Design and Synthesis of Active Filter

References:

H. Lam, "ANALOG AND DIGITAL FILTERS: Design and Realization"

W. K. Chen, "Passive and Active Filters: Theory and Implementations"

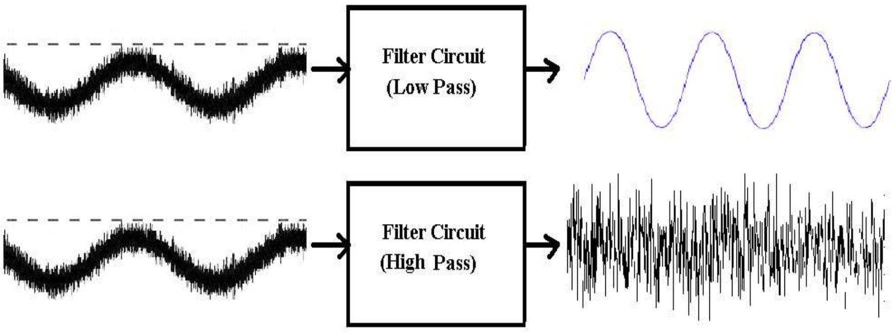
V. K. Aatre, "Network Theory And Filter Design"

What is a Filter?

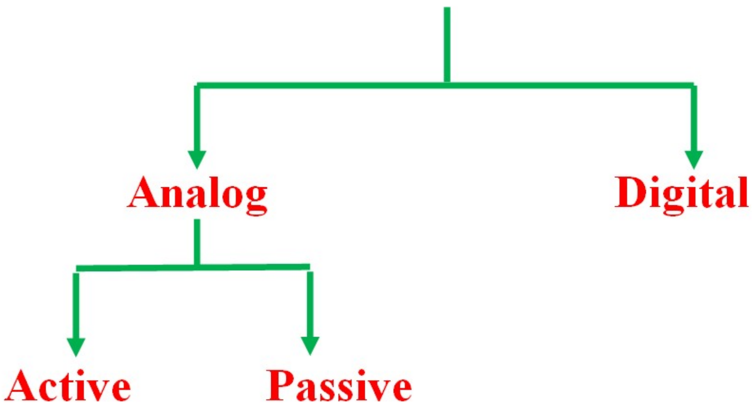
- ***An electric filter is a frequency selective circuit that passes a specified band of frequencies and blocks or attenuates signals of frequencies outside this band.***

- ***Filter modifies signals in frequency domain – both in amplitude and phase.***

What is a Filter?

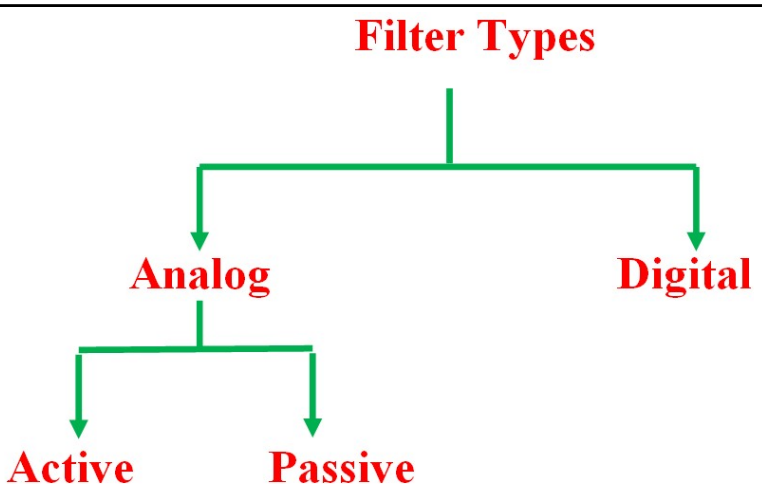


Filter Types



•***Analog filters*** *are designed to process analog signals.*

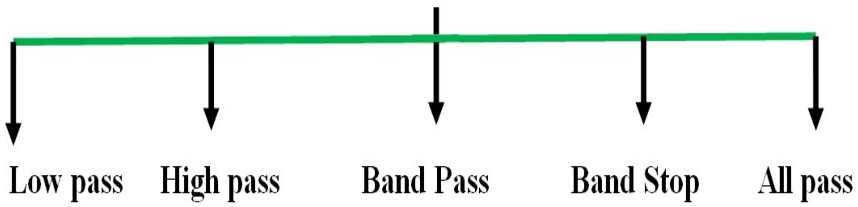
•***Digital filters*** *process analog signals using digital techniques.*



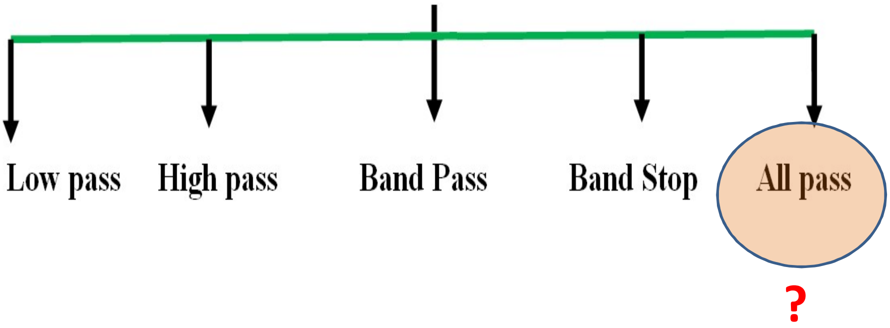
•*Elements used in the passive filters :* **resistors, inductors, capacitors.**

•*Elements used in the active filters :* **Transistors** *or* **Op-Amps** *in addition to* **resistors** *and* **capacitors**

Most commonly used filters

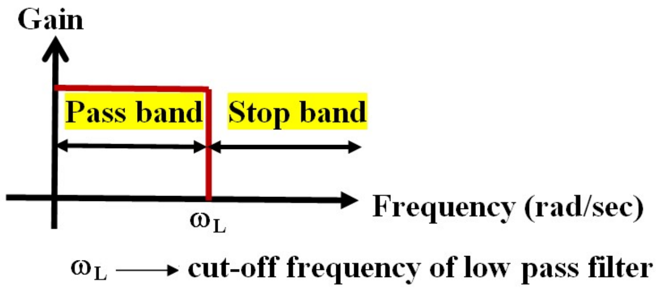


Most commonly used filters



Low pass Filter

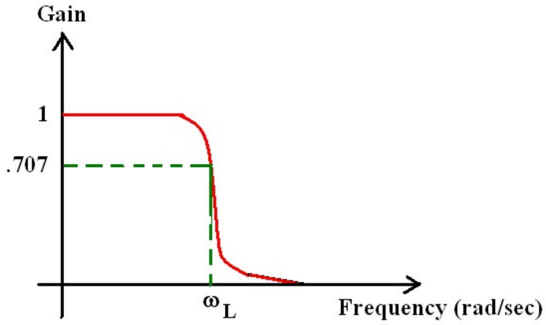
- **Passes low frequency** components and attenuates high frequency components



