Electrical Measurements

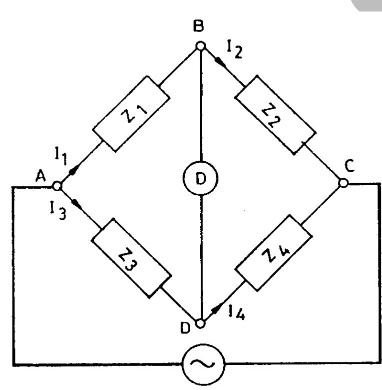
AC Bridges

In the alternating current bridge are the best and most used method for the precise measurement of self and mutual inductance and capacitances. The supply is from an alternating current source. If the supply is at normal commercial frequencies then a vibration galvanometer is used, whereas at high frequencies a headphone is employed as detector. The alternating current bridge networks may be classified in the following three classes:

i) Wheatstone impedance networks without mutual inductance.
ii) Wheatstone networks in which there are mutual inductances between appropriate pairs of branches.
iii) Networks which are not of Wheatstone form and in which there are usually mutual inductances.

Wheatstone Networks:

The difference from the dc Wheatstone bridge is that the four branches have, in general, impedance in place of resistances as shown in figure.



General Four Arm AC Bridge

Under the balance condition, if Z_1 is the unknown impedance then

$$Z_1 = \frac{Z_2}{Z_3} Z_4 \text{ ……………………………….(1)}$$

In general, if R and X represent the resistance and reactance of the branches then

$$(R_1 + jX_1) = \frac{(R_2+jX_2)\,(R_4+jX_4)}{R_3+jX_3} = A + jB \text{ ……………………….(2)}$$

Now, to make the resistance A independent of any adjustment in B and the reactance B independent of any adjustment in A, there are two ways:

a) One quantity in the numerator either Z_2 or Z_4 is made adjustable to bring the balance and bridges of such type are known as ratio bridges.

b) The only quantity in the denominator Z_3 is made adjustable and the bridges of such type are termed as product bridges.

Ratio Bridges:

To fulfill the above condition the ratio arms are taken such that Z_2 / Z_3 is real (or imaginary). If this ratio is real then,

$$\frac{R_2}{R_3} = \frac{X_2}{X_3} = K \text{ ....(3)}$$

Where, $K_{(real)} = \frac{Z_2}{Z_3}$ and as R_2 and R_3 are positive K must also be positive.

Thus, X_2 and X_3 must be reactance of same kind, i.e. both inductive or both capacitive. Also, since, $\frac{X_2}{R_2} = \frac{X_3}{R_3}$, the phase angles $\phi_2 = \phi_3$. From equation (1)

$$\frac{Z_1}{Z_2}\angle(\varphi_1 - \varphi_4) = \frac{Z_2}{Z_3}\angle(\varphi_2 - \varphi_3) \text{ ....(4)}$$

$\therefore \quad \phi_1 - \phi_4 = \phi_2 - \phi_3 = 0$

$\phi_1 = \phi_4$

Hence, Z_1 and Z_4 must have same time constant and

$R_1 + jX_1 = K(R_4 + jX_4)$

When the ratio $\frac{Z_2}{Z_3}$ is imaginary

$$\frac{Z_2}{Z_3} = \frac{R_2 + jX_2}{R_3 + jX_3} = jK$$

From which $\quad -\frac{R_2}{X_3} = \frac{X_2}{R_3} = K, \quad$ Where K may be positive or negative.

Thus, X_2 and X_3 must be reactance of opposite nature, i.e. if one is inductive then the other must be capacitive. Also

$$\frac{X_2}{R_2} = -\frac{R_3}{X_3}$$

or, $\quad \tan \phi_2 = - \cot \phi_3 = \tan (\phi_3 \pm 90^0)$

or, $\quad \phi_2 - \phi_3 = \pm 90^0$ .... (5)

Hence, from equation (4)

$\phi_1 - \phi_4 = \phi_2 - \phi_3 = \pm 90^0$

Also, $R_1 + jX_1 = K(R_4 + jX_4) = -KX_4 + jKR_4 = \frac{X_4R_2}{X_3} + j\frac{X_2R_4}{R_3}$ ……………………(6)

Which implies that X_4 must be the reactance of same kind as X_3, and X_1 of same kind as X_2. Thus, as X_2 and X_3 are of opposite kind X_1 and X_4 must also be of opposite kind.

Product Bridges:

In this case the adjustable branch is Z_2 and Z_2Z_4 must either real or imaginary. Let

$$Z_2Z_4 = (R_2 + jX_2)(R_4 + jX_4) = K_{(real)}$$

From which

$$R_2R_4 - X_2X_4 = K$$

and $R_2X_4 + R_4X_2 = 0$

or, $\frac{X_2}{R_2} = -\frac{R_4}{X_4}$

or, $\tan \phi_2 = -\tan \phi_4$

or, $\phi_2 + \phi_4 = 0$

Since R_2 and R_4 are positive, X_2 and X_4 must be reactance of opposite kinds

Now, $\phi_1 + \phi_3 = \phi_2 + \phi_4 = 0$ ………………………………(7)

$$R_1 + jX_1 = \frac{K}{R_3 + jX_3} = \frac{KR_3}{R_3^2 + X_3^2} - j\frac{KX_3}{R_3^2 + X_3^2} = KG_3 + jKB_3 \quad \text{…………………………(8)}$$

This means that X_1 and X_3 are reactances of opposite kinds, i.e. X_1 has same sign as susceptance B_2. It is important to mention here that in real product bridges the resistance in the unknown branch is balanced by conductance in the adjustable branch and its reactance is balanced by the susceptance of same kind. In particular, if the product arms have pure resistances, which is usual in practice, then

$$K = R_2R_4$$

And $R_1 = R_2R_4G_3$

$X_1 = R_2R_4B_2$ …………………………………(9)

