Electrical Measurements

Strain Gages

The *strain gauges* is basically a device used for measuring mechanical surface strain and is one of the most extensively used electrical transducers. Its popularity stems from the fact that it can detect and convert force or *small mechanical displacements* into electrical signals. Many other quantities such as torque, pressure, weight and tension etc, which involve effects of force or displacement, can also be measured by strain gauges. Furthermore, if the mechanical displacements under measurement have a time-varying form, such as vibrational motion, signals with frequencies of up to 100 kHz can be detected or measured.

The applications of strain gauges may be broadly classified into two areas (i) applications where the gauges measure strain as the primary objective of measurement as in the case of stress analysis of machines and structures and (ii) applications where measurement of strain is utilized in transducers as a measure of another force associated variable. Strain can be measured by using various techniques such as electrical, mechanical and optical ones. So strain gauges, on the basis of operating technique employed may be classified into mechanical, optical or electrical strain gauges.

Mechanical gauges are used for strain measurement only and also in cases where the point of measurement is accessible for visual operation. Optical strain gauges are very similar to mechanical strain gauges except that the magnification is achieved with multiple reflectors using mirrors or prisms and measurement accuracy is high and independent of temperature variations. The *electrical strain gauge* is based upon the measurement of change in resistance, capacitance or inductances that are proportional to the strain transferred from the specimen to the basic gauge element. The *electrical resistance strain gauges* over the years has become most extensively used device and this is what is usually meant when the term *strain gauges* is used. The measurement of resistance change is the method most widely used commercially for determination of strain because it is easy to apply to a wide variety of surfaces and because it is reliable, accurate and easily interfaced with electronics. For measurement of strain most commercially available strain gauges are attached directly to the surface under test or measurement such as a structural member or an element of a transducer.

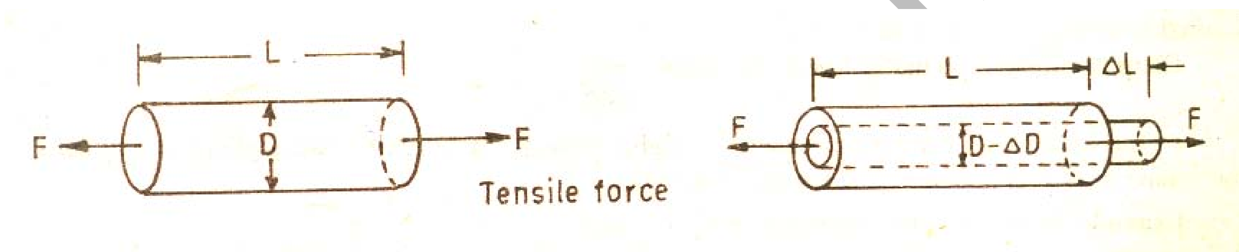
The resistance of wire changes as a function of strain increasing with tension and reducing with compression. Sensitivity varies from material to material. This change in resistance can be measured accurately with Wheatstone bridge. Developed on this principle the electrical resistance strain gauges is essentially metal wire or foil subjected to the same strain as that of the specimen being tested achieved through suitable bonding of the gauge to the specimen. Another class of strain gauge which is of recent origin is the semiconductor type *piezo-resistive* *strain gauge.* Such a gauge has the advantage of high sensitivity small size and adaptability for both static and dynamic measurements.

Operating Principle of Resistance Strain Gages

Let us consider a strain gauge made of circular wire. The wire has the dimensions: length = L, area = A, diameter = D before being strained. The resistivity of the material is ρ.

Resistance of the unstrained gauge, $R = \rho \frac{L}{A}$

Let a tensile stress s be applied to the wire. This produces a positive strain causing the length to increase and area to decrease. Thus when the wire is strained there are changes in its dimensions. Let ΔL = change in length, ΔA = change in area, ΔD = change in diameter and ΔR = change in resistance.



In order to find how ΔR depends upon the material physical quantities, the expression for R is differentiated with respect to stress. Thus we get:

$$\frac{\partial R}{\partial s} = \frac{\rho}{A}\frac{\partial L}{\partial s} - \frac{\rho L}{A^2}\frac{\partial A}{\partial s} + \frac{L}{A}\frac{\partial \rho}{\partial s} \text{……………………(1)}$$

Dividing the above equation by resistance $R = \rho \frac{L}{A}$, we have

$$\frac{1}{R}\frac{\partial R}{\partial s} = \frac{1}{L}\frac{\partial L}{\partial s} - \frac{1}{A}\frac{\partial A}{\partial s} + \frac{1}{\rho}\frac{\partial \rho}{\partial s} \text{……………………..(2)}$$

It is evident from equation (2) that per unit change in resistance is due to:

(i) Per unit change in length = $\frac{\Delta L}{L}$

(ii) Per unit change in area = $\frac{\Delta A}{A}$

(iii) Per unit change in resistivity = $\frac{\Delta \rho}{\rho}$

Area $\qquad A = \frac{\pi}{4} D^2$

$$\therefore \qquad \frac{\partial A}{\partial s} = 2.\frac{\pi}{4} D.\frac{\partial D}{\partial s} \text{…………………(3)}$$

or, $\frac{1}{A}\frac{\partial A}{\partial s} = \frac{\left(\frac{2\pi}{4}\right)D}{\left(\frac{\pi}{4}\right)D^2}\frac{\partial D}{\partial s} = \frac{2}{D}\frac{\partial D}{\partial s}$ ....(4)