Ideal Characteristics of Low pass filter

Low pass Filter

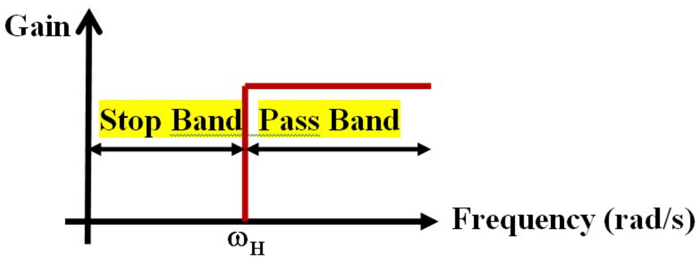
- Passes low frequency components and attenuates high frequency components



Actual Characteristics of Low pass filter

High pass Filter

- Passes high frequency components and attenuates low frequency components

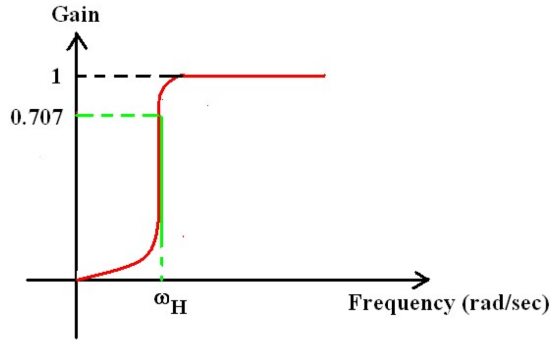


$\omega_H \longrightarrow$ cut-off frequency of high-pass filter

Ideal Characteristics of High pass filter

High pass Filter

- Passes high frequency components and attenuates low frequency components

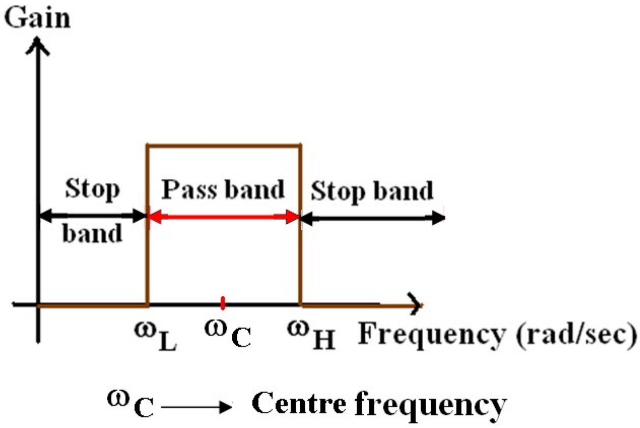


Actual Characteristics of High pass filter

Band pass Filter

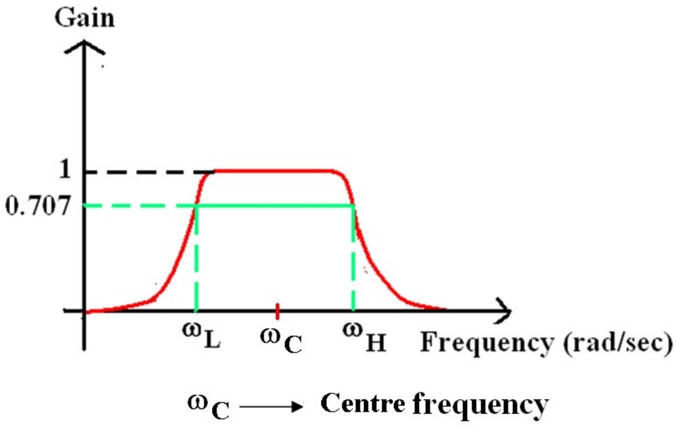
- Passes certain band of frequencies and attenuates frequencies outside this band.
- Has pass-band between two stop-bands
- Bandwidth= $(\omega_H - \omega_L)$

Band pass Filter



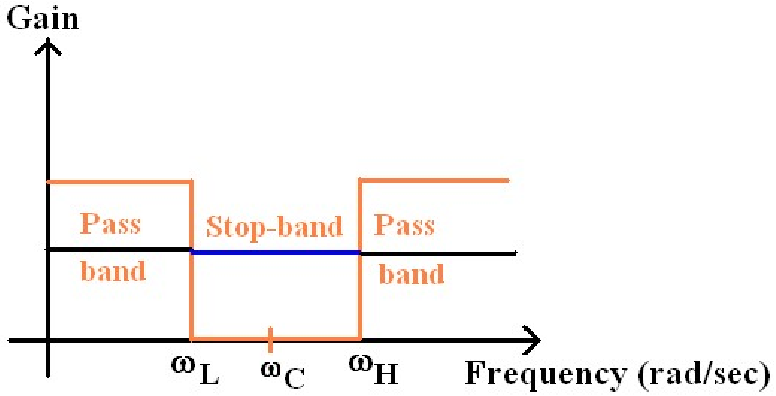
Ideal Characteristics of Band pass filter

Band pass Filter



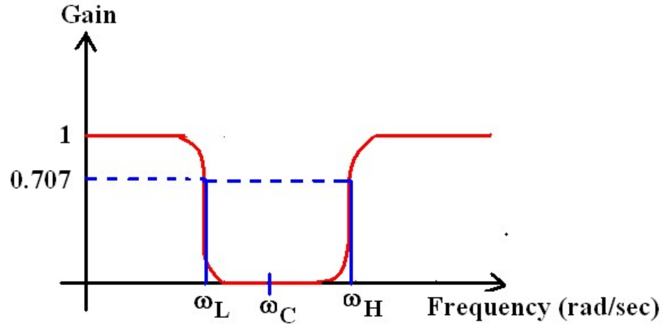
Actual Characteristics of Band pass filter

Band stop Filter



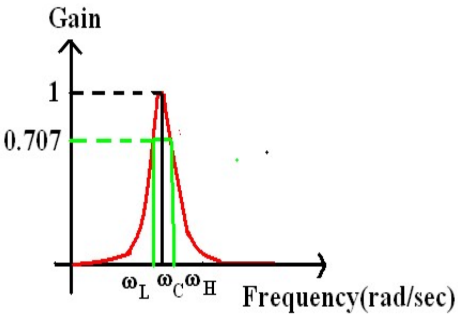
Ideal Characteristics of Band stop filter

