ADAPTIVE CONTROL

What is Adaptive Control?

An 'Adaptive Control System' is one in which the controller parameters are adjusted automatically in such a way as to compensate for variations in the characteristics of the process it controls.

The various types of adaptive control systems differ only in the way the controller parameters are adjusted.

Why Adaptive Control?

Adaptive control of industrial processes are necessary for the following reasons.

1. Most of the industrial processes are non-stationary (i.e. their characteristics change with time). Typical example is the decay of the catalyst activity in a chemical reactor.

   These changes lead to a deterioration in the performance of the linear controller, which was designed using some nominal values of the process parameters, thus requiring adaptations of the controller parameters.
2. Most industrial processes are non linear. Therefore the linearized models that are used to design linear controllers depend on the particular steady state (around which the process is linearized).

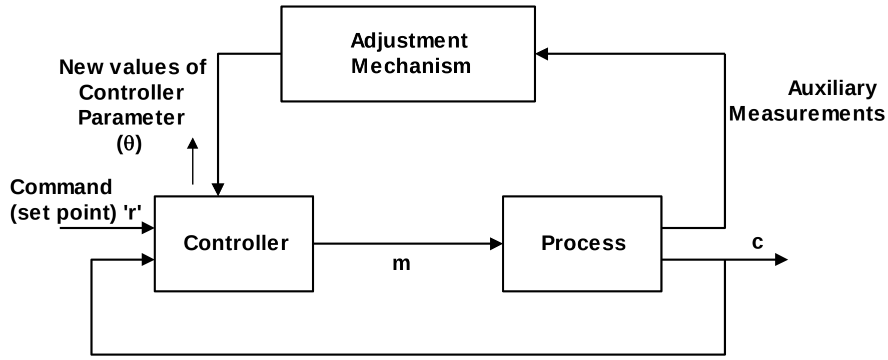
   Example : Liquid flow system

Basic Structures of Adaptive Control System

Three main basic control system structures relevant to the design of adaptive control systems are:

- Parameter Scheduling Control (PSC) or Gain Scheduling Control.
- Model Reference Adaptive Control (MRAC).
- Self-Tuning Regulators (STR) or Model Identification Adaptive Systems (MIAS).

Parameter Scheduling Control (PSC) or Gain Scheduling Control

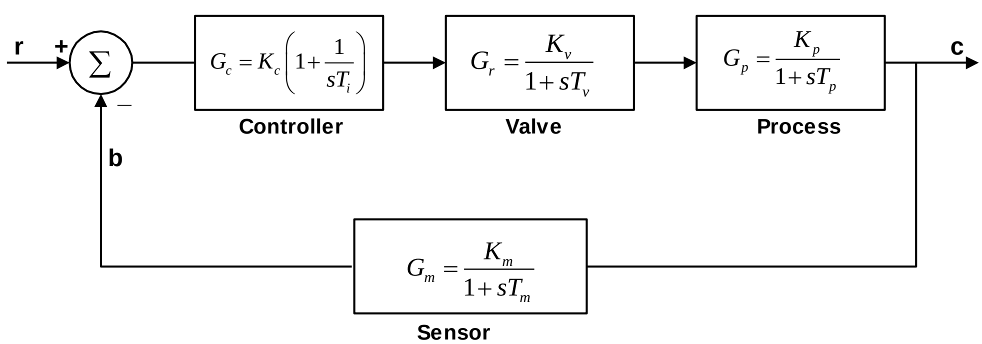


In many control problems it is possible to find auxiliary process variables (other than the plant outputs used for feedback) that correlate well with the changes in process dynamics. From the measurements of these auxiliary variables, the controller parameters can then be adjusted in a predetermined manner as functions of the auxiliary variables, to compensate for the changes in process conditions. This strategy has been originally applied to the adaptation of controller gain factors, and thus has been referred to as "Gain Scheduling".

The advantage of parameter scheduling is that the controller parameters can be changed quickly (as quickly as the auxiliary measurement) in response to changes in process dynamics. It is convenient especially if the process dynamics depend in a well known fashion on a relatively few easily measurable variables.

The disadvantage of gain scheduling is that it is an open-loop adaptation scheme with no real learning or intelligence. Moreover, for a large complex system, the extent of design required for its implementation can be enormous.

Example: Gain Scheduling



[The technique is called gain scheduling because, usually, the steady state gain of the controller is scheduled].