If the product Z_2Z_4 is imaginary then

$$Z_2Z_4 = (R_2 + jX_2)(R_4 + jX_4) = jK$$

From which,

$$R_2R_4 - X_2X_4 = 0$$

And $X_2R_4 + R_2X_4 = K$

∴ $\frac{X_2}{R_2} = \frac{R_4}{X_4}$

or, $\tan \phi_2 = -\tan (\pm 90^0 - \phi_4)$

or, $\phi_2 + \phi_4 = 90^0$

or, $\phi_1 + \phi_3 = \phi_2 + \phi_4 = 90^0$ ………………………………(10)

Thus, the two product branches have the same kind of impedances. From balance condition

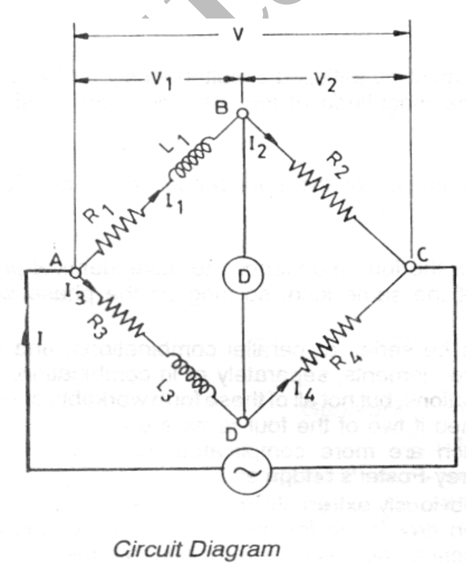
$$R_1 + jX_1 = \frac{jK}{(R_3 + jX_3)} = jK(G_3 + jB_3)$$

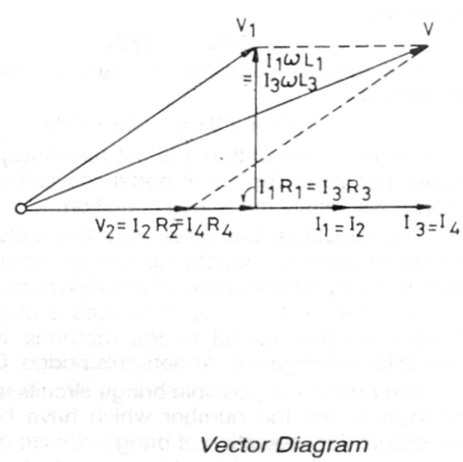
∴ $R_1 = -KB_3$

And $X_1 = KG_3$ …………………………………………(11)

Measurement of Inductance:

1. **Maxwell's Bridge:**
   This method is very suitable for accurate measurement of medium resistances. In this method unknown inductance is determined by comparing it with a standard self inductance.





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Such a bridge circuit is shown in above figure in which L_1 is a unknown self inductance of resistor R_1, L_3 is a known variable inductance of resistor R_3 whose resistance is constant, R_2 and R_4 are pure resistances and D is a detector. The magnitude of L_3 should be of the same order as that of L_1. The bridge is balanced by varying L_3 and one of the resistances R_2 or R_4. The bridge can also be balanced by keeping R_2 and R_4 constant and by varying the resistance of any one of the other two arms by connecting an additional resistance in that arm.

When the bridge is balanced, the current flowing through detector D is zero and

$$I_1 = I_2; \quad I_3 = I_4$$

Potential difference across arm AB = Potential difference across arm AD = V_1

i.e. $I_1Z_1 = I_3Z_3 = V_1$

or, $I_1(R_1 + j\omega L_1) = I_3(R_3 + j\omega L_3)$ ……………(1)

Potential difference across arm BC = Potential difference across arm CD = V_2

i.e. $I_2R_2 = I_4R_4 = V_2$

$I_1R_2 = I_3R_4$ ……………………………….(2)

Dividing expression (1) by (2) we have

$$\frac{R_1+j\omega L_1}{R_2} = \frac{R_3+j\omega L_3}{R_4}$$

$$\frac{R_1}{R_2} + \frac{j\omega L_1}{R_2} = \frac{R_3}{R_4} + \frac{j\omega L_3}{R_4}$$

Equating the real and imaginary parts of both sides separately we have

$$\frac{R_1}{R_2} = \frac{R_3}{R_4}$$

$R_1 = \frac{R_2}{R_4} R_3$ ………………………..(3)

And $$\frac{\omega L_1}{R_2} = \frac{\omega L_3}{R_4}$$

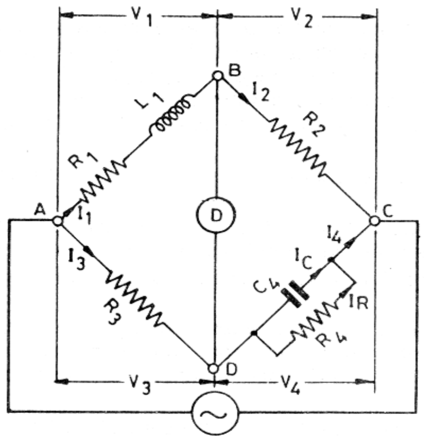
$L_1 = \frac{R_2}{R_4} L_3$ ………………………..(4)

Thus the value of unknown inductance L_1 can be determined.

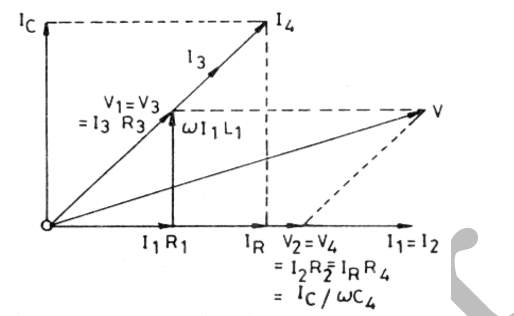
Vector diagram for the balanced condition of the bridge is shown in figure in which currents I_1 and I_2 are made in phase with currents I_3 and I_4 by adjusting the impedances of the various branches. Here one point is to be noted that inductances L_1 and L_3 should be placed at a distances from each other and leads used in the bridge arms should be twisted together properly to avoid loops, otherwise loops introduce inductance in the bridge circuit which may cause error during measurement.

