $p = vi = iR\,i = i^2R = \frac{v^2}{R}$

Wattage essential : connect 100 Ω, 2W carbon resistor across 110 V source — flame, smoke, breakage.

Ratio of current to voltage : $\frac{i}{v} = \frac{i}{Ri} = \frac{1}{R} = G$ (conductance)
unit: mho, ℧ or Siemen (S)
Same ckt. symbol.
Absorbed power: $vi = v^2G = \frac{i^2}{G}$

→ All in terms of instantaneous values. ∴ i_L & v_L through across resistor varies with time in same manner.

$R = 10\,\Omega$, $v = 2\sin 100t$ V $\Rightarrow i = v/R = 0.2\sin 100t$ A
power: $0.4\sin^2 100t$ V ⟶ diff. variation with time. (always +ve)

Short circuit : R = 0 Ω ⟶ ∴ v = 0 V, i any value
Open circuit : R = ∞ Ω ⟶ v any value, i = 0 A.

a) R if v = −8V, i = −5 mA = 1.60 kΩ
b) absorbed power if i = −5A, R = 2.2 Ω = 55W
c) i if R = 8 Ω, pabsorbed = 200 mW = ±158.1 mA
d) G if v = 2.5 V and i = 100 mA = 40 mS

Kirchhoff's laws

Consider simple networks in which two/more simple ckt. elements are interconnected by electrical conductors (leads) with 0 resistance

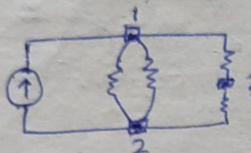
→ Perfectly conducting

∴ Lumped constant network.

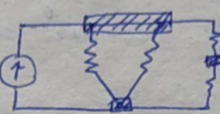
Actual: distributed constant network.

So, ckt. leads kept small

Node: A pt. at which ~~two~~ or more elements have a common connection



not 4



3 nodes

Path: Start at one node in network, through element to another node, through another element to 3^{rd} node and so on w/o encountering a node more than once.

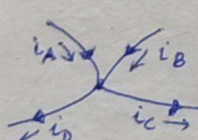
Closed path or loop: if start at node & back to node through a path.

Branch: Single path in network composed of one element & nodes at either end of element. 5 branches.

Kirchhoff's Current Law: (KCL)

Algebraic sum of ~~the~~ the currents entering any node is zero.

[∵ charge cannot accumulate at any node]



$i_A + i_B - i_C - i_D = 0$. currents entering a node. (1)

$\sum_{n=1}^{N} i_n = 0$.

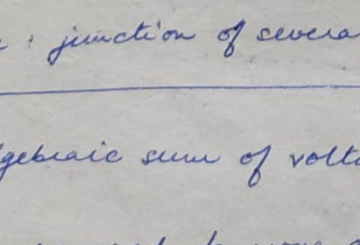
Equally valid for currents leaving the node: $-i_A - i_B + i_C + i_D = 0$

Water: junction of several pipes → node : mass flow rate

KVL:

Algebraic sum of voltages @ any closed path in a ckt. is zero.

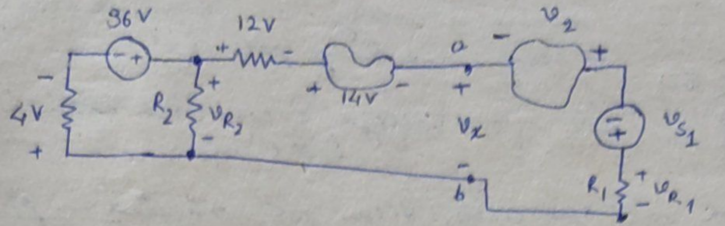
[∵ Energy reqd. to move a charge from one pt. to another is independent of path taken.]



Pott. bet A & B clockwise with − encountered as −v

$v_1 = v_2 - v_3$ i.e. $-v_1 + v_2 - v_3 = 0$.

$\sum_{n=1}^{N} v_n = 0$. (Conservation of energy).



