SWITCHED CAPACITOR FILTER

The essence of switched capacitor is the use of capacitors and analog switches to perform the same function as resistors.

The switches used include two N-channel Metal-Oxide Semiconductor Field-Effect Transistors.

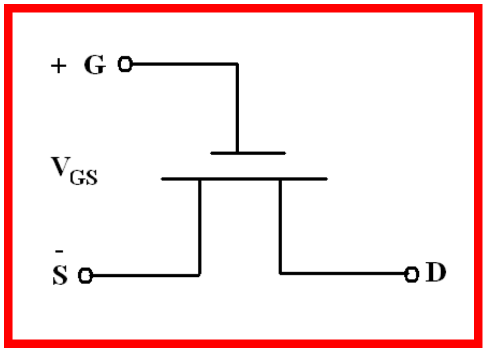
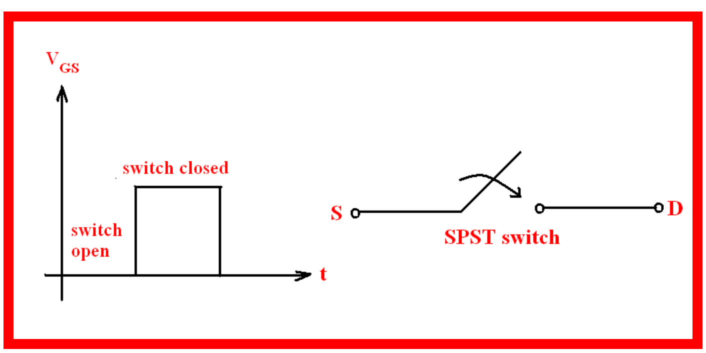


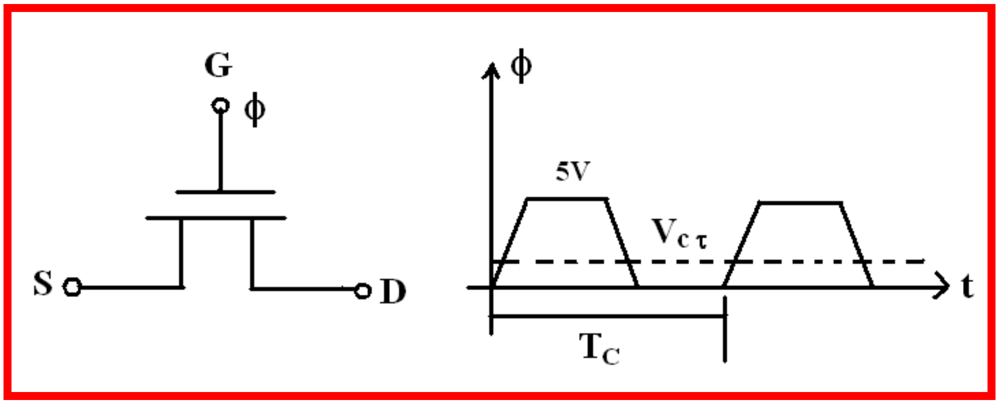
Fig. Simplified model of MOS Switch

The voltage that controls the switching action is V_{GS}, which is voltage between the source and gate. The path of interest is that between S and D, having resistance R_{GS}. The voltage between the source and gate is either zero or a value larger than threshold voltage $V_{c\tau}$ typically of 1or 2V.

Condition	**State**	**Model**
$V_{GS} > V_{c\tau}$	**On**	**Short**
$V_{GS} < V_{c\tau}$	**Off**	**Open**

