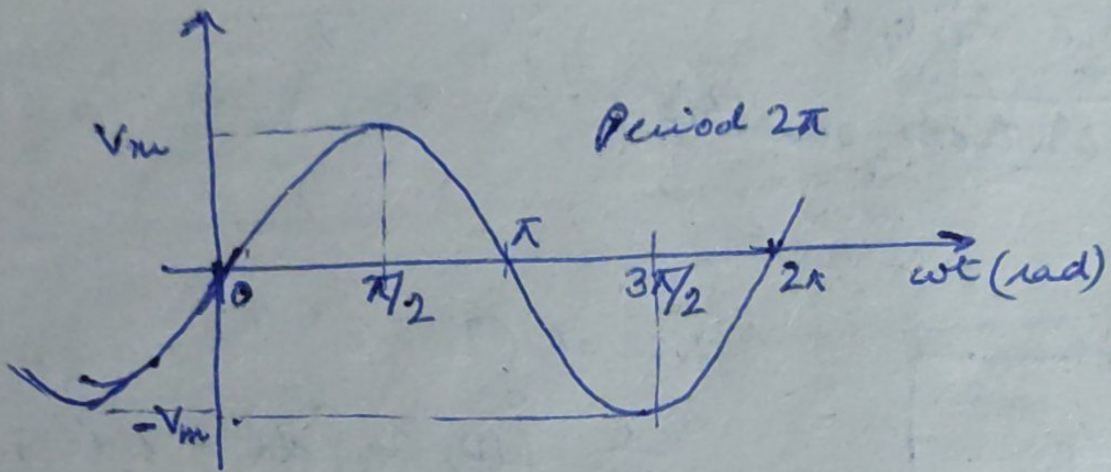
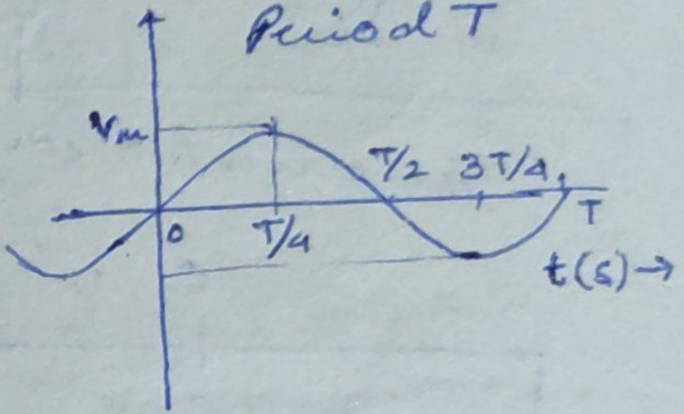
Sinusoidal Steady State Analysis :

Sinusoidally varying voltage $v(t) = V_m \sin\omega t$.

Amplitude V_m

Argument ωt Angular freq. ω ~~Period 2π~~



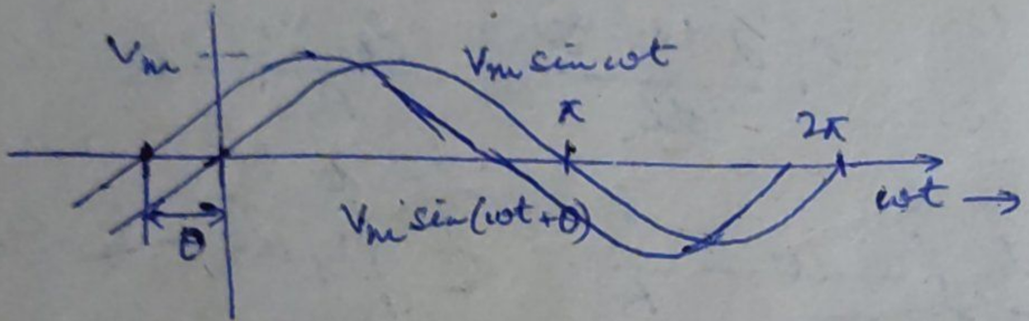


frequency $f = 1/T$. $\omega = \frac{2\pi}{T} = 2\pi f$.

General form: $v_1(t) = V_m \sin(\omega t + \theta)$.

Phase angle θ. ↳ leads $v_2 = V_m \sin \omega t$ by θ rad.

ie v_2 lags $v_1(t)$ by θ rad.



