Digital Controllers

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Digital Controller in a Process Control Loop

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In a processor based digital controller, rapid switching from one algorithm to another *(e.g. a P controller to a PID controller)* **and automatic tuning of controller parameters are possible.**

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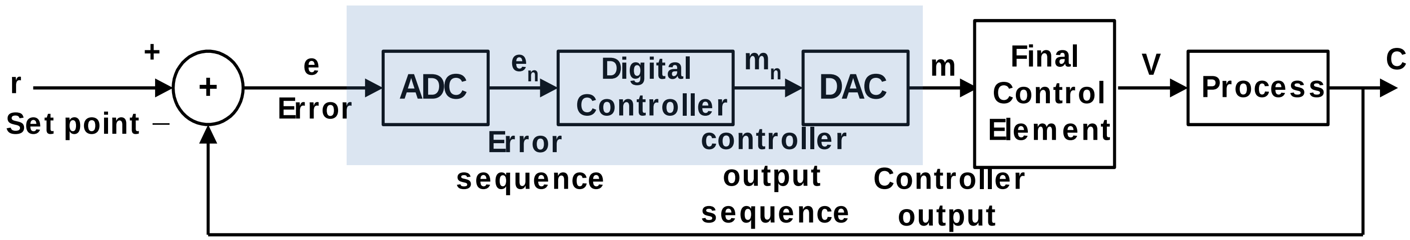

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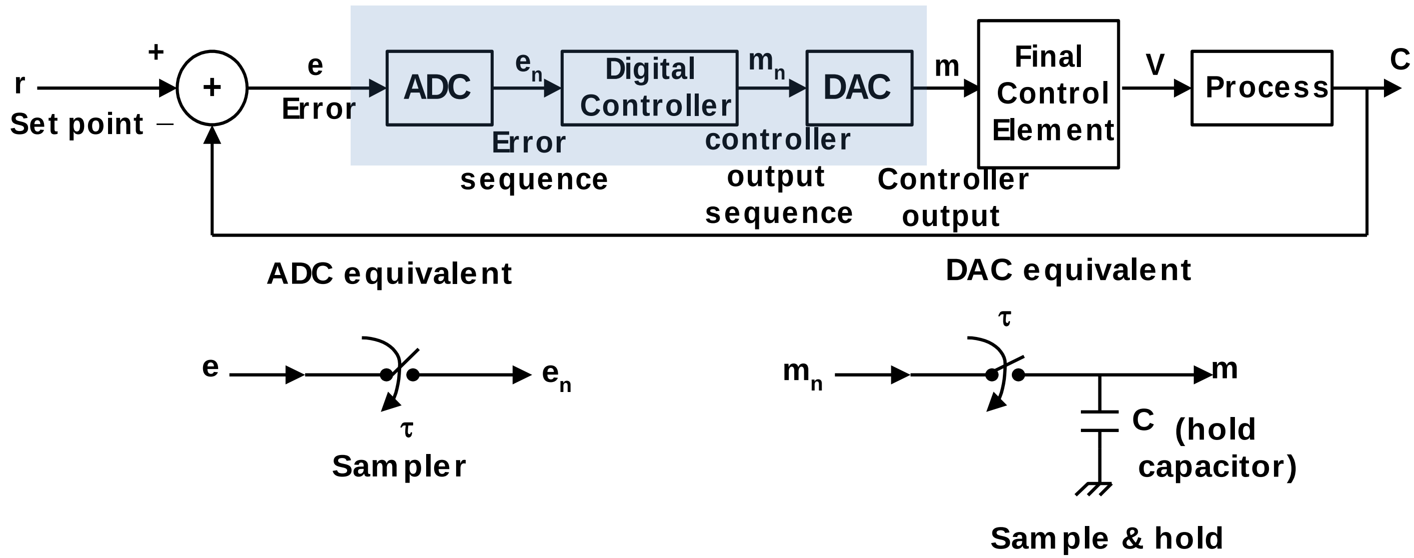

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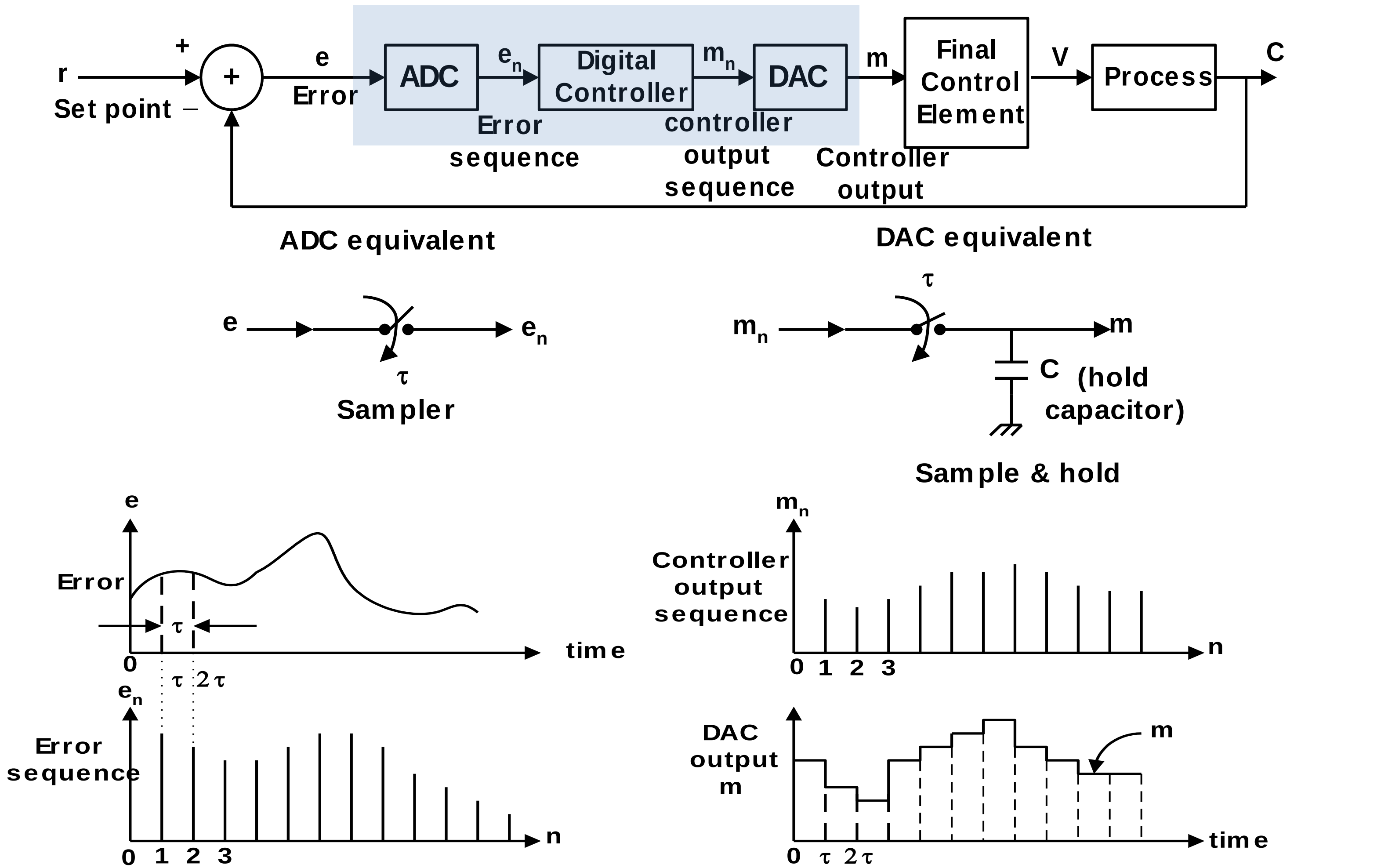

τ → Sampling interval

τ ≥ (ADC conversion time + computation time)

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The error computation may be performed digitally if the set-point is available in digital form (say, from digital keyboard) and the measured variable is digitized with the ADC.

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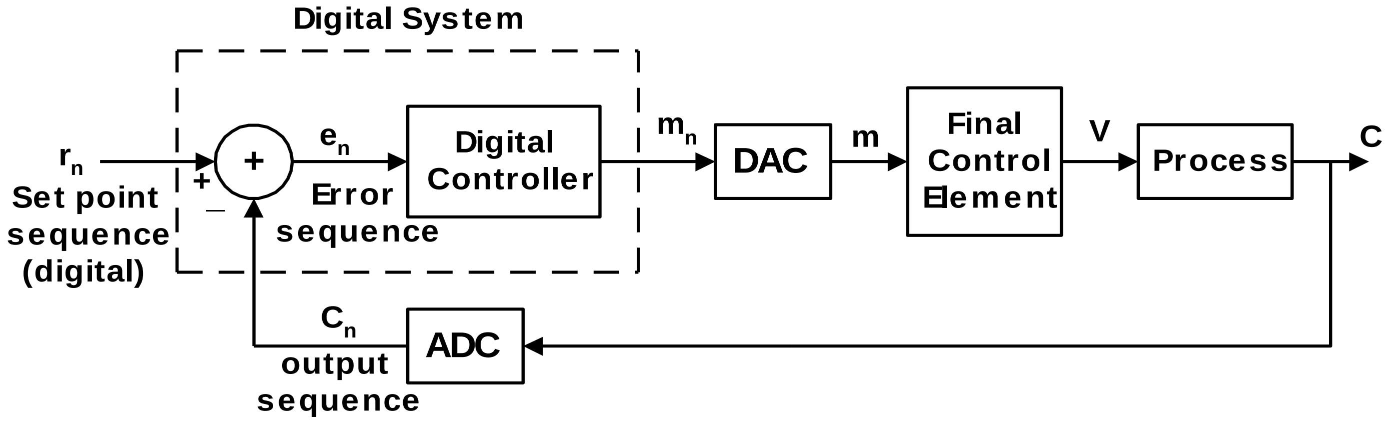

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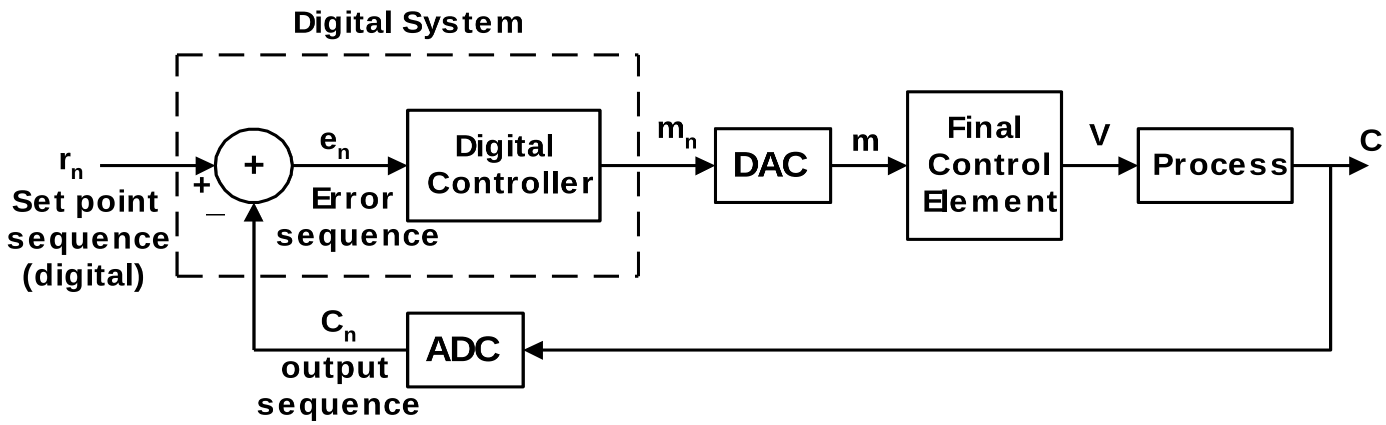

Proper selection of the **sampling interval 'τ'** is necessary for satisfactory operation of the process control loop.

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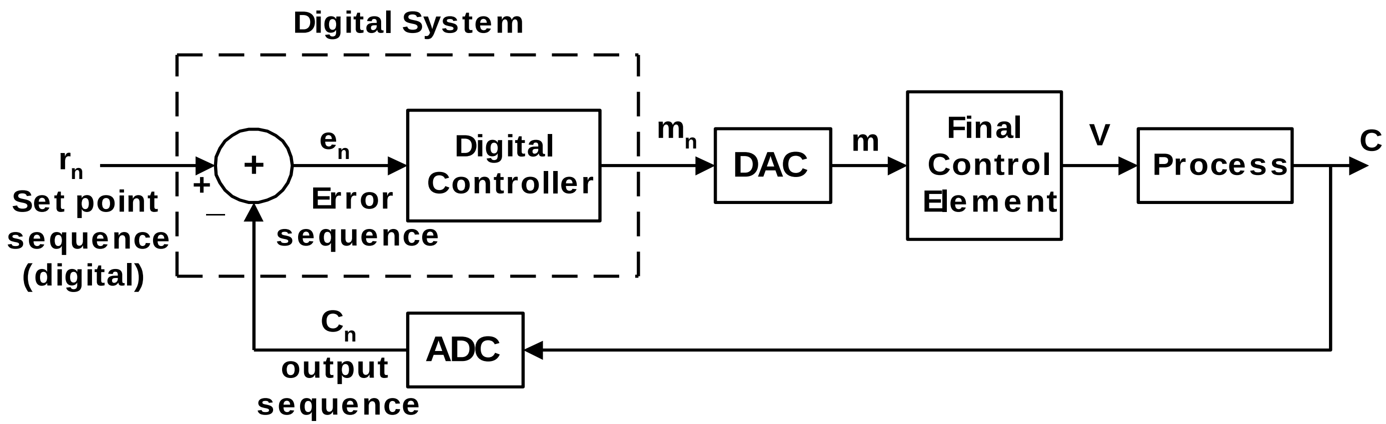

Proper selection of the **sampling interval 'τ'** is necessary for satisfactory operation of the process control loop.

A large 'τ' may lead to **unstable operation** of the loop (because of the extra lag introduced in the loop), whereas a very small 'τ' requires a **high speed digital hardware** (hence high cost) to implement the controller.

Realization of Digital Controllers
(through discrete approximation of analog controllers)

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Proportional (P) Controller

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$$m = K_p e + b$$

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The controller output at the nth instant is given by

$$m_n = K_p e_n + b_n = m_n' + b_n$$

where, K_p = proportional gain,
b_n = fixed bias.

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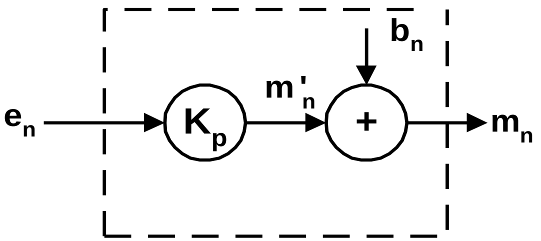
Realization of the P controller:



Realization of Digital Controllers
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Proportional-Integral (PI) Controller

The analog controller output is

$$m = K_p \left[e + \frac{1}{T_i} \int_o^t e.dt \right] + b$$

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$= m' + b$, where T_i is the integral time and

$$m' = K_p \left[e + \frac{1}{T_i} \int_o^t e.dt \right]$$

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Now, $\int_o^{n\tau-\tau} e.dt = \int_o^{(n-1)\tau} e.dt = I_{n-1}$

Then, $I_n = I_{n-1} + \int_{(n-1)\tau}^{n\tau} e.dt$

PI Controller

$$I_n = I_{n-1} + \int_{(n-1)\tau}^{n\tau} e.dt$$

The second term of the above relation represents the area under the curve 'e' for $(n-1)\tau \leq t \leq n\tau$.

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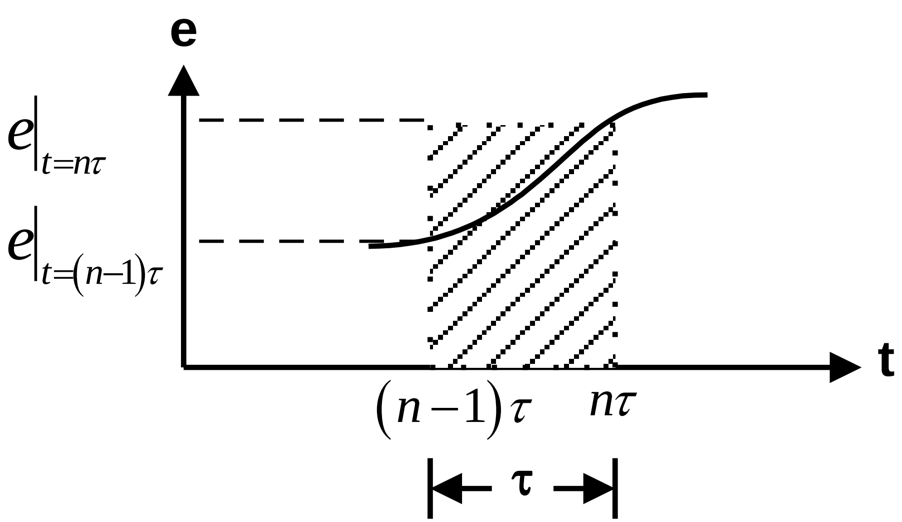
The second term of the above relation represents the area under the curve 'e' for $(n - 1)\tau \leq t \leq n\tau$.