∴ Equations (2) can be written as:

$$\frac{1}{R}\frac{\partial R}{\partial s} = \frac{1}{L}\frac{\partial L}{\partial s} - \frac{2}{D}\frac{\partial D}{\partial s} + \frac{1}{\rho}\frac{\partial \rho}{\partial s} \quad ....(5)$$

Now, Poisson's ratio $v = \frac{lateral\ strain}{longitudinal\ strain} = -\frac{\frac{\partial D}{D}}{\frac{\partial L}{L}}$ ....(6)

or, $\frac{\partial D}{D} = -v\frac{\partial L}{L}$

$$\frac{1}{R}\frac{\partial R}{\partial s} = \frac{1}{L}\frac{\partial L}{\partial s} + v.\frac{2}{L}\frac{\partial L}{\partial s} + \frac{1}{\rho}\frac{\partial \rho}{\partial s} \quad ....(7)$$

For small variations, the above relationship can be written as:

$$\frac{\Delta R}{R} = \frac{\Delta L}{L} + 2v\frac{\Delta L}{L} + \frac{\Delta \rho}{\rho} \quad ....(8)$$

The gauge factor is defined as the ratio of per unit change in resistance to per unit change in length.

Gauge factor $F = \frac{\frac{\Delta R}{R}}{\frac{\Delta L}{L}}$ ....(9)

or, $\frac{\Delta R}{R} = F\frac{\Delta L}{L} = F \times \epsilon$ ....(10)

Where ϵ = strain = $\frac{\Delta L}{L}$

The gauge factor can be written as: = 1 + 2v + $\frac{\Delta \rho}{\rho}/\epsilon$ ....(11)

Therefore the gauge factor are summation of Resistance change due to change in length, Resistance change due to change in area and Resistance change due to piezo-resistive effect.

$$F = \frac{\frac{\Delta R}{R}}{\frac{\Delta L}{L}} = 1 + 2v + \frac{\frac{\Delta \rho}{\rho}}{\frac{\Delta L}{L}}$$

The strain is usually expressed in terms of microstrain. I microstrain = 1 µm/m.

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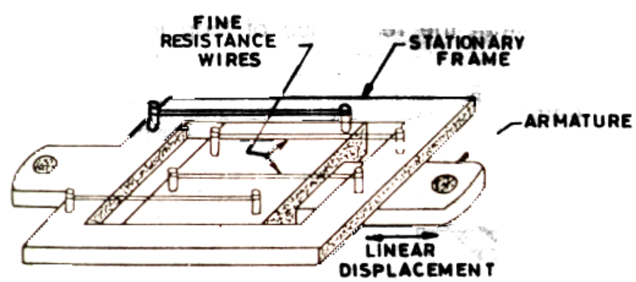
Basic principle of resistance strain gauge is implemented in several different ways:

a. Unbonded metal wire strain gauges
b. Bonded metal wire strain gauges
c. Bonded metal foil strain gauges
d. Vacuum deposited thin metal strain gauges
e. Sputter deposited thin metal strain gauges
f. Bonded semiconductor strain gauges
g. Diffused metal strain gauges.

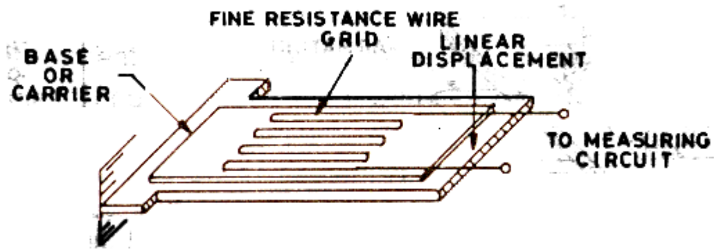
a. <u>Unbonded metal wire strain gauge:</u>

As the name depicted, in such gauges, resistance wire is not bonded on a base, instead of that strain is directly transferred to the resistance wire and so a smaller force is required for changing the wire's length. In such a strain gauge the resistance wires of about 25 micron diameters are stretched between a stationary frame and an armature that is supported in the centre of the frame. Since the resistance wires would buckle under compressive forces, an internal preload, greater than expected external compressive load, is used. Applying an external force increases tension in two wires and reduces it in other two wires causing corresponding variation in resistance of wires which is proportional to the displacement. The actual displacement that can be measured is very small of the order of 50 micron. The length of the wire is 25 mm or less. The transducer will measure forces from ± 40 g to ± 2 kg full scale. The frequency response of the unbonded strain gauge normally depends upon on the mass of the moving parts and practical gauges have resonant frequencies of a few hundred Hz.



