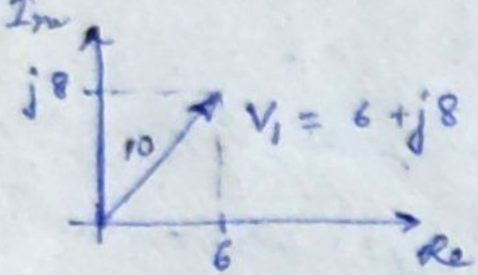
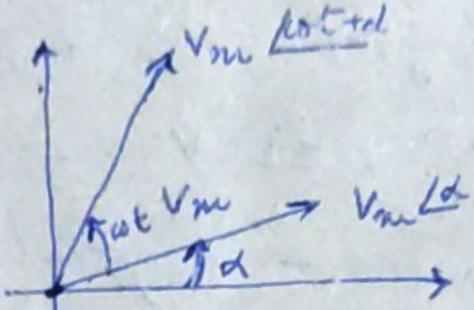
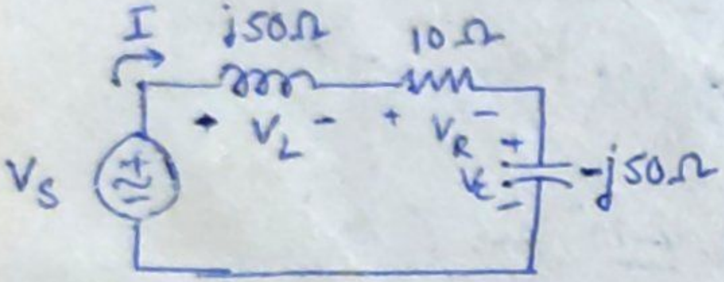
Phasor diagrams:

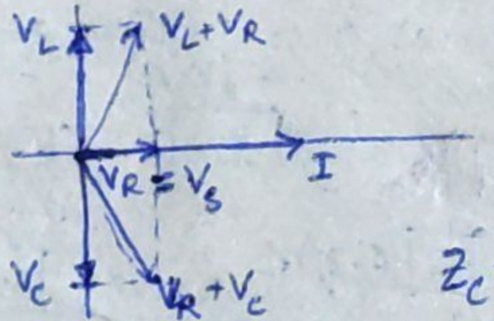


$V_1 = 6 + j8 = 10 \angle 53.1° \; V$



For ~~diff~~. ωt, rotate anticlockwise.



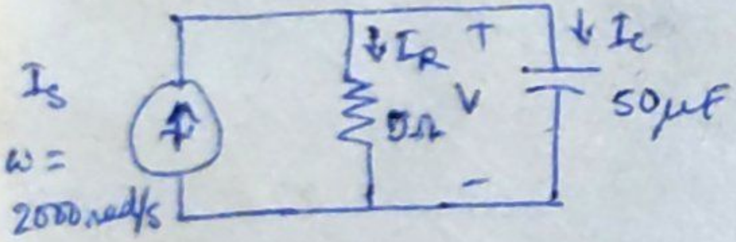


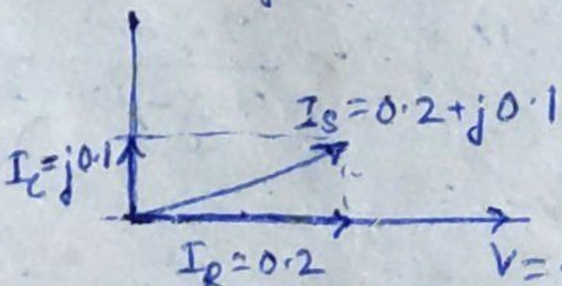
I & V on same plot
magnitudes diff.

$Z_C = -Z_L$ Resonant condn.

Addn & Subtraction: vector rules.
Mul. & div: add & sub. $\angle$, change amplitudes.