This area may be approximated by the shaded rectangle (called the **method of rectangular integration**) as



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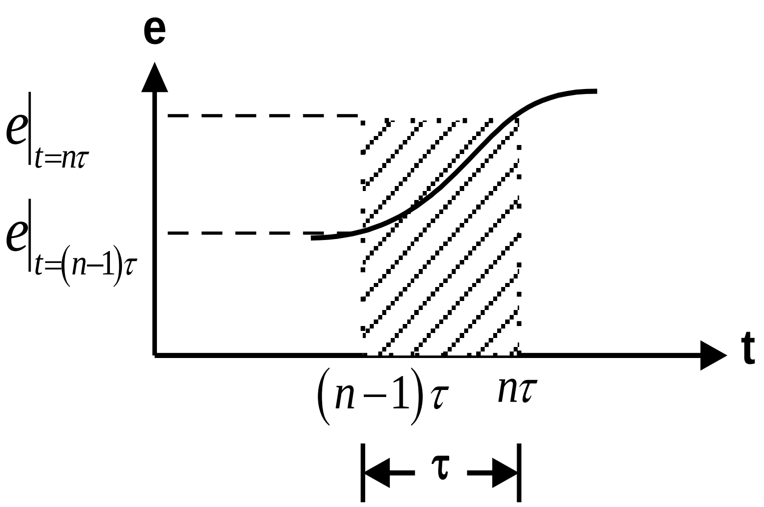

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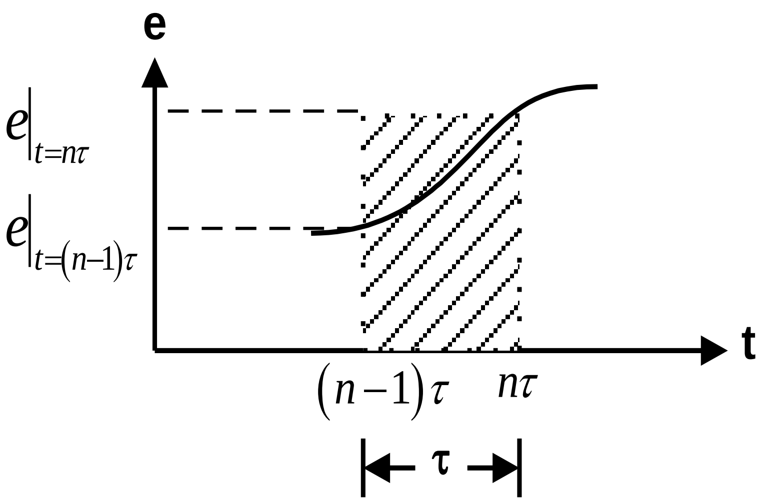

Thus, $$\int_{(n-1)\tau}^{n\tau} e.dt \approx \tau e|_{t=n\tau}$$

Using the notation e_n for $e|_{t=n\tau}$ **,** $$I_n = I_{n-1} + \tau e_n$$

PI Controller

Now, output m′ at the nth instant may be expressed as

$$m'_n = K_p \left[e_n + \frac{I_n}{T_i} \right]$$ where $$I_n = I_{n-1} + \tau e_n$$

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$$m'_{n-1} = K_p \left[e_{n-1} + \frac{I_{n-1}}{T_i} \right]$$

The difference between these two outputs is

$$m'_n - m'_{n-1} = K_p \left[e_n - e_{n-1} + \frac{1}{T_i} \left(I_n - I_{n-1} \right) \right]$$

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Substituting the value of $\boldsymbol{(I_n - I_{n-1})}$ for rectangular integration, $\boldsymbol{[(I_n - I_{n-1}) = \tau e_n]}$

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$$m'_n - m'_{n-1} = K_p\left[e_n - e_{n-1} + \frac{\tau e_n}{T_i}\right]$$

or, $$m'_n = e_n K_p\left(1 + \frac{\tau}{T_i}\right) - K_p e_{n-1} + m'_{n-1}$$

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$$= a_o e_n + a_1 e_{n-1} + m'_{n-1}, \qquad \text{say}$$

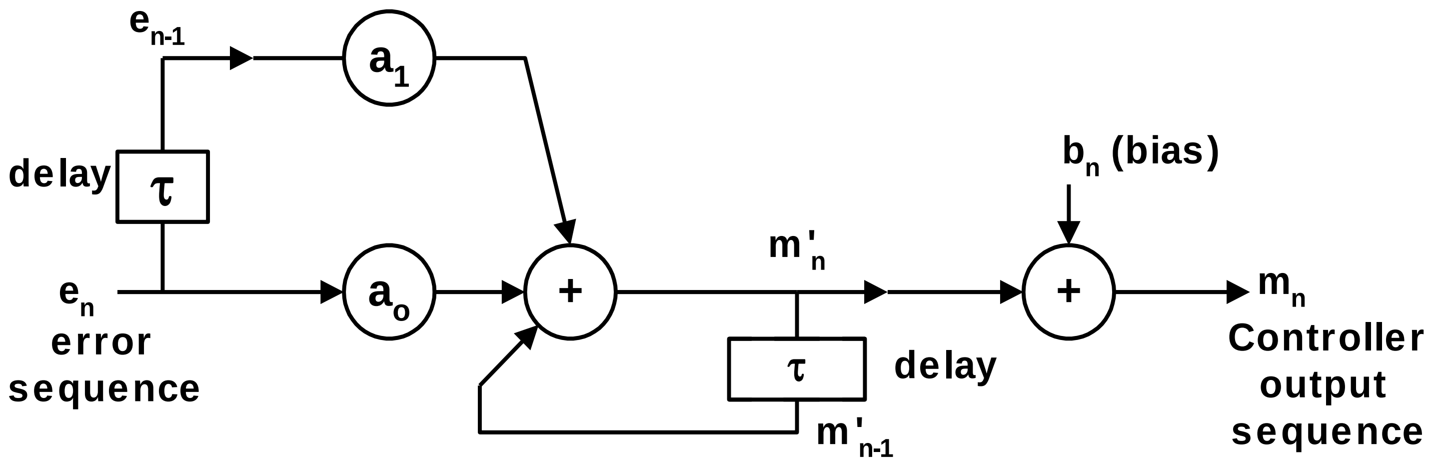
where, $a_o = K_p\left[1 + \dfrac{\tau}{T_i}\right]$ and $a_1 = -K_p$

Realization of the PI Controller

Controller output at the nth instant:

$$m_n = m'_n + b_n$$

$$m'_n = a_0 e_n + a_1 e_{n-1} + m'_{n-1}$$



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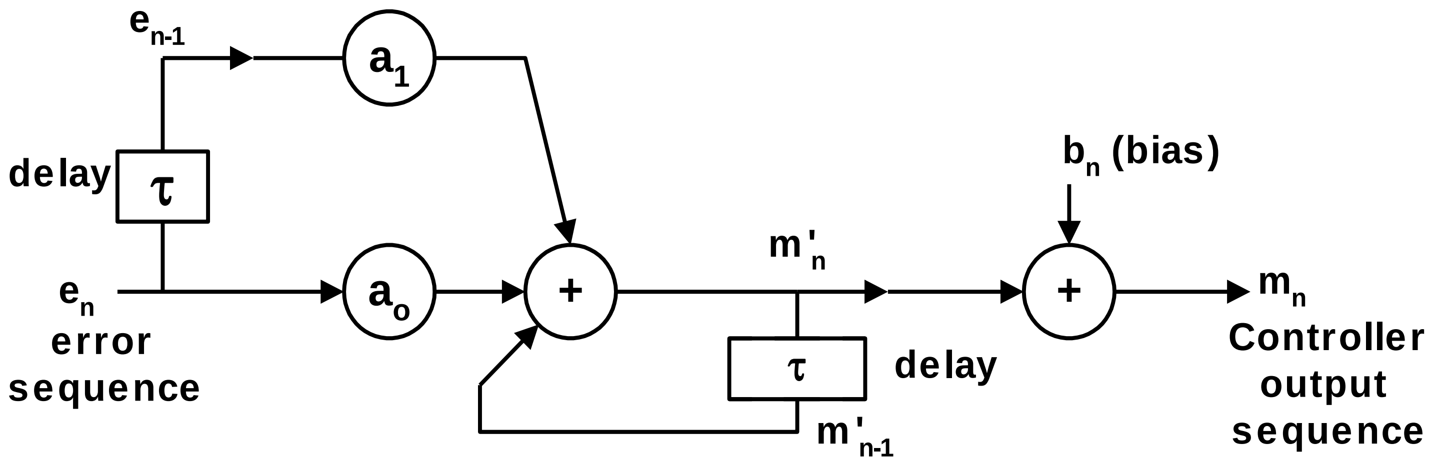

Problem: Develop a digital PI Controller using Trapezoidal rule for integration.

Velocity or incremental form of PI controller

The controller output is proportional to the derivative of a standard PI controller and it may be expressed as (without bias):

$$\Delta m_n = m_n - m_{n-1} = m'_n - m'_{n-1} = a_o e_n + a_1 e_{n-1}$$

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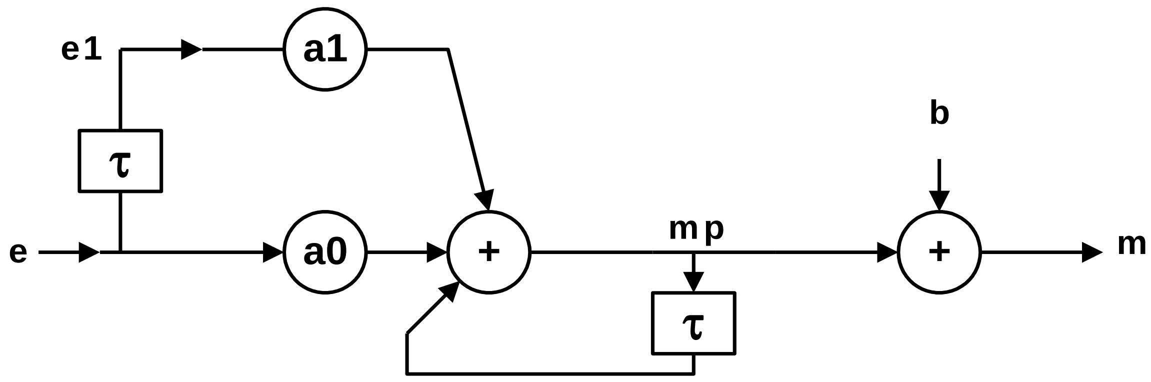
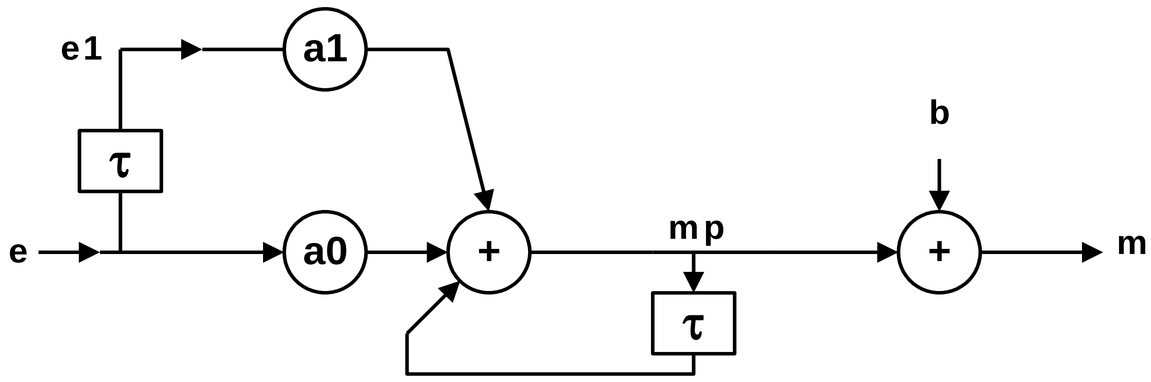
Velocity form of controller is useful when the actuator is some kind of adder (integral action), like a stepping motor.

Software Realization of the PI Controller



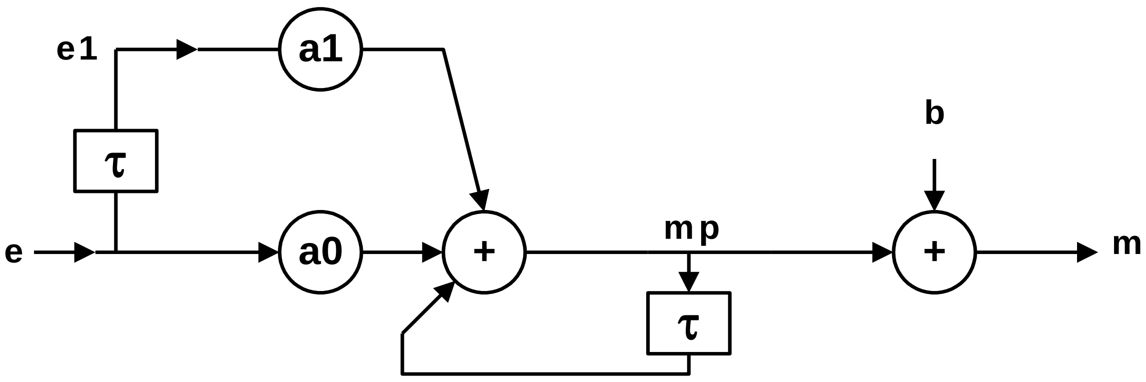

```
# include < studio.h>
void main (void)
      {
      float e = 0, e1, m, mp = 0;
      float a0, a1, b;
      float adc (void) ;        // digitized error
      void dac (float m ) ;   // analog output
      a0 = - - - - - - - - -  ;   // Kp [1 + τ/Ti ]
      a1 = - - - -  --  - - ;   // -Kp
      b = - - - - - - - - - - ;    // bias
```

Software Realization of the PI Controller




```
for (;;)   // continuous loop
      {    // loop time is the sampling interval τ
      e1 = e;
      e = adc ();
      mp = mp + a0*e + a1*e1;
      m = mp + b;
          // provision for saturation
      if (m < 0) m = 0;
      if (m > 100) m = 100;
      dac (m);
      }
}
```

Software Realization of the PI Controller




```
float adc (void)      // Analog-to-digital conversion
      {
      float  v;
      scanf (″%f″, &v);  // for (keyboard) simulation
      return  v;         // (to be replaced for actual
      }                  // realization)

void dac (float m)  // Digital-to-analog conversion
      {
      printf (″%f\n″, m);   // for (VDU) simulation
      }    // (to be replaced for actual realization)
```

Realization of Digital Controllers
(through discrete approximation of analog controllers)

Proportional-Derivative (PD) Controller

The analog controller output is

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where T_d is the derivative time.

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Let the derivative of error 'e' at the nth instant be

$$D_n = \left.\frac{de}{dt}\right|_{t=n\tau}$$

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Let the derivative of error 'e' at the nth instant be $D_n = \left. \frac{de}{dt} \right|_{t=n\tau}$

D_n may be approximated using the **backward difference algorithm** as

$$D_n \approx \frac{e_n - e_{n-1}}{\tau}$$

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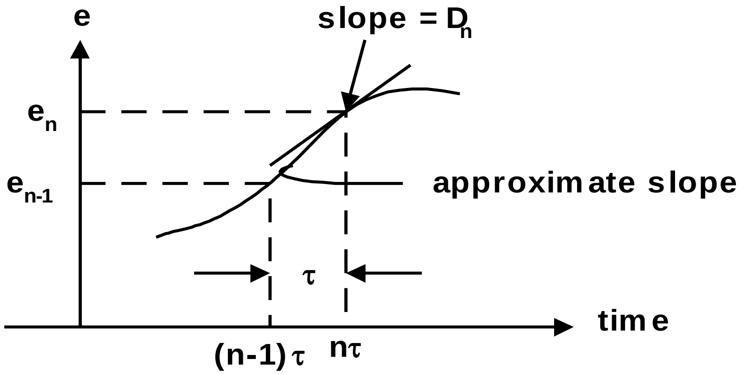

PD Controller

Thus the controller output at the nth instant is

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$$\text{or} \quad m_n = K_p \left[1 + \frac{T_d}{\tau} \right] e_n - \left[\frac{K_p T_d}{\tau} \right] e_{n-1} + b_n$$

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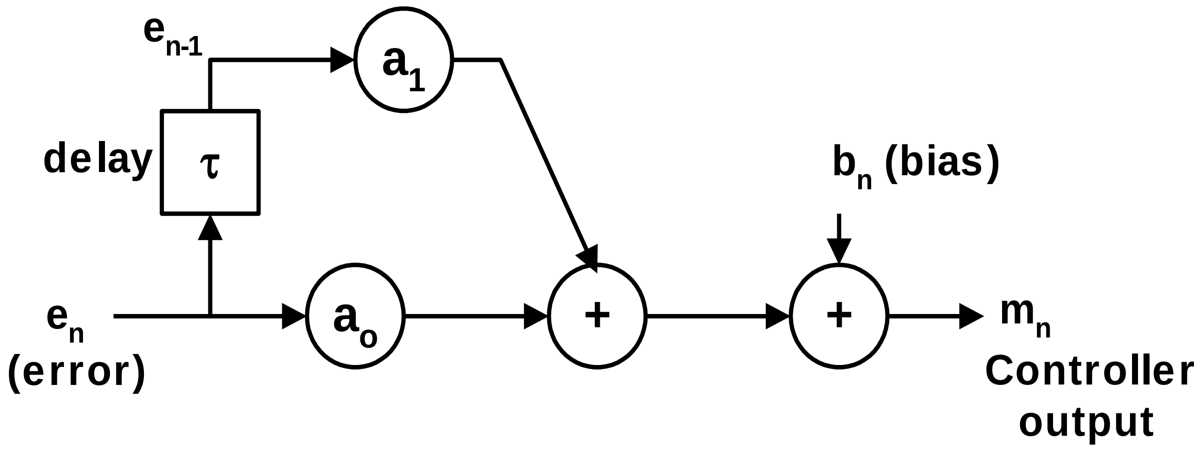
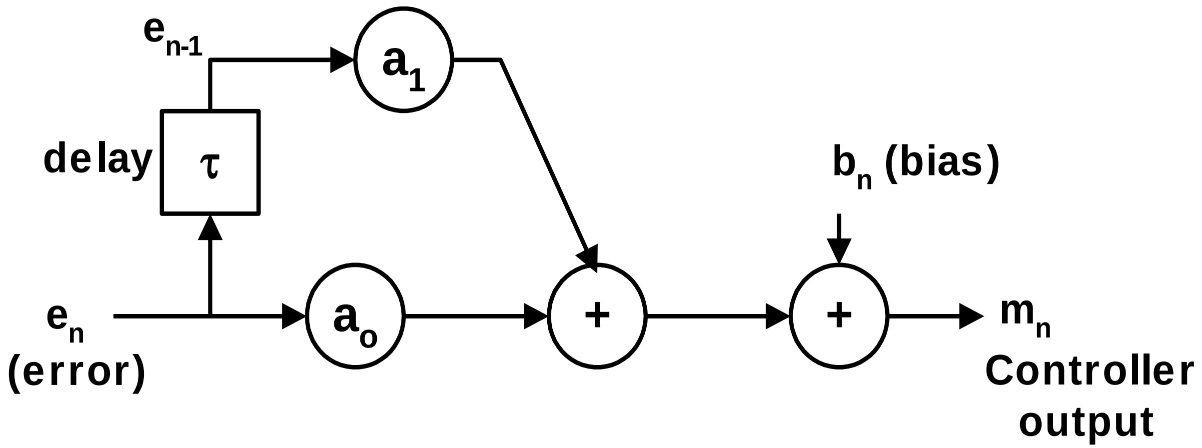
$$= a_o e_n + a_1 e_{n-1} + b_n$$

where $a_o = K_p \left(1 + \frac{T_d}{\tau} \right)$ and $a_1 = -\frac{K_p T_d}{\tau}$

Realization of the PD Controller



Realization of the PD Controller



Problem: Develop a 'c' program for software realization of the PD Controller.