These devices are used mainly for measurement of force, pressure and acceleration rather than for measuring directly displacement or strain.

b. Bonded metal wire strain gauges:



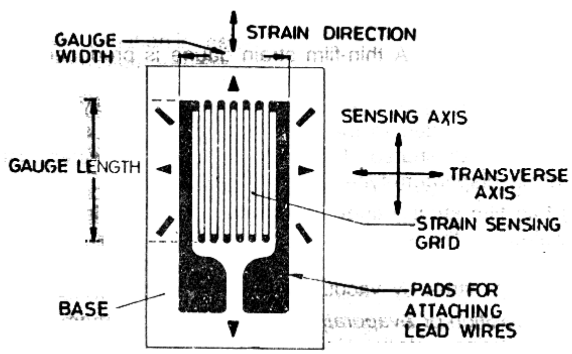
In flat grid type bonded wire strain gauge fine resistance wire of diameter of 25 micron or less is wound back and forth in a grid format with as many loops as possible, laid side by side. This grid is cemented to base or carrier which may be a thin sheet of paper or bakelite or a sheet of teflon. The wire is covered on top with a thin sheet of material in order to protect it from any mechanical damage. The spreading of wire allows a uniform distribution of stress over the grid. A large length to width ratio in the grid structure is also desirable in order to keep the transverse sensitivity minimum. Two connecting leads are soldered or welded to the ends of the grid. The bonded strain gauge is cemented with a special adhesive to the structure whose tensile or compressive strain is to be measured. For some applications, the adhesive used must also be resistant to humidity, temperature and other extreme environmental conditions. Surface area of wire section is kept large than its cross-sectional area in order to avoid stress relaxation and slippage. The wire grid plane is kept as close to the specimen surface as possible for achieving maximum transfer of strain from the specimen and keeping the creep and hysteresis minimum. Because of intimate contact between the wire grid and the strain surface of the specimen and because of practically no strength of wire of its own for resisting any elongation or compression, the wire elongates or compresses exactly the same distance as the specimen. The strain of wire grid is, therefore, exactly the same as the strain of the specimen. The strain of wire grid is measured with the Wheatstone bridge connecting the gauge in one of the four arms while the remaining three arms have standard resistances of nearly equal resistance as that of gauge resistance in the unstrained condition.

c. Bonded metal foil strain gauges:

This class of strain gauges is only an extension of the bonded metal wire strain gauges. The bonded metal wire strain gauges have been completely superseded by bonded metal foil strain gauges. Metal foil strain gauges use identical or similar materials to wire strain gauges and are used today for most general purpose stress analysis applications and for many transducers.

Foil type gauges have a much greater heat dissipation capacity as compared with wire wound strain gauges on account of their surface area for the same volume. For this reason, they can be used for higher operating temperature range. Also the large surface area leads to better bonding.

The sensing elements of foil gauges are formed from sheets less than 0.005 mm thick by photo-etching process, which allow greater flexibility with regard shape. The linear grid gauges are designed with fat end turns. *This local increase in area reduces the transverse sensitivity which is a spurious input since the gauge is designed to measure the strain component along the length of grid elements.*



Foil type of gauges are employed for both stress analysis and as well as construction of transducers. Foil type of gauges are mounted on a flexible insulating *carrier film* about 0.025 mm thick which is made of polymide, glass phenolic etc. typical gauge resistances are 120, 350 and 1000Ω with the allowable gauge current of 5 to 40mA which is determined by the heat dissipation capabilities of the gauge. The gauge factors typically range from 2 to 4.

Minimum practical gauge size is constrained by the manufacturing limitations and handling attachment problems. The smallest gauge sizes are about 0.38 mm long. Foil type of gauges can be applied to curved surfaces; the minimum safe bend radius can be as small as 1.5 mm in some strain gauges.

The maximum measurable strain varies from 0.5 to 4 %. However, special *post yield* gauge devices allow measurement up to 0.1 fatigue life of gauges varies conditions. However 10 million cycles at ±1500 micro strain can be applied to foil gauges without causing failure.

d. <u>Vacuum deposited thin metal strain gauges:</u>

A thin film strain gauge is produced by depositing a thin layer of metal alloy on an elastic metal specimen by means of vacuum deposition or sputtering process. This technique, relatively new and extensively used, produces a strain gauge that is molecularly bonded to the specimen under test, so the draw-backs of the epoxy adhesive bond are eliminated. This technique is most widely used for transducer applications such as in diaphragm type pressure gauges.

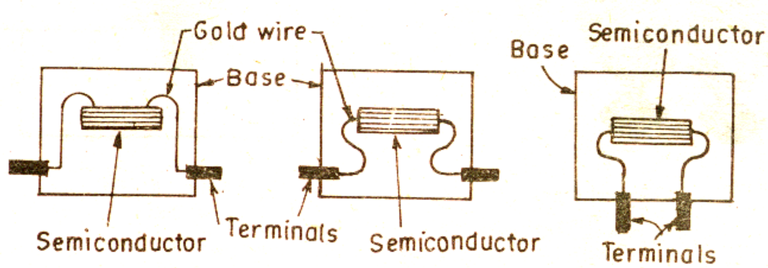
For producing thin film strain gauge transducers, first an electrical insulation such as a ceramic is deposited on the stressed elastic metal member such as a diaphragm or beam and then strain gauge alloy is deposited on the top of the insulation layer. Both layers may be deposited either by vacuum deposition or by sputtering process.

In the vacuum deposition or evaporation process the diaphragm is placed in a vacuum chamber with some insulating material. Heat is applied until the insulating material vaporizes and then condenses, forming a thin dielectric film on the diaphragm. Then suitably shaped templates are placed over the diaphragm and the evaporation-condensation process is repeated with the metallic gauge material, forming the desired gauge pattern on top of the insulating substrate.