Note:
$\cos\theta = \sin(90° + \theta)$
$\sin\theta = \cos(90° - \theta)$
$= \cos(\theta - 90°)$
$\because \cos\theta = \cos(-\theta)$

① Both to be written as sine / cos waves. PREFERRED cos

② V_{m_1}, V_{m_2} assumed +ve

③ diff. in phase expressed as $\angle \leq 180°$ in magnitude.

$$v_1 = V_{m_1} \sin(5t - 30°)$$

$$v_2 = V_{m_2} \cos(5t + 10°) = V_{m_2} \sin(5t + 90° + 10°) = V_{m_2} \sin(5t + 100°)$$

$\therefore v_2$ leads v_1 by $130°$.

Transformation to a phasor : expressed as COSINE wave with PHASE.

Suppose a current $i(t) = I_m \cos(\omega t + \phi)$

$$= \mathrm{Re}\left(I_m e^{j(\omega t+\phi)}\right)$$

$\because$ Euler's identity : $e^{j\theta} = \cos\theta + j\sin\theta$.

Drop (Re) [adding imag. part], then suppress $e^{j\omega t}$ [common to all sinusoids] $\therefore$ $\mathbf{I} = I_m e^{j\phi} \triangleq I_m \angle\phi$ PHASOR

$i(t) \longrightarrow$ TIME DOMAIN REPRESENTATION

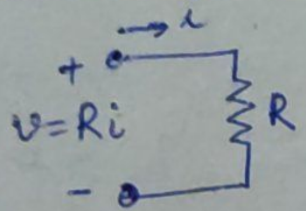
$\mathbf{I} \longrightarrow$ FREQ. DOMAIN REPRESENTATION.

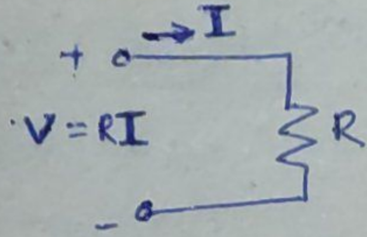
So $v(t) = 100\cos(400t - 30^\circ) \Leftrightarrow \mathbf{V} = 100 \angle -30^\circ$

$i(t) = 5\sin(377t + 150^\circ) = 5\cos(377t - 90^\circ + 150^\circ) = 5\cos(377t + 60^\circ)$

$\Leftrightarrow \mathbf{I} = 5 \angle 60^\circ$

Phasor relationships for R, L and C:





$v(t) = R\,i(t)$.

Apply $V_m e^{j(\omega t+\theta)}$ and assume response $I_m e^{j(\omega t+\phi)}$

$$\therefore V_m e^{j(\omega t+\theta)} = R I_m e^{j(\omega t+\phi)}$$

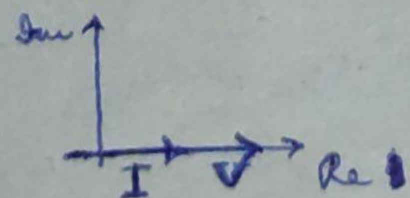
Divide all through by $e^{j\omega t}$. $\therefore V_m e^{j\theta} = R I_m e^{j\Phi}$

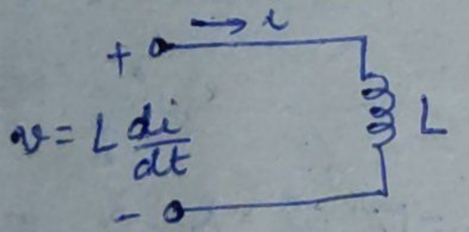
$$\Rightarrow \mathbf{V} = R\,\mathbf{I}$$

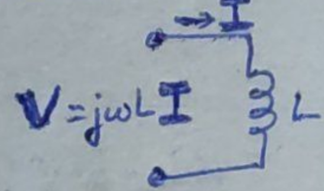
Now equality of θ and ϕ apparent as directly related.

$$i(t) = \frac{v(t)}{R} = \frac{8\cos(100t - 50^\circ)}{4} = 2\cos(100t - 50^\circ)\ \text{A}$$

$$\therefore \mathbf{V} = 8\angle{-50^\circ} \qquad \mathbf{I} = \frac{8\angle{-50^\circ}}{4} = 2\angle{-50^\circ}\ \text{A}$$







$$v(t) = L\frac{di(t)}{dt}.$$

$$\therefore V_m e^{j(\omega t+\theta)} = L\frac{d}{dt} I_m e^{j(\omega t+\phi)}$$

$$= j\omega L\, I_m e^{j(\omega t+\phi)}$$

Suppress $e^{j\omega t}$

$$V_m e^{j\theta} = j\omega L\, I_m e^{j\phi} \qquad \therefore \mathbf{V} = j\omega L \mathbf{I}$$