PD Controller

Provision for anti-derivative kick

To avoid derivative action from a sudden change in set-point, the derivative action is generally derived from the measured output.

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Thus, the controller output may be expressed as,

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using **backward difference** algorithm, the controller output at the nth instant is

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$$= a_o e_n + p_o c_n + p_1 c_{n-1} + b_n$$

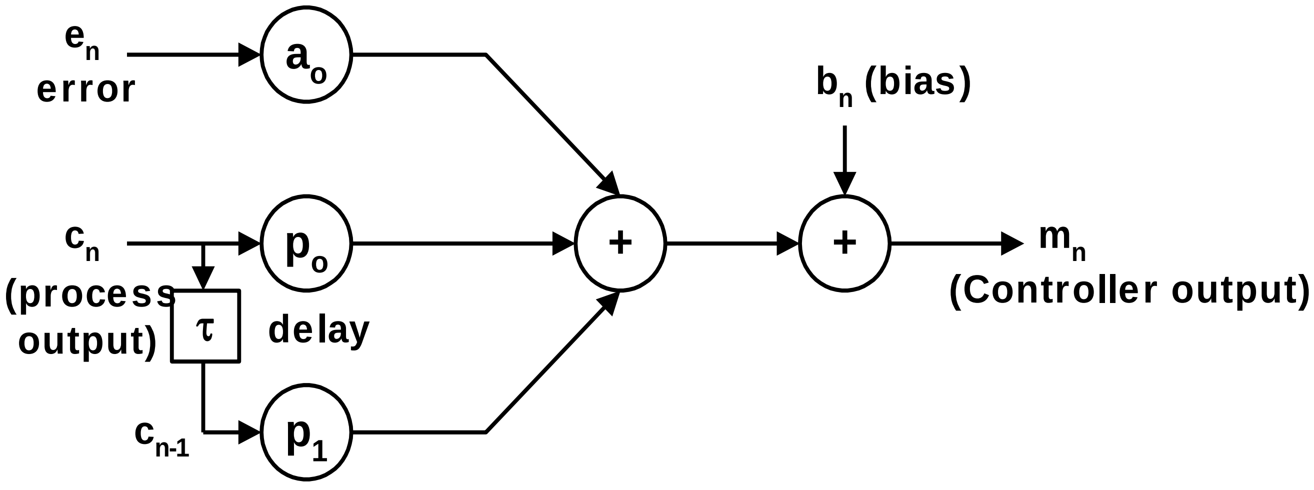
where

$$a_o = K_p$$

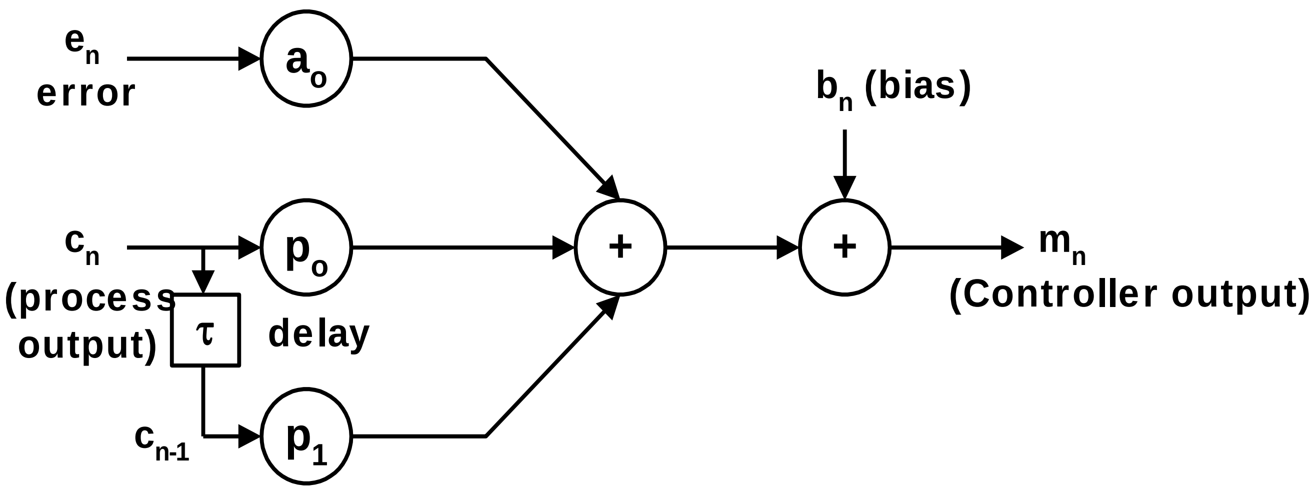
$$p_o = -\frac{K_p T_d}{\tau}$$

$$p_1 = \frac{K_p T_d}{\tau} = -p_o$$

Realization of PD Controller with anti-derivative kick



Realization of PD Controller with anti-derivative kick



Problem: Develop a ‘c’ program for software realization of the PD Controller with anti-derivative kick

Realization of Digital Controllers
(through discrete approximation of analog controllers)

Proportional-Integral-Derivative (PID) Controller

The analog controller output is

$$m = K_p \left[e + \frac{1}{T_i}\int_o^t e\,dt + T_d \frac{de}{dt} \right] + b$$

$$= m' + b \quad \text{(say)}$$

where $$m' = K_p \left[e + \frac{1}{T_i}\int_o^t e\,dt + T_d \frac{de}{dt} \right]$$

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$$= m' + b \quad \text{(say)}$$

where $$m' = K_p\left[e + \frac{1}{T_i}\int_o^t e\,dt + T_d\frac{de}{dt}\right]$$

The controller output (without bias) at the nth instant, using backward difference algorithm, is

$$m'_n = K_p\left[e_n + \frac{I_n}{T_i} + T_d\left(\frac{e_n - e_{n-1}}{\tau}\right)\right]$$

PID Controller

The controller output (without bias) at the nth instant is

$$m'_n = K_p\left[e_n + \frac{I_n}{T_i} + T_d\left(\frac{e_n - e_{n-1}}{\tau}\right)\right]$$

The controller output at the (n-1)th instant is

$$m'_{n-1} = K_p\left[e_{n-1} + \frac{I_{n-1}}{T_i} + T_d\left(\frac{e_{n-1} - e_{n-2}}{\tau}\right)\right]$$

PID Controller

The controller output (without bias) at the nth instant is

$$m'_n = K_p\left[e_n + \frac{I_n}{T_i} + T_d\left(\frac{e_n - e_{n-1}}{\tau}\right)\right]$$

The controller output at the (n-1)th instant is

$$m'_{n-1} = K_p\left[e_{n-1} + \frac{I_{n-1}}{T_i} + T_d\left(\frac{e_{n-1} - e_{n-2}}{\tau}\right)\right]$$

Subtracting,

$$m'_n - m'_{n-1} = K_p\left[e_n - e_{n-1} + \frac{I_n - I_{n-1}}{T_i} + \frac{T_d}{\tau}\left(e_n + e_{n-2} - 2e_{n-1}\right)\right]$$

PID Controller

$$m'_n - m'_{n-1} = K_p\left[e_n - e_{n-1} + \frac{I_n - I_{n-1}}{T_i} + \frac{T_d}{\tau}\left(e_n + e_{n-2} - 2e_{n-1}\right)\right]$$

Using rectangular integration algorithm, $I_n - I_{n-1} = \tau e_n$

PID Controller

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Using rectangular integration algorithm,

$$I_n - I_{n-1} = \tau e_n$$

then,

$$m'_n - m'_{n-1} = K_p\left[e_n - e_{n-1} + \frac{\tau}{T_i}e_n + \frac{T_d}{\tau}\left(e_n - 2e_{n-1} + e_{n-2}\right)\right]$$

PID Controller

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or, $m'_n = e_n\left(1 + \frac{\tau}{T_i} + \frac{T_d}{\tau}\right)K_p - e_{n-1}\left(\frac{2T_d}{\tau} + 1\right)K_p + e_{n-2}\left(\frac{K_p T_d}{\tau}\right) + m'_{n-1}$

PID Controller

$$m'_n - m'_{n-1} = K_p\left[e_n - e_{n-1} + \frac{I_n - I_{n-1}}{T_i} + \frac{T_d}{\tau}\left(e_n + e_{n-2} - 2e_{n-1}\right)\right]$$

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or, $$m'_n = a_o e_n + a_1 e_{n-1} + a_2 e_{n-2} + m'_{n-1}$$

PID Controller

$$m'_n = a_o e_n + a_1 e_{n-1} + a_2 e_{n-2} + m'_{n-1}$$

where, $$a_o = K_p \left(1 + \frac{\tau}{T_i} + \frac{T_d}{\tau} \right)$$

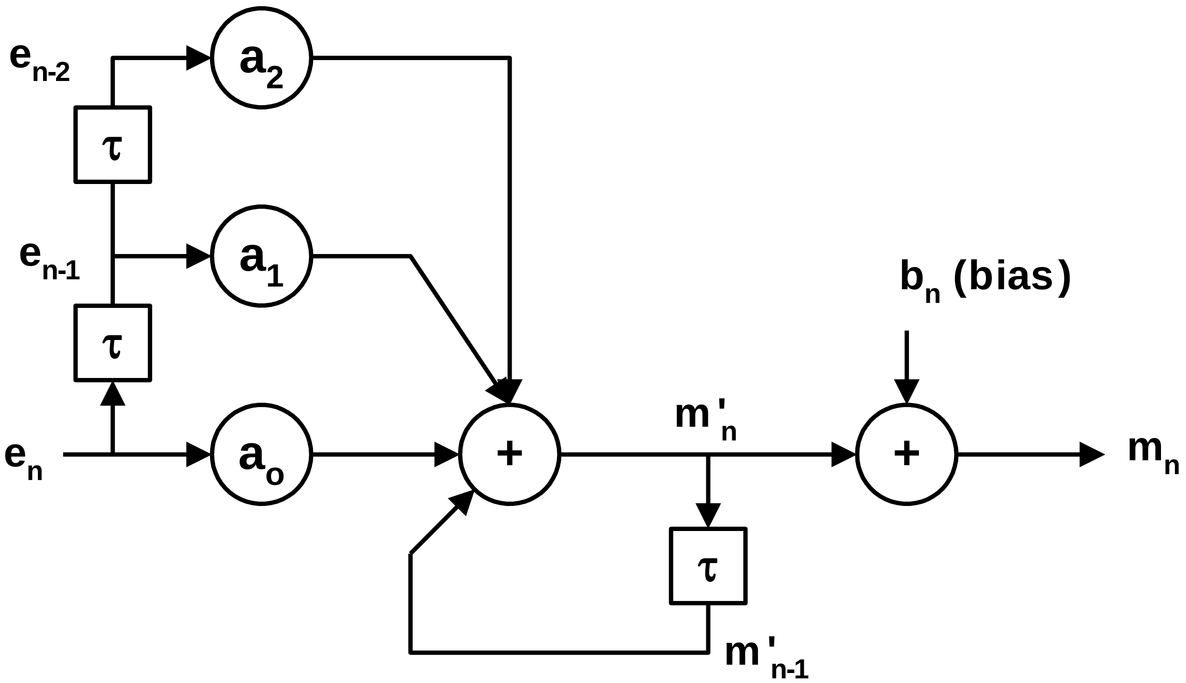
$$a_1 = -K_p \left(\frac{2T_d}{\tau} + 1 \right)$$

and $$a_2 = \frac{K_p T_d}{\tau}$$

Realization of the PID Controller

$$m'_n = a_o e_n + a_1 e_{n-1} + a_2 e_{n-2} + m'_{n-1}$$

and $m_n = m'_n + b_n$



PID Controller

Problems:

1. Develop a digital PID controller using trapezoidal rule for integration
2. Develop a program in ‘C’ for software realization of the PID controller
3. Modify the above controller to provide anti-derivative kick feature

Techniques used for anti-integral windup

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- **By saturating or limiting the integral value**

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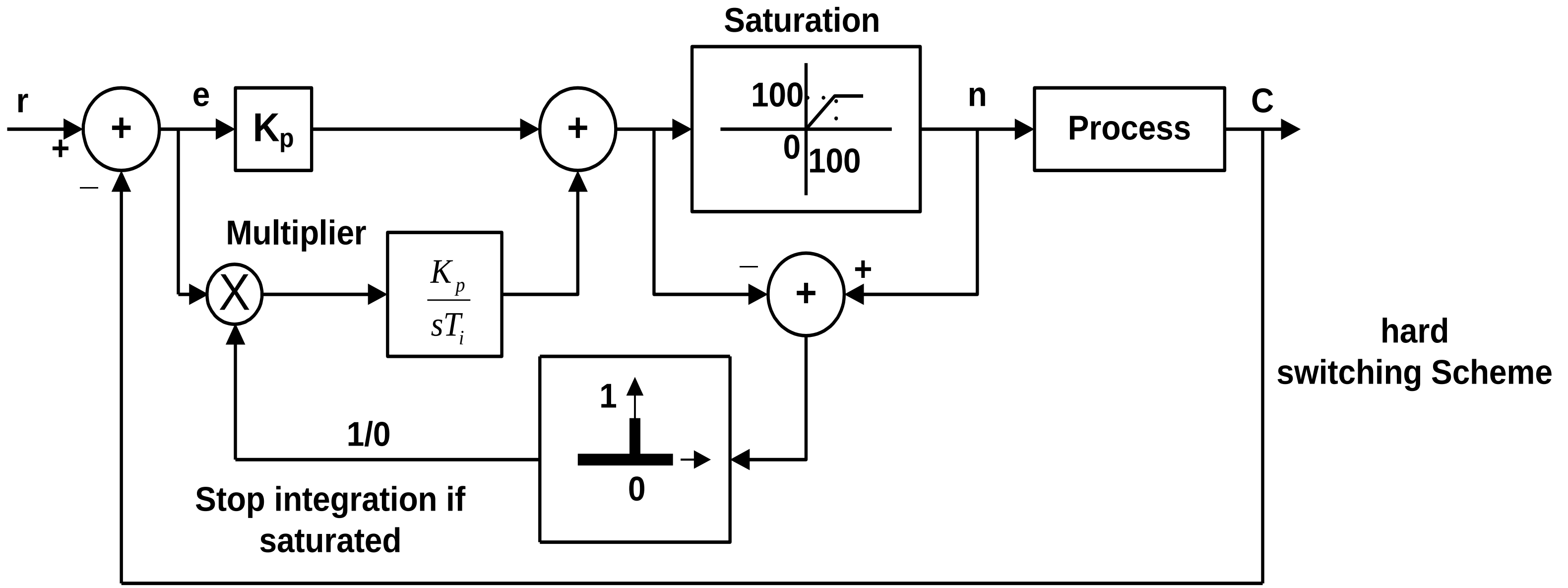
Techniques used for anti-integral windup

- **By saturating or limiting the integral value**
- **By resetting the integral value to zero**
- **By omitting the integral term**
- **By adaptive adjustment of controller parameters**