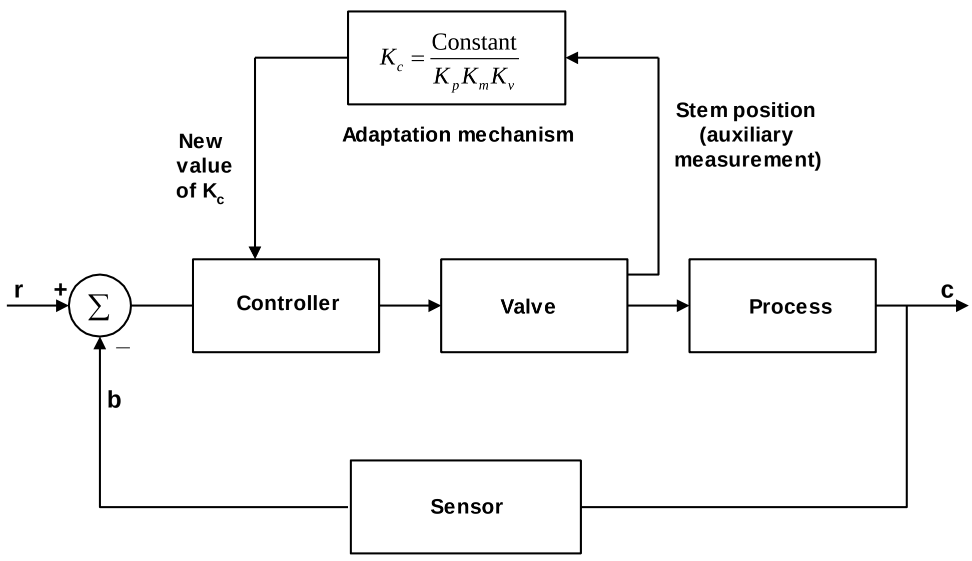
In the control loop shown above, the control valve or another of its components may exhibit non-linear character. The gain of the non-linear component will then depend on the current steady state. Suppose that we want to keep the total gain of the overall system constant, i.e.,

$$K_{overall} = K_c K_v K_p K_m = \text{Constant}$$

Then, as the gain K_v of the non-linear valve changes, the gain of the controller should change as follows:

$$K_c = \frac{\text{Constant}}{K_p K_m K_v} \qquad \text{(i)}$$

Assume that the gain K_p and K_m are known exactly. Moreover the characteristics of the control valve are known well, then its gain K_v can be calculated from the stem position and by measuring the stem position (auxiliary measurement) K_v can be computed. Then equation (i) gives the adaptation mechanism of this simple gain scheduling adaptive controller. The resulting structure is given below.

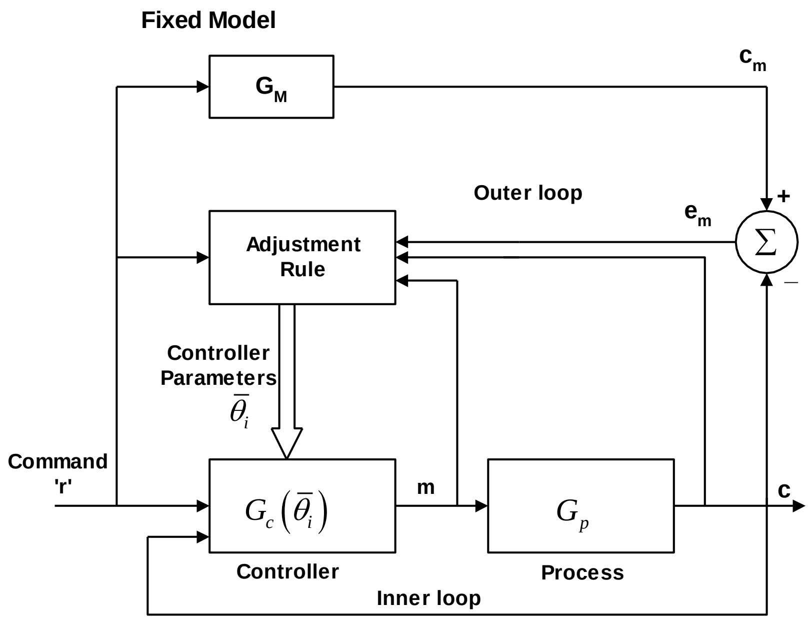


[Here the control valve gain K_v is assumed to be a non-linear one. In the figure the stem position can be measured using a position transducer e.g. LVDT].

Model Reference Adaptive System (MRAS)

Model Reference Adaptive Systems (MRAS) are used to obtain a closed loop response close to that of a given reference model, for given input signal.

