Complex frequency:

Damped sinusoid: $v(t) = V_m e^{\sigma t} \cos(\omega t + \theta)$

Sp. cases: $\sigma = 0, \omega = 0$ ~~sinusoid~~ $v(t) = V_m \cos\theta = V_0$ — constant voltage

$\sigma = 0$, general sinusoid $v(t) = V_m \cos(\omega t + \theta)$

$\omega = 0$, exponential voltage $v(t) = V_m e^{\sigma t} \cos\theta = V_0 e^{\sigma t}$

Any fn. may be written in general as

$\underline{f}(t) = \underline{K} e^{st}$ $\quad s$ = complex frequency $= \sigma + j\omega$

$\therefore \cos(\omega t + \theta) = \frac{1}{2}\left(e^{j(\omega t + \theta)} + e^{-j(\omega t + \theta)}\right)$

so $v(t) = V_m \cos(\omega t + \theta) = \underline{K}_1 e^{s_1 t} + \underline{K}_2 e^{s_2 t} = \left(\frac{1}{2} V_m e^{j\theta}\right) e^{j\omega t} + \left(\frac{1}{2} V_m e^{-j\theta}\right) e^{-j\omega t}$

$[$ s.t. $K_2 = K_1^*, \; S_2 = S_1^*]$ $\quad = K_1 e^{s_1 t} + (K_1 e^{s_1 t})^*$

~ly for $v(t) = V_m e^{\sigma t} \cos(\omega t + \theta)$

So,

voltages	associated complex frequencies
$v(t) = 100$	$s = 0$
$v(t) = 5e^{-2t}$	$s = -2 + j0$
$v(t) = 2 \sin 500t$	$s_{1,2} = \pm j500$ (complex conjugate understood for a physical sys)
$v(t) = 4e^{-3t} \sin(6t + 10^\circ)$	$s_{1,2} = -3 \pm j6$

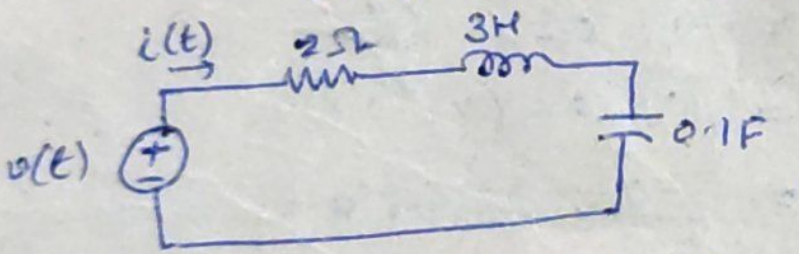
Damped sinusoid forcing fn.:

$$v(t) = V_m e^{\sigma t} \cos(\omega t + \theta) = Re\left(V_m e^{\sigma t} e^{j(\omega t + \theta)}\right)$$

$$= Re\left(V_m e^{j\theta} e^{st}\right)$$

$\therefore$ Omit Re and suppress e^{st} as in sinusoidal SS response

Ex.



$$v(t) = 60 e^{-2t} \cos(4t + 10^\circ).$$

Req. $i(t) = I_m e^{-2t} \cos(4t + \phi)$.

Now $v(t) = Re\left(60 e^{j10^\circ} \cdot e^{(-2+j4)t}\right)$

$$= Re\left(\underline{V} e^{st}\right)$$

$\therefore \underline{V} = 60 \angle 10^\circ \qquad s = -2 + j4$

Dropping Re,

$$60 \angle 10^\circ e^{st} = 2\underline{I} e^{st} + 3s\underline{I} e^{st} + \frac{1}{0.1s} \underline{I} e^{st}$$

$$\therefore 60 \angle 10^\circ = 2\underline{I} + 3s\underline{I} + \frac{10}{s}\underline{I}$$

$$\therefore \underline{I} = \frac{60 \angle 10^\circ}{2 + 3s + 10/s}$$

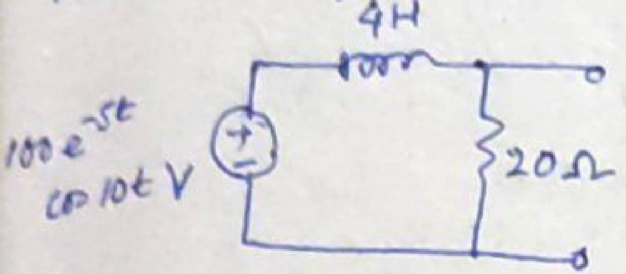
substitute for $s = -2 + j4$ and solve.

$$\underline{I} = 5.37 \angle -106.6^\circ$$

$$\therefore i(t) = 5.37 e^{-2t} \cos(4t - 106.6^\circ).$$

	R	L	C
$\underline{Z}(s)$	R	sL	$1/sC$
$\underline{Y}(s)$	$1/R$	$\frac{1}{sL}$	sC

All techniques as discussed in ss analysis usable.