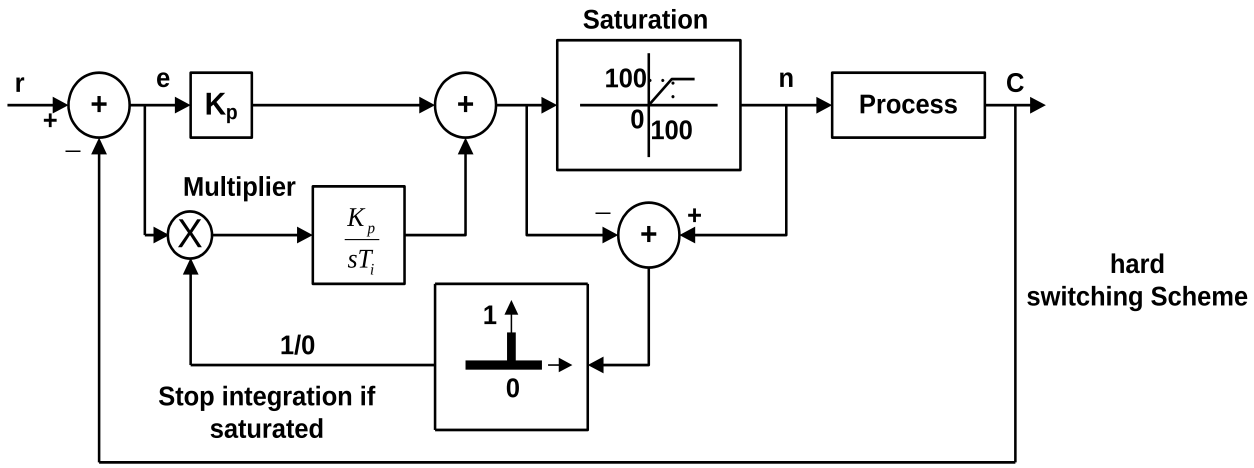
Some anti-integral windup schemes

Stop integration when PI/PID controller internal output (prior to the saturation block) exceeds the saturation limits

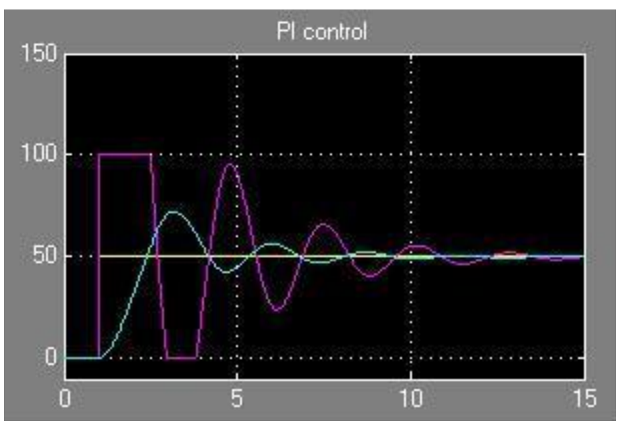


Scheme for PI Controller

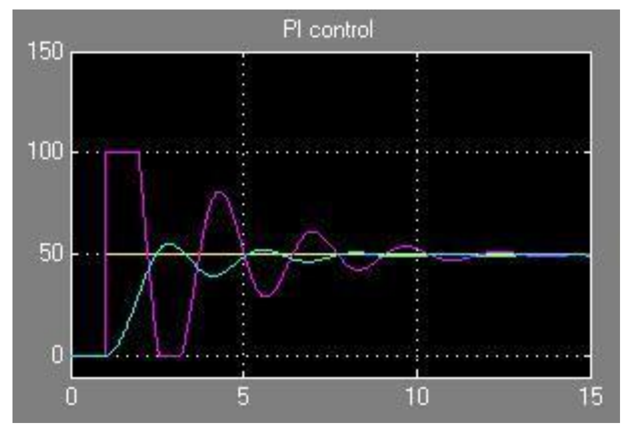
Performance of anti-integral windup scheme



Scheme for PI Controller



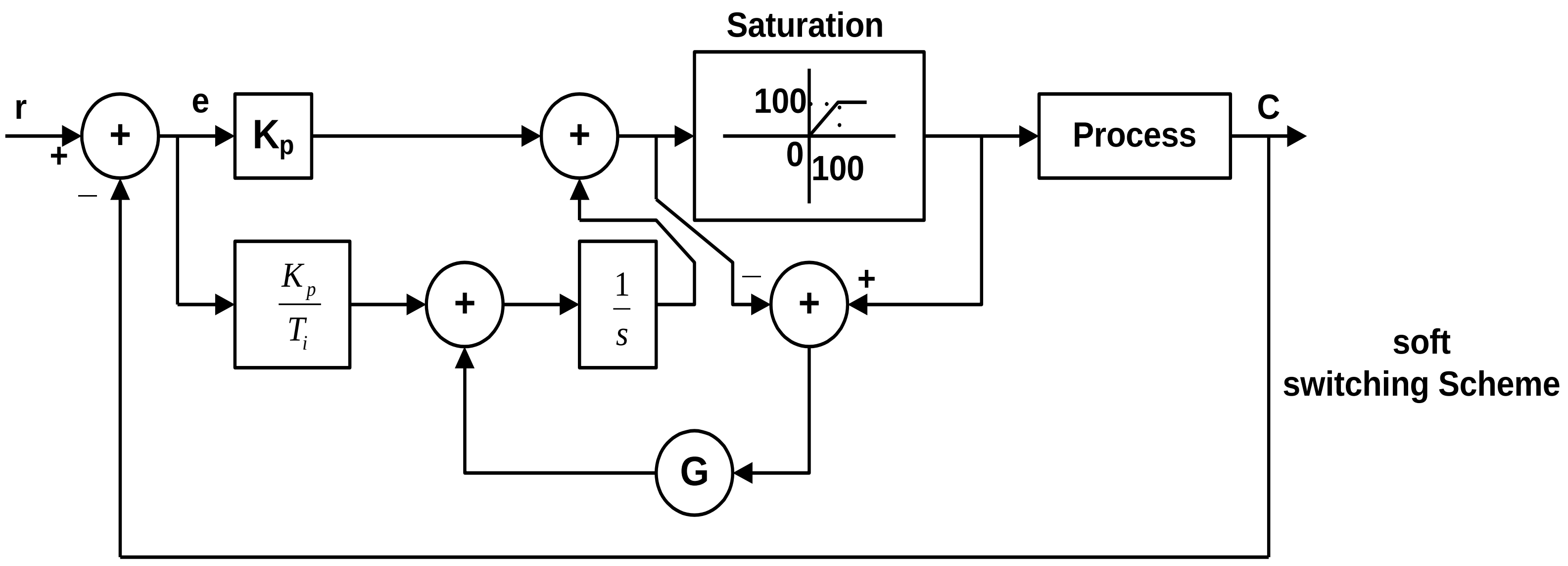
PI control without anti-integral windup



PI control with anti-integral windup

Some anti-integral windup schemes

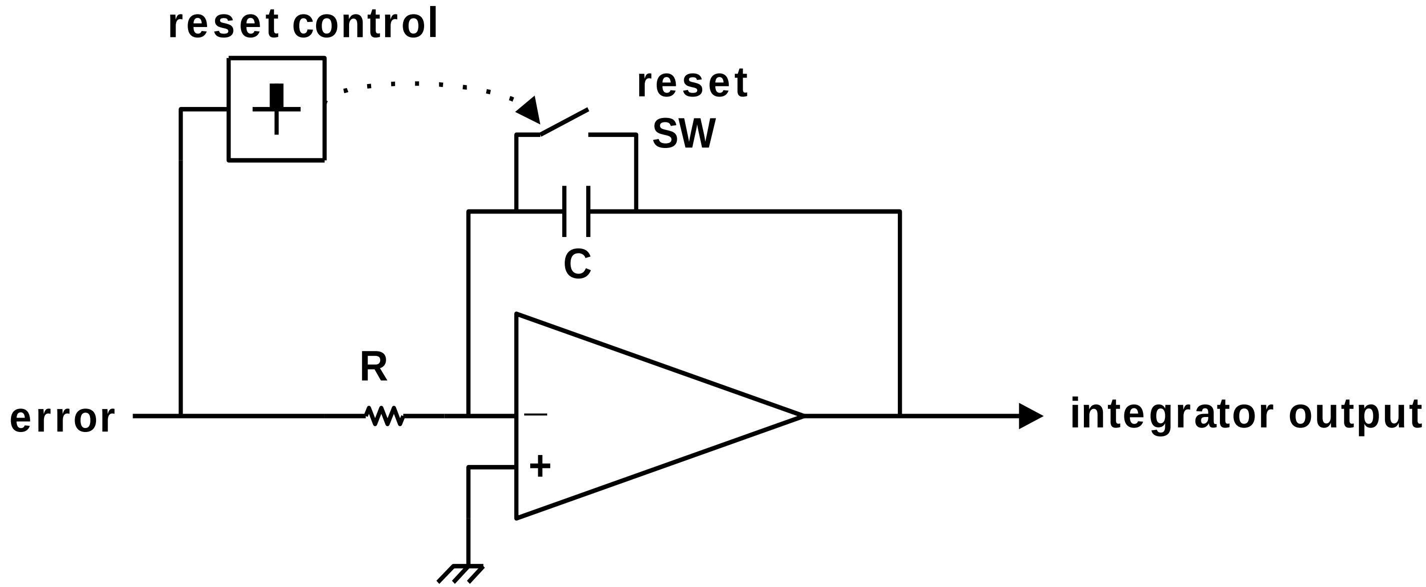
Reduce integration gradually as PI/PID controller internal output exceeds the saturation limits



G : a constant

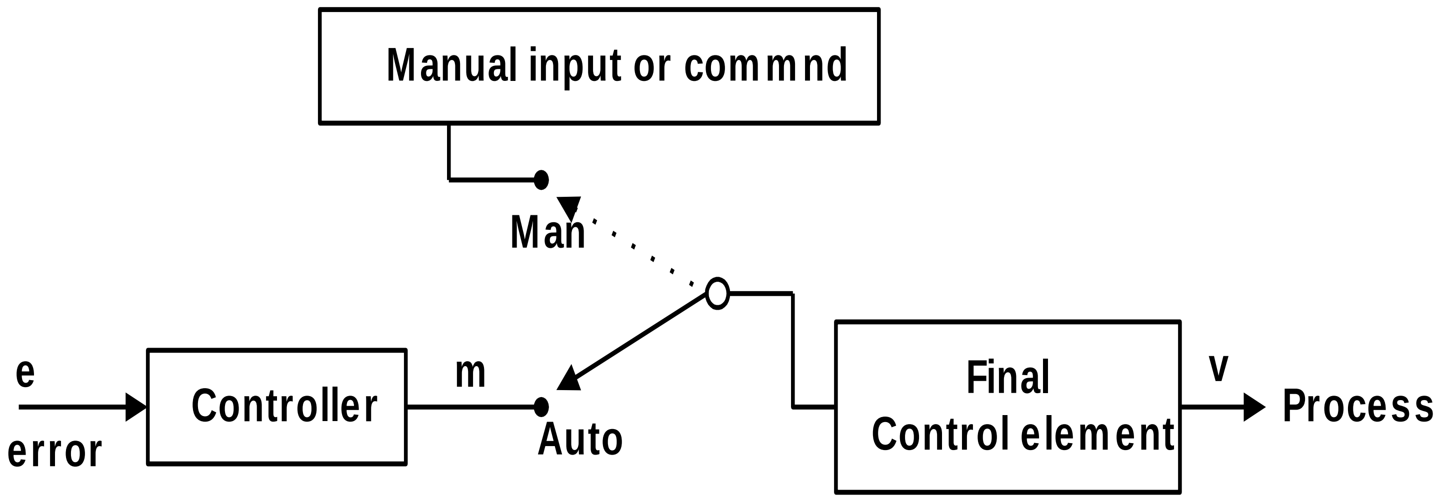
Some anti-integral windup schemes

Clegg integrator – the integrator is set to zero (reset) when the error crosses zero

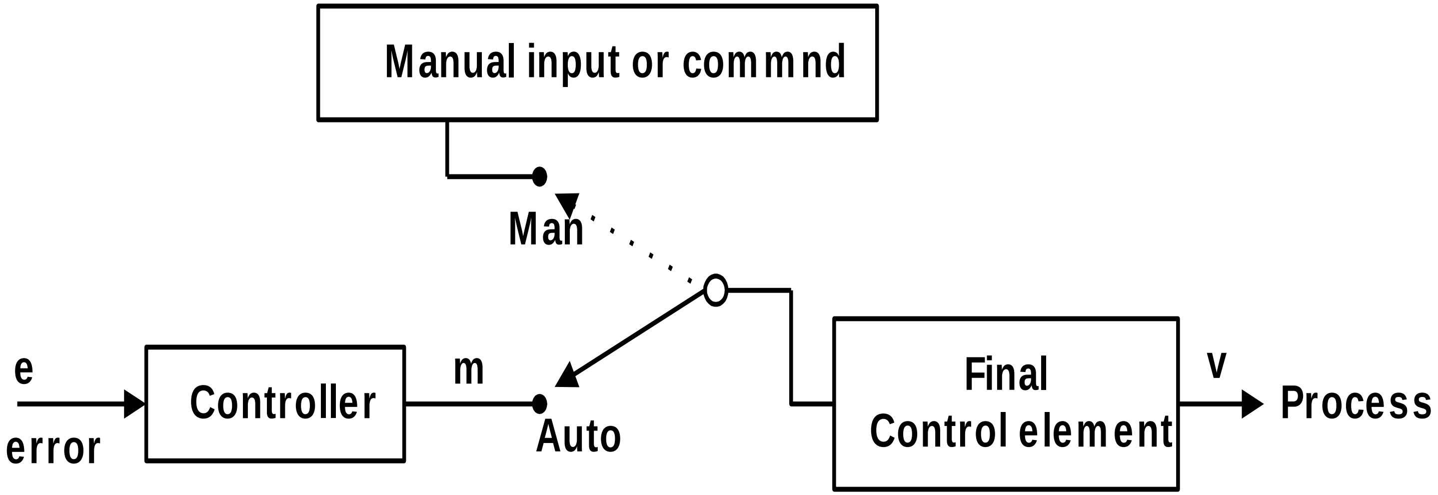


Automatic/Manual modes of Operations

- Automatic mode – means automatic closed loop operation
- Manual mode – means open loop manual control

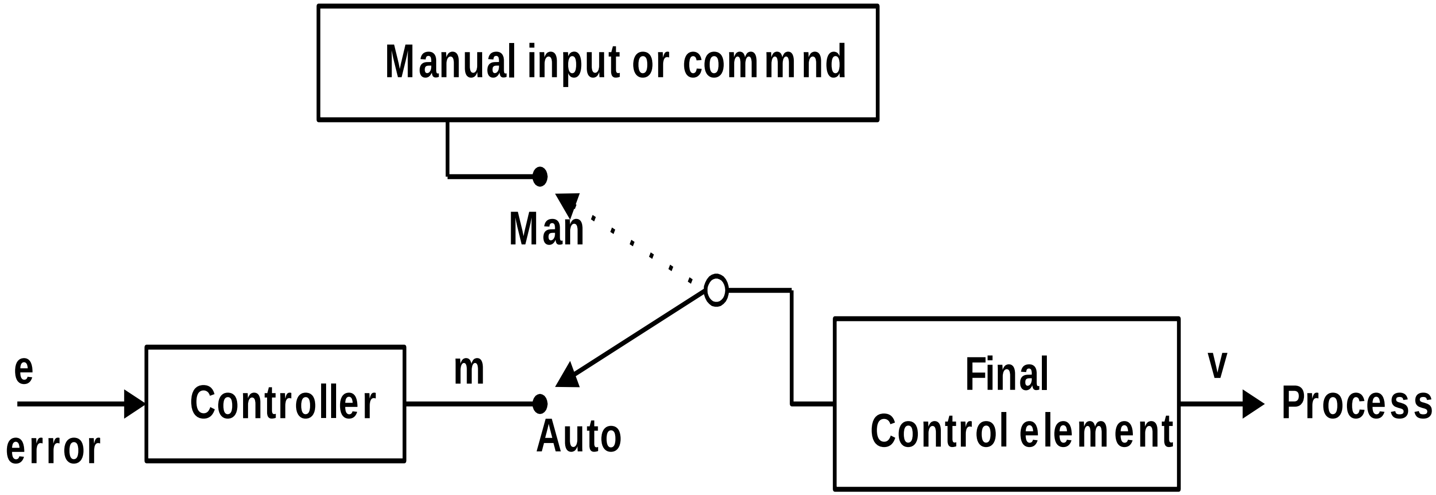


Automatic/Manual modes of Operations



If there is any difference between the controller output and the manual command, a *bump* occurs in the process output when the switch position is altered.