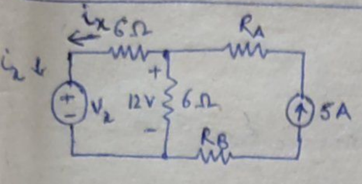
v_{R_2} : $4 - 36 + v_{R_2} = 0$

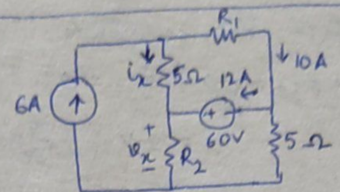
$\therefore v_{R_2} = 32V$

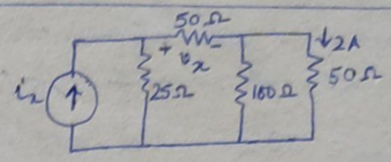
v_x : $-32 + 12 + 14 + v_x = 0$

$v_x = 6V$

or $4 - 36 + 12 + 14 + v_x = 0$



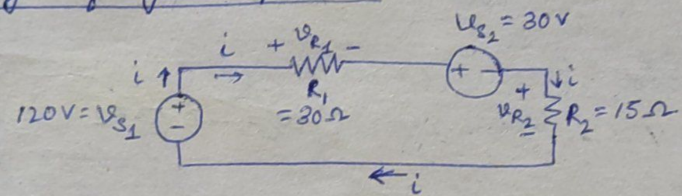




	a)	b)	
Branches	6	6	5
Nodes	5	4	3
i_x	3 A	−4 A	13 A
v_x	−6 V	50 V	150 V

Analysis of single loop circuit :

KCL ∴ same i through whole of ckt.



All elements that carry same current are in series (not equal but same)

$v = Ri$ assumes i enters terminal at which $+v$ ref. located ∴ p + v...

KVL : $-v_{S_1} + v_{R_1} + v_{S_2} + v_{R_2} = 0$

$\Rightarrow -v_{S_1} + R_1 i + v_{S_2} + R_2 i = 0$ $\quad \therefore i = \dfrac{v_{S_1} - v_{S_2}}{(R_1 + R_2)}$

~~[illegible]~~ $\therefore i = \dfrac{120 - 30}{30 + 15}$ A = 2 A. $v_{30} = 2 \cdot 30 = 60V$, $v_{15} = 2 \cdot 15 = 30V$.

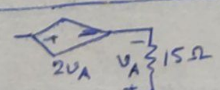
$P_{120V} = 120(-2) = -240W$ (delivered ∴)

$P_{30} = 30(2) = 60W$ and so on.

Power balance exists.

If i considered in opp. direction → check $i_2 = -i$.

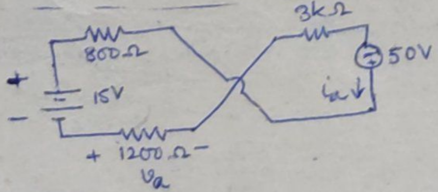
Say $v_{S_2} = 2v_A$



$\therefore -120 + v_{30} + 2v_A - v_A = 0$. $v_A = -15i$, $v_{30} = 30i$

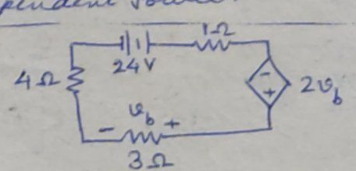
$\therefore i = 8A$

Dependent source.



$i_a = 7mA$, $v_a = 8.4V$,

power supplied by 15V battery : −105 mW.

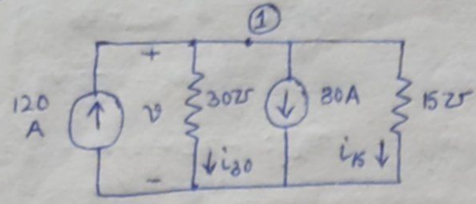


Power absorbed by each element.

$P_{4\Omega}$: 576 $\quad P_{24V}$: −288, $P_{1\Omega}$: 144

P_{2v_b} : −864 $\quad P_{3\Omega}$: 432 W

Single-node-pair circuit → Companion (dual) of single loop ckt.



Assume a voltage across any element – assign arbitrary polarity.

KVL → voltage across each branch identical.

All elements having common voltage across them are connected in parallel

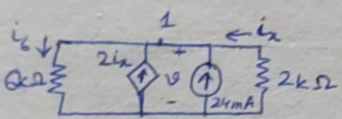
Passive sign convention → select two currents.

