NYQUIST STABILITY:

Basic philosophy:

$$\frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)H(s)}$$

$$F(s) = 1 + G(s)H(s) = 0 \quad ; \quad GH(s) = \frac{N(s)}{D(s)}$$

$$= 1 + \frac{N(s)}{D(s)} = \frac{N'(s)}{D(s)}$$

∴ ① CL poles of system = Zeros of $F(s)$ = ROOTS of charac. eqn.

② OL T.F. $GH(s)$ poles = poles of $F(s)$

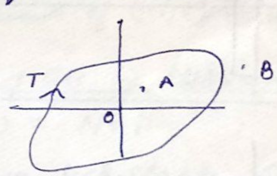
③ For asymptotic stability of CL sys.,
NO restriction on poles/zero of OL T.F. $GH(s)$

BUT Poles of CL T.F. = Zeros of $F(s)$ in LHS.

Finding zeros of $F(s)$ is difficult but using $GH(s)$ roots, infer stability.

ANALYTIC FN $F(s)$ in s-plane if fn. & all its derivatives exist ⇒ no singular pts. ⇒ no POLES.

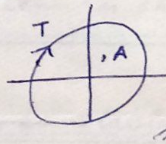
✓ENCIRCLED : A pt. is encircled by a closed path if it is found inside the path. Usually CW (convention for +ve traversal)



O, A encircled
B not.

ENCLOSED

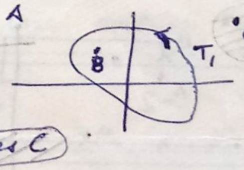
A point or region is said to be enclosed by a closed path if it on the RT. of a path when the path is traversed in CW direction.



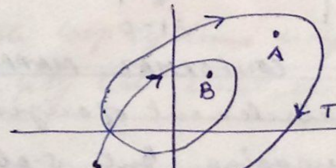
T encloses A

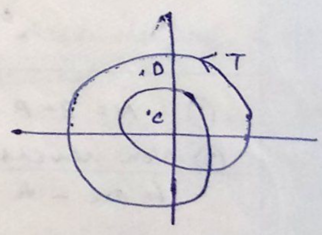
T_1 DOES NOT Enclose B

while encloses C



No. of encirclements



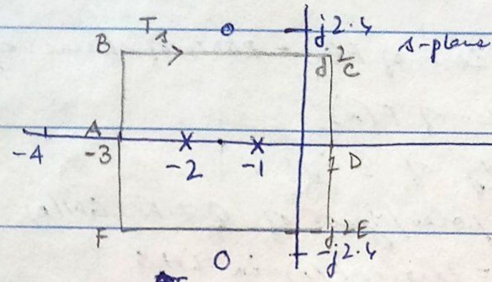


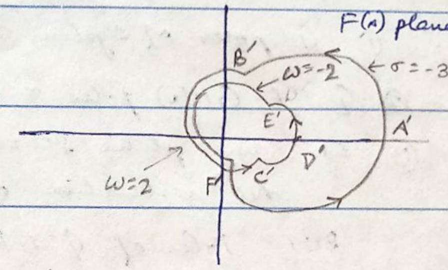
For B $N = +2$
A $N = +1$
C $N = -2$
D $N = -1$

Conformal mapping :

Say $GH(s) = \dfrac{6}{(s+1)(s+2)}$

$\therefore F(s) = 1 + GH(s) = \dfrac{(s+1.5+j2.4)(s+1.5-j2.4)}{(s+1)(s+2)} = 0$

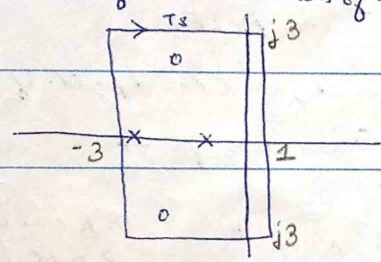


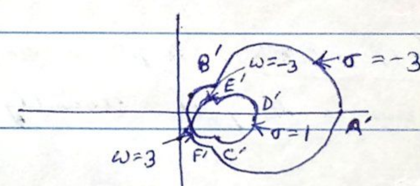


$s = \sigma + j\omega$	$A = -3+j0$	$B = -3+j2$	$C = 1+j2$	$D = 1+j0$	$E = 1-j2$	$F = -3-j2$
$F(s) = u+jv$	$4+j0$	$0.7+j0.9$	$1.11-j0.577$	$2+j0$	$1.11+j0.577$	$0.7-j0.9$

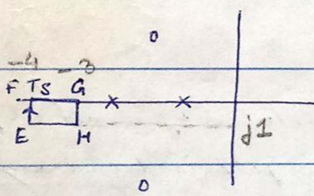
$\therefore$ No. of encirclement of origin $N = -2 = \boxed{Z-P}$ as in T_s → hence can be used as CHECK for ANALYTICAL FN. in specified region

$= 0 - 2$

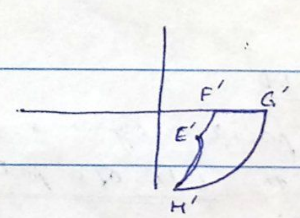


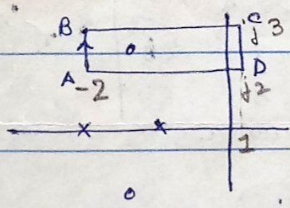


$N = 0 = Z - P = 2 - 2$

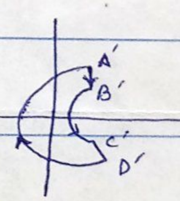


$N = 0$





$N = +1$
$= Z - P$
$= 1 - 0$

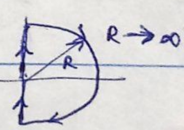


CONFORMAL MAPPING.