$j\omega C = j\,0.1\,\mho$; $\frac{1}{R} = 0.2\,\mho$





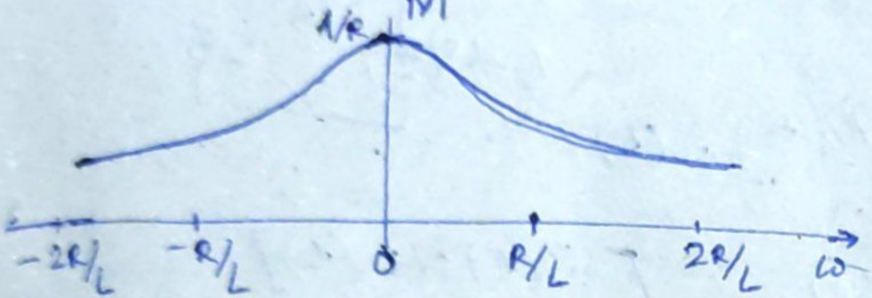
$V = 1 \angle 0°$ (assume it)

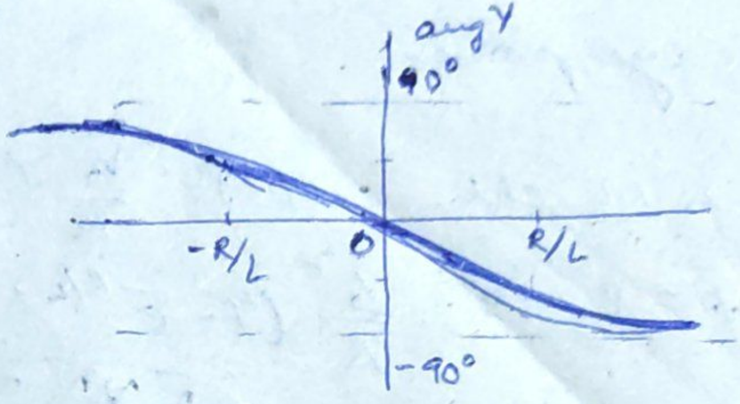
Even if I_S known, yet same technique, then scale suitably.

Response as fn. of ω :

f. Consider series RL ckt. $\therefore\ I = \dfrac{V_S}{R+j\omega L}$ $\therefore\ Y = \dfrac{1}{R+j\omega L}$

$\therefore |Y| = \dfrac{1}{\sqrt{R^2+\omega^2L^2}}$ arg. $Y = -\tan^{-1}\dfrac{\omega L}{R}$





$\omega = -100$ rad/s $\Rightarrow v(t) = 50\cos(\omega t + 30°)$

$v(t) = 50\cos(-100t + 30°)$

at $\omega = \pm R/L = \pm 1/\tau \rightarrow |Y| = 0.707|Y_{max}|$, arg $Y = 45°$.

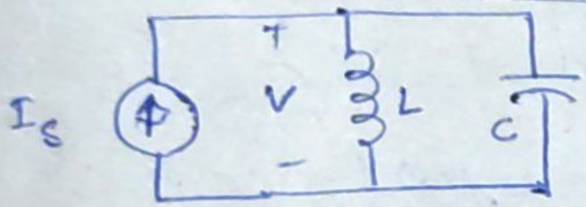
$\downarrow$

avg power $= 0.5$ max. power.

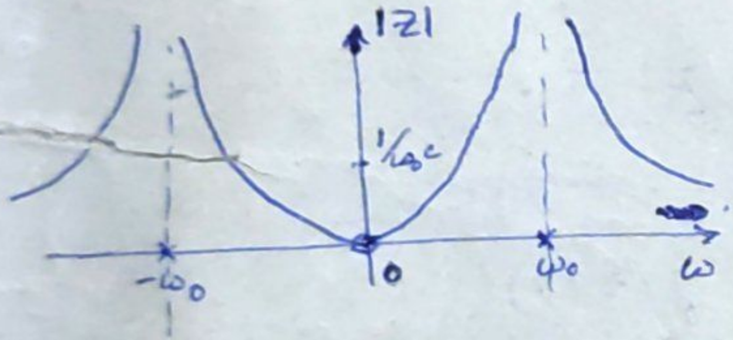
$\therefore$ Half power frequency. $\longrightarrow$ $\therefore$ For same V, $I = \frac{1}{\sqrt{2}} I_{max}$ $\therefore$ avg. power $= (0.707)^2 P_{max} = 0.5 P_{max}$.