KCL at node 1 : $-120 + i_{30} + 30 + i_{15} = 0$.

$i_{30} = 30v$, $i_{15} = 15v$. ∴ $v = 2V$, $i_{30} = 60A$, $i_{15} = 30A$.

$p_{30} = 120W$. $p_{15} = 60W$. $p_{120A} = -240W$, $p_{30A} = 60W$.

~~EXACT~~ DUALS → ~~source elements~~ values same , else DUALS.



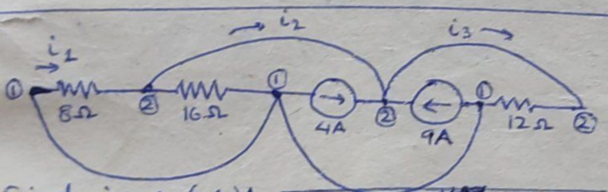
Node 1 (KCL) : $-i_x - 24\times10^{-3} - 2i_x + i_6 = 0$.

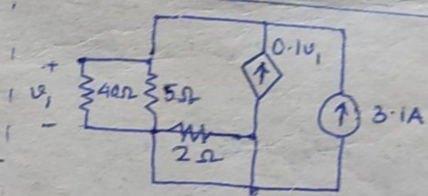
$i_6 = \frac{v}{6000}$, $i_x = \frac{-v}{2000}$

∴ $\frac{v}{6000} - 0.024 + \frac{3v}{2000} = 0$.

$10v = 24\times6 = 144V$ ∴ $v = 14.4V$; $i_x = -0.0072A$

$p_{24} = 14.4(0.024) = 0.346W$

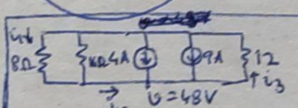




Find i_1 : (−6)A

i_2 : −9A

i_3 : 4A



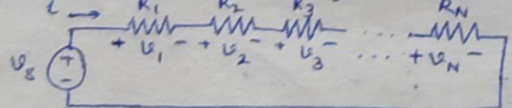
$\frac{v}{8} + \frac{v}{16} + \frac{v}{12} = 13$

L-R power absorbed

15.4W ; 123W ; 0W ; −61.5W ; −76.9W

Equivalent resistances :

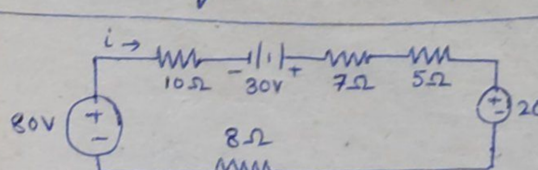
Series combination of N resistors:



KVL: $v_s = v_1 + v_2 + \ldots v_N = i(R_1 + R_2 + \ldots + R_N)$
$= iR_{eq}$.

$R_{eq} = \sum_{i=1}^{N} R_i$

1. Condn : Not specifically require i, v or p associated with indiv. resistor.
2. Order of elements not important
3. Sev. voltage sources may be replaced by eq. voltage source



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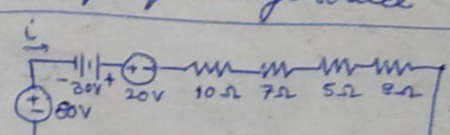

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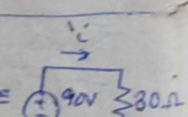
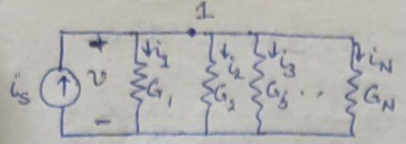


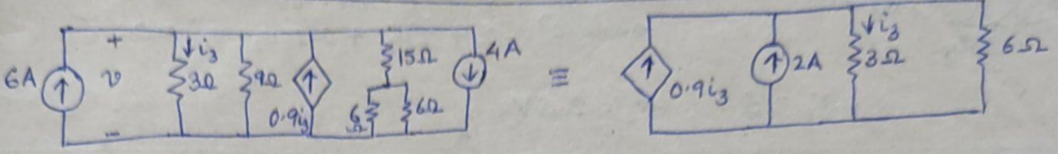
$i = 3A$.