2. Maxwell-Wein Bridge:

In this method of measurement of self inductance the unknown self-inductance is compared with a standard variable capacitance the circuit being shown below.



Circuit Diagram



Vector Diagram

In the circuit L_1 is unknown self inductance and R_1 is unknown resistance of the inductor, R_2, R_3 and R_4 are known non-inductive resistances and C_4 is a standard variable capacitor.

Impedance $Z_1 = R_1 + j\omega L_1$

Impedance $Z_2 = R_2$

Impedance $Z_3 = R_3$

Impedance $Z_4 = \frac{1}{\frac{1}{R_4}+\frac{1}{jX_C}} = \frac{1}{\frac{1}{R_4+j\omega C_4}} = \frac{R_4}{1+j\omega C_4 R_4}$

For balance condition of bridge

$Z_1 Z_4 = Z_2 Z_3$

$\frac{(R_1+j\omega L_1)R_4}{1+j\omega C_4 R_4} = R_2 R_3$

$R_1R_4 + j\omega L_1R_4 = R_2R_3 + j\omega C_4R_2R_3R_4$

Equating real and imaginary quantities

$R_1R_4 = R_2R_3$

or, $R_1 = \frac{R_2}{R_4} R_3$ ……………………….(1)

And $j\omega L_1R_4 = j\omega C_4R_2R_3R_4$

$L_1 = R_2R_3C_4$ ……………………..(2)

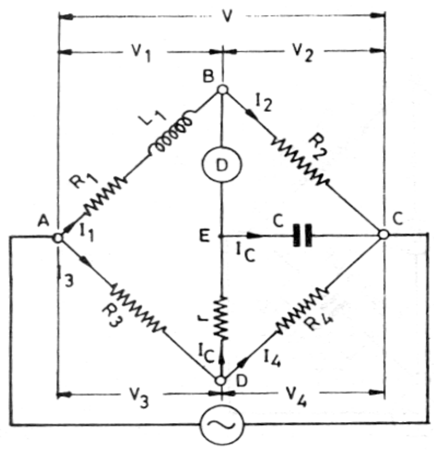
The bridge is preferably balanced by varying C_4 and R_4 which gives independent settings.

The Q-factor of the inductor is given by $\frac{\omega L_1}{R_1}$ and at balance condition

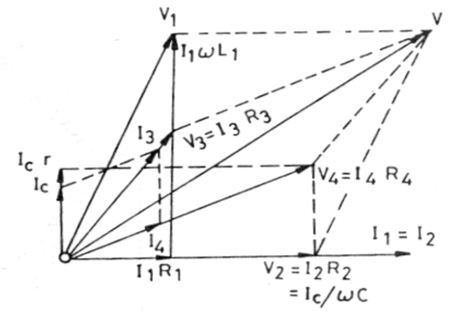
$Q = \frac{\omega L_1}{R_1} = \omega C_4 R_4$ ……………..(3)

3. **Anderson Bridge:**

This method is one of the commonest and best bridge methods for precise measurement of inductance over a wide range. In this method the unknown self inductance is measured in terms of known capacitance and resistance by comparison. It is actually a modification of the Maxwell-wein Bridge and is an example of a more complicated bridge network. The circuit is shown below, in which L_1 is self inductance and R_1 is the resistance of the coil under test, R_2, R_3, R_4 r are known non-inductive resistances and C is a standard known capacitor.



Circuit Diagram



Vector Diagram

The bridge is preliminary balanced for steady current by adjusting R_2, R_3 and R_4 and using an ordinary galvanometer as detector and then bridge is balanced in alternating current by varying r and using vibration galvanometer or telephone depending upon the supply frequency.

When the bridge is balanced

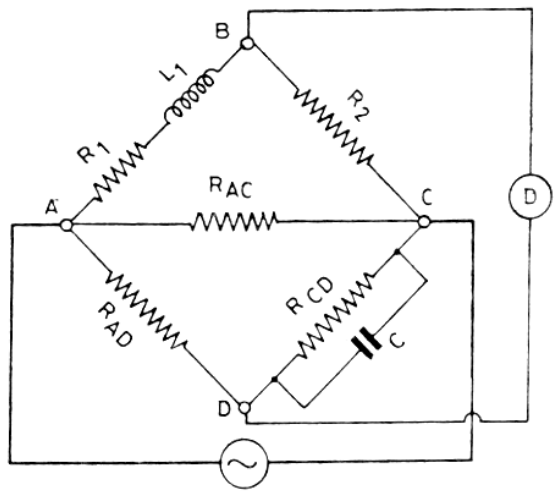
$I_1 = I_1$; $I_r = I_c$; $I_3 = I_4 + I_c$; $V_1 = V_3 + I_c r$; $V_2 = V_4 + I_c r$

From the theory of star-delta transformation

$R_{AD} = R_3 r\left(\frac{1}{R_4} + \frac{1}{R_3} + \frac{1}{r}\right)$

$R_{CD} = R_4 r\left(\frac{1}{R_4} + \frac{1}{R_3} + \frac{1}{r}\right)$

$R_{AC} = R_3 R_4\left(\frac{1}{R_4} + \frac{1}{R_3} + \frac{1}{r}\right)$



Hence the Anderson bridge is reduced to an equivalent Maxwell-wein bridge is shown in above figure. Using the relation derived for the Maxwell-wein bridge we have

$$R_1 = \frac{R_2 R_{AD}}{R_{CD}} = \frac{R_2 R_3 r\left(\frac{1}{R_4}+\frac{1}{R_3}+\frac{1}{r}\right)}{R_4 r\left(\frac{1}{R_4}+\frac{1}{R_3}+\frac{1}{r}\right)} = \frac{R_2 R_3}{R_4} \quad \ldots\ldots\ldots\ldots(1)$$

And $L_1 = CR_2R_{AD} = CR_2R_3r\left(\frac{1}{R_4}+\frac{1}{R_3}+\frac{1}{r}\right)$

$$L_1 = \frac{CR_2}{R_4}\left[r(R_3+R_4)+R_3R_4\right] \quad \ldots\ldots\ldots\ldots(2)$$

Problem

1. An ac bridge is connected as follows: Branch AB is an inductive resistor, branches BC and ED are variable resistors, branches CD and DA are non-reactive resistors of 400Ω each and branch CE is a condenser of 2μF capacitance. The supply is connected to A and C and the detector to B and E. Balance are obtained when the resistance of BC is 400Ω and that of ED is 500Ω. Determine the resistance and inductance of AB.