Note $\angle$ of factor $j\omega L = +90^\circ$

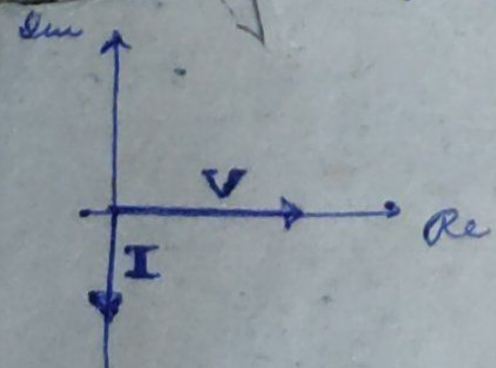
$e^{j\phi} = \cos\phi + j\sin\phi$

phase $\angle = \phi = \tan^{-1}\left(\frac{Im}{Re}\right)$

if $\cos\phi = 0 \quad \sin\phi = 1$

$\Rightarrow \tan\phi = \infty \quad \therefore \phi = 90^\circ$

$\therefore$ **V** leads **I** by 90°

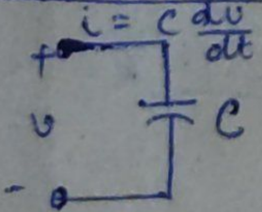


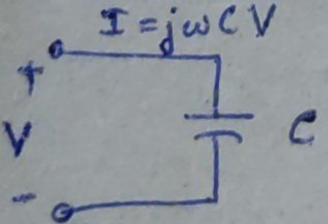
For $\omega = 100$ rad/s,

$$\mathbf{I} = \frac{\mathbf{V}}{j\omega L} = \frac{8\angle -50^\circ}{j\,100(4)} = -j\,0.02\angle -50^\circ$$

$$= 0.02\angle -50^\circ - 90^\circ = 0.02\angle -140^\circ$$

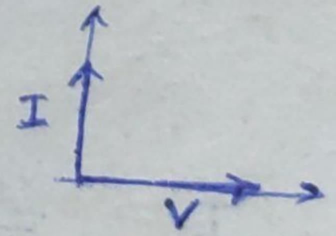
$i(t) = 0.02\cos(100t - 140^\circ)$ A.





$$i(t) = C\,\frac{dv(t)}{dt}$$

$$I = j\omega C V$$



$V = 8\angle -50^\circ$ V then $I = j\,100(4)(8\angle -50^\circ) = 3200\angle 40^\circ$ A.

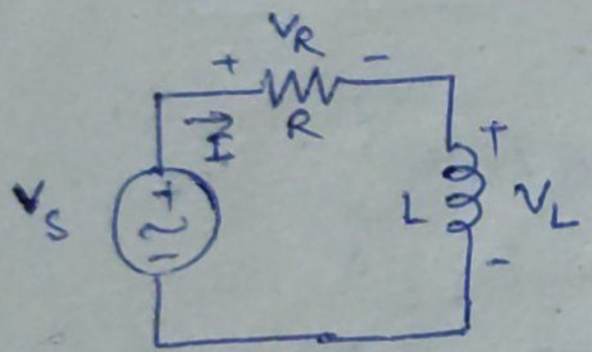
Not practical (4-F capacitor!)

KVL : $v_1(t) + v_2(t) + \dots + v_N(t) = 0$.

Replace by $V_i e^{j\omega t}$, suppress $e^{j\omega t}$

$$V_1 + V_2 + \dots + V_N = 0.$$

~ly KCL holds.



$$V_R + V_L = V_s$$

$$\therefore RI + j\omega L I = V_s$$

$$\Rightarrow I = \frac{V_s}{R + j\omega L}$$

Say source $= V_m \angle 0^\circ$

$$= \frac{V_m}{\sqrt{R^2 + \omega^2 L^2}} \angle -\tan^{-1}\frac{\omega L}{R}$$

Time domain	Freq. domain
$v = Ri$	$V = RI$
$v = L\frac{di}{dt}$	$V = j\omega L I$
$v = \frac{1}{C}\int i\,dt$	$V = \frac{1}{j\omega C} I$

Impedance.