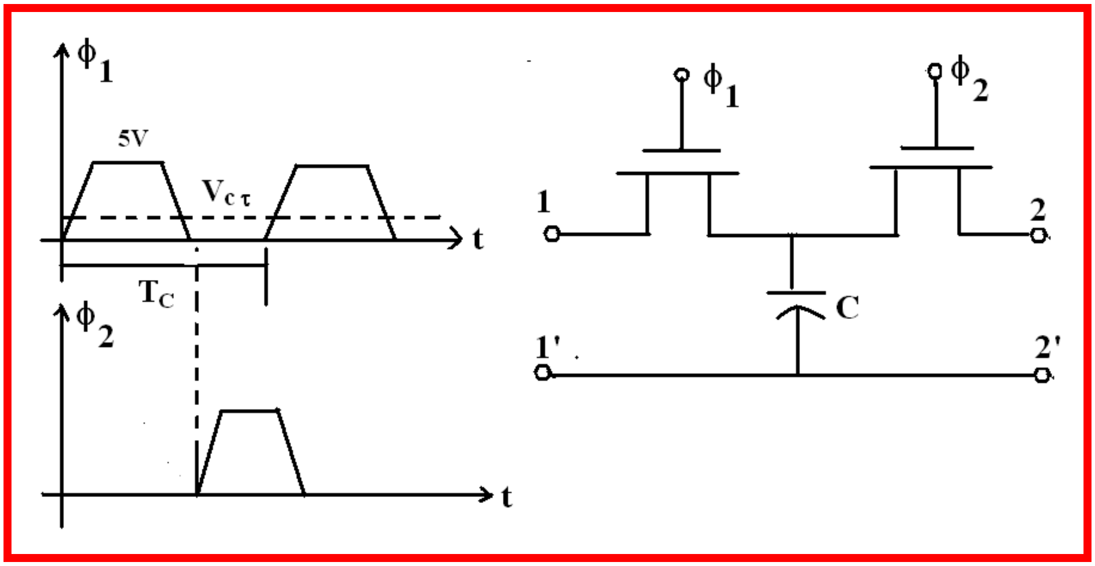


The voltage waveform that is used to activate the MOS switch is shown in the Fig. below

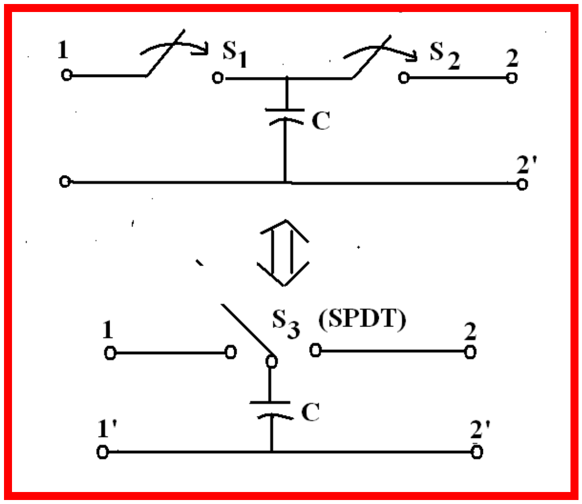


The quantity $f_c = 1 / T_c$ is known as the clock frequency.

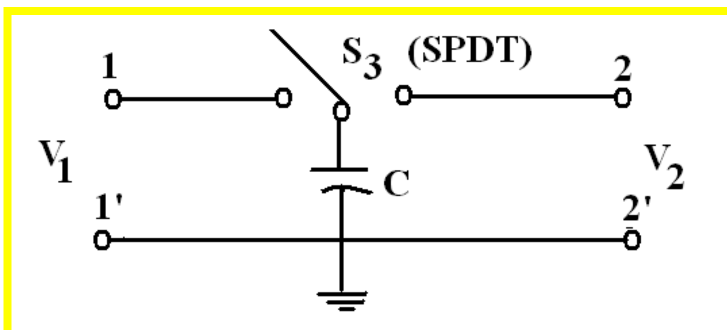
If two MOS switches are connected in series as shown in the Fig. and driven by the two waveforms as shown in the Fig. then there will be no direct connection from 1 to 2.



The above conception gives rise to SPDT (Single pole double throw) switch. The switch is shown below. The SPDT switch is implemented with help of two SPST switches.



SWITCHED CAPACITOR OPERATION



- **When the switch is thrown to the left, the capacitor will charge up to V_1.**
- **When the switch is thrown to the right, the capacitor will discharge down to or charge up to V_2.**
- **As a result of these consecutive switching events, there will be a net charge transfer given by**

$$\Delta Q = C(V_1 - V_2)$$

- If the switch back and forth at a rate of f_{clk} cycles/sec, then the charge transferred in one second transfer is

$$f_{clk}.\Delta Q = f_{clk}.C(V_1 - V_2)$$

The above equation has the units of current,

$$I = f_{clk}.C(V_1 - V_2)$$

If f_{clk} is much higher than the frequency of the voltage waveforms, then the switching process can be taken to be essentially continuous, and the switched capacitor circuit can then be modeled as an equivalent resistance as shown in the Fig. below. The value of the equivalent resistance is given by