In the sputtering process, a thin dielectric layer is again deposited in vacuum over the entire diaphragm surface; however, this detailed mechanism of deposition is quite different from the evaporation process. In the process the gauge or insulating material is held at negative potential and the target (transducer diaphragm or beam) is held at a positive potential. Molecules of gauge or insulating material are ejected from the negative electrode owing to the impact of positive gas ions (argon) bombarding the surface. The ejected molecules are accelerated toward the transducer diaphragm or beam and strike the target area with kinetic energy of magnitude several times greater than that possible with any other deposition process. This produces superior adherence to the specimen under test.

e. Bonded semiconductor strain gauges:

Semiconductor strain gauges are used a very high gauge factor and small envelopes are required. The resistance of the semiconductors changes in applied strain. Unlike in the case of metallic gauges where the change in resistance is mainly due to change in dimensions when strained, the semiconductor strain gauge depend for their action upon **piezo-resistive effect** i.e. the change in the value of the resistance due to change in resistivity.



Semiconducting materials such as silicon and germanium are used as resistive materials for semiconductor strain gauges. A typical strain gauge consists of a strain sensitive crystal material and leads that are sandwiched in a protective matrix. The production of these gauges employs conventional semiconductor technology using semiconducting wafers or filaments which have a thickness of 0.005 mm and bonding them on suitable insulating substrates, such as Teflon. Gold leads are generally employed for making the contacts.

f. Diffused metal strain gauges:

The diffused strain gauges are primarily used in transducers. The diffusion process used in IC manufacture is employed. In presence transducers, for example, the diaphragm would be silicon rather than metal and the strain gauge effect would be realized by depositing impurities in the diaphragm to form an intrinsic strain gauge. This type of construction may allow lower manufacturing costs in some designs as a large number of diaphragms can be made on a single silicon wafer.

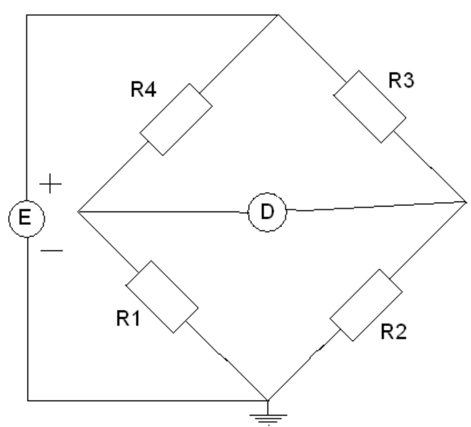
Table: Characteristics of some resistance strain gauge materials:

Material	**Trade name**	**Approx Gauge factor**	**Approx resistivity at 20^{0}C μΩ-cm**	**Temp Co-efficient of resistance $^0C^{-1}X10^6$**	**Remarks**
55% Cu, 45% Ni	Advance Constant Copel	2.0	49	11	Constant over wide range of strain; low temp use below 360^0 C
4% Ni, 12% Mn, 84% Cu	Manganin	0.47	44	20	Constant over wide range of strain; low temp use below 360^0 C
80% Ni, 20% Cu	Nichrome V	2.0	108	400	Suitable for high temp use up to 800^0 C
36% Ni, 8% Cr, 0.5% Mo, 55.5% Fe	Isoelastic	3.5	110	450	Used for low temp up to 300^0 C
67% Ni, 33% Cu,	Monel	1.9	40	1900	Useful up to 750^0 C
74% Ni, 20% Cr, 3% Al, 3% Fe	Karma	2.4	125	20	Useful up to 750^0 C
95% Pt, 5% Ir	-	5.0	24	1250	Useful up to 1000^0 C
Silicon semiconductor	-	-100 to +150	10^9	90,000	Brittle but has high gauge factor, not suitable for large strain measurements.

Most commercial gauges employ constantan or isoelastic for materials of construction.

Most [wire] strain gauges employ either a mitrocellulose cement or a phenolic resin for the bonding agent with a thin paper backing to maintain the wire configuration. Such gauges may be used up to 150^0 C. A Bakelite mounting is usually employed for temperatures up to 260^0 C. Foil gauges are manufactured by an etching process similar to that used with printed circuit boards and use base materials of paper, Bakelite and epoxy film. (Epoxy cements are also employed for both wire and foil gauges.)

Measurement of resistance strain-gauges outputs



The Detector voltage $E_D = E\left(\frac{R_1}{R_1+R_4} - \frac{R_2}{R_2+R_3}\right)$ …………………(1)

At balance, $E_D = 0$

Let R_1 be the strain gauge and a high impedance readout device is employed so that the bridge operates as a voltage sensitive deflection circuit.

Let the bridge be balanced at zero strain conditions and that a strain on the gauge of ϵ results in a change in resistance ΔR_1.