$\boxed{\dfrac{V}{I} = \text{Impedance } Z}$ (complex quantity dimension Ω). $= |Z|\angle\theta = R + j\underbrace{X}_{\text{Reactance}}$,

Cannot be transformed to time domain.

Inductor: time domain : inductance L | Capacitor C

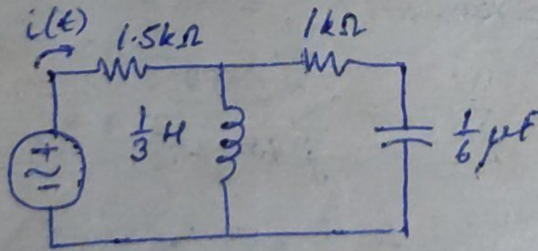
freq. domain : impedance $j\omega L$ | $\dfrac{1}{j\omega C}$

Series combination $Z_L + Z_c = Z_{eq} = j\omega L + \dfrac{1}{j\omega C} = j\left(\omega L - \dfrac{1}{\omega C}\right)$

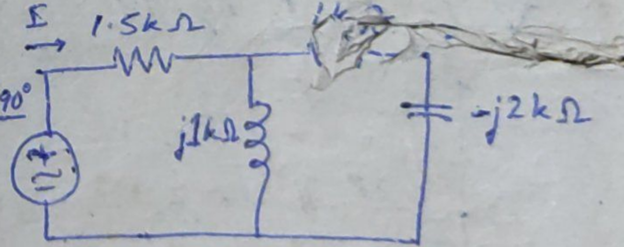
Parallel $Z_{eq} = Z_L \parallel Z_c = \dfrac{Z_L \cdot Z_c}{Z_L + Z_L}$.

RLC ckt.

40 sin 3000t V



⇔



$$Z_{eq} = 1.5 + \frac{j1(1-j2)}{j1 + 1 - j2} = 1.5 + \frac{2+j1}{1-j1}$$

$$= 1.5 + \frac{1+j3}{2} = 2 + j1.5 = 2.5\angle 36.9^\circ \text{ k}\Omega.$$

$$I = \frac{V_s}{Z_{eq}} = \frac{40\angle -90^\circ}{2.5\angle 36.9^\circ} = 16\angle -126.9^\circ \text{ mA}.$$

$\therefore i(t) = 16\cos(3000t - 126.9^\circ)$ mA.

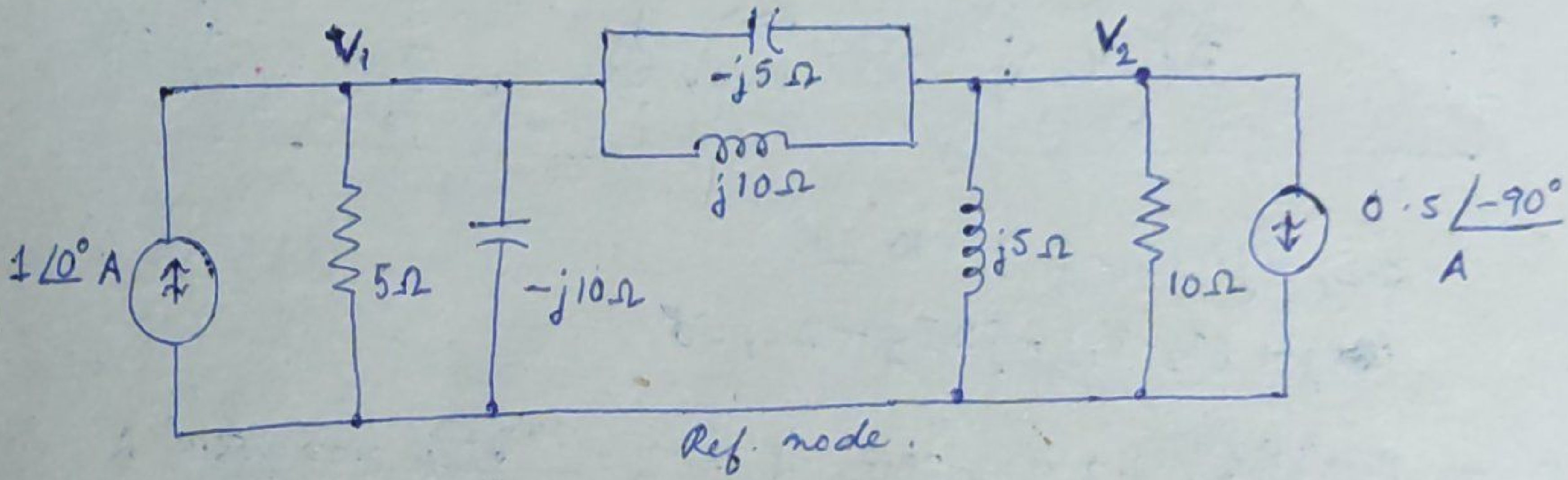
Use current division in freq. domain to obtain i_c.