$$R_{eq} = \frac{V_1 - V_2}{I} = \frac{1}{Cf_{clk}}$$

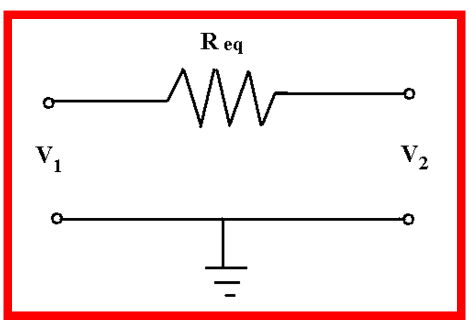


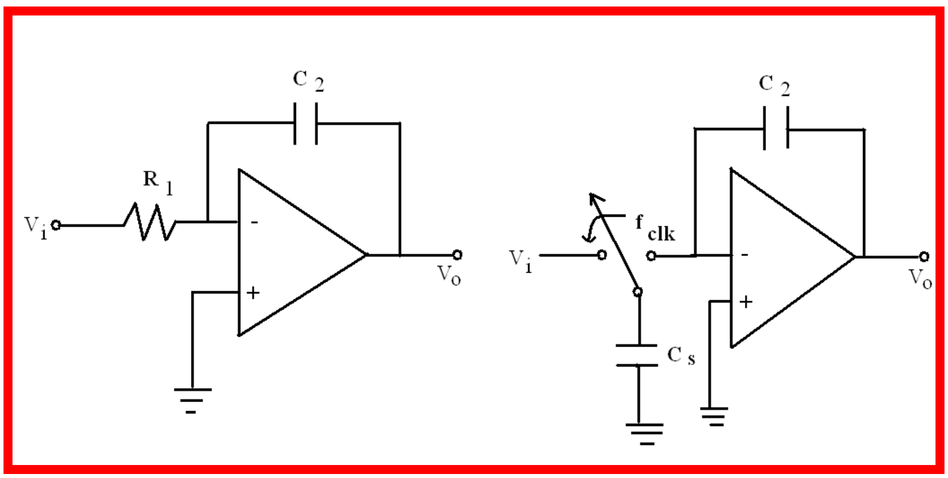
Fig. Equivalent resistor model for switched capacitor circuit

The equivalent resistance has features which make it advantageous when realized in integrated-circuit form:

- ***Resistor takes up excessive silicon die area. With switched capacitor circuit high-value resistors can be implemented in a very little silicon area. For example 1MΩ resistor can be realized with 10pF capacitor at a clock frequency of 100khz.***
- ***Very accurate time constants can be realized because this time constant is depend on ratio of capacitances and inversely proportional to the clock frequency. The ratio of capacitors value can be set accurately and not absolute values (which vary between manufacturing runs).***
- ***The -3dB frequency depends on ratio of capacitances, not on an RC product. The tolerances for ratios are much easier to control than the tolerances for products.***

APPLICATIONS OF SWITCHED CAPACITOR CIRCUITS

Switched Capacitor Integrator



- Input resistor of conventional RC integrator is replaced by the equivalent switched capacitor circuit.
- The transfer function is

$$\frac{V_o(s)}{V_i(s)} = -\frac{1}{R_1 C_2 s} = -\frac{1}{\tau s} = -\frac{f_{clk} C_s}{s C_2}$$

Where τ = integrator time constant = $R_1 C_2$ and

$$R_1 = \frac{1}{f_{clk} C_s}$$

Non-inverting switched capacitor integrator

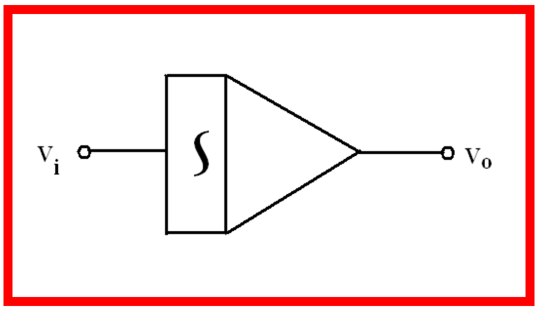


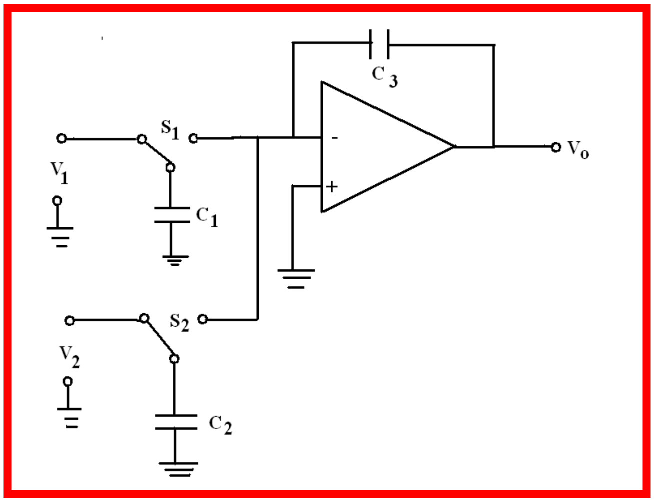
Fig. Symbolic representation of Non-inverting switched capacitor integrator.