$s = -5 + j10 \qquad \underline{V}_S = 100\angle 0^\circ.$

$\underline{Z}_L(s) = 4(-5+j10) = -20 + j40.$

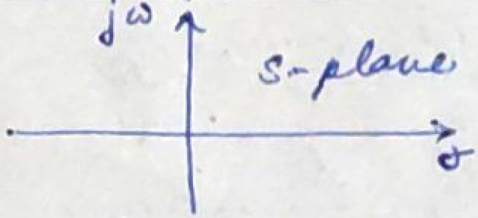
$\underline{Z}_R(s) = 20.$

$\therefore \underline{Z}_{th} = \underline{Z}_L \parallel \underline{Z}_R = 20 + j10.$

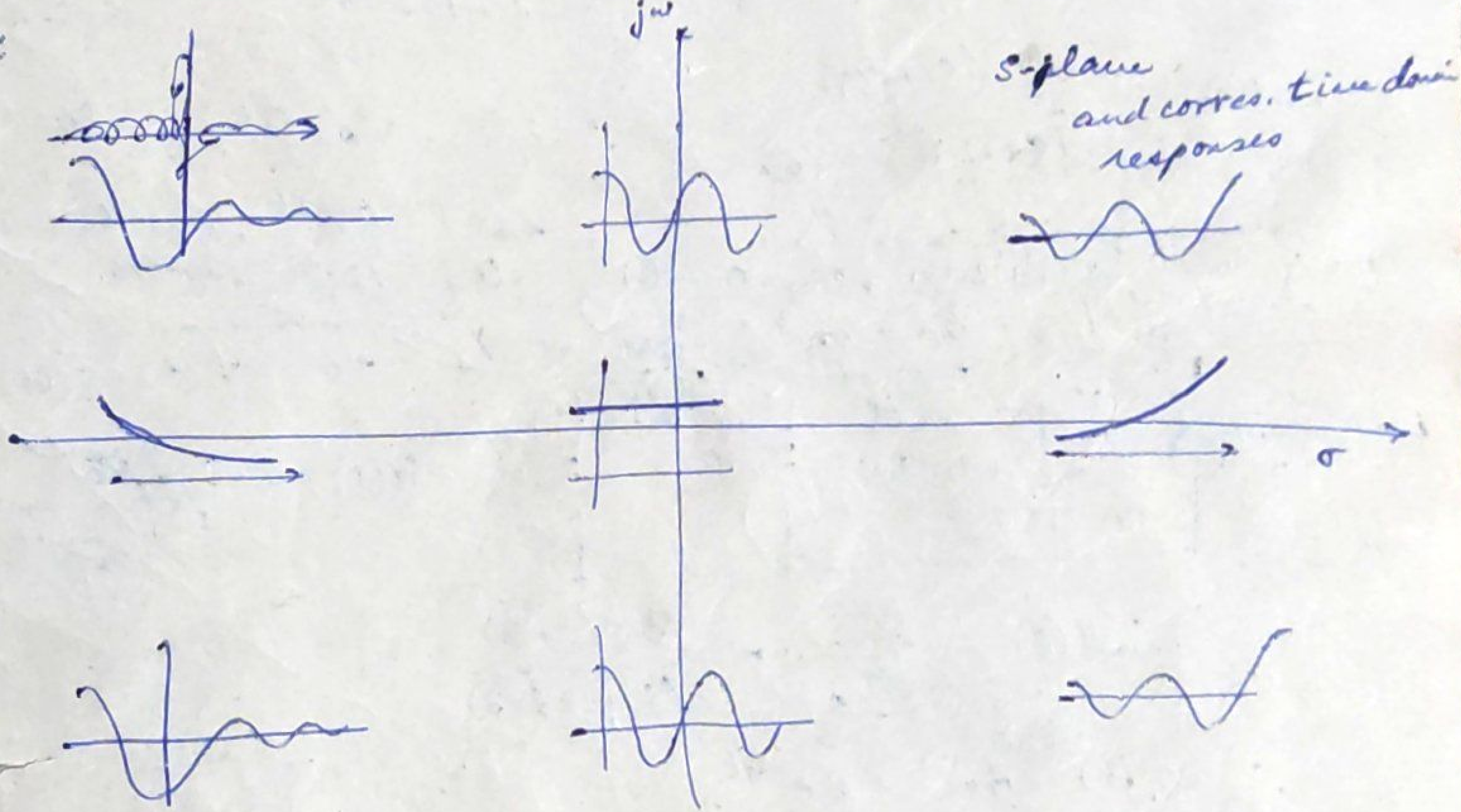
$\underline{V}_{oc} = 100\angle 0^\circ \cdot \dfrac{20}{20 - 20 + j40} = -j50.$