A block diagram representation of the Model Reference Adaptive Control (MRAC) system is shown next.



The MRAS can be thought of as having two loops:- an inner loop which represents the basic control system consisting of the process and the controller, and an outer loop (adaptation loop) that adjusts the parameters of the controller in such a way as to drive the error (e_m) between the model output (c_m) and the process output (c) to zero.

The key problem in the scheme is to obtain an adjustment mechanism that tunes the adjustable controller parameters (represented by vector $\bar{\theta}_i$) to drive the error $e_m = c_m - c$ to zero. For this purpose, **performance criteria** are chosen which are suited for further analytical use. Minimizing the performance criteria and possible additional requirements then result in the **adaptation law.**

MRAC System Employing "Local Parameter Optimization" Through Gradient Method

This MRAC method assumes that the to be tuned parameters θ_i (i = 1,2,…..,p) of the controller are already close to the correct values. If a simple gradient method is used for optimization, then each parameter θ_i is changed proportionally to the performance criterion I.

That is,

$$\Delta\theta_i = -K_i \frac{\partial I}{\partial \theta_i} \qquad (1)$$

$$i = 1, 2, ..., p$$

K_i are positive constants called **adaptation gain,** which have to be appropriately chosen.

From equation (1) it follows that

$$\frac{d\theta_i}{dt} = -K_i \frac{\partial}{\partial t}\frac{\partial I}{\partial \theta_i}$$

or,
$$\frac{d\theta_i}{dt} = -K_i \frac{\partial}{\partial \theta_i}\frac{\partial I}{\partial t} \qquad (2)$$

If the performance criterion ISE is considered for minimization, then,

$$I = \int_0^{t_1} e_m^2(t)\,dt$$

$$\therefore \frac{d\theta_i}{dt} = -2K_i e_m(t)\frac{\partial e_m(t)}{\partial \theta_i}$$

or,
$$\frac{d\theta_i}{dt} = 2K_i e_m(t)\frac{\partial c(t)}{\partial \theta_i} \qquad (3)$$

$\frac{\partial c(t)}{\partial \theta_i}$ is known as the **parameter sensitivity** of the process output signal.

Hence the rate of change of controller parameter θ_i with time is proportional to the product of the model error signal and the parameter sensitivity of the output signal.

Equation (3) is referred to as the “MIT-rule”. An MRAS based on this rule was originally proposed by H.P. Whitaker of MIT, for adaptive control of air-crafts.

Integration of both sides of equation (3) leads to

$$\theta_i = 2K_i \int_0^{t_1} e_m(t)\frac{\partial c(t)}{\partial \theta_i}\,dt \qquad (4)$$

Self Tuning Regulator (STR) or Model Identification Adaptive System (MIAS)

The model identification adaptive systems (MIAS) determine a process model using measured input and output signals and identification algorithm. From the estimates of the process model parameters, controller parameters are calculated in the same way as if these estimated parameters were the two parameters, in accordance with a controller design method which has been preprogrammed.

The block diagram representation of a control system employing self-tuning controller (STC) is shown below.

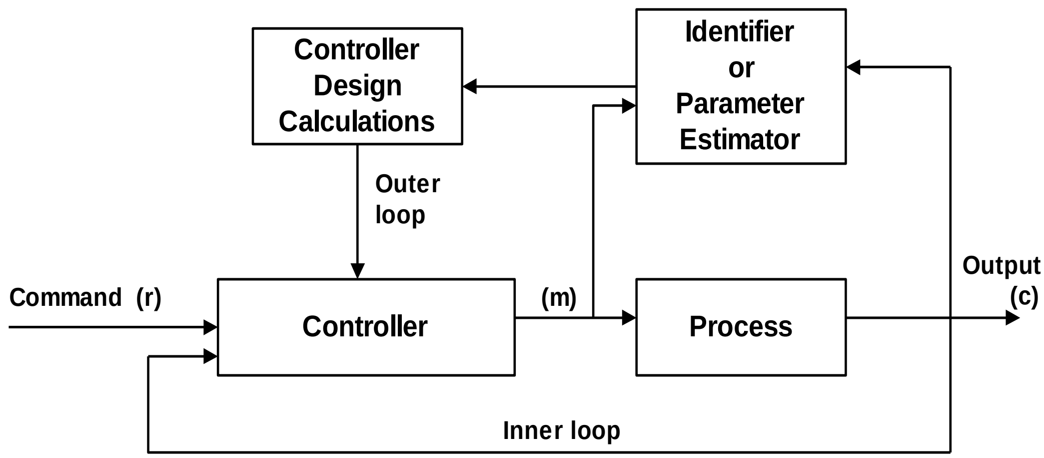


Fig.: Block diagram of a self-tuning regulator.