The transfer function is:

$$\frac{V_0(s)}{V_i(s)} = \frac{1}{\tau s}$$

NOTE : The non-inverting SC integrators find most important application in the fabrication of switch capacitor filter.

Summing Integrator

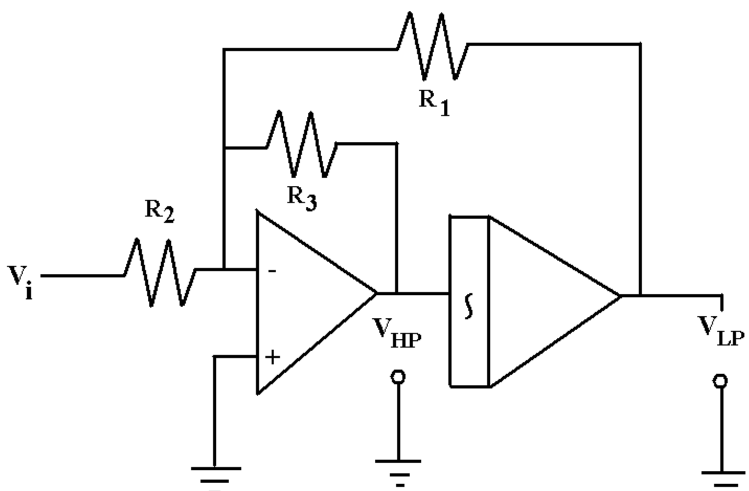


- **The response is given by**

$$V_o(s) = -\frac{V_1(s)}{\tau_1 s} - \frac{V_2(s)}{\tau_2 s}$$

Where $\tau_1 = \frac{C_3}{f_{clk}\,C_1}$ **and** $\tau_2 = \frac{C_3}{f_{clk}\,C_2}$

First order SC Filter



- **This consists of SC integrator preceded by an additional summing amplifier with three supplementary resistors.**

- **The same arrangement simultaneously provides a high pass and low pass filter.**

For Low pass filter

$$V_{LP} = -\frac{V_i}{R_2} R_1 - \frac{V_{HP}}{R_3} R_1$$

$$= -\frac{R_1}{R_2} V_i - \frac{R_1}{R_3} V_{HP}$$

Since $V_{LP} = \frac{1}{\tau s} V_{HP}$

$$V_{LP} = -\frac{R_1}{R_2} V_i - \frac{R_1}{R_3} \tau s V_{LP}$$

$$V_{LP}\left(1 + \tau s \frac{R_1}{R_3}\right) = -\frac{R_1}{R_2} V_i$$

Or

$$\frac{V_{LP}}{V_i} = -\frac{R_1/R_2}{1 + \tau s \frac{R_1}{R_3}}$$

The above equation represents the low-pass filter with cut-off frequency depends upon the switching frequency.

For high-pass filter

$$V_{HP} = -\frac{R_3}{R_2}V_i - \frac{R_3}{R_1}V_{LP}$$

Now $V_{LP} = \frac{1}{\tau s}V_{HP}$

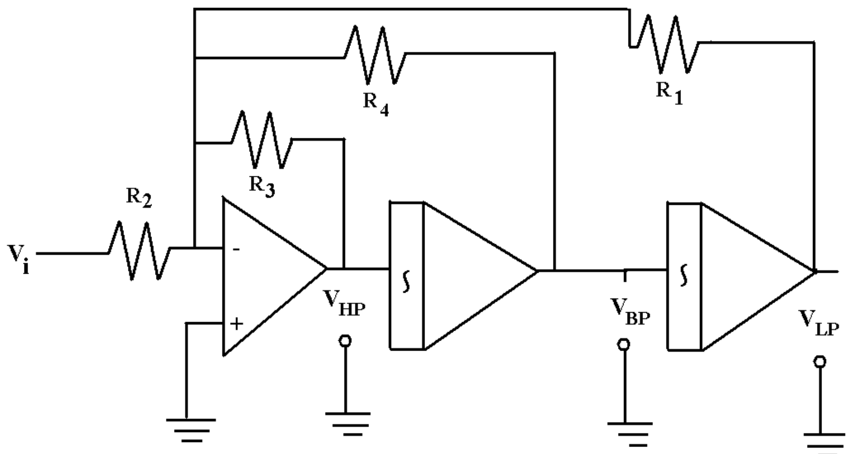
$$V_{HP} = -\frac{R_3}{R_2}V_i - \frac{R_3}{R_1}\frac{1}{\tau s}V_{HP}$$

