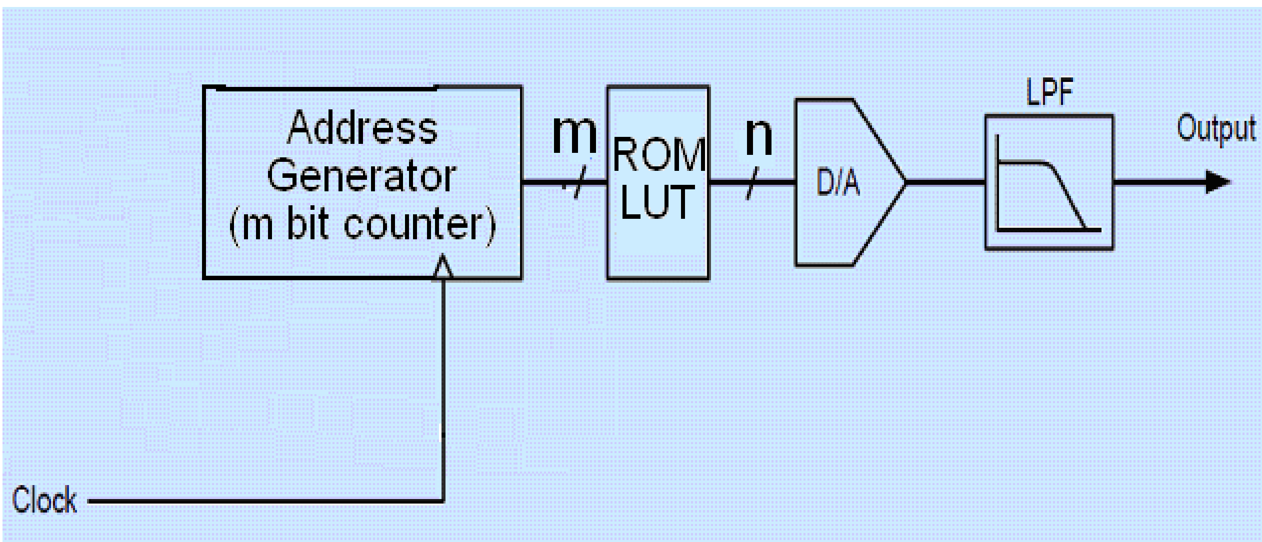
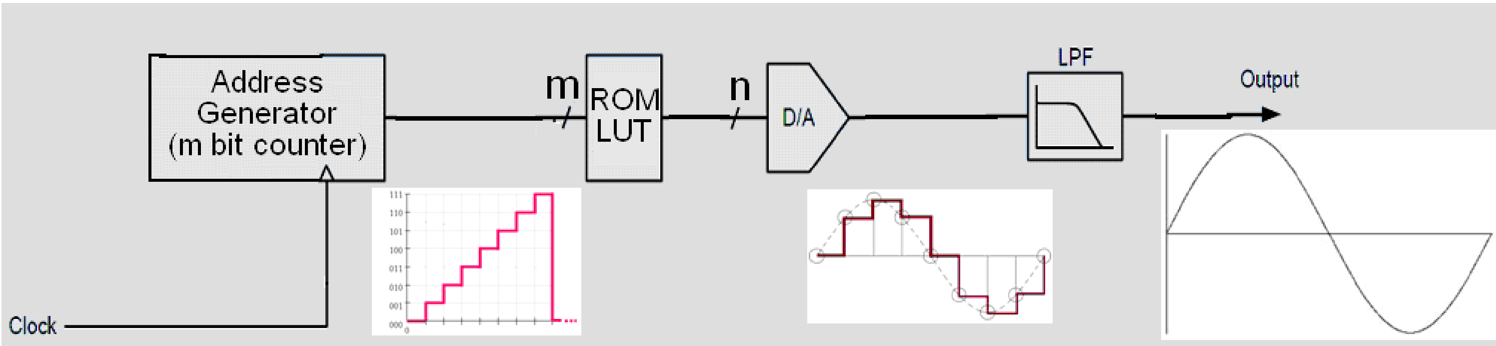
Waveform-synthesizer (Digital)