Note: Always use freq. domain or time domain ALTOGETHER.

$\boxed{Y = \text{admittance} = 1/Z} = \underbrace{G}_{\text{(conductance)}} + j\underbrace{S}_{\text{susceptance}}$.

Sinusoidal Steady State Response :

Nodal analysis



KCL at left node : $\dfrac{V_1}{5} + \dfrac{V_1}{-j10} + \dfrac{V_1 - V_2}{-j5} + \dfrac{V_1 - V_2}{j10} = 1 + j0$

right node : $\dfrac{V_2 - V_1}{-j5} + \dfrac{V_2 - V_1}{j10} + \dfrac{V_2}{j5} + \dfrac{V_2}{10} = -(-j0.5)$

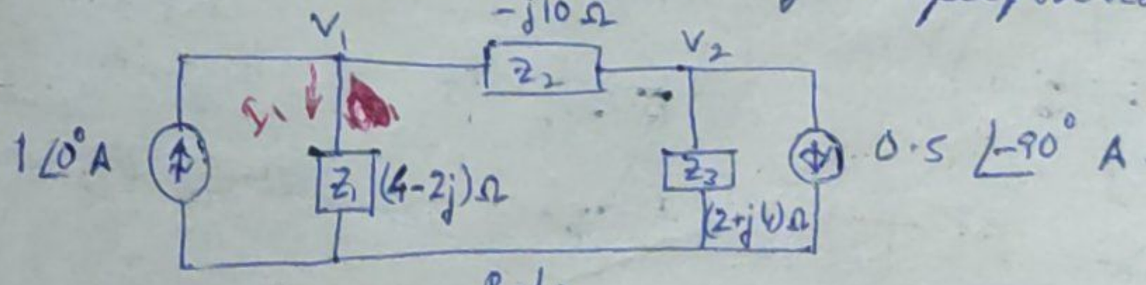
$\longrightarrow \quad V_1 = 1 - j2\,V \quad , \quad V_2 = -2 + j4\,V$

$\quad = 2.24 \angle -63.4^\circ\,V \qquad = 4.47 \angle 116.6^\circ\,V$

Both sources assumed to be operating at same frequency ω [if known, then L, C can be found] $\rightarrow$ $v_1(t) = 2.24\cos(\omega t - 63.4^\circ)\,V$; $v_2(t) = 4.47\cos(\omega t + 116.6^\circ)\,V$

All assumed linear elements ∴ linearity, superposition hold.

∴ Redraw as



Superposition

V_{1L} (due to left source) $= 1\angle 0^\circ \cdot (4-j2) \cdot \dfrac{2-j10+j4}{4-2j-j10+2+j4}\,V = 2-j2\ V$

$V_{1R} = (-0.5\angle -90^\circ)\cdot(4-j2)\cdot \dfrac{2+j4}{4-2j-j10+2+4j} = -1$

$V_1 = V_{1L} + V_{1R} = 1-j2\,V.$

Say Th. equivalent for $-j10\,\Omega$ impedance.

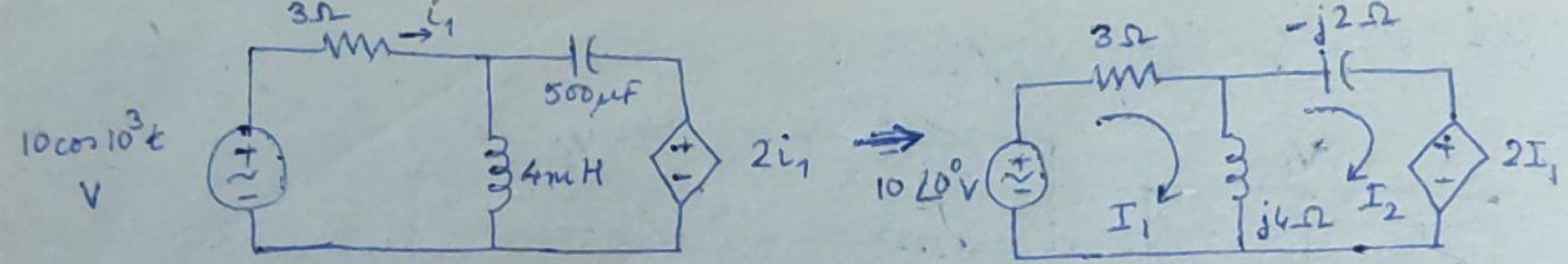
V_{oc} (left +ve) $= [1\angle 0^\circ(4-2j) - (-0.5\angle -90^\circ)(2+j4)]\,V = 6-j3\ V$