We immediately obtain a voltage due to the strain as

$$\Delta E_D = E\left(\frac{R_1+\Delta R_1}{R_1+\Delta R_1+R_4} - \frac{R_2}{R_2+R_3}\right) \text{ ………………..(2)}$$

If all the bridge arm resistances are equal we get

$$R_1 = R_2 = R_3 = R_4$$

Then we get,

$$\frac{\Delta E_D}{E} = \frac{1+\frac{\Delta R_1}{R_1}}{2+\frac{\Delta R_1}{R_1}} - \frac{1}{2} = \frac{\frac{\Delta R_1}{R_1}}{4+2\left(\frac{\Delta R_1}{R_1}\right)} = \frac{1}{4} \times \left(\frac{\Delta R_1}{R_1}\right) \times \frac{1}{\left(1+\frac{\Delta R_1}{2R_1}\right)} \cong \frac{1}{4} \times \left(\frac{\Delta R_1}{R_1}\right) \times \left(1 - \frac{\Delta R_1}{2R_1}\right)$$

$$\frac{\Delta E_D}{E} \cong \frac{\Delta R_1}{4R_1} \text{ ....(3)}$$

Equation (3) expresses the resistance changes as a function of the voltage unbalance at the detector ΔE_D.

The bridge circuit may also be used as a current sensitive device.

$$i_g = \frac{E\left[\frac{R_1}{R_1+R_4} - \frac{R_2}{R_2+R_3}\right]}{R_g + \left[\frac{R_1 R_4}{R_1+R_4} - \frac{R_2 R_3}{R_2+R_3}\right]} = \frac{E(R_1R_3 - R_2R_4)}{R_1R_2R_4 + R_1R_3R_4 + R_1R_2R_3 + R_2R_3R_4 + R_g(R_1+R_4)(R_2+R_3)} \text{ ....(4)}$$

At balance $I_g = 0$,

$\therefore$ $R_1R_3 = R_2R_4$ ....(5)

Let the galvanometer current ΔI_g results from a change in resistance ΔR_1. Denominator of equation (4) is not very sensitive to small changes in R_1 and hence very nearly a constant (C say)

Thus,

$$\Delta I_g = \frac{E}{C}[(R_1 + \Delta R_1)R_3 - R_2R_4] \text{ ....(6)}$$

From (5) & (6) we get

$$\Delta I_g = \frac{E}{C}\Delta R_1 R_3 \text{ ....(7)}$$

$$= \frac{E}{C}R_1R_3.\frac{\Delta R_1}{R_1} = F\epsilon = Constant \times \epsilon \text{ ....(9)}$$

Thus the, deflection current may be taken as a direct indication of the strain imposed on the gauge.

Example 1

A resistance strain gauge with R = 12Ω and F = 2.0 is placed in a equal arm bridge. The power voltage is 4.0 V. Galvanometer resistance is 100Ω. Calculate the detector current in μA/μinch of strain.

Solution:

$$\Delta I_g = \frac{E}{C}R_1R_3.F\epsilon$$

Where C = $R_1R_2R_4 + R_1R_3R_4 + R_1R_2R_3 + R_2R_3R_4 + R_g(R_1 + R_4)(R_2 + R_3)$

We have, $R_1 = R_3 = R_2 = R_4 = 100\Omega$

$C = 4 \times (120)^3 + (100)(240)^2 = 1.267 \times 10^7$

Current sensitivity

$\frac{\Delta I_g}{\epsilon} = \frac{(4.0)(120)^2(2.0)}{1.267\times10^7} = 9.08 \times 10^{-3}\,\frac{A}{in} = 9.08 \times 10^{-3}\ \mu A/\mu inch$

Example 2

For the gauge and bridge above, calculate the voltage indication for a strain of 1.0 μ inch/m, when a high impedance detector is used.

Solution:

We have, $R_1 = R_3 = R_2 = R_4 = 100\Omega$

Under these conditions

$\frac{\Delta E_D}{E} \cong \frac{\Delta R_1}{4R_1} = \frac{1}{4} \times F \times \epsilon = \frac{1}{4} \times 2.0 \times 1 \times 10^{-6} = 0.5 \times 10^{-6}$

$\Delta E_D = (4.0) \times (0.5 \times 10^{-6}) = 2.0\ \mu V$

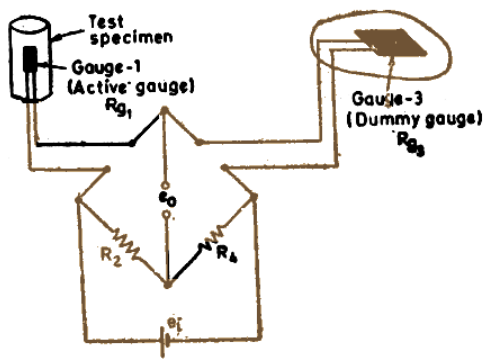
Error sources and compensation:

In practical applications, strain measurements are carried out over long periods of time and so the gauges are subjected to variation in ambient temperature. Temperature variations cause the major errors in the measurement of strains by strain gauges. Temperature changes cause the change in resistance in two ways. The resistance of the wire grid of strain gauge changes with the change in temperature. Another aspect of temperature effect is found in the possible differential expansion of the gauge and the underlying material. This can cause a change in strain and resistance in the gauge even through material is not subject to an external load. These temperature effects can be compensated in various ways such as by using special inherently temperature-compensated gauges. Such gauges are designed to be employed on a specific material and have expansion and resistance properties such that the two effects very nearly cancel each other. This method can only be used for limited ranges of temperature and strain levels. Another approach to this problem is by proper placement of strain gauges in the bridge circuits.