KCL at node 1: $i_s = i_1 + i_2 + i_3 + \dots + i_N = (G_1 + G_2 + \dots + G_N)v = G_{eq}\cdot v$

∴ $G_{eq} = \sum_{i=1}^{N} G_i \Rightarrow \frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \dots + \frac{1}{R_N}$.

For two resistors, $R_{eq} = \frac{R_1 R_2}{R_1 + R_2}$.



∴ $-0.9 i_3 - 2 + i_3 + \frac{v}{6} = 0 \Rightarrow 0.1 i_3 + \frac{v}{6} = 2 \Rightarrow \frac{v}{30} + \frac{v}{6} = \frac{v}{5} = 2$

$\Rightarrow v = 10V$ ∴ $i_3 = \frac{v}{3} = 3.33A$

∴ $0.9 i_3 = \frac{9}{10} \times \frac{10}{3} = 3A$.

∴ Power delivered = $3 \times 10W = 30W$.

Power dissipated in 15Ω resistor → v = 10V across ∴ i = ? → power.

Parallel of two voltage sources / series of two current sources → not physically possible. (KVL/KCL) → only permissible when terminal voltage identical at each instant

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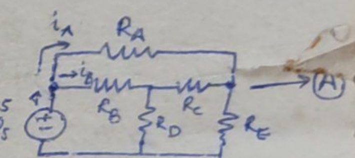
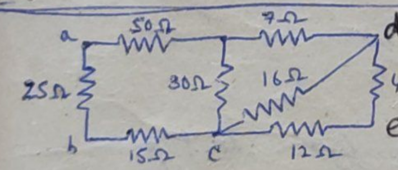
Single loop  → in series as well as in parallel..

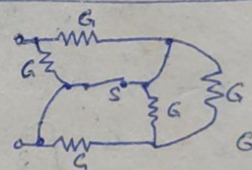
All ckts. may not have elements in series / parallel





$R_{de} = 4 \| [\{((15+25+50)\|30)+7\}\|16) + 12]$
$= \{[29.5 \| 16] + 12\} \| 4$

$R_{ab} = 18.75\Omega$, $R_{ac} = 24\Omega$, $R_{bc} = 12.75\Omega$

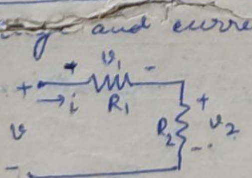


$\frac{2}{5}G + G = \frac{7G}{5} \times 10 = 14G$

G = 10 m℧

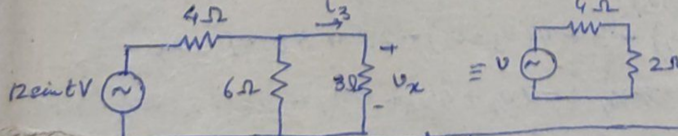
terminal conductance if (i) S open 14m℧
(ii) S closed 20m℧ (iii) replaced by G: 16.25m℧.

Voltage and current division:

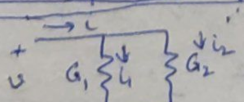


$v_2 = R_2 \cdot i = R_2 \cdot \frac{v}{R_1 + R_2} = \frac{R_2}{R_1 + R_2} \cdot v$; $v_1 = \frac{R_1}{R_1 + R_2} v$.

$v_i = \frac{R_i \cdot v}{\sum_{i=1}^{N} R_i}$ for resistances in series.



∴ $v_2 = \frac{2}{6} \times v = 4\sin t$ V.

Dual: 

∴ $i_2 = G_2 \cdot v = \frac{G_2}{G_1 + G_2} i = \frac{R_1}{R_1 + R_2} \cdot i$

$i_1 = \frac{R_2}{R_1 + R_2} i$, $i_i = \frac{G_i\, i}{\sum_{i=1}^{N} G_i}$

$i_3 = \cancel{\frac{2}{3} \cdot 4\sin t\, A = \frac{8}{3}\sin t\ A}$

$i_3 = \frac{6}{9} \cdot i = \frac{6}{9} \cdot \frac{v}{4+2} = \frac{v}{9} = \frac{12}{9}\sin t = \frac{4}{3}\sin t$ A.

*Not always applicable → say for (A). — only for ∥ branches (same pair of nodes)