Band stop Filter

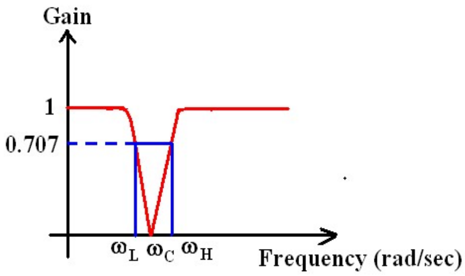


Actual Characteristics of Band stop filter

Narrow Band-pass filter: known as Tuned Filter
Narrow Band-stop filter: known as Notch Filter



Tuned Filter



Notch Filter

Filter Transfer Function

$$H(s) = \frac{a_m s^m + a_{m-1} s^{m-1} \ldots\ldots\ldots\ldots a_1 s + a_0}{s^n + b_{n-1} s^{n-1} \ldots\ldots\ldots b_1 s + b_0}$$

$$= \frac{\sum_{i=0}^{m} a_i s^i}{s^n + \sum_{i=0}^{n-1} b_i\, s^i}$$

m = number of **zeros** and *n* = number of **poles**

First Order Filter

$$H(s) = \frac{a_1 s + a_0}{s + b_0}$$

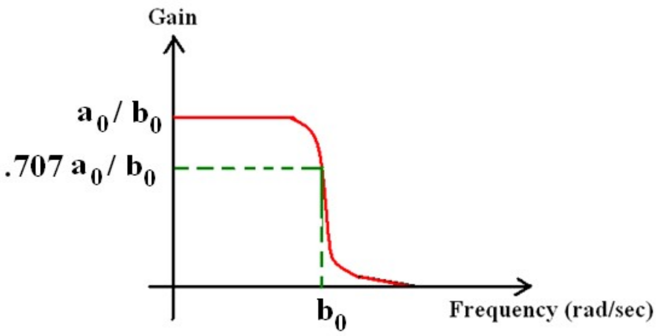
For Low-pass Filter:

$$H_{LP}(s) = \frac{a_0}{s+b_0}$$

Pass-band gain = (a_0 / b_0)
Cut-off frequency = b_0 rad/sec

First Order Filter

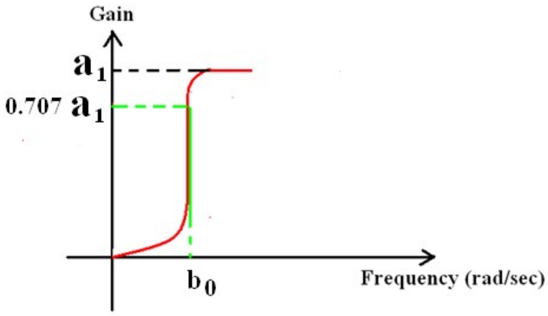
For Low-pass Filter:



Pass-band gain = (a_0 / b_0)
Cut-off frequency = b_0 rad/sec

First Order Filter

For High-pass Filter: $H_{HP}(s) = \frac{a_1 s}{s + b_0}$



Pass-band gain = a_1
Cut-off frequency = b_0 rad/sec

Second Order Filter

$$H(s) = \frac{a_2 s^2 + a_1 s + a_0}{s^2 + b_1 s + b_0}$$

For Low-pass Filter:

$$H_{2LP}(s) = \frac{a_0}{s^2 + b_1 s + b_0}$$

Pass-band gain = a_0 / b_0
Cut-off frequency of the filter = $\sqrt{b_0}$ rad/s

Second Order Filter

$$H(s) = \frac{a_2 s^2 + a_1 s + a_0}{s^2 + b_1 s + b_0}$$

For High-pass Filter:

$$H_{2HP}(s) = \frac{a_2 s^2}{s^2 + b_1 s + b_0}$$

Pass-band gain = a_2
Cut-off frequency of the filter = $\sqrt{b_0}$ rad/s

Second Order Filter

$$H(s) = \frac{a_2 s^2 + a_1 s + a_0}{s^2 + b_1 s + b_0}$$

For Band-pass Filter:

$$H_{2BP}(s) = \frac{a_1 s}{s^2 + b_1 s + b_0}$$

Pass-band gain = a_1 / b_1
Centre frequency of the filter = $\sqrt{b_0}$ rad/s

Second Order Filter

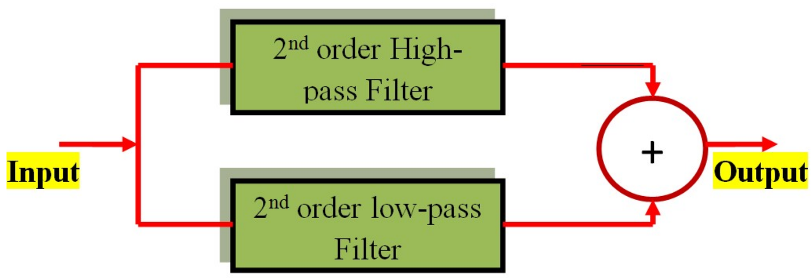
$$H(s) = \frac{a_2 s^2 + a_1 s + a_0}{s^2 + b_1 s + b_0}$$

For Band-stop Filter:

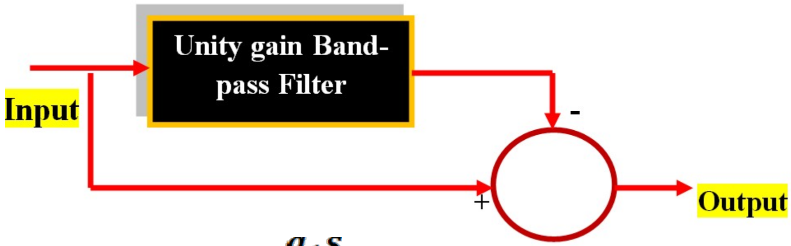
$$H_{2BS}(s) = \frac{a_2(s^2 + b_0)}{s^2 + b_1 s + b_0} \qquad [a_2 = a_0 / b_0]$$

Pass-band gain = a_2
Centre frequency of the filter = $\sqrt{b_0}$ rad/s

Schematic of Band-stop Filter:



Schematic of **Band-stop Filter:**



$$H_{BS}(s) = 1 - \frac{a_1 s}{s^2 + b_1 s + b_0}$$

$a_1 / b_1 = 1$

$$H_{BS}(s) = \frac{s^2 + b_0}{s^2 + b_1 s + b_0}$$

Pass-band gain = 1

Centre frequency of the filter $= \sqrt{b_0}$ rad/s

Filter Circuit Components

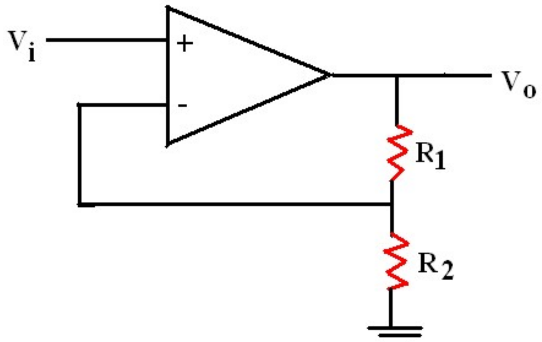
Basic Building Blocks:

Resistors, Capacitors & 0p-Amps

Filter Circuit Components

Secondary Building Block:

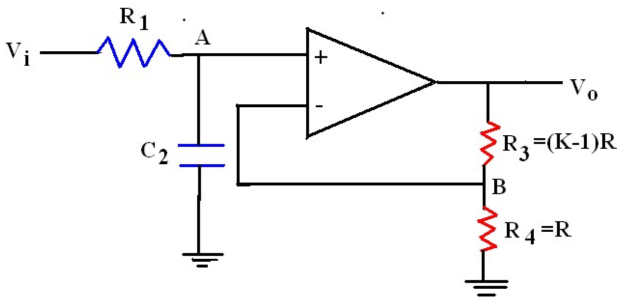
Voltage Controlled Voltage Source (VCVS)



$$V_o = KV_i, \qquad K = \frac{R_1 + R_2}{R_2} = VCVS\ gain$$

First Order Low-pass Filter

$$H_{1LP}(s) = \frac{a_0}{s + b_0}$$

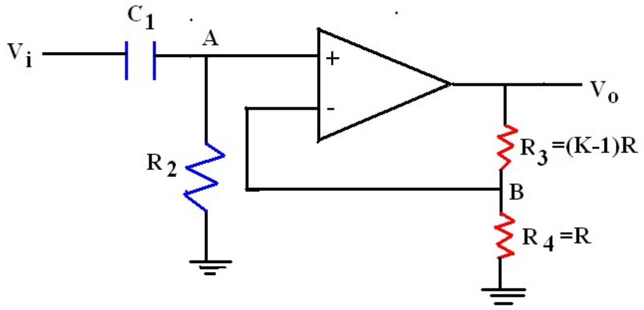


$$H_{1LP}(s) = \frac{K/R_1C_2}{s + \frac{1}{R_1C_2}}$$

Pass-band gain = a_0/b_0 = K
Cut-off frequency $b_0 = 1/R_1C_2$

First Order High-pass Filter

$$H_{1HP}(s) = \frac{a_1 s}{s + b_0}$$

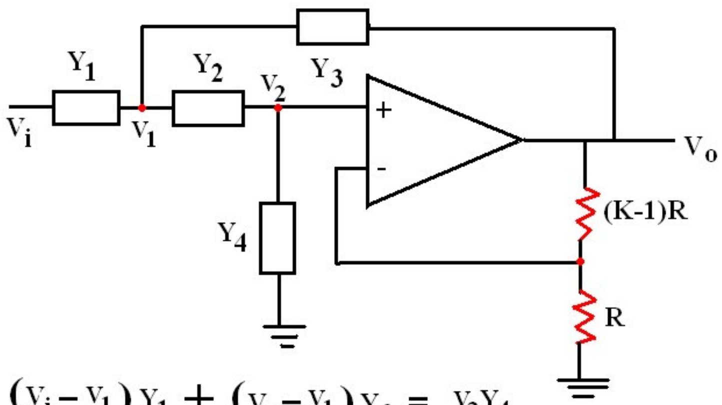


$$H_{1HP}(s) = \frac{Ks}{s + {}^{1}/_{C_1 R_2}}$$

Pass-band gain = a_1=K

Cut-off frequency $b_0 = 1 / R_2C_1$

Second Order Filter Circuit

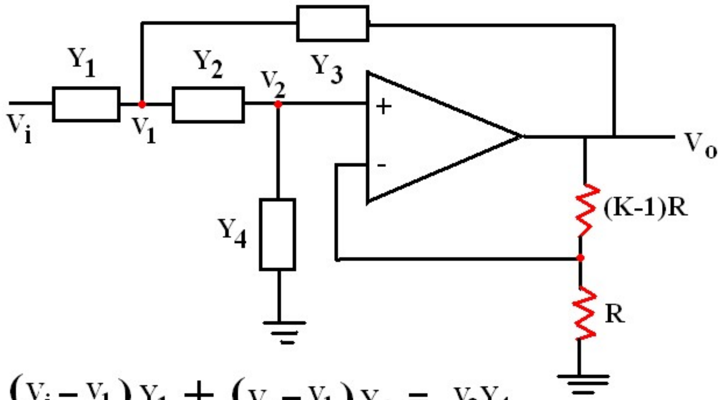


$$(V_i - V_1)Y_1 + (V_o - V_1)Y_3 = V_2 Y_4$$

$$V_2 = \frac{Y_2}{Y_2 + Y_4} V_1$$

$$V_2 = \frac{V_o}{K}$$

Second Order Filter Circuit

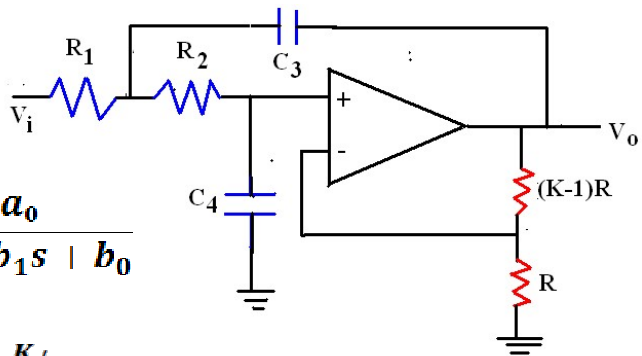


$$(V_i - V_1)Y_1 + (V_o - V_1)Y_3 = V_2 Y_4$$

$V_2 = V_0/K$ & $(V_1 - V_2)Y_2 = V_2 Y_4$

$$\frac{V_0}{V_i} = \frac{KY_1Y_2}{Y_3Y_4 + Y_2Y_3(1-K) + Y_4(Y_1 + Y_2) + Y_1Y_2}$$

Second Order Low-pass Filter



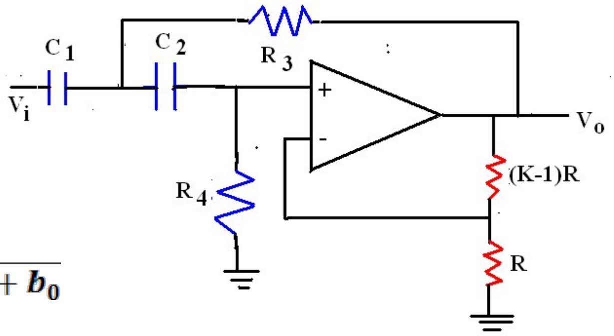
$$H_{2LP}(s) = \frac{a_0}{s^2 + b_1 s + b_0}$$

$$H_{2LP}(s) = \frac{K/_{R_1R_2C_3C_4}}{s^2 + s\left[\frac{1-K}{R_2C_4} + \frac{R_1 + R_2}{R_1R_2C_3}\right] + \frac{1}{R_1R_2C_3C_4}}$$