Temperature compensation through bridge arrangement:

a. Use of Dummy gauge:

One of the ways in which temperature error can be eliminated by using adjacent arm compensating gauge is to use a **dummy gauge** in the adjacent arm. In this arrangement gauge 1 is installed on the test specimen (active gauge) while gauge 3 (dummy gauge) is installed on a like piece of material and is not subjected to any strain. The gauges installed on the test piece and the dummy gauges are at the same temperature. A gauge is called **dummy gauge** is case it is not subjected to any strain. **Active gauge** is one which is subjected to strain.



Initially when the bridge is balanced,

$$\frac{R_1}{R_3} = \frac{R_2}{R_4}$$

Suppose a change in temperature occurs, the resistance R_1 and R_3 change by an amount ΔR_1 and ΔR_3 respectively.

Hence for balance,

$$\frac{R_1+\Delta R_1}{R_3+\Delta R_3} = \frac{R_2}{R_4}$$

or, $\frac{R_4}{R_2}(R_1 + \Delta R_1) = (R_3 + \Delta R_3)$

or, $\frac{R_4}{R_2}R_1 + \frac{R_4}{R_2}\Delta R_1 = (R_3 + \Delta R_3)$

but, $\frac{R_4}{R_2}R_1 = R_3$

$\therefore \frac{R_4}{R_2}\Delta R_1 = \Delta R_3$

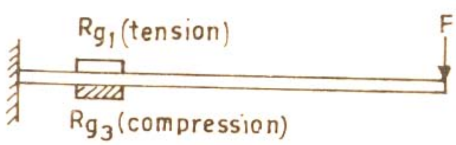
Suppose $R_4 = R_2$

This requires that $\Delta R_1 = \Delta R_3$

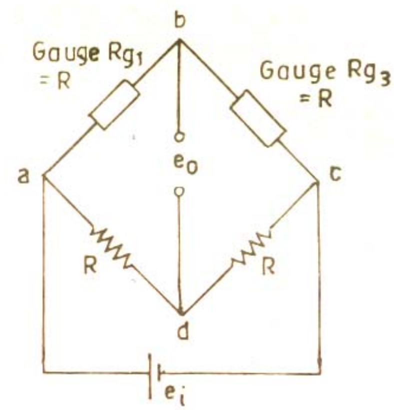
It means that for the bridge to remain insensitive to variations in temperature the gauges R_1 and R_3 should have their resistances change by equal amount when subjected to variation in temperature. Therefore the active gauge R_1 and the dummy gauge R_3 should be identical.

b. <u>Use of two active gauges in adjacent arms:</u>

In certain applications, where equal and opposite strains are known to exist, it is possible to attach two similar gauges in such a way that one gauge experiences a positive strain and the other a negative strain. Thus instead of having an arrangement wherein one gauge acts as the active gauge and the other as the dummy gauge, prepare a arrangement wherein both the gauges are active gauges.



Two gauges used for measurement of strain.



Adjacent arm compensation using two active gauges.

In the above figure shows the two gauges mounted on a cantilever. The gauge R_{g1} is on top of the cantilever and hence experiences tension or a positive strain. The R_{g3} is at bottom surface of the cantilever and hence experiences a compression or a negative strain. There are two active gauges in the four arm bridge and hence it is called Half Bridge.

The temperature effects are cancelled out by having $R_2 = R_4$ and using two identical gauges in the opposite arms of the bridge.

Suppose, $R_{g1} = R_{g3} = R_2 = R_4 = R$

When no strain is applied both point's b and d are at the same potential $e_i/2$ and the value of the output voltage $e_0 = 0$

When the arrangement is subjected to strain, the resistance of gauge 1 increases and that of gauges R_{g3} decreases.

Resistance of gauge R_{g1} when strained is $R(1 + \Delta R/R)$

Resistance of gauge R_{g3} when strained is $R(1 - \Delta R/R)$

Now $R_2 = R_4 = R$

$\therefore$ Potential of point d is $e_i/2$

∴ Potential of point b is $\frac{R\left(1+\frac{\Delta R}{R}\right)}{R\left(1+\frac{\Delta R}{R}\right)+R\left(1-\frac{\Delta R}{R}\right)} \times e_i = \frac{\left(1+\frac{\Delta R}{R}\right)}{2} e_i$

∴ Change in output voltage when strain applied is,

$$\Delta e_0 = \frac{1+\frac{\Delta R}{R}}{2} e_i - \frac{e_i}{2} = \frac{\frac{\Delta R}{R}}{2} e_i = \frac{F\epsilon}{2} e_i$$

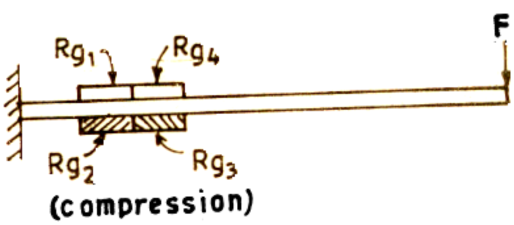
Thus the output voltage from a half bridge is twice that from a quarter bridge and therefore the sensitivity is doubled. In addition the temperature effects are cancelled. The gauge sensitivity of a half bridge is, $S_g = 2KR_gF$.