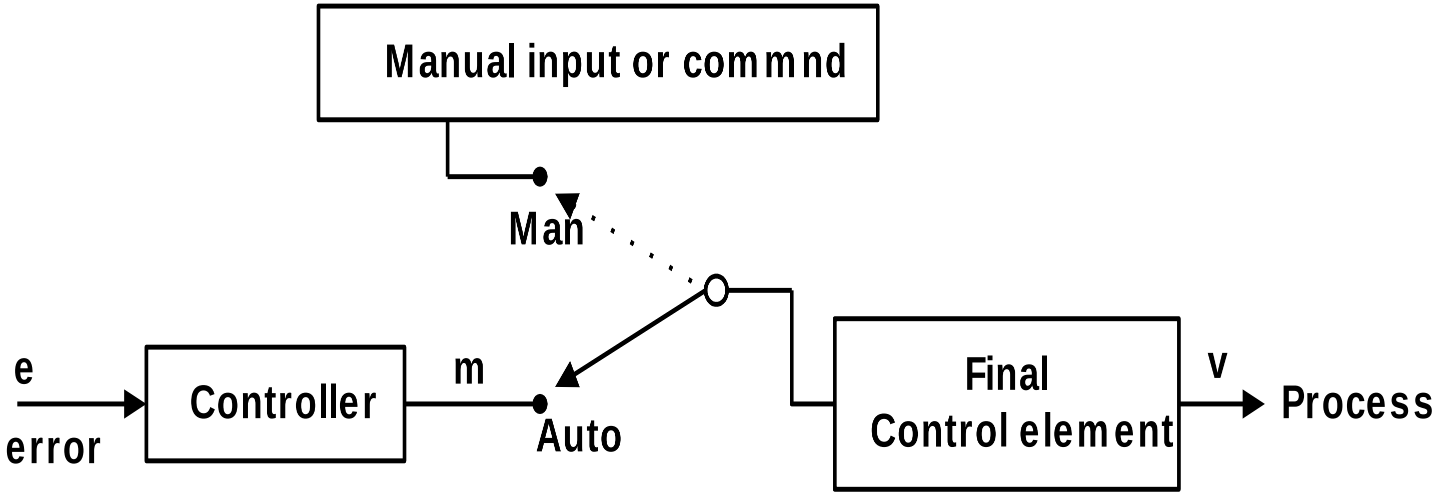
Automatic/Manual modes of Operations



If there is any difference between the controller output and the manual command, a *bump* occurs in the process output when the switch position is altered.

To provide *'bump-less transfer'* from auto-to-manual change over, special arrangements may be made for *'set-point initialization'*.

Automatic/Manual modes of Operations



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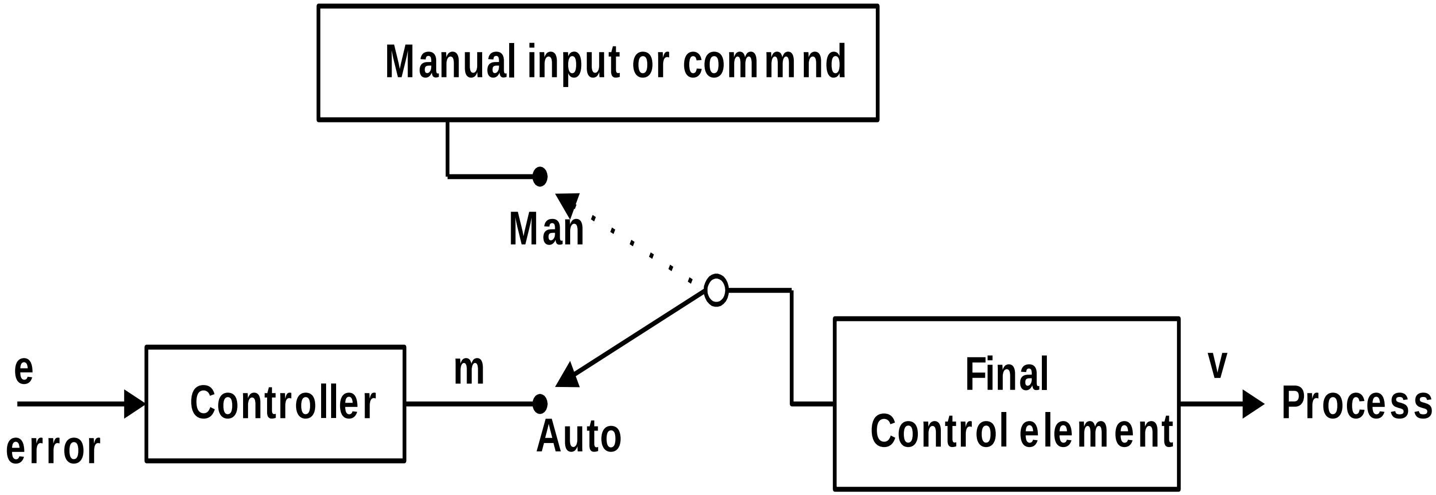
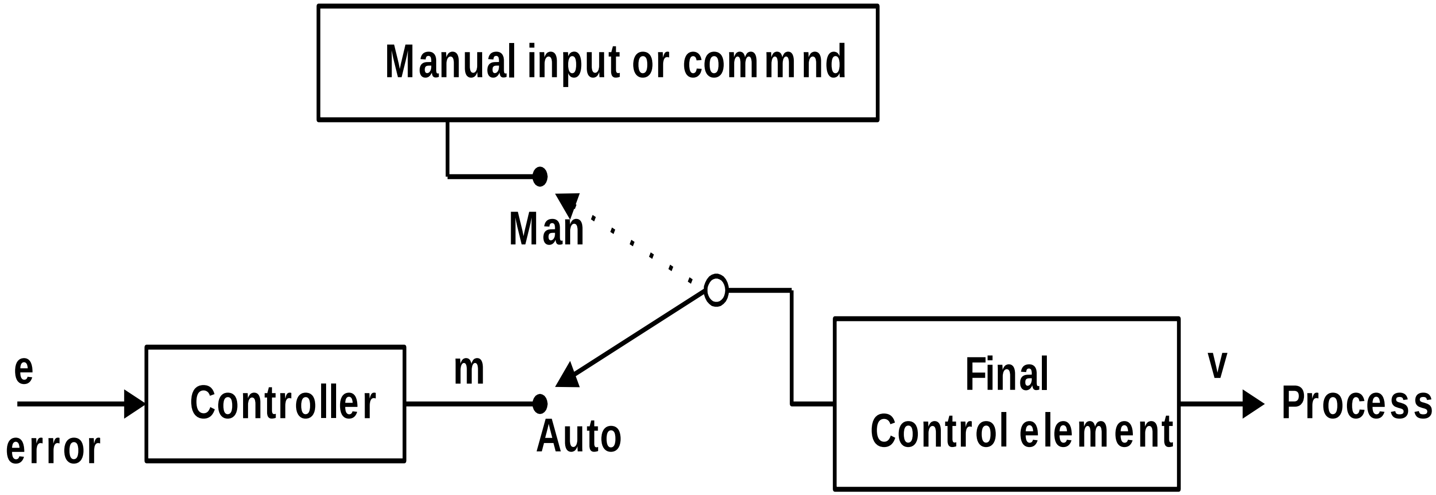
The manual command is driven to equal the controller output when the loop is in AUTO mode.

Automatic/Manual modes of Operations



When the loop is in MANual mode, if there is a steady error existing due to any difference between the set-point of the controller and the process output (under manual control), integral term, in case of PI and PID controllers, may wind-up to a large value, and consequently anti-integral wind-up is necessary for such situations.

Automatic/Manual modes of Operations



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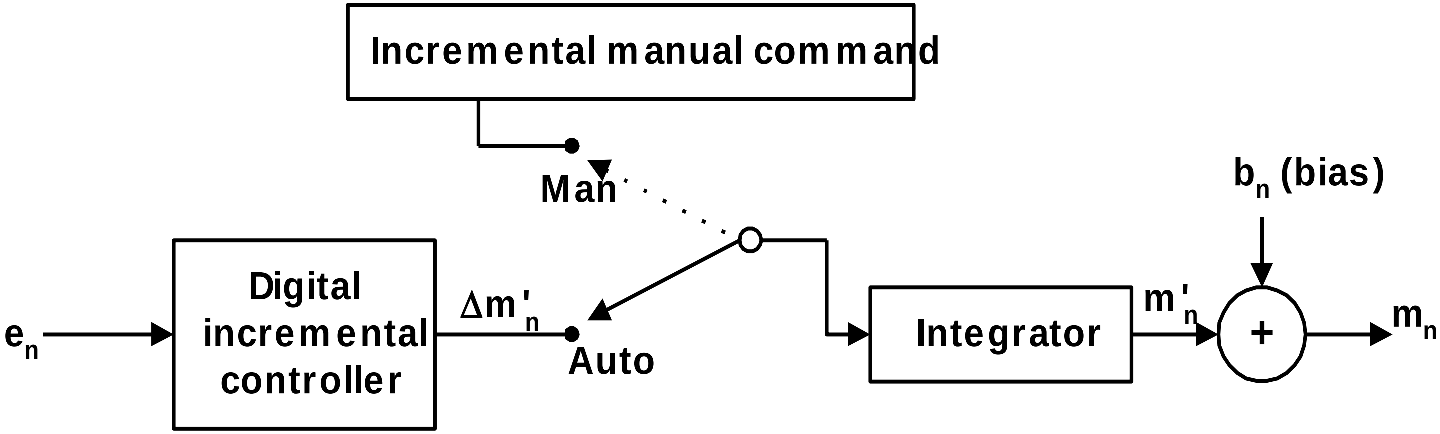
To provide *bump-less transfer* for all the operating modes, *incremental or velocity* from of controller is used with an additional integrator.

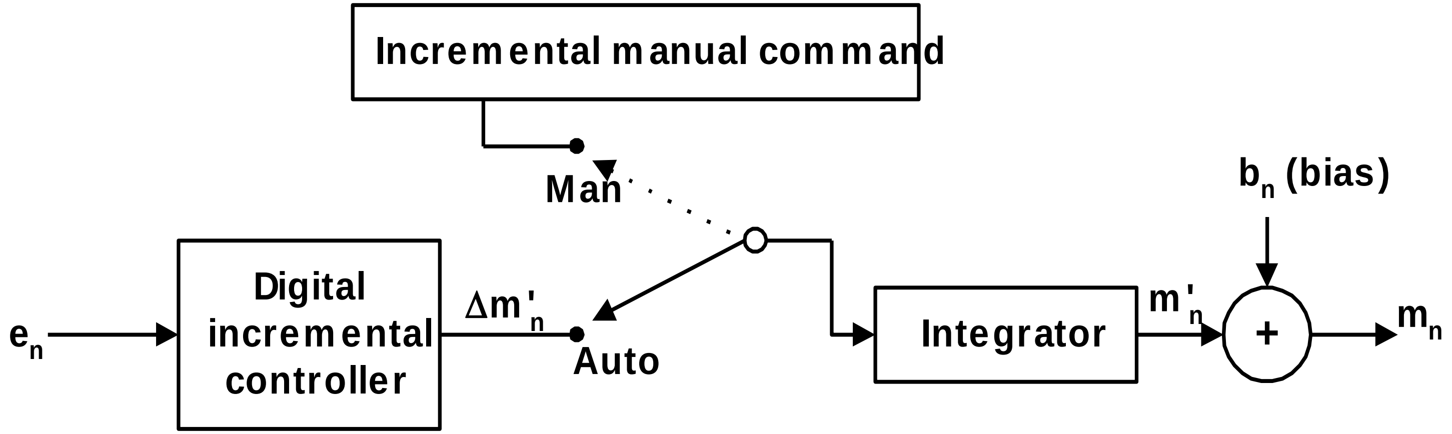
Automatic/Manual modes of Operations

Scheme for bump-less transfer



Automatic/Manual modes of Operations

Scheme for bump-less transfer



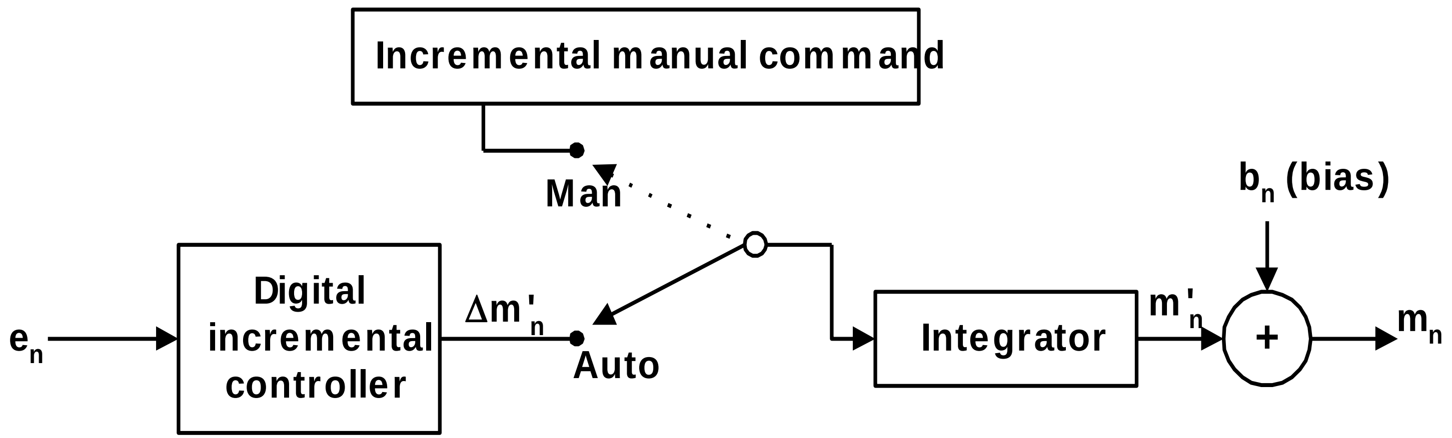
The incremental controller output (without bias) at the nth instant may be expressed as

$$\Delta m'_n = m'_n - m'_{n-1}$$

$$= e_n K_p \left(1 + \frac{\tau}{T_i} + \frac{T_d}{\tau}\right) - e_{n-1} K_p \left(\frac{2T_d}{\tau} + 1\right) + e_{n-2} \frac{K_p T_d}{\tau}$$ **for a PID controller**

Automatic/Manual modes of Operations

Scheme for bump-less transfer



The incremental controller output (without bias) at the nth instant may be expressed as

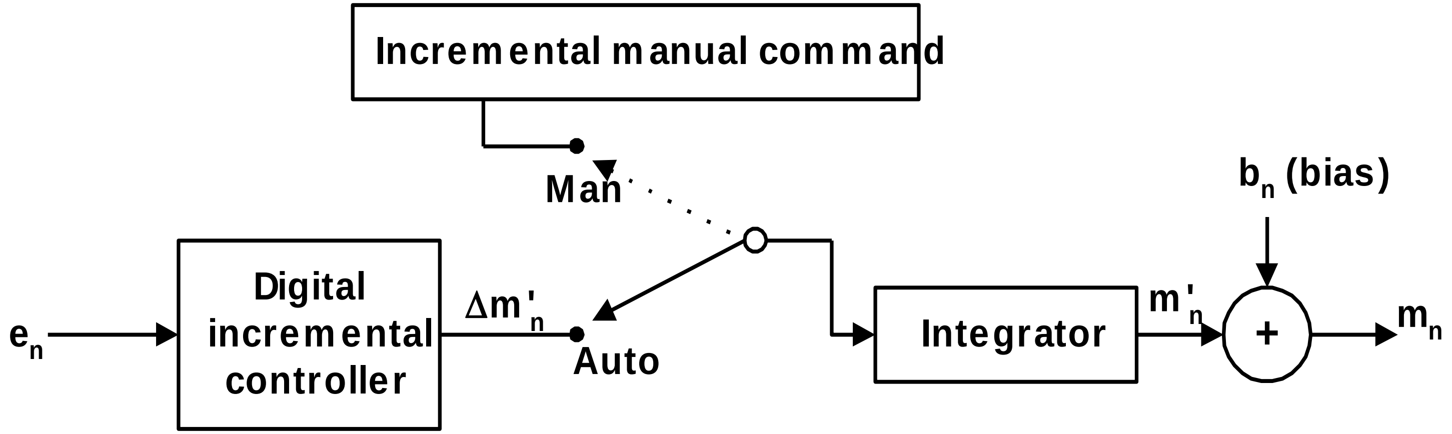
$$\Delta m_n' = m_n' - m_{n-1}'$$

$$= e_n K_p \left(1 + \frac{\tau}{T_i} + \frac{T_d}{\tau}\right) - e_{n-1} K_p \left(\frac{2T_d}{\tau} + 1\right) + e_{n-2} \frac{K_p T_d}{\tau}$$ **for a PID controller**

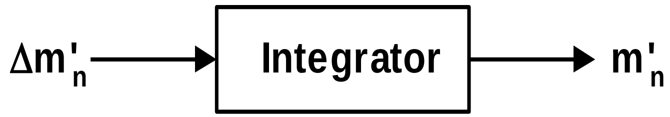
$$= a_o e_n + a_1 e_{n-1} + a_2 e_{n-2}$$