$R_2 = 400\Omega$, $R_3 = 400\Omega$, $R_4 = 400\Omega$, $r = 500\Omega$, $C = 2\mu F = 2 \times 10^{-6} F$

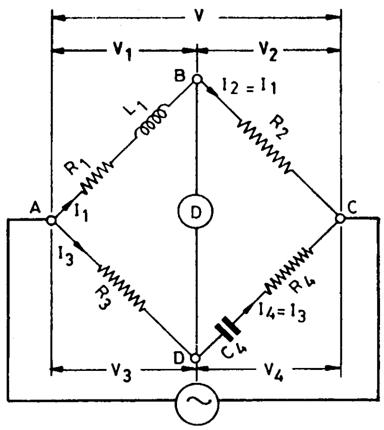
$$\therefore \quad R = \frac{R_2 R_3}{R_4} = \frac{400 \times 400}{400} = 400\Omega$$

$$L = \frac{CR_2}{R_4}\left[r(R_3+R_4)+R_3R_4\right]$$

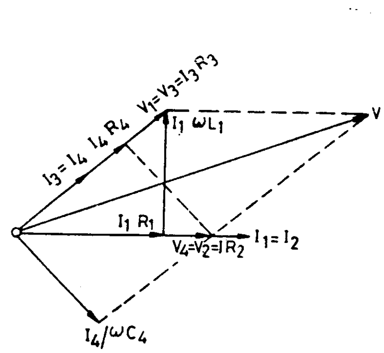
$$= 2 \times 10^{-6} \times \frac{400}{400}(500(400+400)+400 \times 400) = 1.12 \text{ Henry}$$

4. **The Hey Bridge:**

This is another modification of the Maxwell-Wein Bridge, which may be used to advantage if the phase angle of the inductor $(tan^{-1}\frac{\omega L}{R})$ under test is large.



Circuit Diagram



Vector Diagram

The Hay Bridge

The circuit arrangement is shown in above figure. In this arrangement L_1 is self-inductance and R_1 is the resistance of the coil under test R_2, R_3, R_4 are known non-inductive resistances and C_4 is a standard variable capacitance. The bridge is balanced by varying R_4 and C_4. It is often more convenient to use a capacitor of fixed value and to make R_4 and either R_2 or R_3 adjustable.

When the bridge is balanced

$I_2 = I_1$; $I_4 = I_3$; $V_1 = V_3$ and $V_2 = V_4$

Since $V_1 = I_1Z_1 = I_1(R_1 + j\omega L_1)$ and $V_3 = I_3R_3$

$\therefore I_1(R_1 + j\omega L_1) = I_3R_3$ ……………………..(1)

And since $V_2 = I_2R_2 = I_1R_2$

$V_4 = I_4Z_4 = I_3\left(R_4 - \frac{j}{\omega C_4}\right)$

$\therefore I_1R_2 = I_3\left(R_4 - \frac{j}{\omega C_4}\right)$ …………………..(2)

Dividing expression (1) by (2) we have

$$\frac{R_1 + j\omega L_1}{R_2} = \frac{R_3}{R_4 - \frac{j}{\omega C_4}}$$

$$R_1R_4 + \frac{L_1}{C_4} + j\left(\omega L_1 R_4 - \frac{R_1}{\omega C_4}\right) = R_2R_3 \quad \text{………………………………..(3)}$$

Separating real and imaginaries, we have

$$R_1R_4 + \frac{L_1}{C_4} = R_2R_3 \quad \text{……………………………………….(4)}$$

And, $\omega L_1 R_4 - \frac{R_1}{\omega C_4} = 0$

Or, $R_1 = \omega^2 C_4 L_1 R_4$ ………………………………….(5)

Solving expressions (4) and (5) we get

$$L_1 = \frac{R_2 R_3 C_4}{1 + \omega^2 {C_4}^2 {R_4}^2} \quad \text{……………………….(6)}$$

And $R_1 = \omega^2 C_4 R_4 \cdot \frac{R_2 R_3 C_4}{1 + \omega^2 {C_4}^2 {R_4}^2} = \frac{R_2 R_3 R_4 {C_4}^2 \omega^2}{1 + \omega^2 {C_4}^2 {R_4}^2}$ ……………………..(7)

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Expression (6) and (7) indicates that the balance condition is a function of frequency. However, the frequency need not be accurately known to determine inductance since ω appears only in a term which will be small compared to unity in cases where use of Hey bridge is indicated (i.e. where Q factor of coil is large).

Q-factor of the coil = $\frac{\omega L_1}{R_1} = \frac{\frac{\omega R_2 R_2 C_4}{1+\omega^2 {C_4}^2 {R_4}^2}}{\frac{R_2 R_3 R_4 {C_4}^2 \omega^2}{1+\omega^2 {C_4}^2 {R_4}^2}} = \frac{1}{\omega R_4 C_4}$

Now the term $\omega^2 {C_4}^2 {R_4}^2$ in the denominators of equation (6) and (7) has the value of 0.01 if Q=10 and even more smaller for higher values of Q so the term $\omega^2 {C_4}^2 {R_4}^2$ can be dropped without causing an appreciable error. In case, this term is to be included in calculations of L_1 and R_1 then it is of such minor importance that it may be computed with sufficient accuracy from an approximate value of frequency. Here it should be noted if the term $\omega^2 {C_4}^2 {R_4}^2$ is excluded from the equation, then it is same for L_1 as for the Maxwell Bridge.