$Z_{th} = z_1 + z_3 = (6+j2)\,\Omega$

$I_{12} = \dfrac{6-j3}{Z_{th}+Z_l} = (0.6+j0.3)\ A$ (from 1 to 2)

$\therefore I_1 = I_{SL} - I_{12} = 0.4-j0.3 \Rightarrow V_1 = I_1 Z_1 = 1-j2\ V$

~ can use Norton's or even source transformations.



Mesh Analysis

$$3I_1 + j4(I_1 - I_2) = 10\angle 0^\circ$$

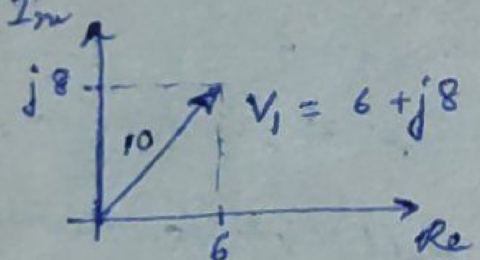
$$j4(I_2 - I_1) - j2I_2 + 2I_1 = 0.$$

$$I_1 = \frac{14+j8}{13}\,A = 1.24\,\angle 29.7^\circ\ A \quad ; \quad I_2 = \frac{20+j30}{13}\,A = 2.77\,\angle 56.3^\circ\ A$$

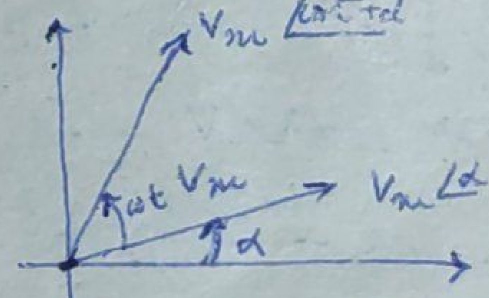
$$\therefore\ i_1(t) = 1.24\cos(10^3 t + 29.7^\circ)\,A \quad ; \quad i_2(t) = 2.77\cos(10^3 t + 56.3^\circ)\,A.$$

Note: Same techniques applicable for any complex frequency in general.

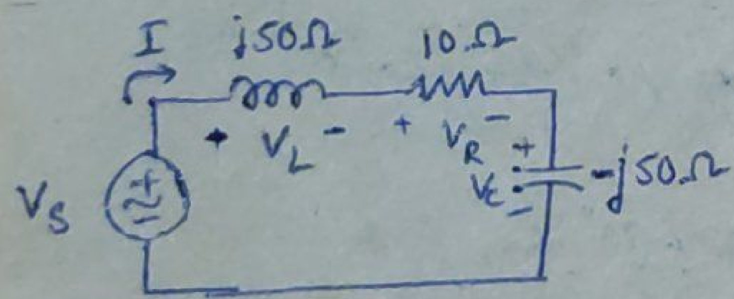
Phasor diagrams :

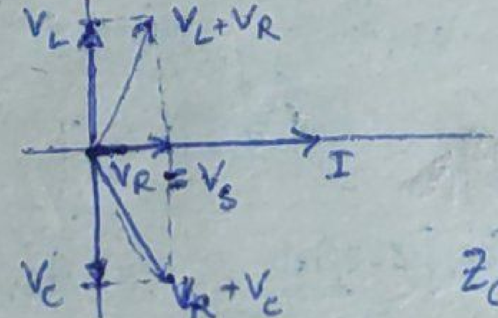


$V_1 = 6 + j8 = 10 \angle 53.1° \ V$



For diff. ωt , rotate anticlockwise.