Automatic/Manual modes of Operations

Scheme for bump-less transfer

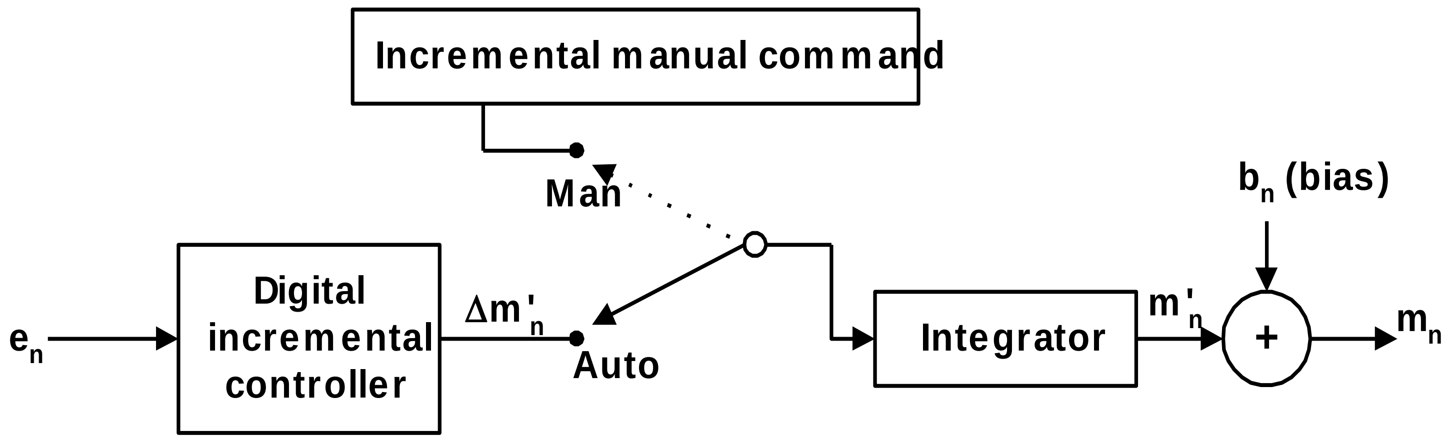


The integrator may be represented as



Automatic/Manual modes of Operations

Scheme for bump-less transfer



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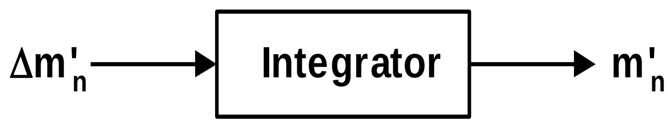
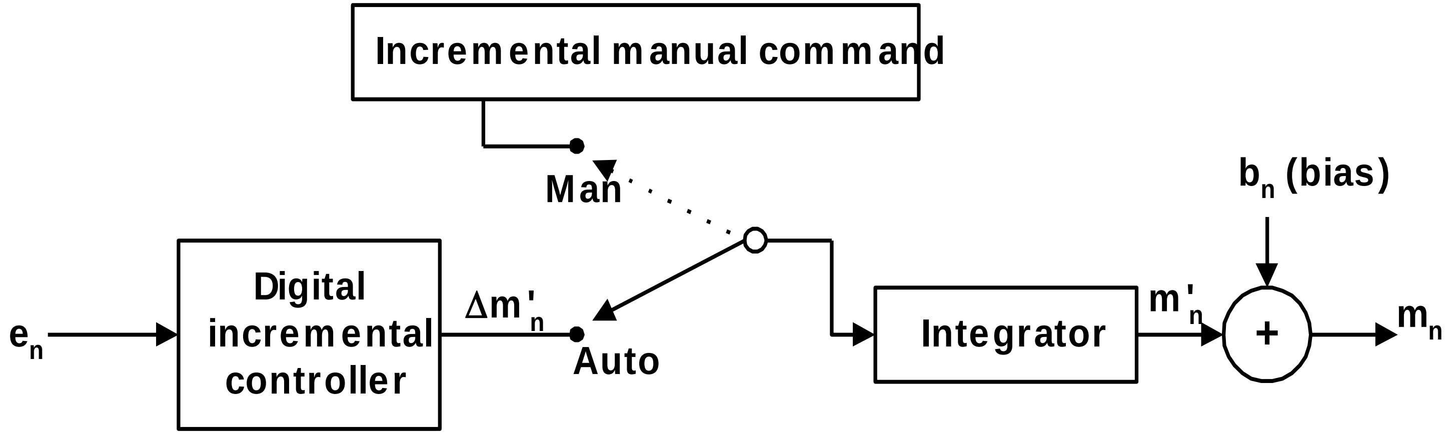


The integrator output may be represented as

$$
\begin{aligned}
m'_n &= m'_n - m'_{n-1} + m'_{n-1} \\
&= \Delta m'_n + m'_{n-1}
\end{aligned}
$$

Automatic/Manual modes of Operations

Scheme for bump-less transfer



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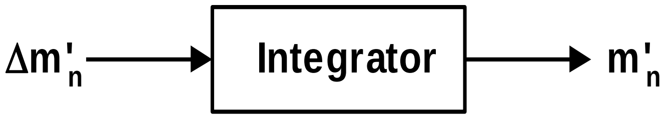
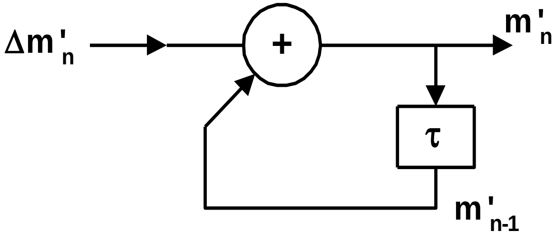


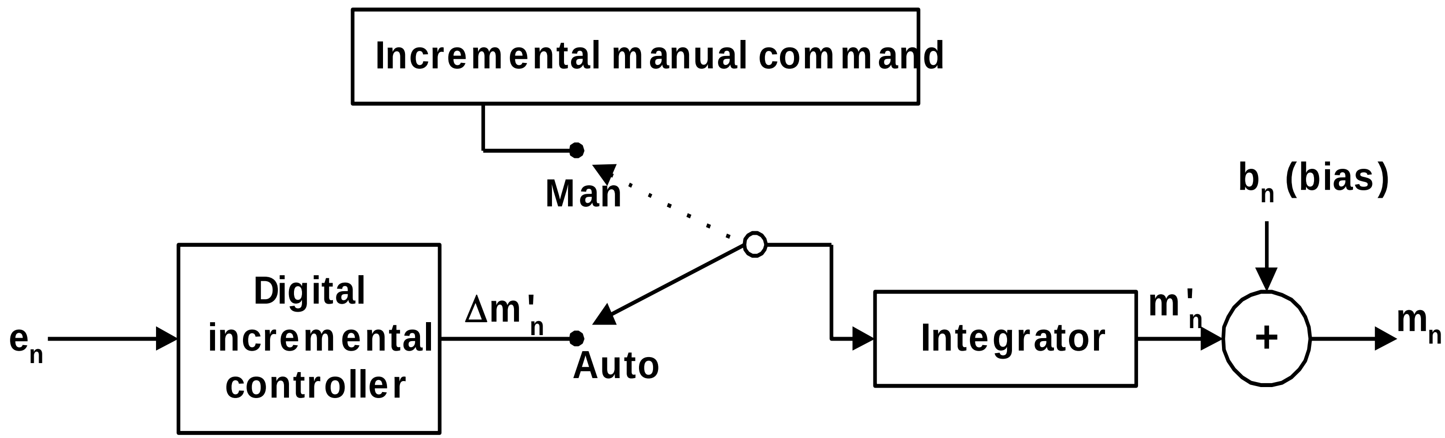
The integrator output may be represented as

$$m'_n = m'_n - m'_{n-1} + m'_{n-1}$$
$$= \Delta m'_n + m'_{n-1}$$



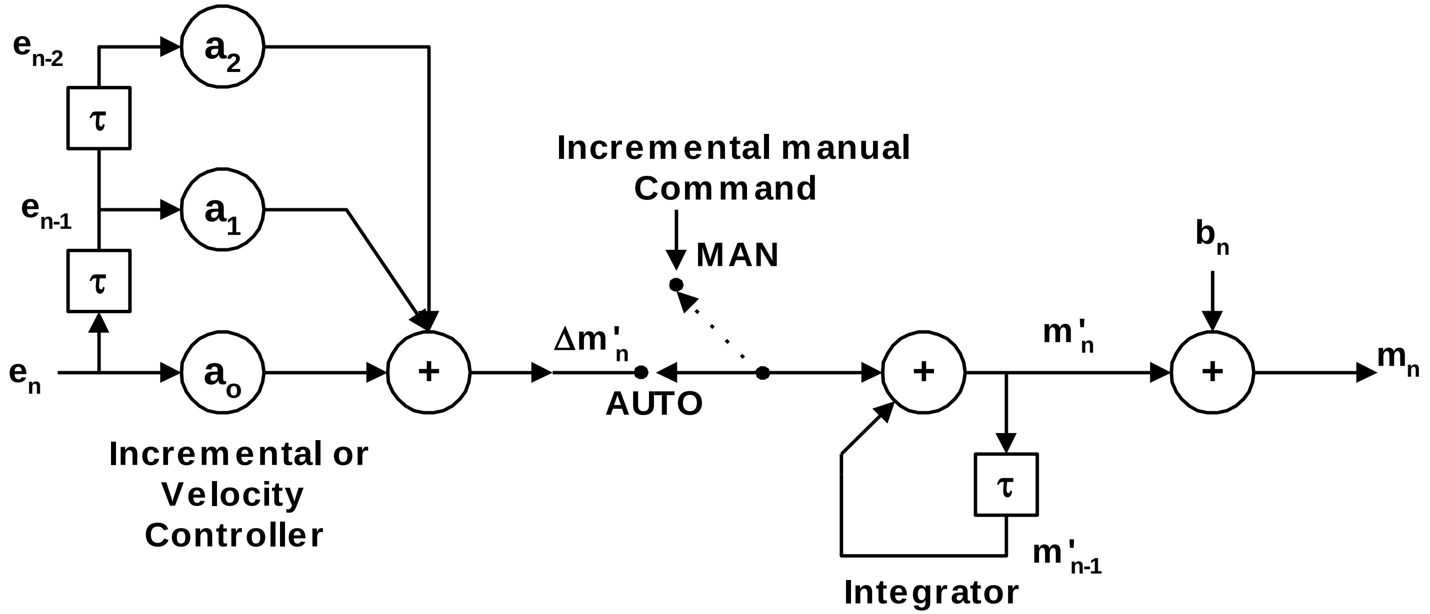
Automatic/Manual modes of Operations

Scheme for bump-less transfer



The presence of integrator at the output ensures a smooth output variation even when the actual manual command is different from the actual controller output under closed-loop control.

Realization of the incremental type PID Controller



Automatic tuning of PID Controllers – the Relay autotuner

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Suitable for processes with non-zero dead-time.

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K_c is the gain margin of the process loop.

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Controller parameters K_p, T_i and T_d are calculated according to *Ziegler – Nichols (Z-N) rule* for a stable time response.

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Z-N settings

Controller	K_p	T_i	T_d
P – Controller	$0.5\ K_c$		
PI – Controller	$0.45\ K_c$	$T_c/1.2$	
PID – Controller	$0.6\ K_c$	$T_c/2$	$T_c/8$

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⇐ **gain margin: 2** (P – Controller row)

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The critical gain K_c and critical time period T_c are determined from an experiment with relay (switching element) feedback.

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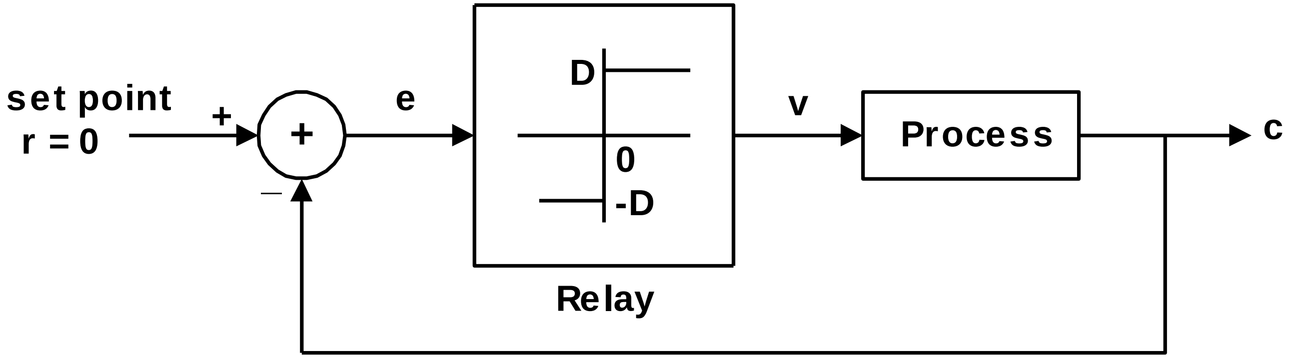

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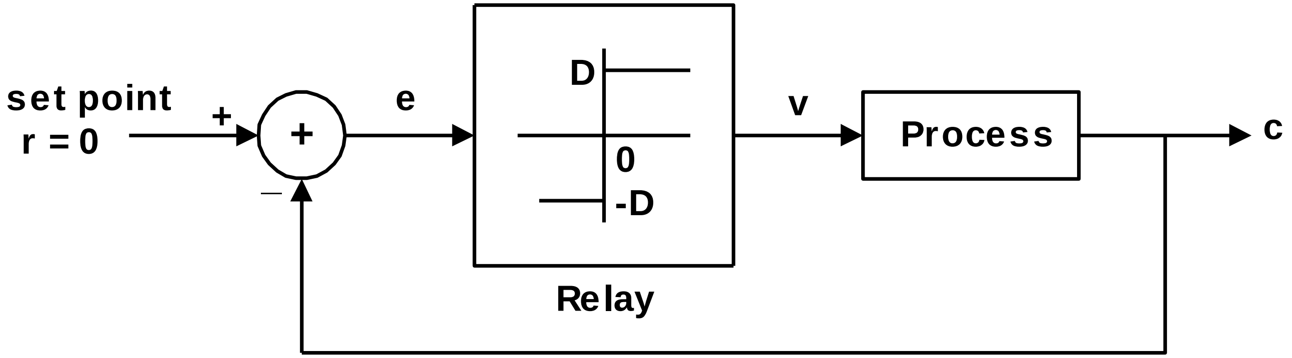

✓**The relay control provides ON / OFF control of the process.**

Automatic tuning of PID Controllers – the Relay autotuner

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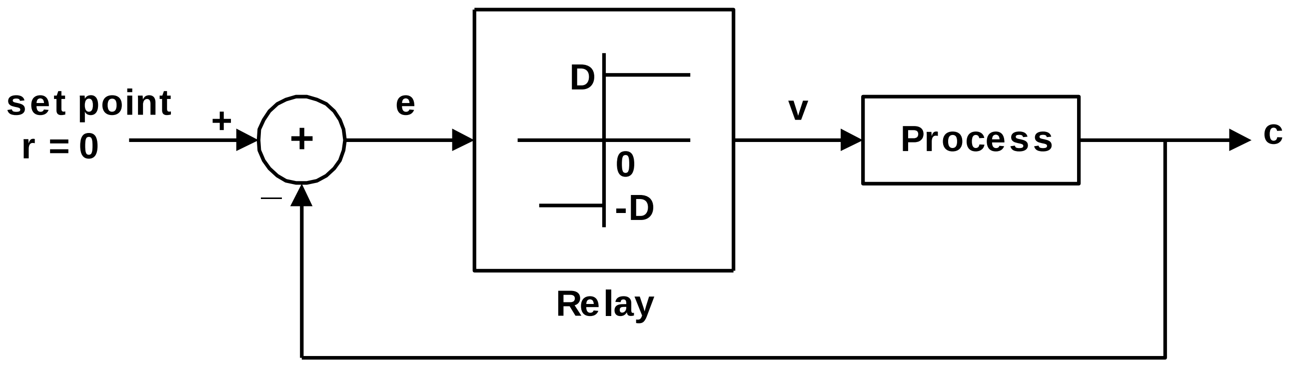
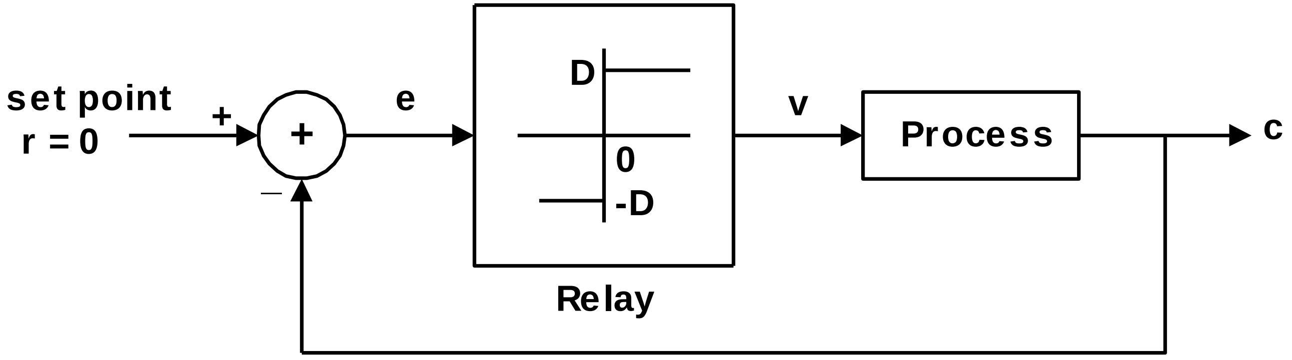


✓**The relay control provides ON / OFF control of the process.**

✓**The input 'r' is set to zero.**

Automatic tuning of PID Controllers – the Relay autotuner

The critical gain K_c and critical time period T_c are determined from an experiment with relay (switching element) feedback.



✓The relay control provides ON / OFF control of the process.

✓The input 'r' is set to zero.

✓The output 'c' oscillates around a mean value of zero (limit-cycle oscillations).