Then for some specific load, desired response found in freq. domain then transformed back to time domain.

complex frequency plane :

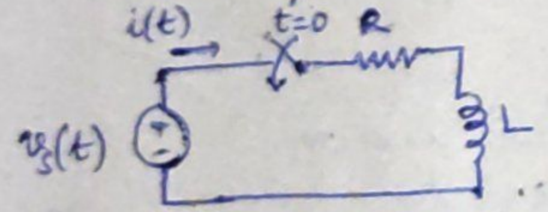


Responses :



$\underline{Y}(s) = \frac{1}{s+3}$ $\therefore$ $|\underline{Y}(s)| = \frac{1}{\sqrt{(\sigma+3)^2+\omega^2}}$ So for particular ω, σ-response may be obtained & vice versa.

Natural response and the s-plane



$$i(t) = i_x(t) + i_f(t).$$

$$\underline{I}_f(\underline{s}) = \frac{\underline{V}_s}{R + \underline{s}L}$$

Transfer function $\underline{H}(s) = \dfrac{\underline{I}_f(\underline{s})}{\underline{V}_s} = \dfrac{1}{L(R/L + \underline{s})}$

Natural response when $\underline{V}_s = 0$ but only when operating at a pole of $\underline{H}(\underline{s})$ is it possible to have nonzero $\underline{I}_f(s)$.

So say $\underline{I}(\underline{s}) = A$ at $\underline{s} = -R/L + j0$ $\longrightarrow$ Natural response.

$\therefore\ i_n(t) = \mathrm{Re}\left(A e^{-Rt/L}\right) = A e^{-Rt/L}$

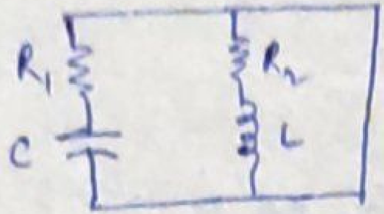
$\therefore\ i(t) = i_n(t) + i_f(t)\ \ldots$

General network: say $\dfrac{\underline{V}_2(\underline{s})}{\underline{V}_s} = \underline{H}(\underline{s}) = k\,\dfrac{(\underline{s}-\underline{s}_1)(\underline{s}-\underline{s}_3)\ldots}{(\underline{s}-\underline{s}_2)(\underline{s}-\underline{s}_4)\ldots}$

So poles at $\underline{s}_2, \underline{s}_4 \ldots$ $\therefore\ v_{2n}(t) = \underline{A}_2 e^{\underline{s}_2 t} + \underline{A}_4 e^{\underline{s}_4 t} + \ldots$

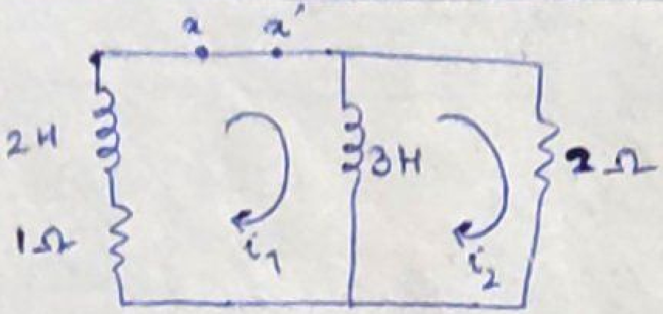
Important: Location of virtual source for evaluating natural response.

Say



$\therefore$ virtual source on $R_1 C$ link would yield $\underline{H}(\underline{s}) = 0$ since it produces no s-s i in $R_2 L$ link.

Ex. 1.