$L_1 = \frac{R_2 R_3 C_4}{1+\omega^2 {C_4}^2 {R_4}^2}$

But $Q = \frac{1}{\omega R_4 C_4}$

So, $L_1 = \frac{R_2 R_3 C_4}{1+\left(\frac{1}{Q}\right)^2}$

For the value of Q greater than 10, the term $\omega^2 {C_4}^2 {R_4}^2$ will be smaller than $\frac{1}{100}$ and so can be neglected.

Therefore $L_1=R_2R_3C_4$ and it is the same as for the Maxwell Bridge.

The bridge has the advantage of requiring only a relatively low-value resistor for R_4 where as for large inductance low resistance coils, the Maxwell-wein would require a parallel resistance R_4 of very high value, perhaps 10^5 or 10^6Ω. Resistance boxes of such high values are very costly.

This bridge is not suited for the measurement of low Q-factor of the inductors, because in that case, the term $\omega^2 {C_4}^2 {R_4}^2$ in the denominator becomes very important. And then it is required to know the bridge frequency to a value accurate limit. Moreover with low value of Q-factor, it gives poor convergence in balancing.

Problem:

1. The four arms of a Hey bridge are arranged as follows: AB is a coil of unknown impedance, BC is a non-inductive resistor of 1000Ω, CD is a non-inductive resistor of 833Ω in series with a standard capacitor of 0.38µF. DA is non-inductive resistor of 16800Ω. If the supply frequency is 50Hz, determine the inductance of the resistance at the balanced condition.

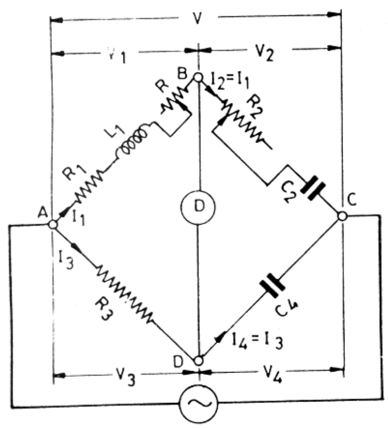
R_2 = 1000Ω; R_3 = 16800Ω; R_4 = 833Ω; C_4 = 0.38µF;

∴ $L_1 = \frac{R_2 R_3 C_4}{1+\omega^2 {C_4}^2 {R_4}^2} = \frac{1000\ X\ 16800\ X\ 0.38\ X\ 10^{-6}}{1+(2\pi\ X\ 50)^2\ X\ (0.38\ X\ 10^{-6})^2\ X\ (833)^2}$ = 6.321 H.

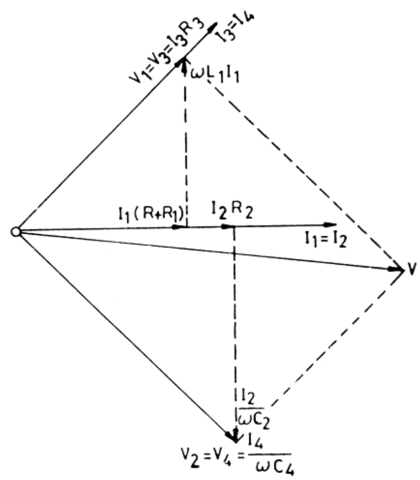
$R_1 = \frac{R_2 R_3 R_4 {C_4}^2 \omega^2}{1+\omega^2 {C_4}^2 {R_4}^2} = \frac{1000\ X\ 16800\ X\ 833\ X\ (2\pi\ X\ 50)^2\ X\ (0.38\ X\ 10^{-6})^2}{1+(2\pi\ X\ 50)^2\ X\ (0.38\ X\ 10^{-6})^2 (833)^2}$ = 197.487Ω

5. The Owen Bridge:

The bridge circuit also determines the unknown inductance in terms of resistance and capacitance. The advantage of this method is that the inductances over a wide range can be determined by this method by employing.



Circuit Diagram



Vector Diagram

The arrangement of the bridge as shown in above figure in which the unknown inductance L_1 whose resistance is R_1 is connected in series with another variable non-inductive resistor R. The capacitance C_2 is also connected in series with a non-inductive resistor and C_4 is known standard capacitor. The bridge is balanced by successively varying R_1 and R_2 in the circuit.

At balance $$\frac{R_1 + j\omega L_1 + R}{R_2 - \frac{j}{\omega C_2}} = \frac{R_3}{-\frac{j}{\omega C_4}}$$

Or, $$-\frac{j}{\omega C_4}(R_1 + j\omega L_1 + R) = R_3\left(R_2 - \frac{j}{\omega C_2}\right)$$

Equating real and imaginary quantities, separately

$$L_1 = R_2R_3C_4$$

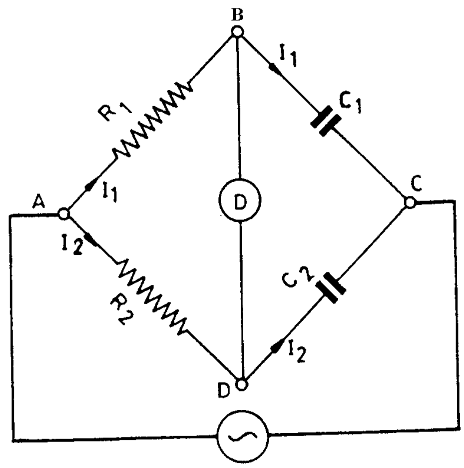
And $$R_1 = \frac{R_3C_4}{C_2} - R$$

Since the expression obtained for unknown quantities R_1 and L_1 does not have ω, so the balancing of bridge is independent of frequency and wave form.

Measurement of Capacitance:

1. **De Sauty's Bridge:**

   This bridge is used to determine the unknown capacitance by comparing it with known standard capacitor. The circuit is shown in below figure in which C_1 is a standard capacitor of known magnitude and R_1 and R_2 are known non-inductive resistances. The bridge is balanced by adjustment of either R_1 or R_2.