c. Use of four active gauges:

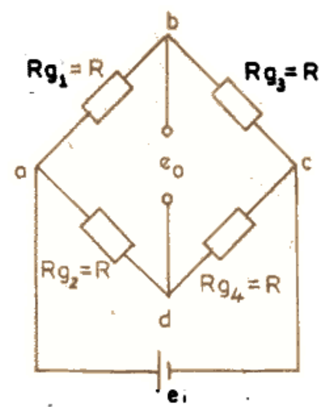
In this type of bridge shows a cantilever using four strain gauges for the measurement of strain. All the four gauges are similar and have equal resistance when strained i.e.

$$R_{g1} = R_{g2} = R_{g3} = R_{g4} = R$$

Since the bridge has four active gauges with one gauge in each of the four arms, it is called a full bridge.



Use of four strain gauges for measurement of strain.



Bridge circuit for measurement of strain four using active gauges.

When no strain is applied the potential of points b and d are both equal to $e_i/2$ and hence the output voltage $e_0 = 0$.

When strained, the resistances of various gauges are:

For R_{g1} and R_{g4} : $R(1 + \Delta R/R)$ and for R_{g2} and R_{g3} : $R(1 - \Delta R/R)$

Potential of b when strain is applied = $\frac{R\left(1+\frac{\Delta R}{R}\right)}{R\left(1+\frac{\Delta R}{R}\right)+R\left(1-\frac{\Delta R}{R}\right)} \times e_i = \frac{\left(1+\frac{\Delta R}{R}\right)}{2} e_i$

Potential of d when strain is applied = $\frac{R\left(1-\frac{\Delta R}{R}\right)}{R\left(1-\frac{\Delta R}{R}\right)+R\left(1+\frac{\Delta R}{R}\right)} \times e_i = \frac{\left(1-\frac{\Delta R}{R}\right)}{2} e_i$

Therefore change in output voltage $\Delta e_0 = \frac{1+\frac{\Delta R}{R}}{2} e_i - \frac{1-\frac{\Delta R}{R}}{2} e_i = \frac{\Delta R}{R} = F\epsilon e_i$

Four active arm bridges are extensively used when strain gauges are used as secondary transducers to give maximum sensitivity combined with full temperature compensation.

Gautam Sarkar, Electrical Engineering Department, Jadavpur University

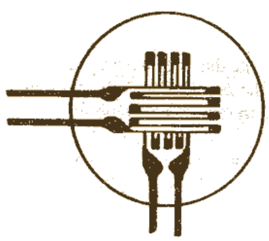
This effect of increasing the number of active gauges is the same if a low impedance detector is used.

The gauge sensitivity of a full bridge $S_g = 4KR_gF$

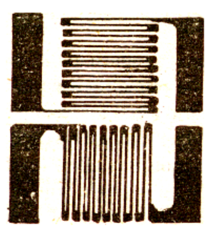
Strain gauge Rosettes

Simultaneous measurement of strains in more than one direction can be accomplished by placing single element gauges at the proper locations. However to simplify this task and provide greater accuracy, multi-element a *rosette* gauges are available.

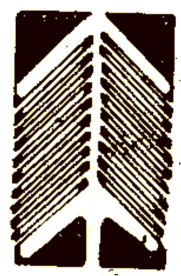
Two element rosettes, shown below, are often used in force transducers. [The gauges are wired in a Wheatstone bridge circuit to provide maximum output. For stress analysis, the axial and transverse elements may have different resistances that can be so selected that the combined output is proportional to stress while the output of the axial element alone is proportional to strain.]



90° Planer (foil)

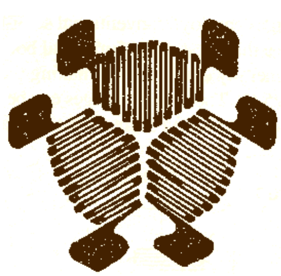


90° Planar (foil)

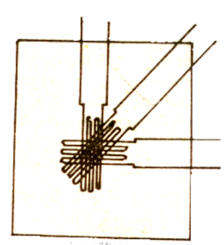


45° Planar (foil)

Three- element rosettes are often used to determine the direction and magnitude of principal strains resulting from complex-structural loading. The most popular types have 45^0 or 60^0 angular displacements between the sensing elements. 45^0 rosettes provide greater angular resolutions but 60^0 rosettes are used when the direction of principal strains is unknown.



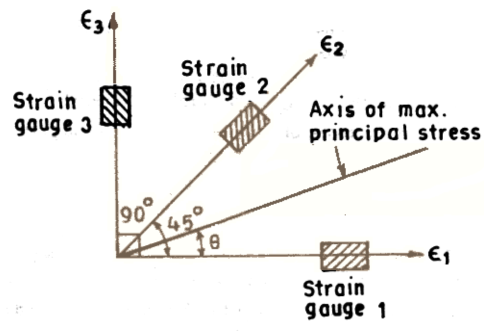
60° Planer (foil)



45° Planer (wire)

<u>Some calculations with rosettes</u>

a. <u>For Rectangular rosettes:</u>



Rectangular Strain Gauge Rosette.