Pass-band gain = a_0/b_0 = K=VCVS gain

Cut-off frequency = $\sqrt{b_0} = \frac{1}{\sqrt{R_1R_2C_3C_4}}$

Second Order High-pass Filter



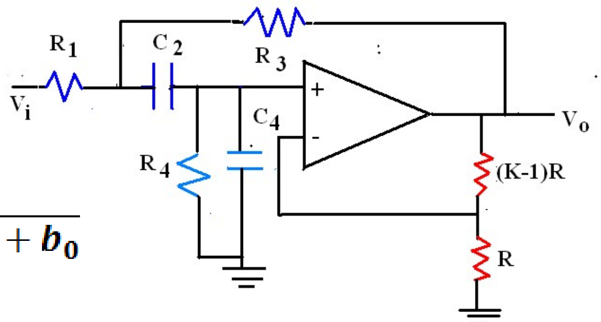
$$H_{2HP}(s) = \frac{a_2 s^2}{s^2 + b_1 s + b_0}$$

$$H_{2HP}(s) = \frac{K s^2}{s^2 + s\left[\frac{1-K}{R_3 C_1} + \frac{C_1 + C_2}{C_1 C_2 R_4}\right] + \frac{1}{C_1 C_2 R_3 R_4}}$$

Pass-band gain = a_2 =K = VCVS gain

Cut-off frequency = $\sqrt{b_0} = 1/\sqrt{C_1 C_2 R_3 R_4}$

Band-pass Filter Circuit

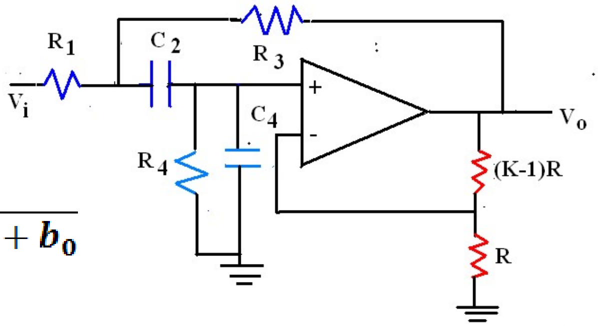


$$H_{DP}(s) = \frac{a_1 s}{s^2 + b_1 s + b_0}$$

$$H_{BP}(s) = \frac{Ks/R_1C_4}{s^2 + s\left[\frac{1}{R_3C_2} + \frac{1}{R_4C_4} + \frac{1}{R_1C_2} + \frac{1}{R_1C_4} + \frac{1-K}{R_3C_4}\right] + \frac{1}{R_4}\left[\frac{1}{R_1} + \frac{1}{R_3}\right]\frac{1}{C_2C_4}}$$

Pass-band gain = a_1/b_1 = $\frac{K/R_1C_4}{\left[\frac{1}{R_3C_2} + \frac{1}{R_4C_4} + \frac{1}{R_1C_2} + \frac{1}{R_1C_4} + \frac{1-K}{R_3C_4}\right]}$

Band-pass Filter Circuit



$$H_{BP}(s) = \frac{a_1 s}{s^2 + b_1 s + b_0}$$

$$H_{BP}(s) = \frac{Ks/R_1C_4}{s^2 + s\left[\frac{1}{R_3C_2} + \frac{1}{R_4C_4} + \frac{1}{R_1C_2} + \frac{1}{R_1C_4} + \frac{1-K}{R_3C_4}\right] + \frac{1}{R_4}\left[\frac{1}{R_1} + \frac{1}{R_3}\right]\frac{1}{C_2C_4}}$$

Center/ cut-off frequency = $\sqrt{b_0} = \sqrt{\frac{1}{R_4}\left[\frac{1}{R_1} + \frac{1}{R_3}\right]\frac{1}{C_2C_4}}$

Filter Approximation Techniques

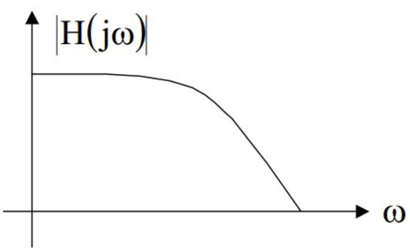
Why?

Ideal characteristic is impossible to achieve

Why?

Commonly used filter approximation techniques:

1) Butterworth Filter

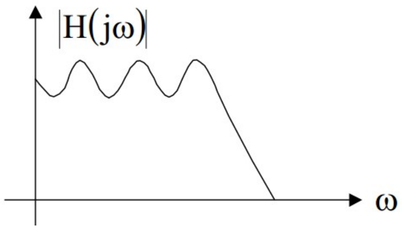


Why?

Commonly used filter approximation techniques:

1) Butterworth Filter

2) Chebyshev Filter



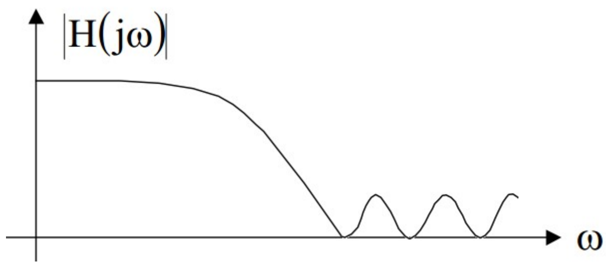
Why?

Commonly used filter approximation techniques:

1) Butterworth Filter

2) Chebyshev Filter

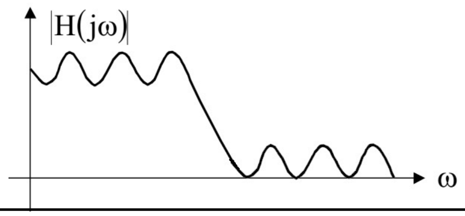
3) Inverse-Chebyshev Filter



Why?

Commonly used filter approximation techniques:

1) Butterworth Filter
2) Chebyshev Filter
3) Inverse-Chebyshev Filter
4) **Elliptic (Cauer) Filter**

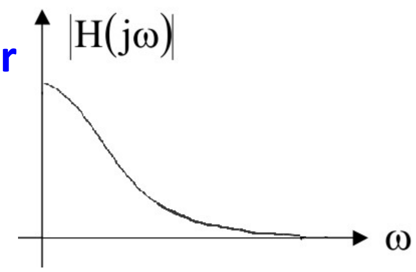


Why?