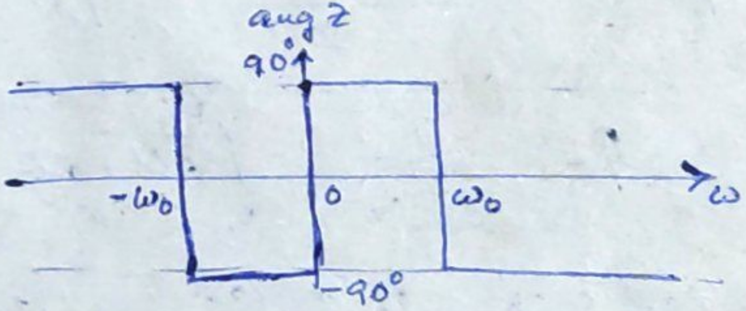
only due to R

2. Parallel LC ckt. driven by sinusoidal source:



$$V = I_S \cdot \frac{(j\omega L)\cdot(1/j\omega C)}{j\omega L - j(1/\omega C)}$$





$$Z = \frac{L/C}{j(\omega L - 1/\omega C)} = -j\cdot\frac{1}{C}\cdot\frac{\omega}{\omega^2 - 1/LC}.$$

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

$$\therefore\ |Z| = \frac{1}{C}\,\frac{|\omega|}{|(\omega-\omega_0)(\omega+\omega_0)|}$$

Critical frequencies at which responses 0 or ∞.

0 → zeros (0)

∞ → poles (x)

note: $\omega = \pm\infty$ is a zero

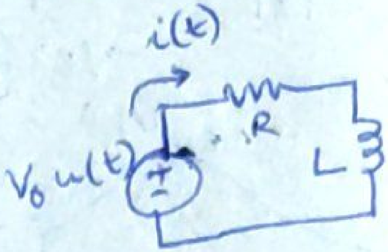
Power :

Instantaneous power (freq. to be limited else distortion in speakers, → audio systems)

↓ used to calculate

average power.

$$p = vi$$



Say series RL ckt. $\therefore i(t) = \frac{V_0}{R}\left(1 - e^{-R/L\, t}\right) u(t)$.

supplied $p = vi = \frac{V_0^2}{R}\left(1 - e^{-R/L\, t}\right) u(t)$. $\quad [u^2(t) = u(t)]$.

$p = p_R + p_L$. $\qquad i^2 R = p_R \qquad L\frac{di}{dt} = v_L$

$\cdot \frac{V_0}{R} e^{-R/L\, t} u(t) \cdot i_L = p_L$

$\therefore p_L = \frac{V_0^2}{R} e^{-R/L\, t}\left(1 - e^{-R/L\, t}\right)$

Let $v(t) = V_m \cos\omega t \qquad \therefore i(t) = I_m \cos(\omega t + \phi)$

$$I_m = \frac{V_m}{\sqrt{R^2 + \omega^2 L^2}} \qquad \phi = -\tan^{-1}\frac{\omega L}{R}.$$

$\therefore p = vi = V_m I_m \cos(\omega t + \phi) \cdot \cos\omega t \qquad$: supplied by source

$$= \frac{V_m I_m}{2}\left[\cos\phi + \cos(2\omega t + \phi)\right]$$

$\therefore$ Avg. power $= \frac{V_m I_m \cos\phi}{2} \quad$ [2nd term has avg. 0]