3 strain gauges are oriented as shown (Rectangular Rosettes)

3 strains measured are ϵ_1, ϵ_2, ϵ_3

The principal strains for these situations are

$$\epsilon_{max}\,,\epsilon_{min} = \frac{\epsilon_1+\epsilon_2}{2} \pm \frac{1}{\sqrt{2}}\sqrt{(\epsilon_1-\epsilon_2)^2+(\epsilon_2-\epsilon_3)^2}$$

The principal stresses are

$$б_{max}\,,б_{min} = \frac{E(\epsilon_1+\epsilon_3)}{2(1-\mu)} \pm \frac{E}{\sqrt{2}(1+\mu)}\sqrt{(\epsilon_1-\epsilon_2)^2+(\epsilon_2-\epsilon_3)^2}$$

The maximum shear stress

$$\tau_{max} = \frac{E}{\sqrt{2}(1+\mu)}\sqrt{(\epsilon_1-\epsilon_2)^2+(\epsilon_2-\epsilon_3)^2}$$

The principal stress axis is located at angle θ,

$$\tan 2\theta = \frac{2\epsilon_2-\epsilon_1-\epsilon_3}{\epsilon_1-\epsilon_3}$$

This is the axis at which the maximum stress $б_{max}$ occurs.

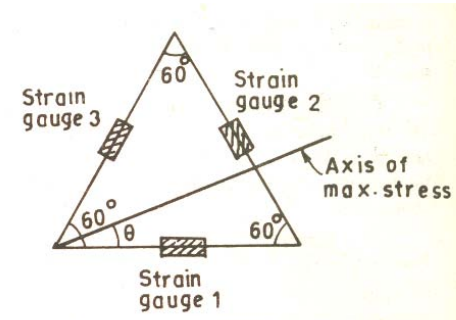
Angle θ lies in the first quadrant

if $\epsilon_2 > \frac{\epsilon_1+\epsilon_2}{2}$

and in the second quadrant

if $\epsilon_2 < \frac{\epsilon_1+\epsilon_2}{2}$

b. For Delta Rosettes:



Delta Strain gauge rosette.

The principal strains for these situations are

$\epsilon_{max}, \epsilon_{min} = \frac{\epsilon_1+\epsilon_2+\epsilon_3}{2} \pm \frac{\sqrt{2}}{3}\sqrt{(\epsilon_1-\epsilon_2)^2+(\epsilon_2-\epsilon_3)^2+(\epsilon_3-\epsilon_1)^2}$

The principal stresses are

$б_{max}, б_{min} = \frac{E(\epsilon_1+\epsilon_2+\epsilon_3)}{3(1-\mu)} \pm \frac{\sqrt{2}E}{3(1+\mu)}\sqrt{(\epsilon_1-\epsilon_2)^2+(\epsilon_2-\epsilon_3)^2+(\epsilon_3-\epsilon_1)^2}$

Again, $\tau_{max} = \frac{\sqrt{2}E}{3(1+\mu)}\sqrt{(\epsilon_1-\epsilon_2)^2+(\epsilon_2-\epsilon_3)^2+(\epsilon_3-\epsilon_1)^2}$

$\tan 2\theta = \frac{\sqrt{3}(\epsilon_3-\epsilon_2)}{2\epsilon_1-\epsilon_2-\epsilon_3}$

θ lies in 1st quadrant when $\epsilon_3 > \epsilon_2$ and in the 2nd quadrant when $\epsilon_2 > \epsilon_3$.

Example:

A rectangular rosette is mounted on a steel plate having E = 29 X 10^6 psi and μ = 0.3. the three strain are measured as ϵ_1 = +500 μin/in; ϵ_2 = +400 μin/in; ϵ_3 = -100 μin/in. Calculate the principal strains and stress and the maximum shear stress. Locate the axis of principal stress.

Solution:

Let $A = \frac{\epsilon_3+\epsilon_1}{2}$ = 200 μin/in

$B = \sqrt{(\epsilon_1-\epsilon_2)^2+(\epsilon_2-\epsilon_3)^2}$ = 510 μin/in

Then $\epsilon_{max} = A + \frac{B}{\sqrt{2}}$ = 561 μin/in

$\epsilon_{min} = A - \frac{B}{\sqrt{2}}$ = -161 μin/in

$б_{max} = \frac{EA}{1-\mu} + \frac{EB}{\sqrt{2}(1+\mu)} = \frac{(29\times10^6)(200\times10^{-6})}{1-0.3} + \frac{(29\times10^6)(510\times10^{-6})}{\sqrt{2}(1+0.3)}$

Б_{max} = 8280 + 8050 = 16330 psi (112.6 MN/m^2)

Б_{min} = 8280 – 8050 = 230 psi (1.59 KN/m^2)

$\tau_{max} = \frac{EB}{\sqrt{2}(1+\mu)}$ = 8050 psi (55.5 MN/m^2)

We also have

$\tan 2\theta = \frac{2\epsilon_2-\epsilon_1-\epsilon_2}{\epsilon_1-\epsilon_3} = \frac{(2)(400)-(500)-(-100)}{(500)-(-100)} = 0.667$

∴ 2θ = 33.7 or 213.7^0

∴ θ = 16.8 or 106.8^0

Here θ = 16.8^0 because, $\epsilon_2 > \frac{\epsilon_1+\epsilon_3}{2}$