Commonly used filter approximation techniques:

1) Butterworth Filter
2) Chebyshev Filter
3) Inverse-Chebyshev Filter
4) Elliptic (Cauer) Filter
5) **Bessel-Thomson Filter**

And many more…



Why?

Commonly used filter approximation techniques:

1) **Butterworth Filter**
2) **Chebyshev Filter**
3) **Inverse-Chebyshev Filter**
4) **Elliptic (Cauer) Filter**
5) **Bessel-Thomson Filter**

And many more…

Why?

Commonly used filter approximation techniques:

1) **Butterworth Filter**
2) **Chebyshev Filter**

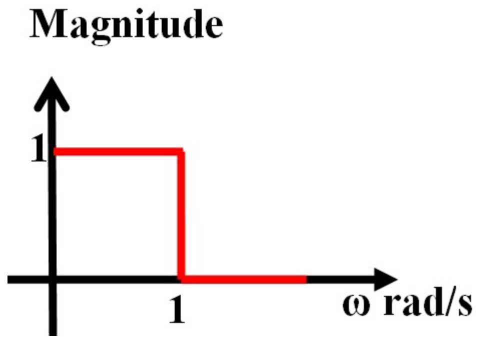
These techniques are based on *normalized ideal low-pass filter*.

The normalized ideal low –pass filter has gain of one in the band of frequencies 0 to 1 rad/s and gain of zero for all frequencies above 1 rad/s

Normalized ideal low-pass filter

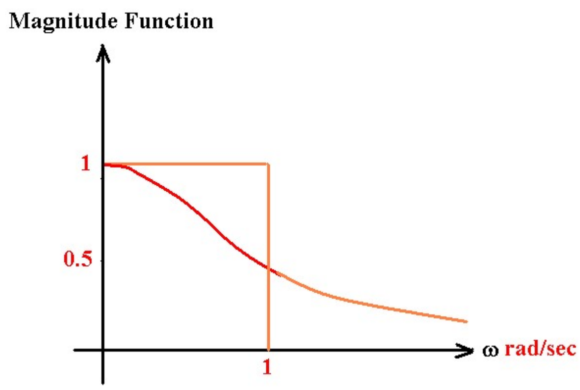
$H(j\omega) = e^{-j\omega}$ $for\ 0 \le \omega \le 1$

$= 0$ $for\ |\omega| > 1$



Butterworth filter Approximation

- Characterized by a ***monotonically decreasing magnitude function*** of ω for $\omega \ge 0$.



Butterworth filter Approximation

The *nth* order normalized low-pass Butterworth filter has a **magnitude function** given by

$$|H(j\omega)|^2 = \frac{1}{1 + B_n^2(\omega)} = \frac{1}{1 + \omega^{2n}}$$

$B_n(\omega)$ is an ***nth* order Butterworth polynomial**

Butterworth Polynomial

- ***n*^th^ order** Butterworth polynomial ***B*$_n$*(ω)*** satisfies the conditions:
  - **$B_n(\omega)$ is an n^{th} order polynomial.**
  - **$B_n(0) = 0$**
  - **$B_n(\omega)$ is maximally flat at $\omega = 0$**
  - **$B_n(1) = 1$**

Butterworth Polynomial

- **$B_n(\omega)$ is an n^{th} order polynomial.**
- $B_n(0) = 0$
- $B_n(\omega)$ is maximally flat at $\omega = 0$
- $B_n(1) = 1$

From 1st condition

$$B_n(\omega) = c_0 + c_1\omega + c_2\omega^2 + \ldots\ldots\ldots\ldots\ldots\ldots + C_n\omega^n$$

Butterworth Polynomial

- $B_n(\omega)$ is an n^{th} order polynomial.
- **$B_n(0) = 0$**
- $B_n(\omega)$ is maximally flat at $\omega = 0$
- $B_n(1) = 1$

From 2nd condition, $c_0 = 0$

$$B_n(\omega) = c_1\omega + c_2\omega^2 + \ldots\ldots\ldots\ldots\ldots\ldots + C_n\omega^n$$

Butterworth Polynomial

- $B_n(\omega)$ is an n^{th} order polynomial.
- $B_n(0) = 0$
- **$B_n(\omega)$ is maximally flat at $\omega = 0$**
- $B_n(1) = 1$

From 3rd condition, as many derivatives as possible of $B_n(\omega)$ are zero at ω =0

$$\frac{dB_n}{d\omega} = c_1 + 2c_2\omega + \cdots \dots \dots \dots \dots . . + nc_n\omega^{n-1} = 0$$

Hence, $c_1 = 0$

Butterworth Polynomial

- $B_n(\omega)$ is an n^{th} order polynomial.
- $B_n(0) = 0$
- **$B_n(\omega)$ is maximally flat at $\omega = 0$**
- $B_n(1) = 1$

Similarly, $\frac{d^2 B_n}{d^2\omega} = 0$ **So $c_2 = 0$**

$\frac{d^{n-1} B_n}{d^{n-1}\omega}$ **= 0** **So $c_{n-1} = 0$**

Therefore, $B_n(\omega) = c_n\omega^n$

Butterworth Polynomial

- $B_n(\omega)$ is an n^{th} order polynomial
- $B_n(0) = 0$
- $B_n(\omega)$ is maximally flat at $\omega = 0$
- **$B_n(1) = 1$**

From 4th condition, $c_n = 1$

Therefore, $B_n(\omega) = \omega^n$

Properties of Butterworth filter magnitude function

For all n, $|H(j0)|^2 = 1, |H(j1)|^2 = 0.5, |H(j\infty)|^2 = 0$

This implies that the dc gain is 1
and
3dB cut-off frequency is at 1 rad/s

Butterworth Pole location

$$|H(j\omega)|^2|_{\omega=s/j} = H(j\omega)H(-j\omega)|_{\omega=s/j} = \frac{1}{1+\omega^{2n}}|_{\omega=s/j}$$
$$= \frac{1}{1+(-1)^n s^{2n}}$$