Automatic tuning of PID Controllers – the Relay autotuner

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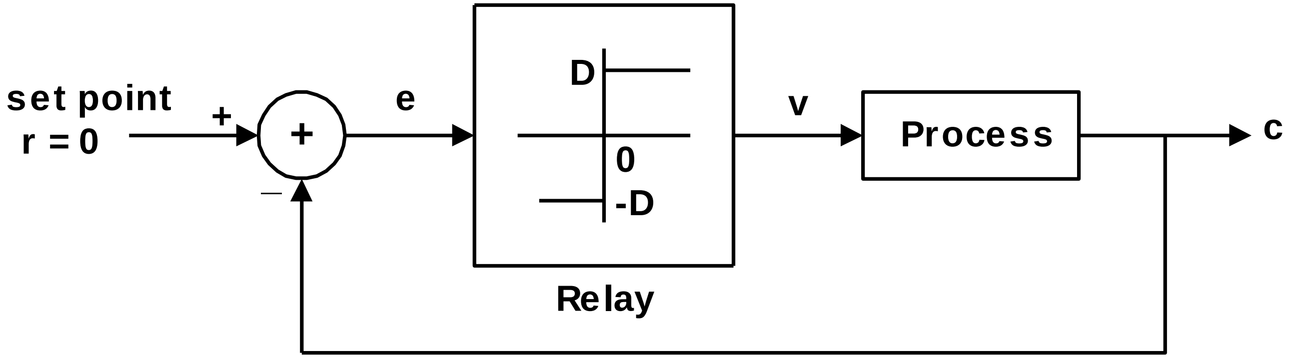


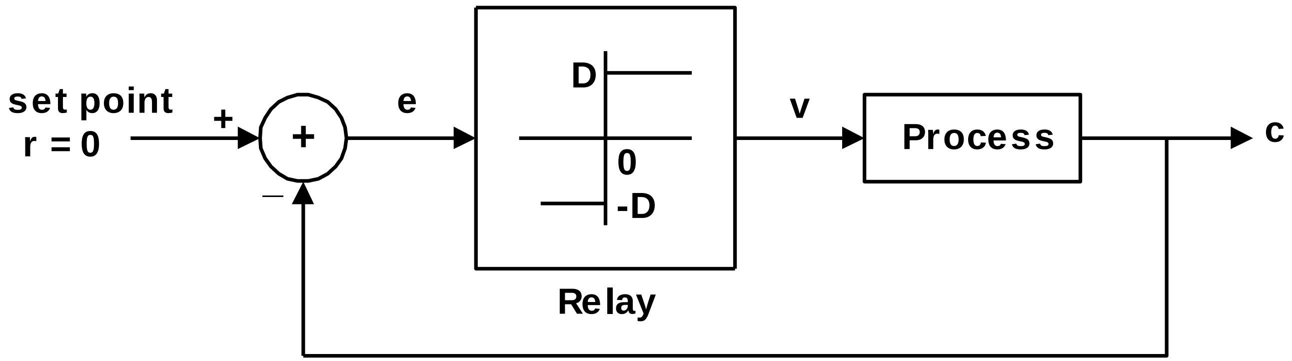
✓The relay control provides ON / OFF control of the process.

✓The input 'r' is set to zero.

✓The output 'c' oscillates around a mean value of zero (limit-cycle oscillations).

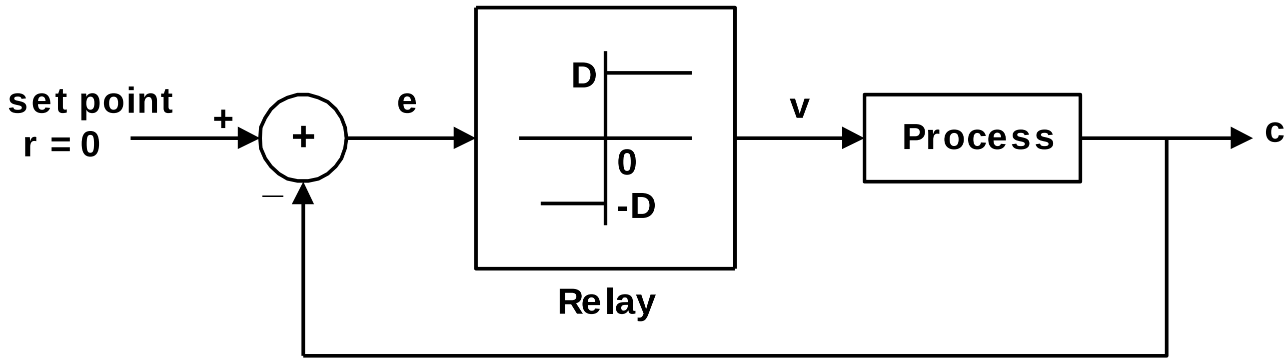
✓The process is driven by a square wave of amplitude 'D'.

Automatic tuning of PID Controllers – the Relay autotuner



Assuming the process to be a low-pass system, the process output 'c' contains mainly the fundamental component.

Automatic tuning of PID Controllers – the Relay autotuner

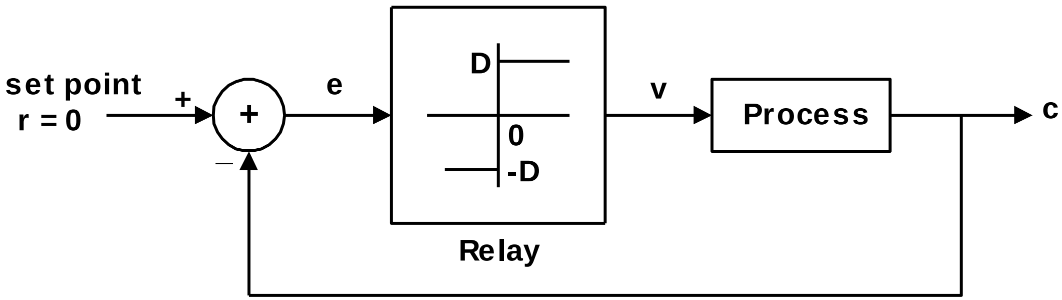


Assuming the process to be a low-pass system, the process output ‘c’ contains mainly the fundamental component.

Thus the error signal ‘e’ becomes sinusoidal,

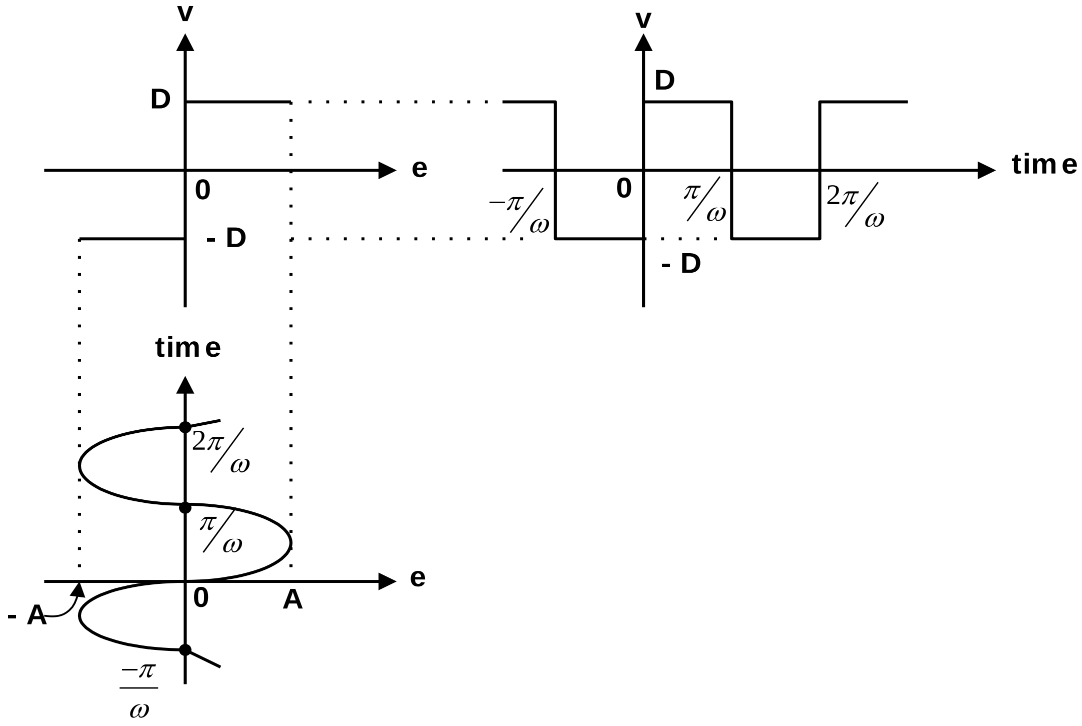
$$e = A \sin \omega t$$

Automatic tuning of PID Controllers – the Relay autotuner



$e = A \sin \omega t$

The relay output 'v' may be found out as follows:



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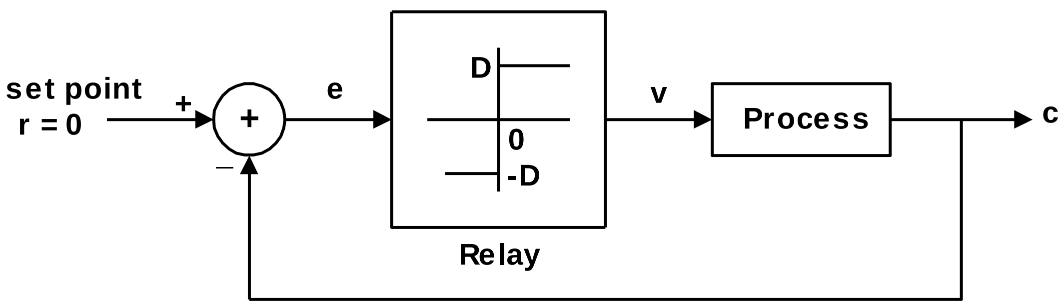

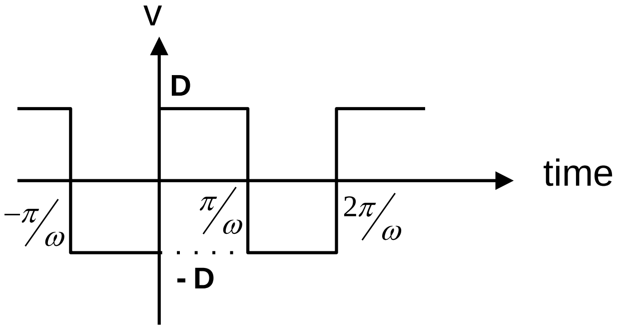

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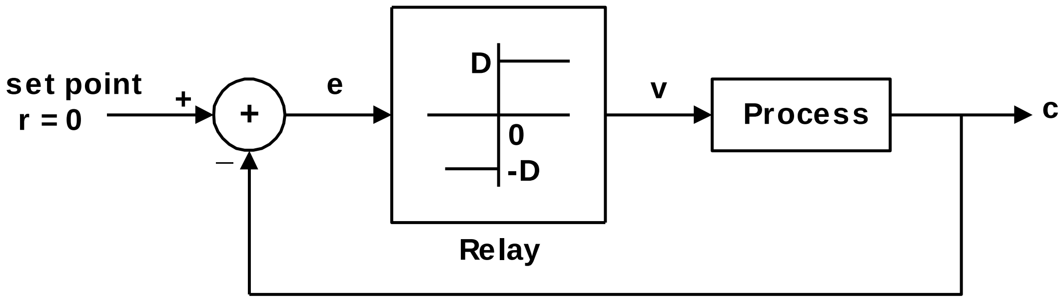
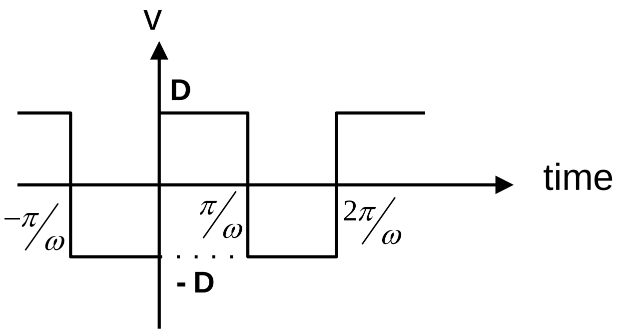
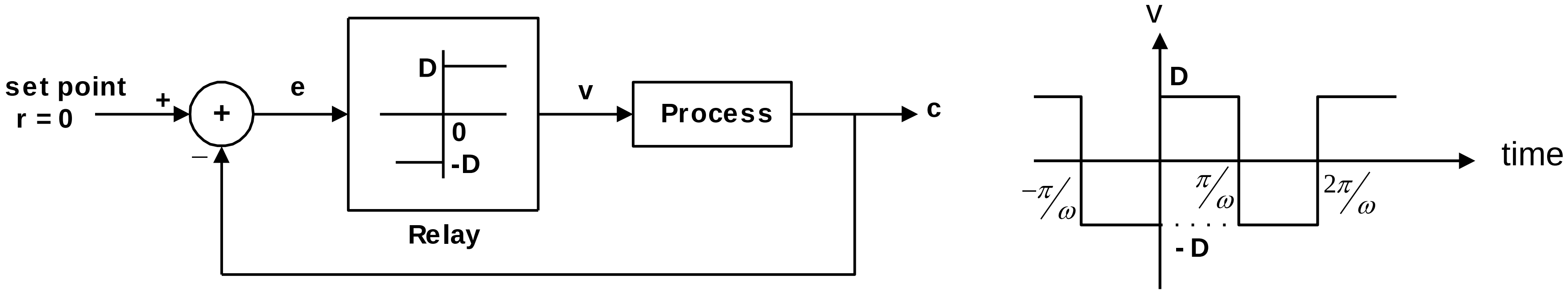




The Fourier series of the relay output (v) may be expressed as:

$$v = \frac{4D}{\pi}\left(\sin\omega t + \frac{1}{3}\sin 3\omega t + \frac{1}{5}\sin 5\omega t + ...\right)$$

Automatic tuning of PID Controllers – the Relay autotuner



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The process practically attenuates all higher harmonics other than the fundamental.

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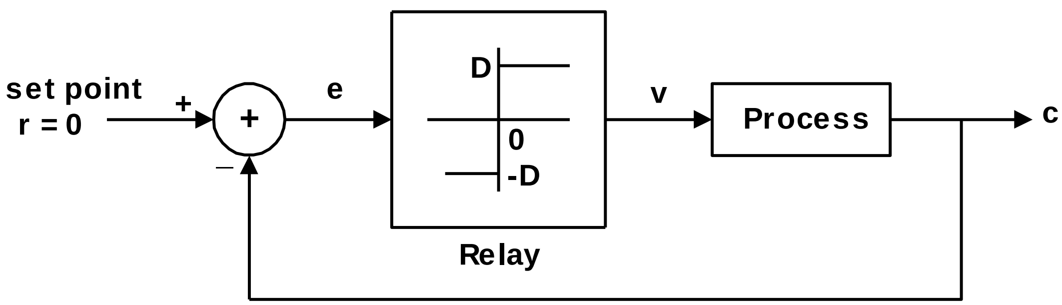

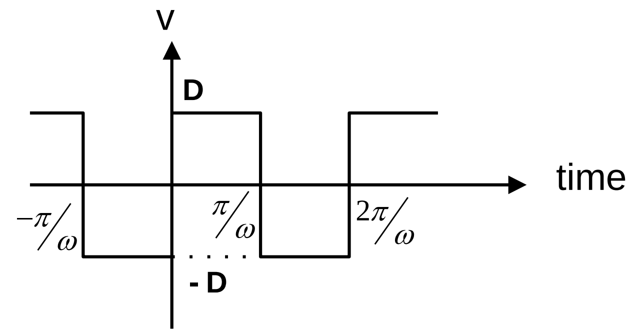

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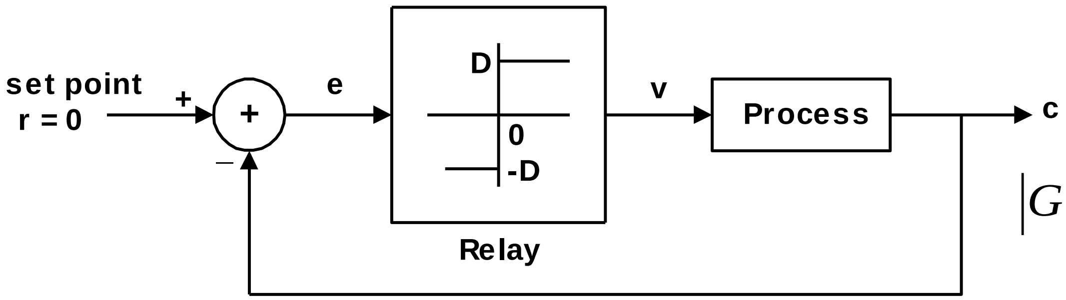
Then the process gain at frequency 'ω' becomes

$$\left|G(\omega)\right| = \frac{output\ \ amplitude}{input\ \ amplitude} = \frac{A}{\frac{4D}{\pi}} = \frac{\pi A}{4D}$$

Automatic tuning of PID Controllers – the Relay autotuner



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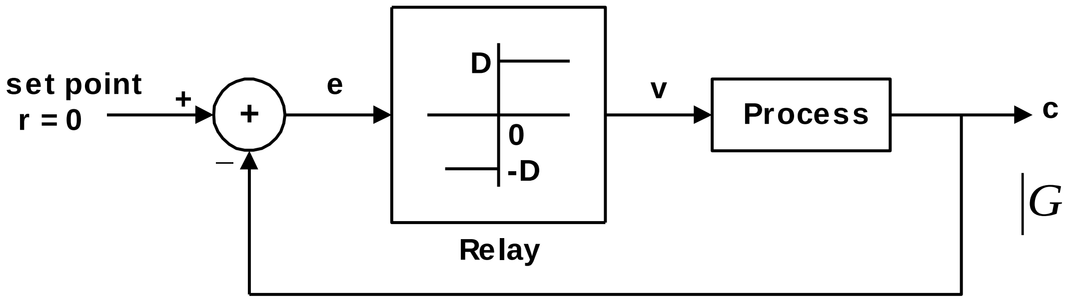

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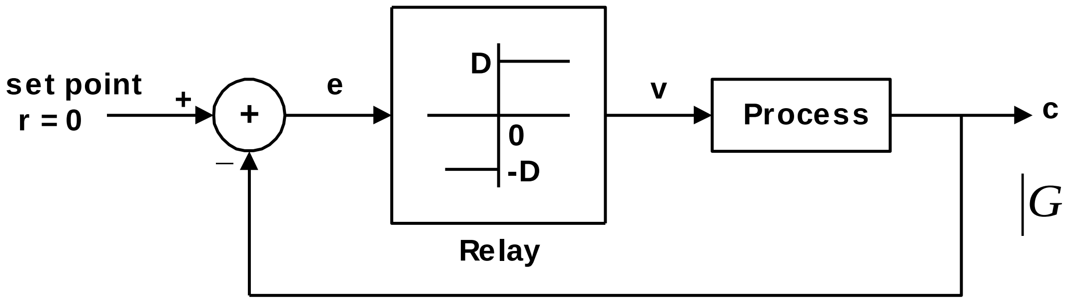
Automatic tuning of PID Controllers – the Relay autotuner



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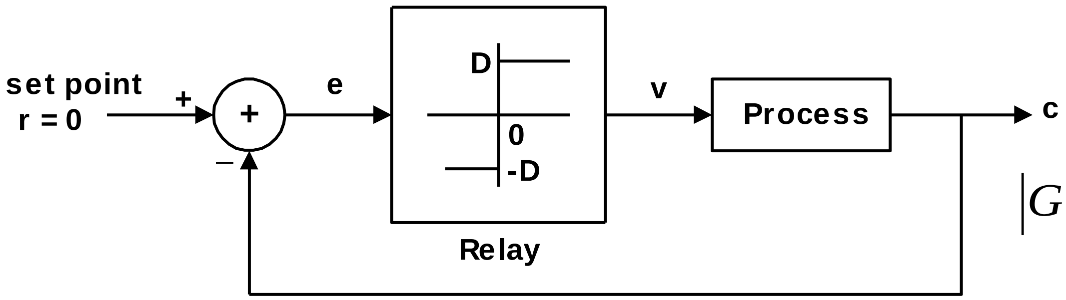
Thus the gain of the relay controller (i.e. the critical gain) at $\omega = \omega_c$ is

$$K_c = \frac{1}{\left|G(\omega_c)\right|} = \frac{4D}{\pi A} \qquad \left[\text{as } K_c . \left|G(\omega_c)\right| = 1\right]$$

Automatic tuning of PID Controllers – the Relay autotuner



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$$K_c = \frac{1}{|G(\omega_c)|} = \frac{4D}{\pi A} \qquad \left[\text{as } \mathrm{K_c}.|\mathrm{G}(\omega_c)| = 1\right]$$

Also $\angle G(\omega_c) = \pi$, as relay phase shift is zero.