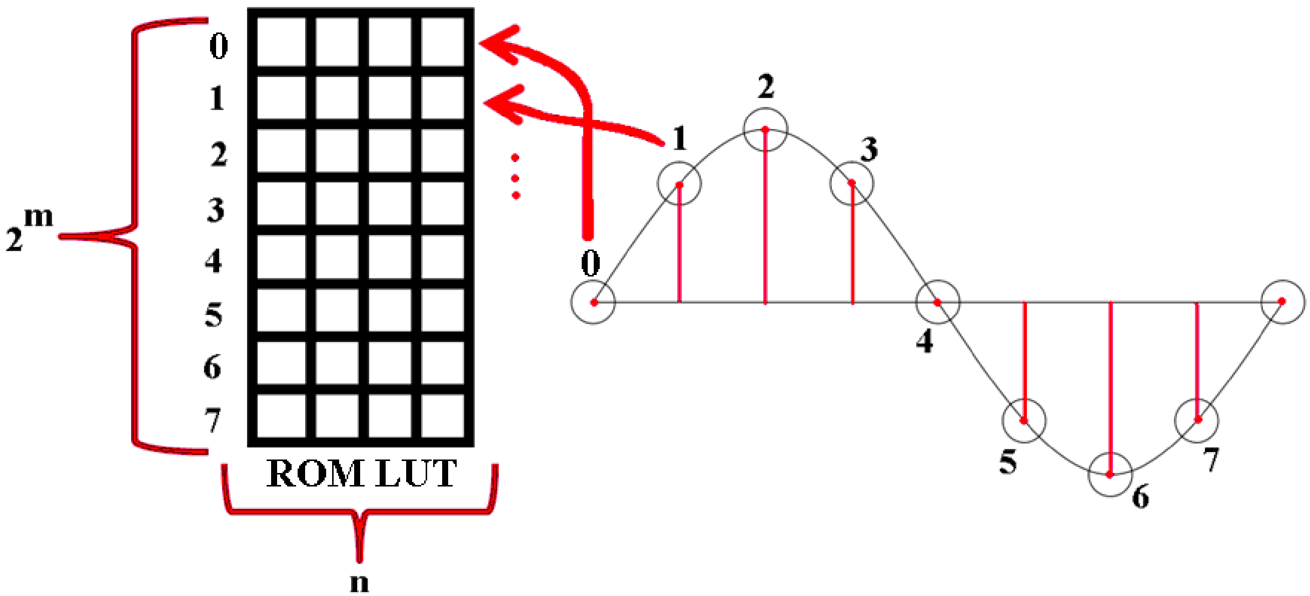
- Must have stable frequency, wave-shape and amplitude.

Frequency Synthesizer → employs **Direct Digital Synthesis (DDS)**

Basic Scheme:

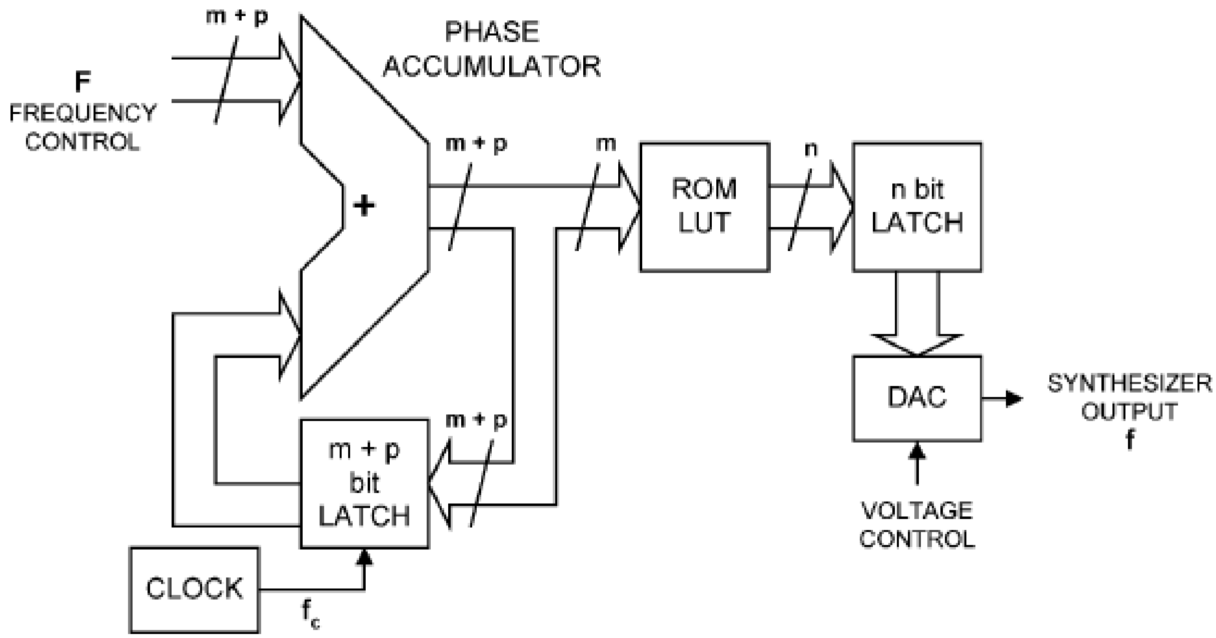






Output frequency, $f = \dfrac{f_c}{2^m}$

Modified Scheme

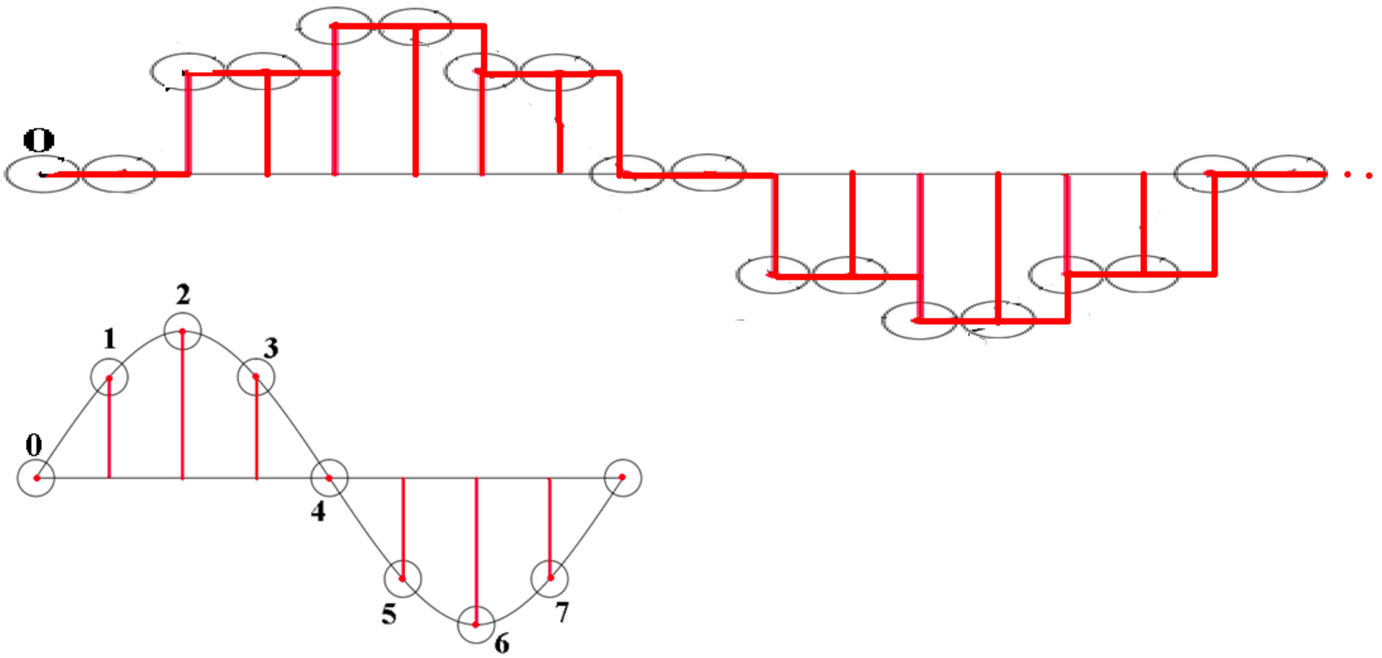


Min. Output frequency, $f_{\min} = \dfrac{f_c}{2^{m+p}}$ when

$$F = 2^{-p}$$

Max. Output frequency, $f_{\max} = \dfrac{f_c}{2}$ when

$$F = 2^{m-1}$$



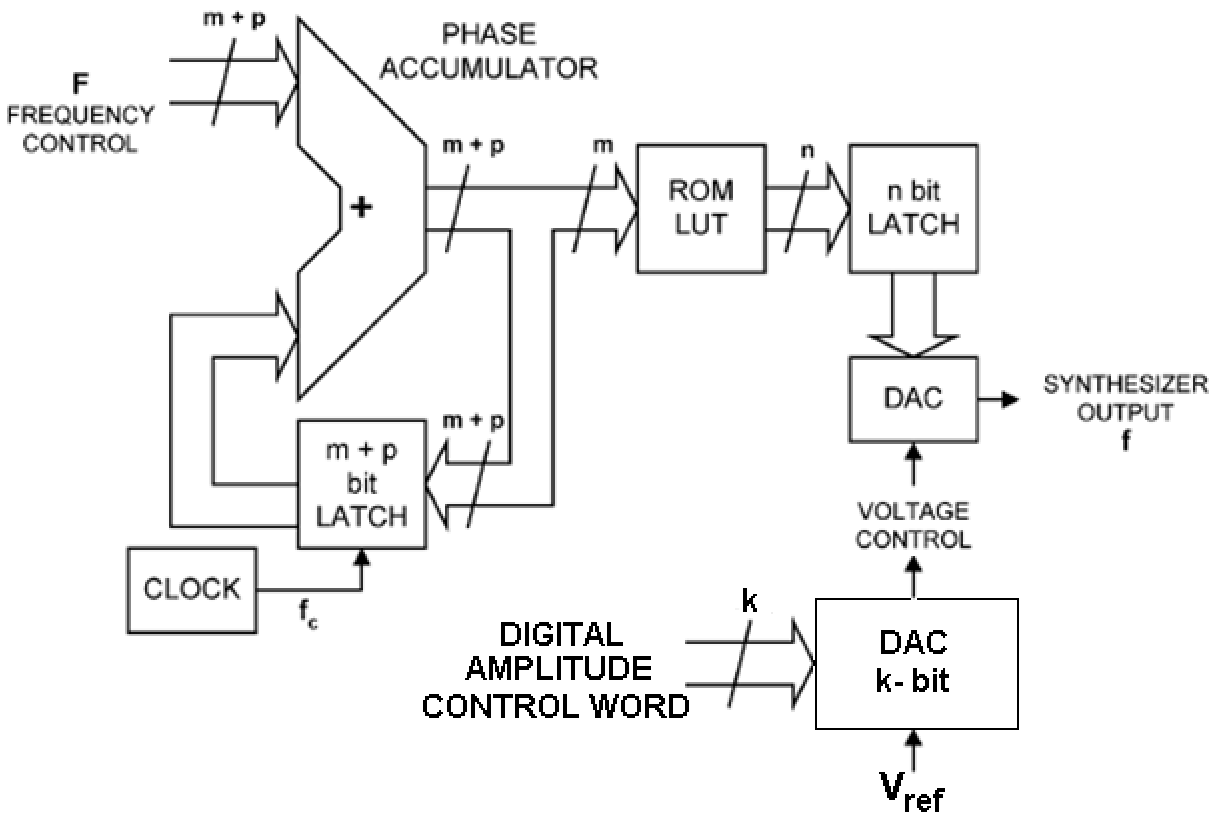
In general,   Output frequency,

$$f = F\,\frac{f_c}{2^m}$$

Advantages:

- The frequency is tunable with sub-hertz resolution
- The phase is digitally adjustable
- Simple design and low parts count

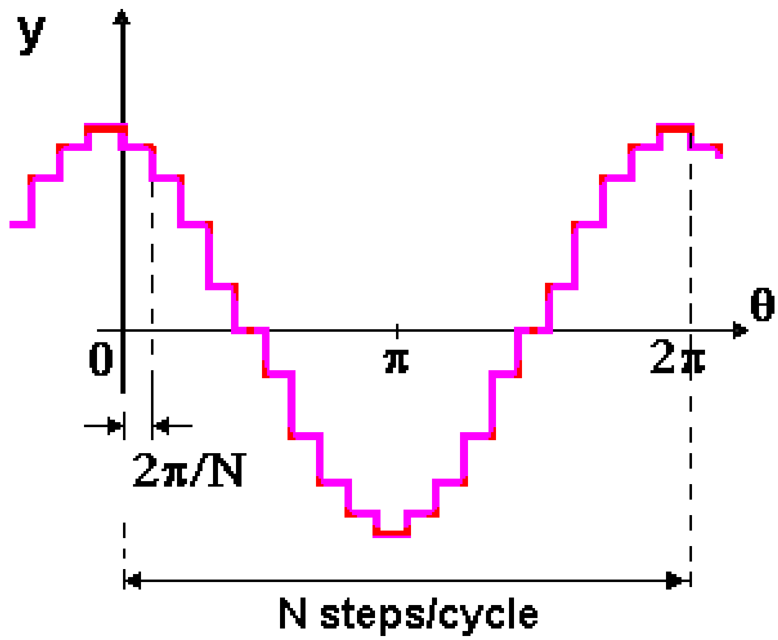
Adjustment of gain:



Problem. :

A digital frequency synthesizer employs a 4.194304 MHz crystal oscillator and gives a 256 step-sinusoid. Determine the maximum and minimum output frequency if the number of fractional bit is 2. Also find out the frequency control word for these cases.