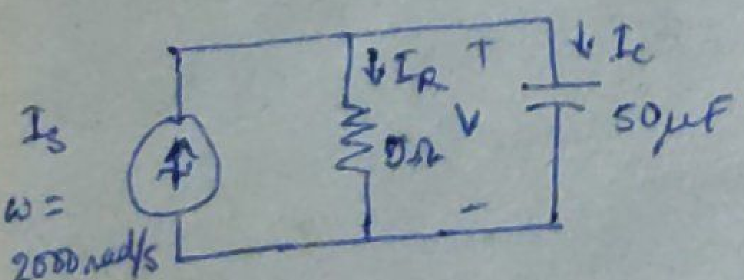


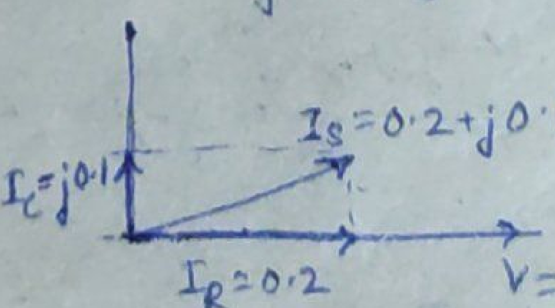
I & V on same plot
magnitudes diff.

$Z_C = -Z_L$ Resonant condn.

Addn & Subtraction : vector rules.
Mul. & div : add & sub. ∠ , change amplitudes.

$j\omega C = j0.1℧$, $\frac{1}{R} = 0.2℧$



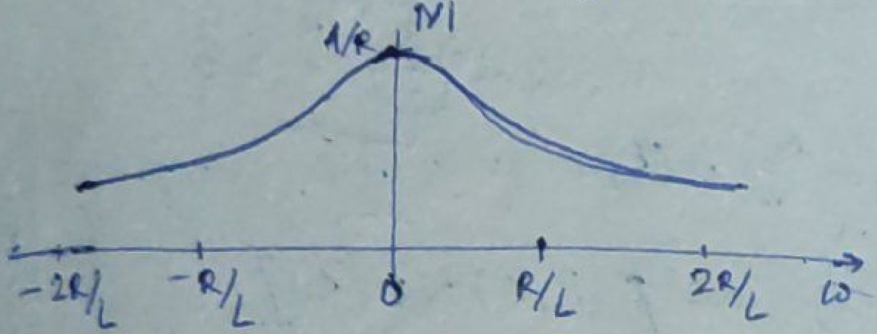


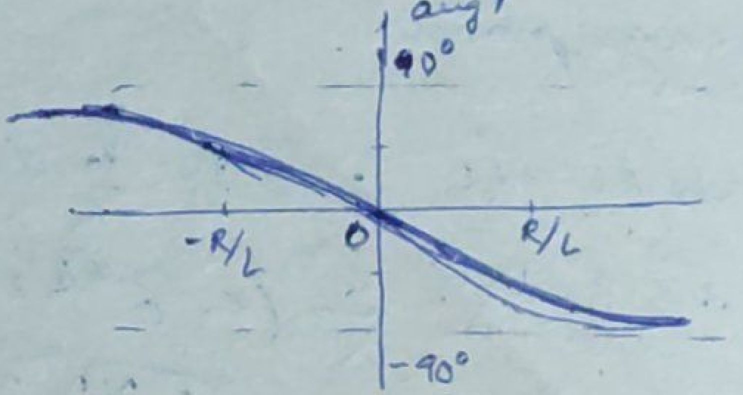
Even if I_S known, yet same technique, then scale suitably.

Response as fn. of ω :

i. Consider series RL ckt. $\therefore I = \dfrac{V_s}{R+j\omega L}$ $\therefore Y = \dfrac{1}{R+j\omega L}$

$\therefore |Y| = \dfrac{1}{\sqrt{R^2+\omega^2L^2}}$ arg. $Y = -\tan^{-1}\dfrac{\omega L}{R}$





$\omega = -100$ rad/s $\Rightarrow v(t) = 50\cos(\omega t + 30°)$

$v(t) = 50\cos(-100t + 30°)$

at $\omega = \pm R/L = \pm 1/\tau \rightarrow |Y| = 0.707|Y_{max}|$, arg $Y = 45°$.

only due to R

$\therefore$ Half power frequency. avg power $= 0.5$ max. power. $\longrightarrow \therefore$ For same V, $I = \frac{1}{\sqrt{2}} I_{max}$ $\therefore$ avg. power $= (0.707)^2 P_{max} = 0.5 P_{max}$