$i_1(0) = i_2(0) = 11A.$

$\underline{H}(\underline{s}) = \underline{I}_1(\underline{s})/\underline{V}_s$

$$\underline{I}_1(\underline{s}) = \frac{\underline{V}_s}{2\underline{s}+1+\dfrac{6\underline{s}}{3\underline{s}+2}} = \frac{3\underline{s}+2}{6\underline{s}^2+13\underline{s}+2}\underline{V}_s.$$

$$\therefore\ \underline{H}(\underline{s}) = \frac{\frac{1}{2}(\underline{s}+2/3)}{(\underline{s}+2)(\underline{s}+1/6)}.$$

$\therefore\ i_1(t) = Ae^{-2t} + Be^{-t/6} \ldots$ ①

Then using i.c. as earlier.

KVL @ perimeter $1i_1 + 2\dfrac{di_1}{dt} + 2i_2 = 0 \ldots$ ②

Solve for $\left.\dfrac{di_1}{dt}\right|_{t=0} = -2A - 1/6\,B = -\dfrac{22+11}{2} \ldots$ ②'

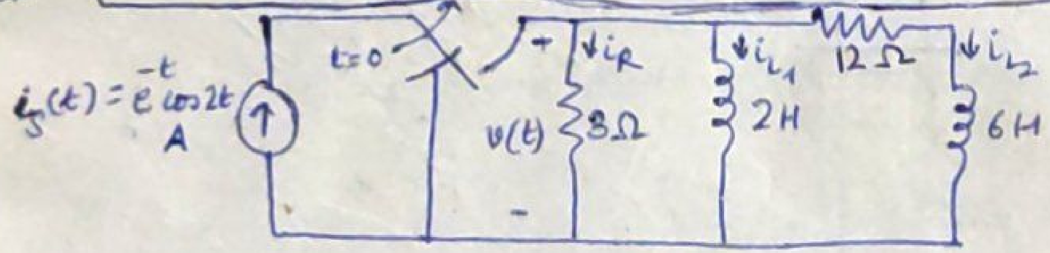
$\therefore\ \dfrac{di_1}{dt} = -2Ae^{-2t} - \dfrac{1}{6}Be^{-t/6} \ldots$ ①'

$i_1(t) = 8e^{-2t} + 3e^{-t/6}$

$i_2(t) = 12e^{-2t} - e^{-t/6}$

Ex 2.

Ex 2. Solve for $\frac{di}{dt}\Big|_{t=0} = -2A - \frac{1}{6}B = -\frac{22+11 \ldots}{2}$ (2)



$v(t) = v_f(t) + v_n(t)$.

$$\underline{V}(\underline{s}) = \frac{\underline{I}_s}{\frac{1}{3} + \frac{1}{2\underline{s}} + \frac{1}{6\underline{s}+12}}$$

$$\therefore \underline{H}(\underline{s}) = \frac{\underline{V}(\underline{s})}{\underline{I}_s} = \frac{3\underline{s}(\underline{s}+2)}{(\underline{s}+1)(\underline{s}+3)}$$

$$\therefore v_n(t) = Ae^{-t} + Be^{-3t}.$$

To find forced response $\underline{I}_s(\underline{s}) = 1$ at $\underline{s} = -1+j2$ multiplied by impedance at $\underline{s} = -1+j2$

$$\Rightarrow \underline{V}(\underline{s}) = \underline{I}_s(\underline{s})\,\underline{H}(\underline{s})\Big|_{\underline{s}=-1+j2} = 1.875\sqrt{2}\angle 45^\circ = 3\frac{(-1+j2)(1+j2)}{j2(2+j2)}$$

$$\therefore v_f(t) = 1.875\sqrt{2}\,e^{-t}\cos(2t+45^\circ).$$

Then use i.c.s. $i_{L_1}(0) = i_{L_2}(0) = 0$. $\therefore$ 1 A through 3Ω resistor.

$$\therefore v(0) = 3 = A + B + \frac{1.875\sqrt{2}}{\sqrt{2}}.$$

$$v(t) = 3i_R = 3i_s - 3i_{L_1} - 3i_{L_2} \quad \therefore \frac{dv}{dt} = 3\frac{di_s}{dt} - 3\frac{di_{L_1}}{dt} - 3\frac{di_{L_2}}{dt}$$

at $t=0$, RHS $= 3(-3) - 3.\left(\frac{3}{2}\right) - 3.\left(\frac{1}{2}\right) = -9 = -5.625 - A - 3B$

$\left(\frac{v}{L}\right)$ $\left(\frac{3}{6}\right)$

$\therefore A = 0$, $B = 1.125$.

$$\therefore v(t) = 1.125e^{-3t} + 1.875\sqrt{2}e^{-t}\cos(2t+45^\circ)$$

$i(0) = 0$
$v = i(R) + L\frac{di}{dt}$
v/L