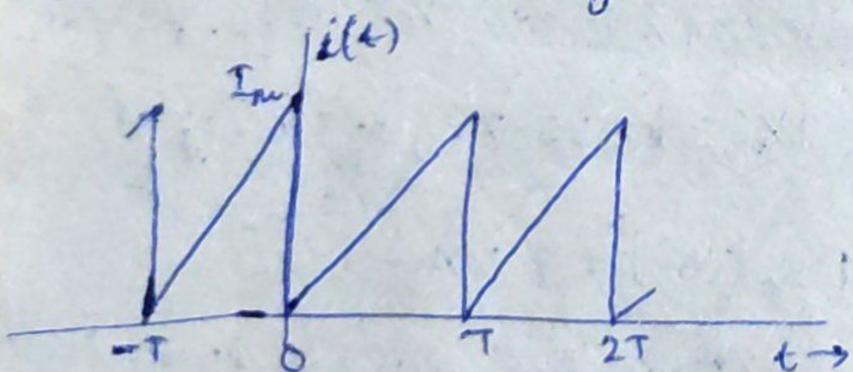
Avg. power : $$P = \frac{1}{t_2 - t_1} \int_{t_1}^{t_2} p(t)\, dt.$$

For a periodic fn. of period T i.e. $f(t) = f(t+T)$

$$P = \frac{1}{T} \int_{t_x}^{t_x+T} p\,dt = \frac{1}{nT} \int_{t_x}^{t_x+nT} p\,dt \qquad n = 1, 2, 3 \ldots$$

$$= \lim_{n\to\infty} \frac{1}{nT} \int_{-nT/2}^{nT/2} p\,dt = \lim_{\tau\to\infty} \frac{1}{\tau} \int_{-\tau/2}^{\tau/2} p\,dt.$$

Ex. Sawtooth waveform :



$$i(t) = \begin{cases} \frac{I_m}{T} t & 0 < t \leq T \\ \frac{I_m}{T}(t - T) & T < t \leq 2T \\ \vdots \end{cases}$$

$$p(t) = i^2 R = \begin{cases} \frac{1}{T^2} I_m^2 R\, t^2 & 0 < t \leq T \\ \frac{1}{T^2} I_m^2 (t-T)^2 R & T < t \leq 2T \\ \vdots \end{cases}$$

So, avg. power delivered to $R\,\Omega$ resistor

$$\therefore P = \frac{1}{T} \int_0^T \frac{I_m^2 R}{T^2} t^2\, dt = \frac{1}{3} I_m^2 R$$

General expression:

$v(t) = V_m \cos(\omega t + \theta)$ $\quad$ $i(t) = I_m \cos(\omega t + \phi)$.

$p(t) = V_m I_m \cos(\omega t + \theta) \cos(\omega t + \phi)$

$= \frac{1}{2} V_m I_m [(\cos(\theta - \phi) + \cos(2\omega t + \theta + \phi)]$

Note: period of v & i = T, period of $p = \frac{1}{2}T$

$\boxed{\text{Avg. power } P = \frac{1}{2} V_m I_m \cos(\theta - \phi)}$ — Sense of diff. immaterial

$\because \cos(\theta) = \cos(-\theta)$.

$v(t) = 4 \cos \frac{\pi t}{6}$ V $\therefore \underline{V} = 4 \angle 0^\circ$ V across $\underline{Z} = 2 \angle 60^\circ\ \Omega$.

$\therefore \underline{I} = 2 \angle -60^\circ$ A, $\quad P = \frac{1}{2} 4 \cdot 2 \cos 60^\circ = 2$ W.

$i(t) = 2 \cos\left(\frac{\pi t}{6} - 60^\circ\right)$ $\quad$ $p(t) = 8 \cos \frac{\pi t}{6} \cdot \cos\left(\frac{\pi t}{6} - 60^\circ\right)$ $\quad$ $\left[\cos A \cos B = \frac{\cos(A+B) + \cos(A-B)}{2}\right]$

$= 2 + 4 \cos\left(\frac{\pi t}{3} - 60^\circ\right)$.

Note: P_R (pure resistor) $= \frac{1}{2} V_m I_m = \frac{1}{2} I_m^2 R, \ \frac{V_m^2}{2R}$

V_m: voltage across resistor, I_m: current through R.