The MIAS consists of two loops: an inner loop consisting of the process and a conventional controller, but with varying parameters, and an outer loop containing an identifier and a design box (representing an on-line solution to a design problem for a system with known parameters) which adjust the controller parameters.

Identification of a Process Model

Let us consider a process that is poorly known. This may mean that the physical or chemical phenomena in the process are poorly understood or that the process parameters are imprecisely known. In the first case the model order is not known; the second case is just a parameter estimation problem with known model order.

Let the process be described by the following linear difference equation of order K.

$$c_n = a_1 c_{n-1} + a_2 c_{n-2} + \ldots\ldots + a_k c_{n-k} + b_1 m_{n-1} + b_2 m_{n-2} + \ldots . + b_k m_{n-k} \qquad (1)$$

where c_i and m_i are the process output and input values at the ith sampling instant and $a_j\left(j=1....K\right)$ and $b_j\left(j=1....K\right)$ are constant but imprecisely known parameters. Order K of the model may or may not be known.

A prespecified change in the input to the process is introduced. Let $\hat{m}_n$ and $\hat{c}_n$ be the measured values of the input and output variables at nth sampling instant with n = 0,1,2,…,N.

The error between the process output values computed from the postulated model [equation (1)] and the measured values, is

$$\in_n = \hat{c}_n - c_n = \hat{c}_n - \left(a_1\hat{c}_{n-1} + a_2\hat{c}_{n-2} + ... + a_k\hat{c}_{n-k} + b_1\hat{m}_{n-1} + b_2\hat{m}_{n-2} + ... + b_k\hat{m}_{n-k}\right) \qquad (2)$$

Best estimate of the process parameters is given by the solution of the following

Least Squares Problem.

Minimize the **mean-square error**

$$P = \frac{1}{N}\sum_{n=1}^{N}\in_n^2 = \frac{1}{N}\sum_{n=1}^{N}\left[\hat{c}_n - a_1\hat{c}_{n-1} - .... - a_k c_{n-k}^2 - b_1\hat{m}_{n-1} - b_2\hat{m}_{n-2} - .... - b_k\hat{m}_{n-k}\right]^2 \qquad (3)$$

One of the numerical methods for solution of this minimization problem is based on the solution of the following set of algebraic equations (necessary conditions to be satisfied at the point where P is a minimum):

$$\frac{\partial P}{\partial a_1} = \frac{\partial P}{\partial a_2} = ... = \frac{\partial P}{\partial a_k} = \frac{\partial P}{\partial b_1} = \frac{\partial P}{\partial b_2} = ... = \frac{\partial P}{\partial b_k} = 0 \qquad (4)$$

If, for the determined values of the process parameters, the value of P is considerably larger than the theoretically possible minimum value of zero, it can be concluded that the assumed model order is unacceptably low, and that a higher order model should be used.

Hence the steps that constitute the experimental identification of a process model, can be summarized as:

1. **Postulate a model for the process.** As a starting point, one can employ first or second order models with or without dead time.
2. **Introduce a Known input change to the process and record its output.** However, normal operating data for the values of input and output variables can also be used.
3. **Estimate the best values of the unknown process parameters** from the record of the input and output by the method of least squares.

Sap. No.	I/P	O/P
0	1.0	0
1	0.6	0.5
2	0.3	0.9
3	0.1	0.91
4	0.0	0.866

$$c_n = a_1 c_{n-1} + b_1 m_{n-1}$$

$$P = \frac{1}{4} \sum_{n=1}^{4} \left(\hat{c}_n - a_1 \hat{c}_{n-1} - b_1 \hat{m}_{n-1} \right)^2$$

$$\frac{\partial P}{\partial a_1} = \sum_{n=1}^{4} 2 \left(\hat{c}_n - a_1 \hat{c}_{n-1} - b_1 \hat{m}_{n-1} \right)\left(-\hat{c}_{n-1} \right) = 0.$$

$$1.8881\, a_1 + 0.661\, b_1 = 2.057$$

$$0.661\, a_1 + 1.46\, b_1 = 1.39996$$

$$a_1 = 0.896$$

$$b_1 = 0.553$$