(1) $\therefore N = Z - P$ (encirclement of origin)

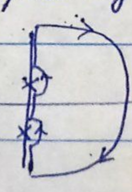
(2) Non reversible mapping But s-plane to F(s) plane is one-to-one mapping

Usually T_s as $j\omega$ axis & RHS-plane



But if pt. of singularity or pole on $j\omega$ axis then small detour.

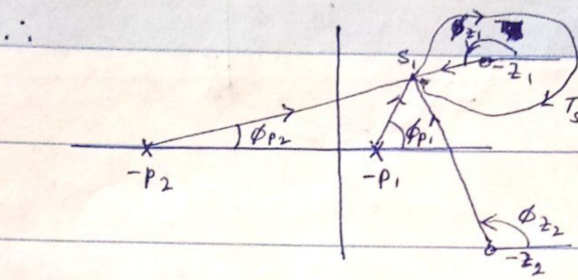
— to keep region enclosed in s-plane s.t. F(s) is analytic — value exists



CAUCHY'S THEOREM:

Say $F(s) = \dfrac{(s+z_1)(s+z_2)}{(s+p_1)(s+p_2)} = |F(s)| \angle(\phi_{z_1} + \phi_{z_2} - \phi_{p_1} - \phi_{p_2})$

$\therefore$



$\therefore$ As s_1 traverses T_s, angles generated by $-p_1, -p_2, -z_2$ after one complete round are 0, while $\angle$ generated by $-z_1$ is 2π radians (clockwise)

$\therefore$ If Z zeros & P poles are encircled, then angle generated is $2\pi(Z) - 2\pi(P) = \phi_F = 2\pi N$ = net angle generated by T_F and hence net encirclement of origin of $F(s)$ plane in CW direction is $N = Z - P$.

Note $GH(s) + 1 = F(s) \Rightarrow GH(s)$ plane $(-1,0)$ pt. $\equiv F(s)$ plane origin

$\therefore$ Polar plot encirclement of $(-1,0)$pt. $\equiv F(s)$ plane encirclement of origin.

$\therefore$ If there are P poles of GH in RHS and N encirclements of $(-1,0)$
$\Rightarrow Z$ of $F(s) = 1 + GH(s) = N + P$

Required to be zero for A stable C system $\Rightarrow N = -P$ required.

NYQUIST STABILITY CRITERION:

a) The Nyquist plot is obtained by mapping the fn. $GH(s)$ – polar plot.
b) The Nyquist contour is chosen to enclose entire RH of s-plane

Then $Z = N + P$

For C system to be stable, $Z = 0 \Rightarrow N = -P$.

<u>Note</u>: Since multiple loop systems may <u>NOT</u> include poles in RHS
$\therefore$ Routh's stability test must be performed.

II

s	F(s)
A = −3+j0	4+j0
B = −3+j3	0·68 + j0·41
C = 1+j3	0·92 − j0·38
D = 1+j0	2 + j0
E = 1−j3	0·92+j0·38
F = −3−j3	0·68 − j0·41

III

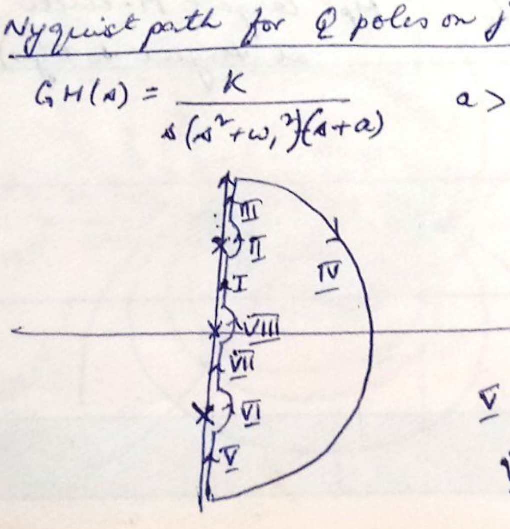
s	F(s)
A = −4+j0	2+j0
B = −3+j0	4 + j0
C = −3+j1	1·6 − j1·8
D = −4−j1	1·6 − j0·6

IV

s	F(s)
A = −2+j2	−0·2 + j0·6
B = −2+j3	0·4 + j0·2
C = 1+j3	0·92 − j0·38
D = 1+j2.	1·11 − j0·58

Nyquist path for Q poles on jω axis

$$GH(s) = \frac{K}{s(s^2+\omega_1^2)(s+a)} \qquad a>0$$



I $\quad s = j\omega \quad j0^+$ to $j\omega_1^-$

II $\quad s = \lim_{\varepsilon\to 0} j\omega_1 + \varepsilon e^{j\theta} \quad \theta \in [-\pi/2, \pi/2]$

III $\quad s = j\omega \quad j\omega_1^+$ to $j\infty$

IV $\quad s = \lim_{R\to\infty} Re^{j\phi} \quad \phi \in [\pi/2, -\pi/2]$

V to VII $\quad$ IPP of III to I

VIII $\quad s = \varepsilon e^{j\theta} \quad \theta \in [-\pi/2, \pi/2]$