Avg. power of a purely reactive device $= 0$ $\quad [\cos(\theta - \phi) = \cos 90^\circ = 0]$.

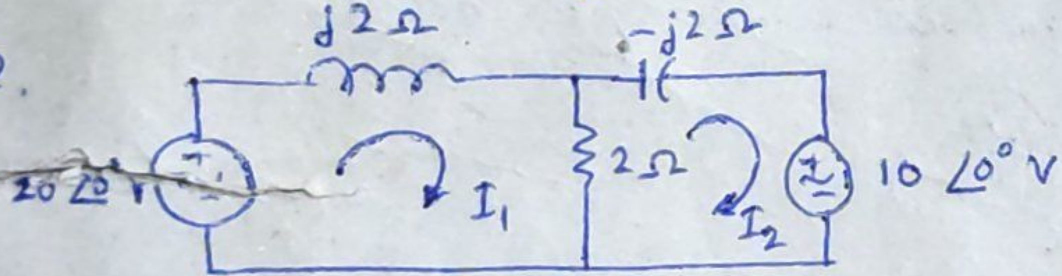
Ex. 1. $Z_L = (8 - j11)\ \Omega$ $\quad I = 5\angle 20^\circ\ A$ $\quad \therefore P = \frac{1}{2}(5^2)8 = 100W$.

$\because$ no power can be absorbed by $-j11\ \Omega$.

say $I = (2+j5)\ A$ then $|I|^2 = 2^2+5^2 = 29$.

$\therefore P = \frac{1}{2} \cdot 29 \cdot 8 = 116\ W$.

2.



$\underline{I}_1 = 5 - j10 = 11.18\angle -63.45^\circ$

$\underline{I}_2 = 5 - j5 = 7.07\angle -45^\circ$

$\underline{I}_{2\Omega} = \underline{I}_1 - \underline{I}_2 = -j5 = 5\angle -90^\circ$

$\therefore I_m = 5$. $\quad P_R = \frac{1}{2} I_m^2 R = \frac{1}{2} 5^2 \cdot 2 = 25W$.

$P_{left} = -\frac{1}{2}(20)(11.18)\cos(0^\circ + 63.45^\circ) = -50\ W$

$P_{right} = \frac{1}{2}(10)(7.07)\cos(0^\circ + 45^\circ) = 25W$.

__Maximum power transfer theorem__: For a Thevenin source V_s, impedance $Z_{th} = R_{th} + jX_{th}$ connected to load $Z_L = R_L + jX_L$, __Avg. power__ is maximum when $R_L = R_{th}$, $X_L = -X_{th}$ i.e. $Z_L = Z_{th}^*$.

Some nonperiodic fns.:

$i(t) = \sin t + \sin \pi t$ → (ratio of periods → irrational)

---

* Note: $i(t) = \sin t + \sin 3.14\, t$ is periodic

$\boxed{2\pi n = \frac{2\pi m}{3.14}}$ is solvable $T = 100\pi$ s.

$P = i^2 R = \lim_{\tau\to\infty} \frac{1}{\tau} \int_{-\tau/2}^{\tau/2} (\sin^2 t + \sin^2 \pi t + 2 \sin t \sin \pi t)\, dt$

delivered to 1Ω resistor

$(\frac{1}{2} - \frac{1}{2}\cos 2t)$ $\quad (\frac{1}{2} - \frac{1}{2}\cos 2\pi t)$ $\quad (\cos A - \cos B)$

avg. $P = \frac{1}{2} + \frac{1}{2} + 0 = 1\,W$.