Or

$$V_{HP}\left(1 + \frac{1}{\tau s}\frac{R_3}{R_1}\right) = \frac{R_3}{R_2}V_i$$

$$\frac{V_{HP}}{V_i} = -\frac{R_3/R_2}{1 + \frac{1}{\tau s}\frac{R_3}{R_1}}$$

Second order switch capacitor Filter



For low-pass Filter

$$V_{LP} = -\frac{R_1}{R_2}V_i - \frac{R_1}{R_4}V_{BP} - \frac{R_1}{R_3}V_{HP}$$

Here

$$V_{BP} = \frac{1}{\tau s}V_{HP}, and\, V_{LP} = \frac{1}{\tau s}V_{BP}$$

$$V_{LP} = --\frac{R_1}{R_2}V_i - \frac{R_1}{R_4}\tau s V_{LP} - \frac{R_1}{R_3}\tau^2 s^2 V_{LP}$$

Or

$$V_{LP}\left(1 + \frac{R_1}{R_4}\tau s + \frac{R_1}{R_3}\tau^2 s^2\right) - -\frac{R_1}{R_2}V_i$$

$$\frac{V_{LP}}{V_i} = -\frac{{R_1}/{R_2}}{1 + \frac{R_1}{R_4}\tau s + \frac{R_1}{R_3}\tau^2 s^2}$$

The above equation represents the second order low-pass filter characteristics.

For High-pass filter

$$V_{HP} = -\frac{R_3}{R_2}V_i - \frac{R_3}{R_4}V_{BP} - \frac{R_3}{R_1}V_{LP}$$

Here

$$V_{BP} - \frac{1}{\tau s}V_{HP}, and\, V_{LP} - \frac{1}{\tau s}V_{BP}$$

$$V_{HP} = -\frac{R_3}{R_2}V_i - \frac{1}{\tau s}\frac{R_3}{R_4}V_{HP} - \frac{1}{\tau^2 s^2}\frac{R_3}{R_1}V_{HP}$$

Or

$$V_{HP}\left(1 + \frac{1}{\tau s}\frac{R_3}{R_4} + \frac{1}{\tau^2 s^2}\frac{R_3}{R_1}\right) = -\frac{R_3}{R_2}V_i$$

$$\frac{V_{HP}}{V_i} = \frac{{R_3}/{R_2}}{1 + \frac{1}{\tau s}\frac{R_3}{R_4} + \frac{1}{\tau^2 s^2}\frac{R_3}{R_1}}$$

For Band-pass filter

$$V_{BP} = -\frac{R_4}{R_2}V_i - \frac{R_4}{R_3}V_{HP} - \frac{R_4}{R_1}V_{LP}$$

Since

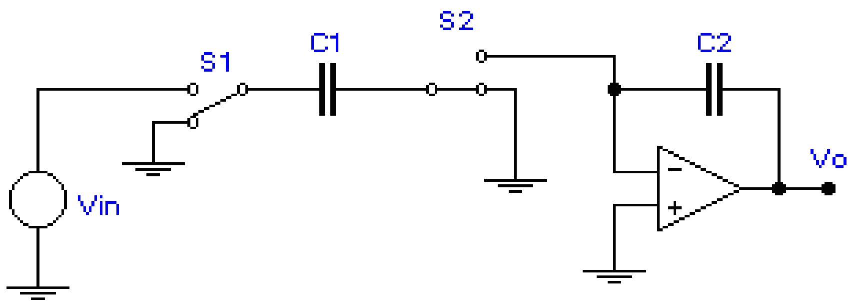
$$V_{BP} = \frac{1}{\tau s} V_{HP}, \; and \; V_{LP} = \frac{1}{\tau s} V_{BP}$$

$$V_{BP} = -\frac{R_4}{R_2} V_i - \frac{R_4}{R_3} \tau s V_{BP} - \frac{R_4}{R_1} \frac{1}{\tau s} V_{BP}$$

Or

$$\frac{V_{BP}}{V_i} = -\frac{R_4 / R_2}{1 + \frac{1}{\tau s} \frac{R_4}{R_1} + \tau s \frac{R_4}{R_3}}$$

How to eliminate the effect of stray capacitance



How to make a Non-inverting integrator