Harmonic Analysis of Waveform Synthesizer Output for Sinusoidal Waveform



$$y(\theta) = A\cos(\frac{2\pi k}{N} + \frac{\pi}{N}) \quad \text{for} \quad \frac{2\pi k}{N} < \theta \leq \frac{2\pi(k+1)}{N}$$

[where, k= 0,1,2,... , N-1]

$$y(\theta) = a_0 + \sum_{r=1}^{\infty} a_r \cos(r\theta) + \sum_{r=1}^{\infty} b_r \sin(r\theta)$$

[where, r= harmonic number]

$$y(\theta) = a_1 \cos\theta + a_3 \cos 3\theta + a_5 \cos 5\theta + \ldots\infty$$

$$a_r = \frac{2}{\pi}\int_0^{\pi} y(\theta)\cos(r\theta)d\theta$$

[where, r=1,3,5,…∞]

$$a_r = \frac{2}{\pi}\left[\sum_{k=0}^{\frac{N}{2}-1} \int_{\frac{2\pi k}{N}}^{\frac{2\pi(k+1)}{N}} A\cos\frac{\pi}{N}(2k+1)\cos(r\theta)d\theta\right]$$

$$a_r = \frac{2A}{\pi}\left[\sum_{k=0}^{\frac{N}{2}-1} \cos\frac{\pi}{N}(2k+1) \int_{\frac{2\pi k}{N}}^{\frac{2\pi(k+1)}{N}} \cos(r\theta)d\theta\right]$$

$$a_r = \frac{2A}{\pi}\left[\sum_{k=0}^{\frac{N}{2}-1} \cos\frac{\pi}{N}(2k+1)\left(\frac{\sin(r\theta)}{r}\bigg|_{\frac{2\pi k}{N}}^{\frac{2\pi(k+1)}{N}}\right)\right]$$

$$a_r = \frac{4A}{\pi r}\sin\frac{\pi r}{N}\left[\sum_{k=0}^{\frac{N}{2}-1} \cos\frac{\pi}{N}(2k+1)\cos\frac{\pi}{N}(2k+1)r\right] \qquad (1)$$

$$a_r = \frac{2A}{\pi r}\sin\frac{\pi r}{N}\left[\sum_{k=0}^{\frac{N}{2}-1} \cos\frac{\pi}{N}(2k+1)(r+1) + \cos\frac{\pi}{N}(2k+1)(r-1)\right]$$

Now, $\cos\theta + \cos 3\theta + \cos 5\theta + \ldots \frac{N}{2} terms = \frac{\sin N\theta}{2\sin\theta}$

$$a_r = \frac{2A}{\pi r}\sin\frac{\pi r}{N}\left[\frac{\sin\pi(r+1)}{2\sin\frac{\pi}{N}(r+1)} + \frac{\sin\pi(r-1)}{2\sin\frac{\pi}{N}(r-1)}\right]$$

[where, r=1,3,5,…∞]

a_r **will be 0 for all *r* except *r = Np±1*** **[where, *p=0,1,2,…∞*]**

For *r=1,*

$$a_r = \frac{4A}{\pi}\sin\frac{\pi}{N}\left[\sum_{k=0}^{\frac{N}{2}-1} \cos^2\frac{\pi}{N}(2k+1)\right]$$

Now, $\cos^2\theta + \cos^2 3\theta + \cos^2 5\theta + \ldots\frac{N}{2}\,terms = \frac{1}{2}\left[\frac{N}{2} + \frac{\sin 2N\theta}{2\sin 2\theta}\right]$

$$a_1 = \frac{4A}{\pi}\sin\frac{\pi}{N}\left[\frac{1}{2}\left(\frac{N}{2} + \frac{\sin 2\pi}{2\sin\frac{2\pi}{N}}\right)\right]$$

$$a_1 = \frac{NA}{\pi}\sin\frac{\pi}{N} \quad \text{for } N=4,5,6,\ldots \qquad (2)$$

In general,

$$a_{Np\pm1} = \frac{4A}{\pi(Np\pm1)}\sin\frac{\pi(Np\pm1)}{N}\left[\sum_{k=0}^{\frac{N}{2}-1} \cos\frac{\pi}{N}(2k+1)\cos\frac{\pi}{N}(2k+1)(Np\pm1)\right]$$

[where, p=0,1,2,…∞]

Typical values for *N=20*,

$$a_1 = 0.9959A$$

$$a_{19} = -0.0524A$$

$$a_{21} = 0.047A$$

$$a_{39} = -0.0255A$$

$$a_{41} = 0.0242A$$

$$\text{THD} = \sqrt{\frac{\text{sum of the powers of all harmonic components}}{\text{power of the fundamental frequency}}} = \frac{\sqrt{\sum_{n=2}^{\infty} a_n^{\ 2}}}{a_1}$$