At balance $I_1R_1 = I_2R_2$

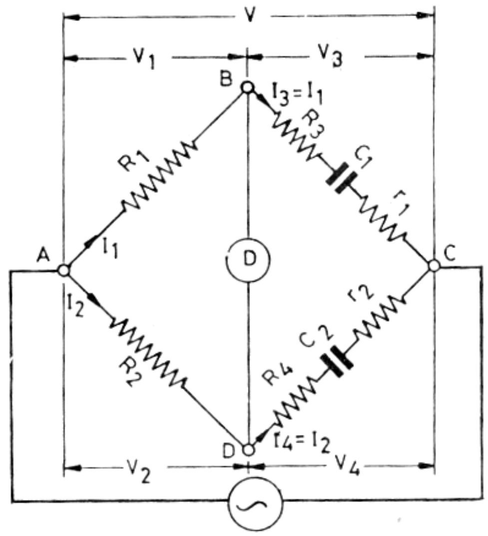
And $-\frac{j}{\omega C_1} I_1 = -\frac{j}{\omega C_2} I_2$

Then, $\frac{R_1}{-j/\omega C_1} = \frac{R_2}{-j/\omega C_2}$

Or, $C_1 = C_2 \frac{R_2}{R_1}$

This is very simple method but it is very difficult rather impossible to obtain a perfect balance if both of the capacitors are not free from dielectric loss. Only in the case of air capacitors a perfect balance can be obtained.

For computing two imperfect capacitors the bridge is modified by connecting resistances in series with them. R_3= and R_4 are the series resistances, where r_1 and r_2 are small resistances representing the loss components of the capacitors. Balance is obtained by variation of the resistances R_1, R_2, R_3, R_4.



At balance $\quad I_1R_1 = I_2R_2$

And $\quad I_1\left(R_3+r_1-\frac{j}{\omega C_1}\right) = I_2\left(R_4+r_2-\frac{j}{\omega C_2}\right)$

Then $\quad \frac{R_3+r_1-\frac{j}{\omega C_1}}{R_1} = \frac{R_4+r_2-\frac{j}{\omega C_2}}{R_2}$

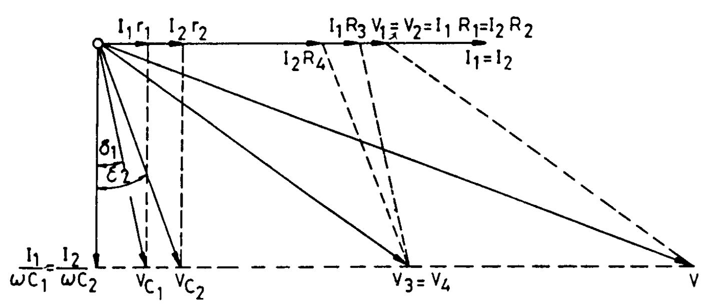
Or, $\quad R_1(R_4+r_2) - \frac{jR_1}{\omega C_2} = R_2(R_3+r_1) - \frac{jR_2}{\omega C_1}$

Equating real and imaginary quantities separately

$$R_1(R_4+r_2) = R_2(R_3+r_1)$$

And $\quad \frac{R_1}{C_2} = \frac{R_2}{C_1}$

Or, $\quad \frac{R_1}{R_2} = \frac{R_3+r_1}{R_4+r_2} = \frac{C_2}{C_1}$



Vector Diagram

The angles δ_1 and δ_2 are the phase angles of capacitors C_1 and C_2 respectively.

$\therefore \quad \delta_1 = \tan\delta_1 = \frac{r_1}{1/\omega C_1} = r_1\omega C_1$

$\delta_2 = \tan\delta_2 = \frac{r_2}{1/\omega C_2} = r_2\omega C_2 \qquad$ (δ_1 and δ_2 are being very small)

$$\delta_2 - \delta_1 = \omega(r_2C_2 - r_1C_1) = \omega(C_1R_3 - C_2R_4) = \omega C_1(R_3 - \frac{R_1R_4}{R_2})$$

From the above expression the phase angle of one capacitor can be determined in terms of the phase angle of the other one.

Problem:

1. In a modified De Sauty bridge measurement the following readings are obtained: R_1=1000Ω; R_2=1000Ω; R_3=2000Ω; R_4=2000Ω; C_1=1μF; f=1000Hz; r=10Ω (res. Of C_1). Find out phase angle errors and unknown capacitance.

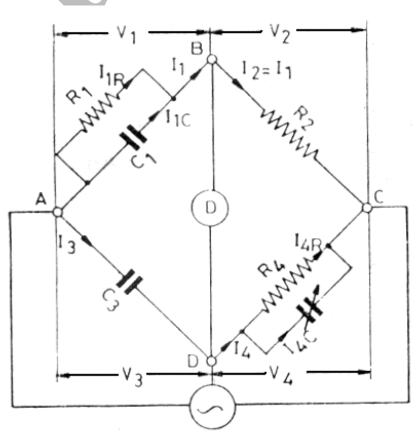
   R_1=1000Ω; R_2=1000Ω; R_3=2000Ω; R_4=2000Ω; $C_1=1X10^{-6}$ F;
   ω = 2πf = 2 X 3.14 X 1000 = 6.280; r_1=10Ω;
   Unknown Capacitance $C_2 = \frac{C_1R_1}{R_2} = 1 \times 10^{-6} X \frac{1000}{1000} = 1 X 10^{-6}$ F = 1μF

   $$r_2 = \frac{R_2(R_3+r_1)-R_1R_4}{R_1} = \frac{1000(2000+10)-1000\ X\ 2000}{1000} = 10\Omega$$

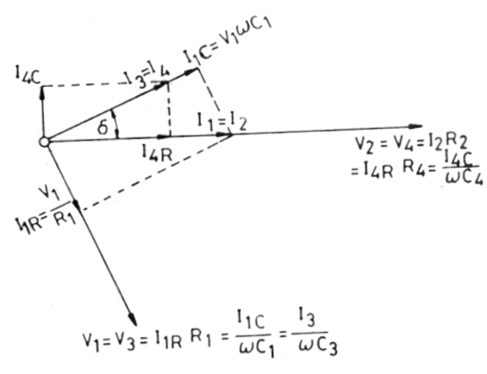
   Phase angle error, $\delta_1 = r_1\omega C_1 = 6280 X 10 X 1 X 10^{-6}$ rad $= 6280 X 10 X 1 X 10^{-6} X \frac{180}{\pi} = 3.6^0$

   Similarly, $\delta_2 = r_2\omega C_2 = 6280 X 10 X 1 X 10^{-6}$ rad $= 6280 X 10 X 1 X 10^{-6} X \frac{180}{\pi} = 3.6^0$

2. **Schering Bridge:**
   The circuit is shown in figure in which C_1 is the capacitor under test, R_1 is an imaginary resistance representing its dielectric loss component; C_3 is a standard capacitor. C_4 is a variable capacitor and R_2 and R_4 are known non-inductive resistors. The resistance R_2 is variable.