Thus $\quad i(t) = I_{m_1} \cos \omega_1 t + \dots + I_{mN} \cos \omega_N t$

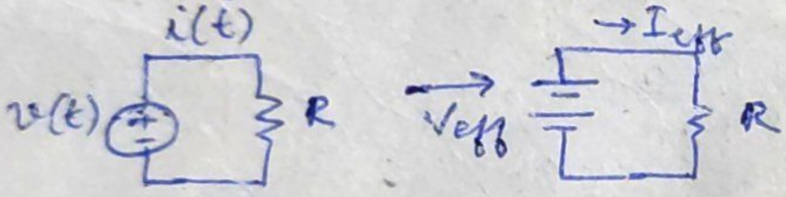
$P = \frac{1}{2}(I_{m_1}^2 + I_{m_2}^2 + \dots + I_{mN}^2) R$ delivered to resistance R.

* So when $\omega_1 \neq \omega_2 \neq \omega_3 \dots \neq \omega_N$, superposition holds for power.

$i_1 = 2\cos 10t - 3\cos 20t$ A → 26 W to 4Ω resistor

while $i_2 = 2\cos 10t - 3\cos 10t$ A → 2 W avg. power to 4Ω resistor.

Effective values of current and voltage: measure of effectiveness of a (voltage) source in delivering power to a resistive load.



Resistor receives same average power

$I_{eff} \longrightarrow$ of any periodic current is equal to the value of the direct current which, flowing through an R-ohm resistor, delivers same power to the resistor as does the periodic current.

$$\therefore \quad P = I_{eff}^2 R = \frac{1}{T} \int_0^T i^2 R \, dt$$

$$\Rightarrow \quad I_{eff} = \sqrt{\frac{1}{T} \int_0^T i^2 dt} \quad \longrightarrow \text{rms.}$$

$\sim$ly for V_{eff}.

For sinusoidal waveform, $i(t) = I_m \cos(\omega t + \phi)$ $\quad T = \frac{2\pi}{\omega}$.

$I_{eff} = I_m / \sqrt{2}$. $\longrightarrow$ real qnty. independent of phase angle

$\therefore \quad i(t) = \sqrt{2} \cos(\omega t + \phi)\, A \longrightarrow I_{eff} = 1\, A \longrightarrow$ same power delivered to resistive load.

For sawtooth waveform, $I_{eff} = I_m / \sqrt{3}$.

$\therefore\ P = \frac{1}{2} I_m^2 R$ avg. power delivered to $R\,\Omega$ resistor by sinusoidal current.

$= I_{eff}^2 R$

$*\ = V_{eff} \cdot I_{eff} \cos(\theta - \phi)$ *

$= \dfrac{V_{eff}^2}{R}.$

<u>Note</u>: V & I may be represented as $V_m \angle\theta$ or $V_{eff} \angle\theta$ — reqd. to check.

waveform composed of a

For a sum of a no. of sinusoids of <u>diff.</u> frequencies (periodic or nonperiodic)

$$P = \frac{1}{2}\left(I_{m_1}^2 + I_{m_2}^2 + \dots + I_{m_N}^2\right) R$$

$$= \left(I_{1eff}^2 + I_{2eff}^2 + \dots + I_{Neff}^2\right) R.$$

$$\therefore\ I_{eff} = \sqrt{\sum I_{i\,eff}^2}$$

Ex. Say sinu. current of 5A rms at 60 Hz. flows thru' 2 Ω resistor
Avg. power of 50 W absorbed $(= 5^2 \times 2)$. 2^{nd} current of 3 A rms
at 120 Hz. absorbed power $= 50 + 3^2 \cdot 2 = 68$ W

But if 3 A rms at 60 Hz., then $8 \le P \le 128$ W depending on
phase. $[(5-3)^2 \cdot 2$ to $(5+3)^2 \cdot 2]$

$$I_{eff} = \sqrt{I_{1eff}^2 + I_{2eff}^2} = 5.83 \text{ A}$$ (1^{st} case) else bet. 2 to 8 A.

Apparent power : due to electric power industry : efficiency of energy transfer related to cost of elec. energy.

Customer provides load $\xrightarrow[\text{in}]{\text{results}}$ poor transmission efficiency.

$\longrightarrow$ greater price for each kWh of elec. energy he actually receives & uses.