The poles of

$H(s)H(-s)$ *are the solution of the equation*

$$1+(-1)^n s^{2n} = 0$$

$$-s^2 = (-1)^{\frac{1}{n}} = e^{\frac{j(2k-1)\pi}{n}}$$

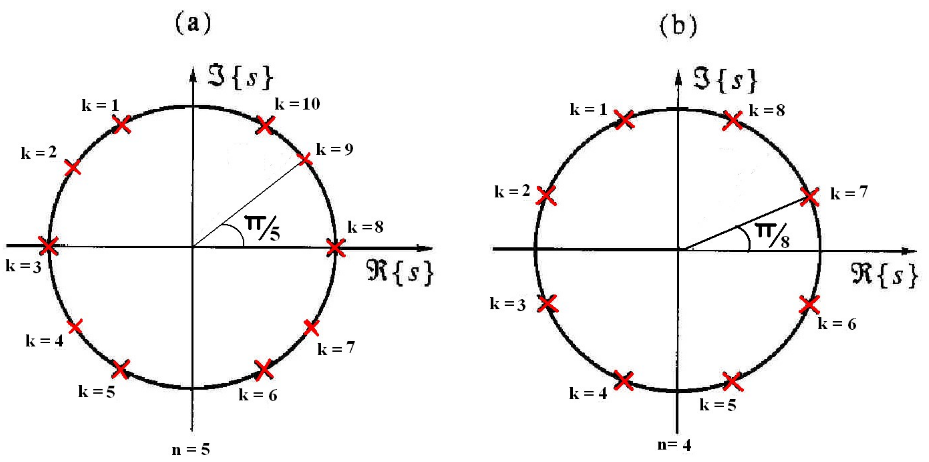
Butterworth Pole location

$$-s^2 = (-1)^{\frac{1}{n}} = e^{\frac{j(2k-1)\pi}{n}}$$

$$s_k = e^{\frac{j(2k+n-1)\pi}{2n}}$$

The pole in the right half-plane corresponds to an unstable system, so pole on the left-half of the s-plane have to be considered

Butterworth Pole location



Butterworth Pole location

Considering poles on the left-hand side of *s*-plane

$$s_k = \cos\left(\frac{2k+n-1}{2n}\right)\pi + j\sin\left(\frac{2k+n-1}{2n}\right)\pi$$

$$= \cos\left(\frac{2k-1}{2n}\pi + \frac{\pi}{2}\right) + j\sin\left(\frac{2k-1}{2n}\pi + \frac{\pi}{2}\right)$$

$$= -\sin\left(\frac{2k-1}{2n}\right)\pi + j\cos\left(\frac{2k-1}{2n}\right)\pi$$

for k = 1,2……n

$$= -\sin\theta_k + j\cos\theta_k$$

where $\theta_k = \frac{2k-1}{2n}\pi$

The transfer function of the *nth* order Butterworth filter

$$H(s) = \prod_{k=1}^{n} \frac{1}{s - s_k} = \prod_{k=1}^{n} \frac{1}{(s - \sigma_k - j\omega_k)}$$

Where $s_k = \sigma_k + j\omega_k,$

$\sigma_k = -\sin\theta_k$ $\omega_k = \cos\theta_k,$

$\theta_k = \frac{2k-1}{2n}\pi$

Therefore, the poles of the Butterworth filter are on unit circle as,

$$|s_k|^2 = \sigma_k^{\,2} + \omega_k^{\,2} = \sin^2\theta_k + \cos^2\theta_k = 1$$

For 1st order (i.e. n=1) normalized low-pass filter

$$H_{1LP}(s) = \frac{1}{s+1}$$

For 2nd order (i.e. n=2) normalized low-pass filter

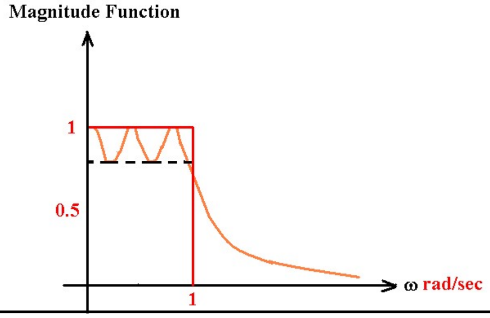
$$H_{2LP}(s) = \frac{1}{s^2 + 1.414s + 1}$$

Chebyshev Filter Approximation Technique

- *equiripplie magnitude function across the pass-band*

and

- *monotonically decreasing magnitude function in the stop-band.*



Chebyshev Filter Approximation Technique

- The *nth* order Chebyshev polynomial is:

$$T_n(\omega) = \cos(n \cos^{-1} \omega) \quad for\ 0 \le \omega \le 1$$
$$= \cosh(n \cosh^{-1} \omega) \quad for\ \omega > 1$$

If, $x = \cos^{-1}\omega$, then $T_n(\omega) = \cos nx$

Remember, cosh *(ju)* = cos *u* & sinh *(ju)* = *j* sin *u*

Chebyshev Filter Approximation

Remember, cosh *(ju)* = cos *u* & sinh *(ju)* = *j* sin *u*

$$T_n(\omega) = \cos(n \cos^{-1} \omega) \quad for\ 0 \le \omega \le 1$$
$$= \cosh(n \cosh^{-1} \omega) \quad for\ \omega > 1$$

If, $x = \cos^{-1}\omega$, then $T_n(\omega) = \cos nx$

$T_0(\omega) = \cos 0 = 1$

$T_1(\omega) = \cos x = \cos(\cos^{-1}\omega) = \omega$

$T_2(\omega) = \cos 2x = 2\omega^2 - 1$

$T_3(\omega) = \cos 3x = -3\omega + 4\omega^3$

$T_4(\omega) = \cos 4x = 1 - 8\omega^2 + 8\omega^4$

Properties of Chebyshev polynomial

- ☞ $|T_n(0)| = 0$ when n is odd
- ☞ $|T_n(0)| = 1$ when n is even
- ☞ $|T_n(1)| = +1$

Recursive formula for Chebyshev Polynomials

$$T_{n+1}(\omega) = 2\,\omega T_n(\omega) - T_{n-1}(\omega)$$
$$T_0(\omega) = 1 \;\&\; T_1(\omega) = \omega$$

Chebyshev Magnitude Response

The ***nth*** order normalized low-pass Chebyshev filter is given by:

$$|H(j\omega)|^2 = \frac{1}{1+\epsilon^2 T_n^2(\omega)}$$

ε is a free parameter that sets the amplitude of the ripple called ***ripple factor***