Circuit Diagram



Vector Diagram

The impedance of the Schering Bridge arms are:

$$Z_1 = \frac{1}{\frac{1}{R_1}+\frac{1}{-j/\omega C_1}} = \frac{1}{\frac{1}{R_1}+j\omega C_1} = \frac{R_1}{1+j\omega C_1 R_1}$$

$$Z_2 = R_2$$

$$Z_3 = -\frac{j}{\omega C_3}$$

$$Z_4 = \frac{R_4}{1+j\omega C_4 R_4}$$

Under balanced conditions

$$Z_1 Z_4 = Z_2 Z_3$$

i.e. $$\frac{R_1}{1+j\omega C_1 R_1} . \frac{R_4}{1+j\omega C_4 R_4} = -\frac{jR_2}{\omega C_3}$$

or, $$\frac{R_1 R_4}{1+j\omega C_1 R_1} . \frac{1-j\omega C_1 R_1}{1-j\omega C_1 R_1} = -\frac{jR_2}{\omega C_3} . (1+j\omega C_4 R_4)$$

or, $$\frac{R_1 R_4 (1-j\omega C_1 R_1)}{1+\omega^2 {C_1}^2 {R_1}^2} = -\frac{jR_2}{\omega C_3} . (1+j\omega C_4 R_4)$$

Equating real terms we have

$$\frac{R_1 R_4}{1+\omega^2 {C_1}^2 {R_1}^2} = \frac{C_4 R_4 R_2}{C_3}$$

or, $$\frac{R_1}{1+\omega^2 {C_1}^2 {R_1}^2} = \frac{C_4 R_2}{C_3}$$

From vector diagram for the capacitor C_1 and R_1 are in parallel

$$\text{Cos } \delta = \frac{I_{1C}}{I_1} = \frac{I_{1C}}{\sqrt{{I_{1c}}^2+{I_{1R}}^2}} = \frac{V_1 \omega C_1}{\sqrt{{V_1}^2 \omega^2 {C_1}^2 + \frac{{V_1}^2}{{R_1}^2}}} = \frac{\omega C_1 R_1}{\sqrt{1+\omega^2 {C_1}^2 {R_1}^2}}$$

or, $$\text{Cos}^2 \delta = \frac{\omega^2 {C_1}^2 {R_1}^2}{1+\omega^2 {C_1}^2 {R_1}^2}$$

Substituting value of $\frac{\omega^2 {C_1}^2 {R_1}^2}{1+\omega^2 {C_1}^2 {R_1}^2} = \text{Cos}^2 \delta$

$$\frac{Cos^2 \delta}{\omega^2 {C_1}^2 R_1} = \frac{C_4 R_2}{C_3}$$

$$C_1 = \frac{C_3 Cos^2 \delta}{\omega^2 C_1 C_4 R_1 R_2}$$

Again at balance condition

$$\tan \delta = \frac{I_{4C}}{I_{4R}} = \frac{V_4 \omega C_4}{V_4/R_4} = \omega C_4 R_4$$

and also tan δ = $\frac{I_{1R}}{I_{1C}} = \frac{V_1/R_1}{V_1\omega C_1} = \frac{1}{\omega C_1 R_1}$

∴ $\omega C_4 R_4 = \frac{1}{\omega C_1 R_1}$

$R_4 = \frac{1}{\omega^2 C_1 C_4 R_1}$

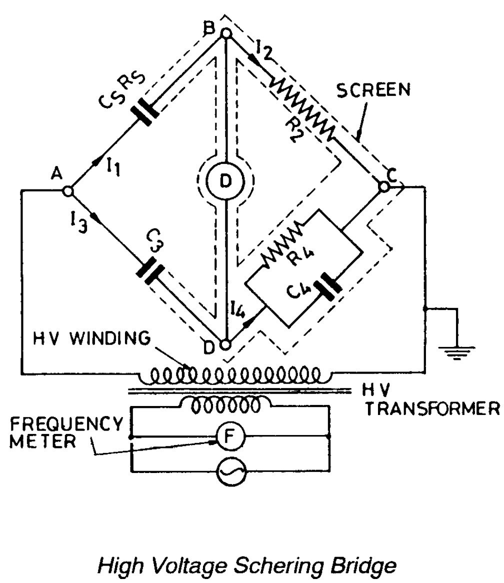
$C_1 = \frac{C_3 R_4 COS^2\delta}{R_2}$

Where δ is known as a loss angle of capacitor and sin δ is known as a power factor of the capacitor.

3. High Voltage Schering Bridge:

This is one of the most important and useful bridge circuits available for measurement of capacitance, dielectric loss of a capacitor and power factor of cables. It is generally used, both for precision measurements of capacitors at low voltage and for study of insulation and insulating structures at high voltages.

The bridge is also used for measurement of capacitance and dissipation factor for insulation and in determining the general properties of condenser, bushings, insulating oil and other insulating materials which in any case have very small capacitances, so for such types of measurements high voltage Schering bridge as shown in figure required. But certain modifications have to be made in Schering Bridge to operate at high voltage.