$v(t) = V_m \cos(\omega t + \theta) \qquad i(t) = I_m \cos(\omega t + \phi)$

v leads i by $(\theta - \phi)$

$$P = \frac{1}{2} V_m I_m \cos(\theta - \phi) = V_{eff} I_{eff} \cdot \cos(\theta - \phi) ..$$

For dc applied voltages & currents, $P = V_{eff} I_{eff} = V_m I_m$.

$\therefore$ Absorbed power is 'apparently' $V_{eff} I_{eff} = P_{apparent}$

Power factor $PF = \dfrac{\text{avg. power}}{\text{apparent power}} = \dfrac{P}{V_{eff} I_{eff}} = \cos(\theta - \phi)$ for sine waves.

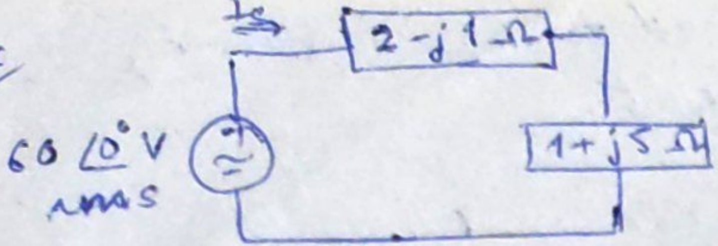
<u>Note</u> : Purely resistive & $X_L = X_C$ loads, $PF = 1$.

Purely reactive $PF = 0$ $\longrightarrow$ worst case.

PF $\rightarrow$ leading or lagging in terms of phase of <u>current</u> w.r.t. voltage.

$\cdot X_L \longrightarrow$ voltage leads current , PF lagging.

Ex



$$I_s = \frac{60}{3+j4}\ A = 12\angle -53.1^\circ\ A\ rms.$$

Source supplies apparent power

$60(12) = 720$ V.A

Avg. power : $720\cos(0+53.1^\circ) = 432$ W [$\theta-\phi$ or $\phi-\theta$ → no difference]

Avg. power received by $(2-j1)\,\Omega = 12^2 \cdot 2$ W $= 288$ W

$(1+j5)\,\Omega = 12^2 \cdot 1$ W $= 144$ W.

$$\text{PF of combined loads} = \frac{P}{V_{eff}\,I_{eff}} = \frac{432}{720} = 0.6.$$

$Z_{eq} = 3+j4 = 5\angle 53.1^\circ\ \Omega$ ∴ PF $= \cos 53.1^\circ = 0.6$.

Combined load inductive ∴ lagging PF.

Ex. Practical importance:

Generator produces 200 V rms at 60 Hz. ; max. power o/p 1 kW.

$\therefore$ rms current of 5A deliverable to load.

But if load req. 1 kW at lagging PF = 0.5 connected to generator

then $200 \cdot I_{eff} \cdot 0.5 = 1000 \Rightarrow I_{eff} = 10$ A. reqd.

Ex. Avg. power of 11 kW at unity PF and 220 V rms. $R_{lines} = 0.2\,\Omega$.

$\therefore \frac{\cancel{11000}^{50}}{\cancel{220}}$ A rms current flows in load and in lines $\therefore$ loss of $50^2 \times 0.2$

$= 500$ W.

$\therefore$ Reqd. to generate 11.5 kW to send 11 kW to customer.

Say req. 11 kW at PF $\angle$ of 60° lagging $\therefore$ 100 A forced $\therefore$ loss $= 100^2 \times 0.2$

$= 2$ kW $\therefore$ 13 kW to be generated.