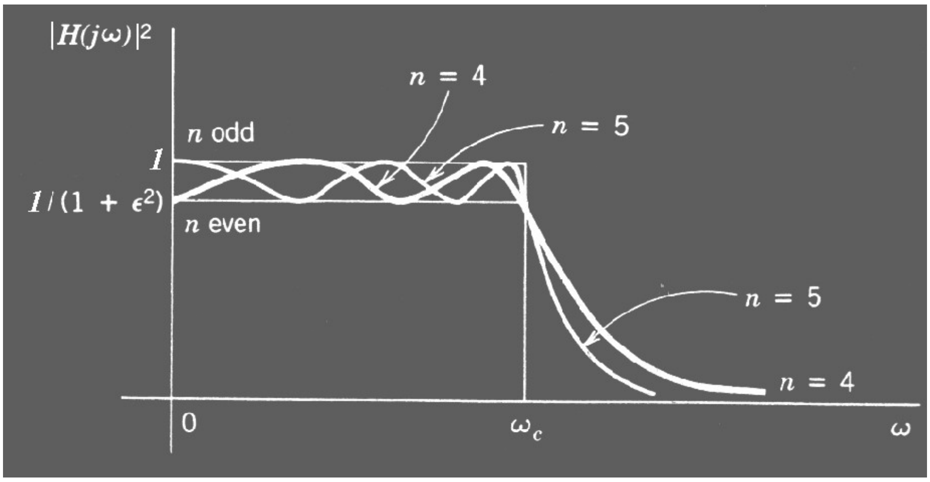
Chebyshev Magnitude Response

1. $|H(j0)|^2$=1 when n is odd and $|H(j0)|^2 = \frac{1}{1+\epsilon^2}$ when n is even.
2. At ω = 1, $|T_n(1)| = +1$ *so* $|H(j0)|^2 = \frac{1}{1+\epsilon^2}$
3. For $|\omega| \leq 1, |H(j\omega)|^2$ oscillates between 1 and $\frac{1}{1+\epsilon^2}$

   and attains a maximum value 1 or minimum value $\frac{1}{1+\epsilon^2}$.
4. If $\frac{1}{1+\epsilon^2} > 0.5$ the 3dB cut-off frequency of the

   Chebyshev filter is larger than 1 radian/sec.

Chebyshev Magnitude Response



Chebyshev Pole Locations

The magnitude function of
nth order normalized low-pass Chebyshev filter is

$$|H(j\omega)|^2 = \frac{1}{1+\epsilon^2\, T_n^2\,(\omega)} = H(j\omega)H(-j\omega)$$

Put $\omega = s/j$, then

$$H(s)H(-s) = \frac{1}{1+\varepsilon^2 T_n^2(s/j)}$$

Chebyshev Pole Locations

$$H(s)H(-s) = \frac{1}{1+\varepsilon^2 T_n^2(s/j)}$$

The pole locations are determined by solving the equation:

$$1+\varepsilon^2 T_n^2(s/j)=0 \text{ or } T_n^2(s/j)=-\frac{1}{\varepsilon^2}=\frac{j^2}{\epsilon^2}$$

$$T_n\left(\frac{s}{j}\right) = \pm\frac{j}{\epsilon} = 0 \pm \frac{j}{\epsilon}$$

Chebyshev Pole Locations

If $s_k = \sigma_k + j\omega_k$ we have

$$\sigma_k = \pm \sin \frac{\pi}{2n}(2k-1) \sinh a$$

$$\omega_k = \cos \frac{\pi}{2n}(2k-1) \cosh a$$

where k= 1,2, ………,2n and $a = \frac{1}{n}\sinh^{-1}\frac{1}{\varepsilon}$

Chebyshev Pole Locations

Considering left-hand-side *s* plane poles,

$$\bar{s}_k = -\sin\frac{\pi}{2n}(2k-1)\sinh a + j\cos\frac{\pi}{2n}(2k-1)\cosh a$$

$= \bar{\sigma}_k + j\bar{\omega}_k$ where k= 1, 2 , …, n

$$\overline{\sigma_k} = -\sinh a \sin\frac{\pi}{2n}(2k-1)$$

$$\overline{\omega_k} = \cosh a \cos\frac{\pi}{2n}(2k-1)$$

Chebyshev Pole Locations

$$\frac{\overline{\sigma_k}}{\sinh a} = -\sin\frac{\pi}{2n}(2k-1)$$

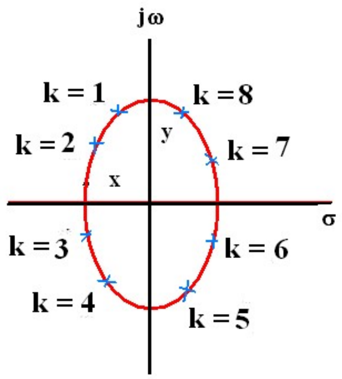
$$\frac{\overline{\omega_k}}{\cosh a} = \cos\frac{\pi}{2n}(2k-1)$$

$$\frac{\overline{\sigma}_k^2}{sinh^2 a} + \frac{\overline{\omega}_k^2}{cosh^2 a} = sin^2\frac{\pi}{2n}(2k-1) + cos^2\frac{\pi}{2n}(2k-1) = 1$$

Hence, poles of Chebyshev filter are on ***s-plane ellipse***

Chebyshev Pole Locations

$$\frac{\overline{\sigma}_k^2}{sinh^2 a} + \frac{\overline{\omega}_k^2}{cosh^2 a} = sin^2\frac{\pi}{2n}(2k-1) + cos^2\frac{\pi}{2n}(2k-1) = 1$$



n = 4

Chebyshev Filter Transfer Function

$$H(s) = \frac{H_0}{\prod_{k=1}^{n}(s - \bar{s}_k)}$$

$$\overline{\sigma_k} = -\sinh a \sin\frac{\pi}{2n}(2k-1)$$

$$\overline{\omega_k} = \cosh a \cos\frac{\pi}{2n}(2k-1)$$

where k= 1,2, ………, n and $a = \frac{1}{n}\sinh^{-1}\frac{1}{\varepsilon}$

Chebyshev Filter Transfer Function

$$H(s) = \frac{H_0}{\prod_{k=1}^{n}(s - \bar{s}_k)}$$

$$H_0 = \frac{1}{\sqrt{1+\varepsilon^2}}\prod_{k=1}^{n}(-\bar{s}_k) \ for\ n\ even$$

$$= \prod_{k=1}^{n}(-\bar{s}_k) \ for\ n\ odd$$

Important Observations

Max value in pass band for $|H(j\omega)|^2 = H_0$

Min value in pass band for $|H(j\omega)|^2 = \frac{H_0}{1+\varepsilon^2}$

Note: at $\omega = \omega_c$ $|H(j\omega)|^2 = \frac{H_0}{1+\varepsilon^2}$

Comparison of Butterworth & Chebyshev Responses

•Butterworth filter is maximally flat filter while Chebyshev fiter has ripple in the passband and monotonically decreasing stopband.

•For Butterworth response, ω=1 rad/s identifies the half-power frequency, but for Chebyshev response ω=1 rad/s identifies the end of the ripple band.

•The normalized cut-off frequency of the Butterworth filter is always at 1 rad/s & Chebyshev filter has normalized cut-off frequency more than 1 rad/s.

•For Butterworth pole locations (on unit circle):

$\sigma_k = -\sin\theta_k$ $\omega_k = \cos\theta_k,$

•For Chebyshev pole locations (on s-plane ellipse):

$\overline{\sigma_k} = -\sinh a \sin\frac{\pi}{2n}(2k-1)$ $\overline{\omega_k} = \cosh a \cos\frac{\pi}{2n}(2k-1)$