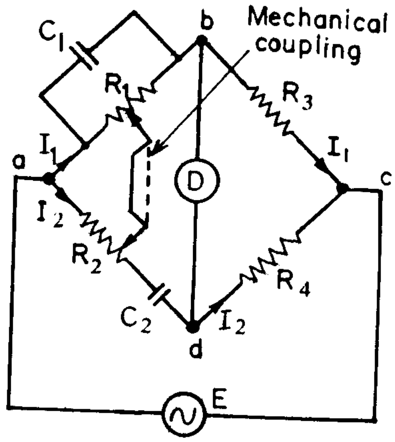
High Voltage Schering Bridge

1. In this case a high voltage supply is obtained from a transformer usually at the power frequency and the detector used is vibration galvanometer.

2. The Schering Bridge is advantageous for high voltage tests due to the fact that the arms AB and AD, each contains only capacitors, which can be specially constructed for high voltage work. Also these two arms will have very high impedances, at power frequencies in comparison with that of the arms BC and CD. Thus the major portion of the source potential difference exists across the arms AB and AD and the potential difference across the arms BC and CD, which contains the control, is small. In order to afford safely to the operator from the high voltage hazards, while carrying out control, junction point C is earthed.
   The arrangement is ideal from the point of view of safety but it is much less sensitive than its conjugate (with the source and detector interchanged). However at high voltages sensitivity, with a good detector will usually be ample for all practical purposes. In precise measurements at low voltages the conjugate bridge is often used since in this case sensitivity will be much greater.
3. In order to prevent dangerous rises of voltage across these arms in case of breakdown of either of the high voltage capacitors; a spark (set to breakdown at about 100 volts) is connected across each of the arms BC and CD.
4. The impedances of arms AB and AD are high so current from the source is less. The arrangement is ideal from the point of view of power consumption but it is much less sensitive so a sensitive detector has to be used.
5. In operating the Schering Bridge at high voltages and in the precise measurements at low voltages it is extremely important that the effect of stray capacitances between the bridge elements be eliminated. This is done by enclosing the vulnerable portions of the network within electrostatic shields and by maintaining these shields at suitable potentials. Such a shielded bridge suitable for high voltage operation is shown in figure. Earth capacitance effect on the galvanometer and leads is eliminated by using Wagner earth device.
6. A capacitor C_3 of fixed value is used which has normally air or compressed gas as dielectric. The dissipation factor with dry and clean gas is practically zero but loss in the insulating supports cannot be avoided. However such losses can be prevented from influencing the measurement by the use of a guard ring from which both the high and low voltage electrodes are supported. Then the current through the high voltage supports passes direct to earth and as the potential difference between the low voltage electrode and the guard ring is very small, the insulation of the low voltage electrode has only a very small effect. In case, solid dielectric standard capacitor is used then the losses must be accurately known.

Measurement of Frequency:

1. **Wien's Bridge:**
   The Wien's Bridge is primarily known as a frequency determining bridge. A Wien's bridge may be employed in a harmonic distortion analyzer, where it is used as notch filter, discriminating against one specific frequency. The Wien's bridge also finds applications in audio and HF oscillators as the frequency determining device,



In the above figure shows a Wien's bridge under balance conditions.

At balance $\left(\frac{R_1}{1+j\omega C_1 R_1}\right) R_4 = \left(R_2 - \frac{j}{\omega C_2}\right) R_3$

or, $\frac{R_4}{R_3} = \frac{R_2}{R_1} + \frac{C_1}{C_2} + j\left(\omega C_1 R_2 - \frac{j}{\omega C_2 R_1}\right)$

Equating the real and imaginary parts,

$\frac{R_4}{R_3} = \frac{R_2}{R_1} + \frac{C_1}{C_2}$ ……………………………………….(1)

and $\omega C_1 R_2 - \frac{j}{\omega C_2 R_1} = 0$

From which $\omega = \frac{1}{\sqrt{R_1 R_2 C_1 C_2}}$

And frequency $f = \frac{1}{2\pi\sqrt{R_1 R_2 C_1 C_2}}$ Hz …………………………..(2)

In most Wien bridges, the components are chosen that $R_1 = R_2 = R$ and $C_1 = C_2 = C$

Then equation (1) reduces to $\frac{R_4}{R_3} = 2$

And equation (2) reduces to $f = \frac{1}{2\pi RC}$

Switches for resistors R_1 and R_2 are mechanically linked so as to fulfill the condition $R_1 = R_2$. As long as C_1 and C_2 are fixed capacitors equal in value and $R_4 = 2R_3$, the Wien's bridge may be used as a frequency determining device, balanced by single control. This control may be directly calibrated in terms of frequency as is evident from equation (3).

This bridge is suitable for measurement of frequencies from 100Hz to 100 KHz. It is possible to obtain an accuracy of 0.1% to 0.5%.

Apparatus Used in Conjunction with AC Bridges:

Now it has become clear that the apparatus required for the construction and use of AC bridges may be classified under the following three headings:

1. Standards of resistance, inductance and capacitance.
2. Sources of alternating current
3. Detectors.

Sources of alternating current: These may be divided into the following classes:

(a) Interrupters
(b) Microphone Hummers
(c) Alternators
(d) Oscillators

(a) Interrupters:

An interrupter is a very common source of alternating current used for bridge measurements. This consists of a small induction coil or transformer and an interrupter. The primary circuit of the transformer contains a battery and an interrupter working at a constant frequency supplies the current to the bridge.

A simple example of interrupters may be an electrically maintained tuning fork of suitable frequency, operating a contact which regularly opens or closes the primary circuit.

The frequency depends on the pitch of the fork and is therefore very constant. The waveform at the secondary terminals is fairly pure. Frequencies which may be obtained lie between 50Hz and about 1000Hz. The disadvantage of interrupters is the small amount of power available.

(b) Microphone Hummers:

The principle utilized in a microphone hummer is that when a microphone, connected in series with a battery is subjected to suitable mechanical vibrations the current through it will be made to vary. This varying current can be fed back in the form of vibrations, produced by the feedback system, to maintain the oscillation of the microphone granules. Thus this principle is similar to that applied in an oscillator.

For an example of the microphone hummers,