Complex power:

$$P = V_{eff} I_{eff} \cdot Re\left[e^{j(\theta-\phi)}\right] = Re\left[V_{eff} I_{eff} e^{j(\theta-\phi)}\right]$$

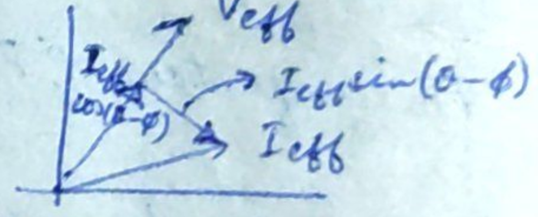
$$= Re\left[\vec{V}_{eff} \vec{I}^{*}_{eff}\right] \qquad \vec{V}_{eff} = V_{eff}\angle\theta, \quad \vec{I}_{eff} = I_{eff}\angle\phi$$

$$\text{or} = Re\left[\underline{V}_{eff} \underline{I}^{*}_{eff}\right]$$

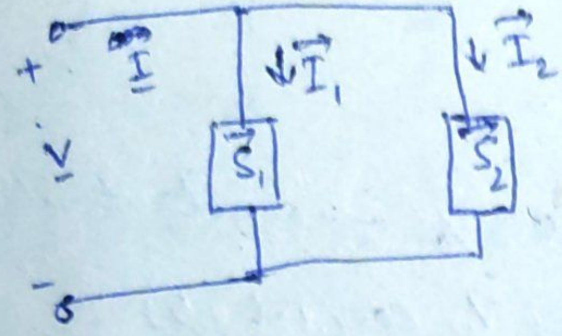
Complex power $\vec{S} = \vec{V}_{eff} \cdot \vec{I}_{eff}^{\,*} = P + jQ$ → real power + reactive power

Apparent power $= |\vec{S}|$ — units var (voltampere reactive)

$\therefore Q = V_{eff} I_{eff} \sin(\theta-\phi)$ → Quadrature power



$$\vec{S} = \vec{V}\vec{I}^{*} = \vec{V}\left(\vec{I}_1 + \vec{I}_2\right)^{*} = \vec{V}\left(\vec{I}_1^{\,*} + \vec{I}_2^{\,*}\right).$$



Industrial consumer operating

~~for~~ 50 kW induction motor at lagging PF 0.8.

connect $\underline{S}_2 = ?$ to raise PF to 0.95 lagging.

by adding purely reactive load [cost effective].

$\vec{V}$ same $\therefore\ \vec{S}_2 \parallel \vec{S}_1$.

$\underline{V} = 230 \angle 0^\circ$ V rms.

$\boxed{\overset{*}{\underline{S}}_1(0.8) = 50\ \angle\cos^{-1}(0.8)\ \text{kVA}}$ (real part of $\underline{S}_1 = |\underline{S}_1|\cos\theta$)

$\therefore\ \underline{S}_1 = 50 + j\,37.5$ kVA.

to achieve PF 0.95, $\underline{S} = \dfrac{50}{0.95}\ \angle\cos^{-1}(0.95) = 50 + j\,16.43$ kVA.

$\therefore\ \underline{S}_2 = -j\,21.07$ kVA [Complex power drawn by corrective load]

$$\underline{I}_2^* = \frac{\underline{S}_2}{\underline{V}} = -j\,\frac{21070}{230} = -j\,91.6 \text{ A}.$$

$\therefore\ \underline{I}_2 = j\,91.6$ A.

$$\therefore\ \underline{Z}_2 = \frac{\underline{V}}{\underline{I}_2} = \frac{230}{j\,91.6}\ \Omega = -j\,2.51\ \Omega$$ (Capacitive ~~load~~ load)

At 60 Hz operating freq., 1056 μF capacitor reqd.

$$* \quad 50 = Re\, \underline{S}_1$$

$$= |\underline{S}_1| . \cos(\theta - \phi)$$

$$y = |S_1| . \sin(\theta - \phi)$$

$$= |S_1| . \sin(\cos^{-1} 0.8)$$

$$= \frac{50}{0.8} . \sin(\cos^{-1} 0.8)$$

$$\underline{S}_1 = \frac{50}{0.8} . \cos(\cos^{-1} 0.8) + j\frac{50}{0.8} \sin \cos^{-1}(0.8)$$

$$= \frac{50}{0.8} \angle \cos^{-1}(0.8)$$