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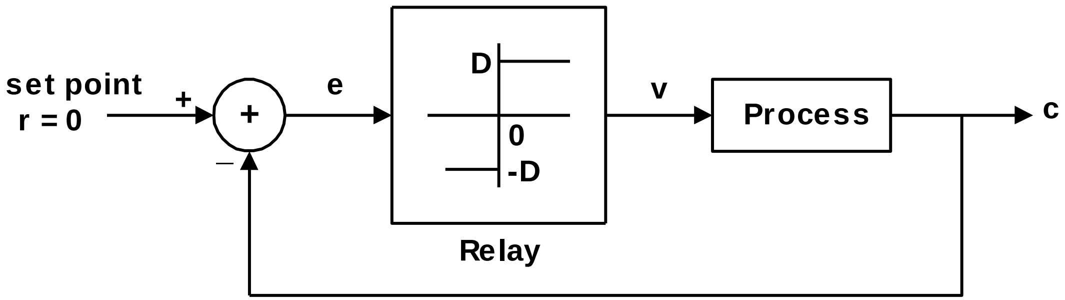
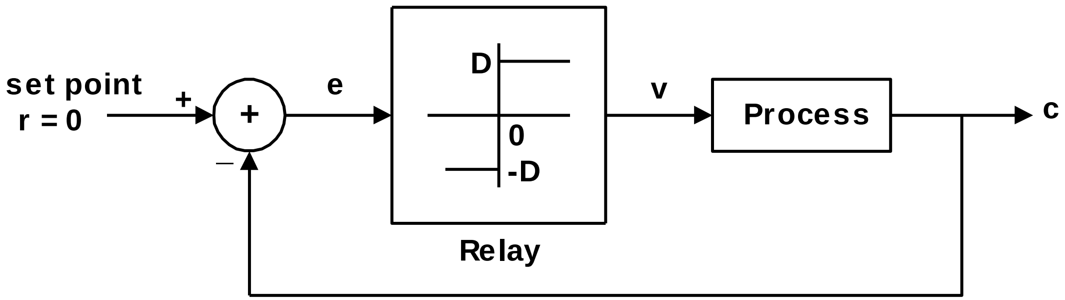
Automatic tuning of PID Controllers – the Relay autotuner



Thus by knowing the relay amplitude 'D' and by measuring the amplitude 'A' of the process output 'c', critical gain K_c may be determined $\left[K_c = \dfrac{4D}{\pi A} \right]$**.**

Automatic tuning of PID Controllers – the Relay autotuner

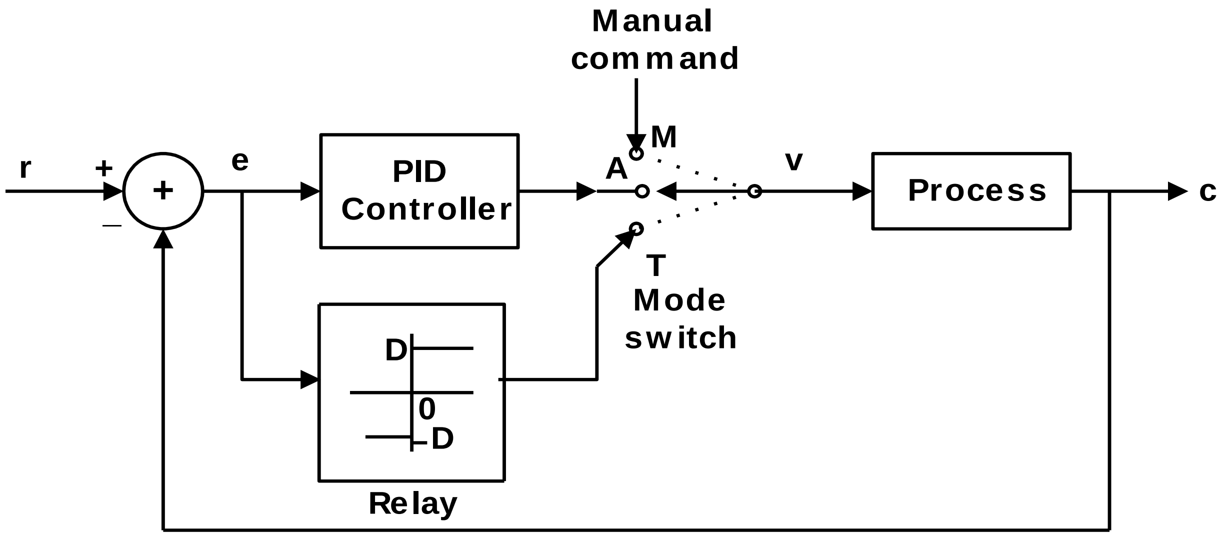


Thus by knowing the relay amplitude 'D' and by measuring the amplitude 'A' of the process output 'c', critical gain K_c may be determined $\left[K_c = \frac{4D}{\pi A}\right]$**.**

T_c **may be estimated by measuring the frequency of the output oscillation** $\left[T_c = \frac{2\pi}{\omega_c}\right]$**.**

Block diagram of the Relay autotuner

(The Satt Control Autotuner by Satt Control, Sweden)



M → Manual position
A → Auto position
T → Tune position

Block diagram of the Relay autotuner

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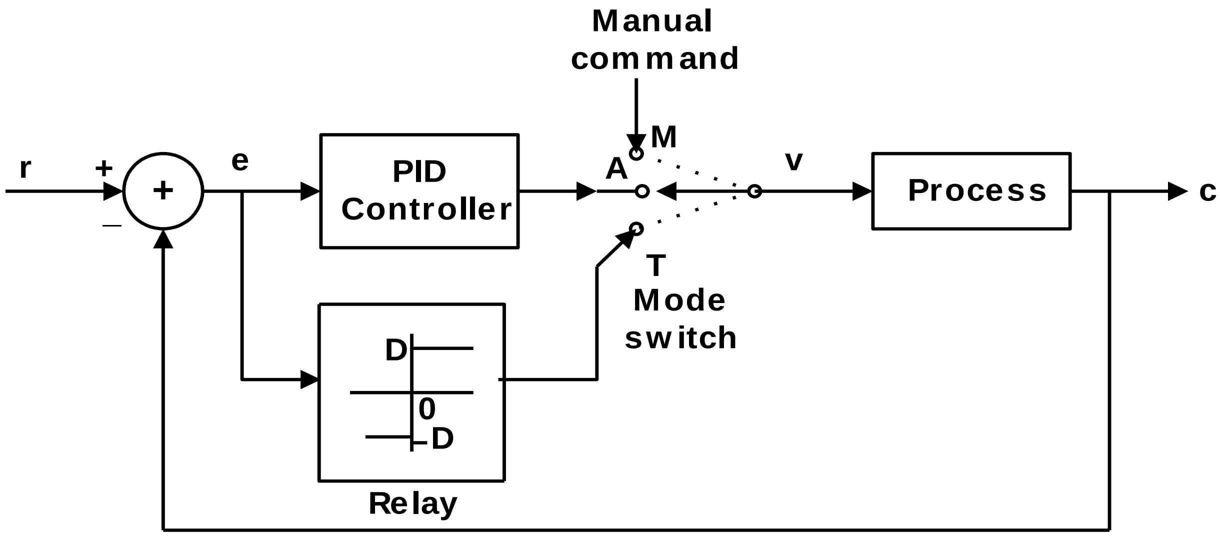

M → Manual position
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At first, the process is brought to equilibrium state (≈ zero error), by setting a constant control signal in manual mode.

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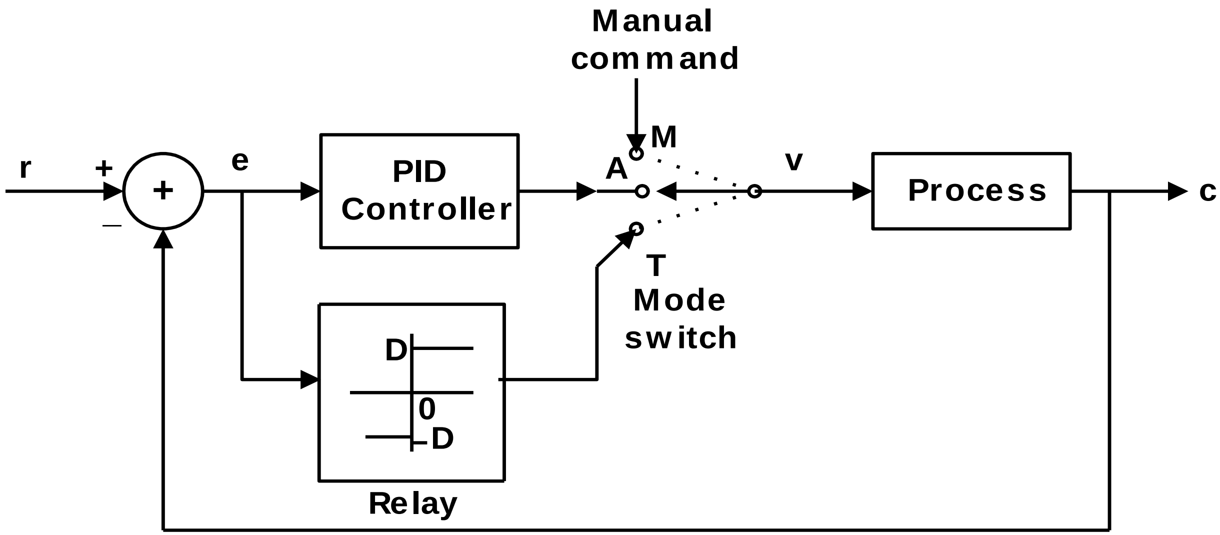

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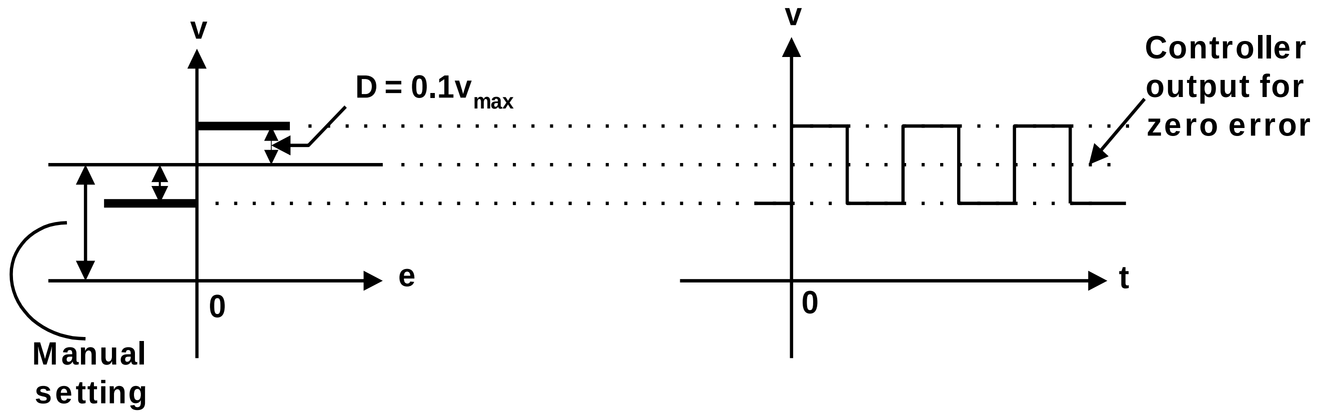
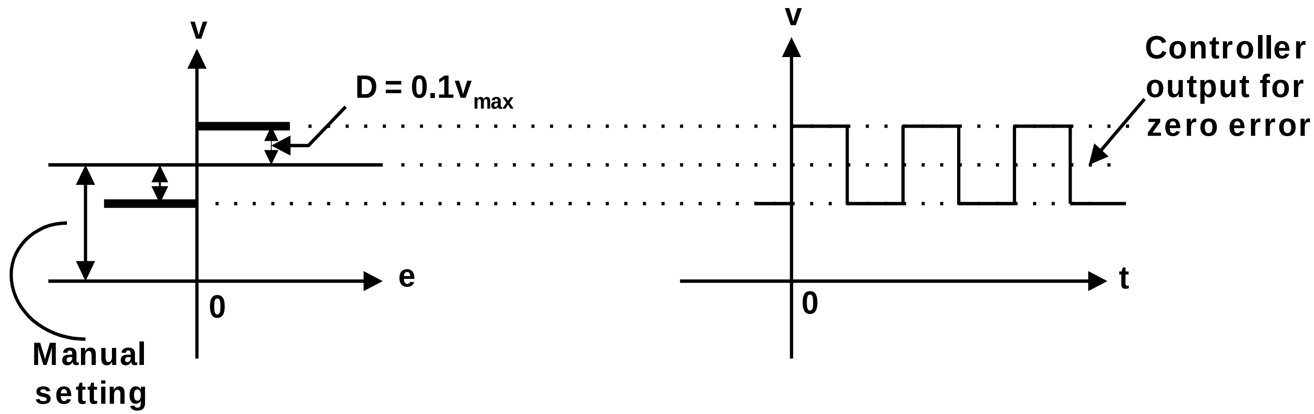
At first, the process is brought to equilibrium state (≈ zero error), by setting a constant control signal in manual mode.

The tuning is then activated by pushing the mode switch to tune position.

Modified relay characteristic for a non-zero set-point



Modified relay characteristic for a non-zero set-point



➢ **The relay amplitude ‘D’ is initially set to 10% of the controller output-range.**

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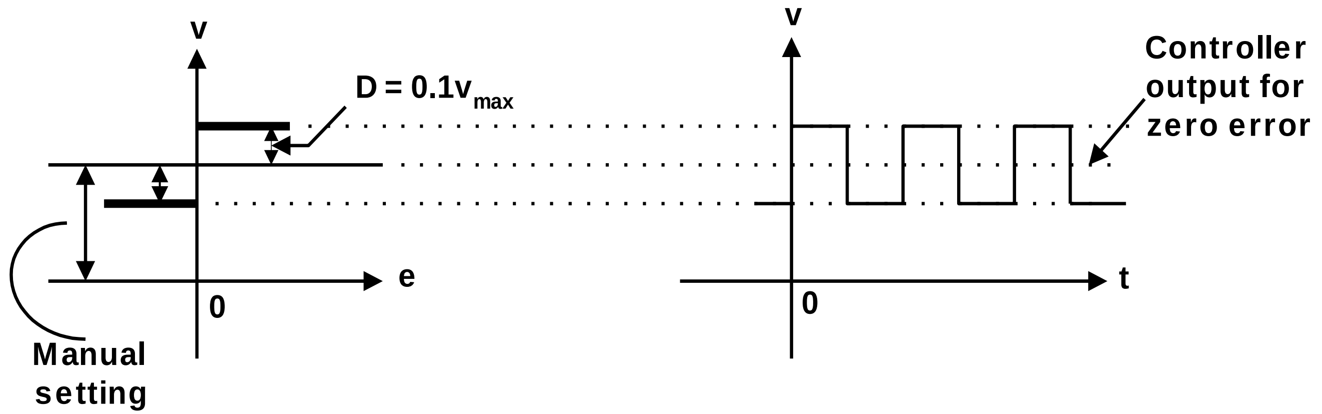
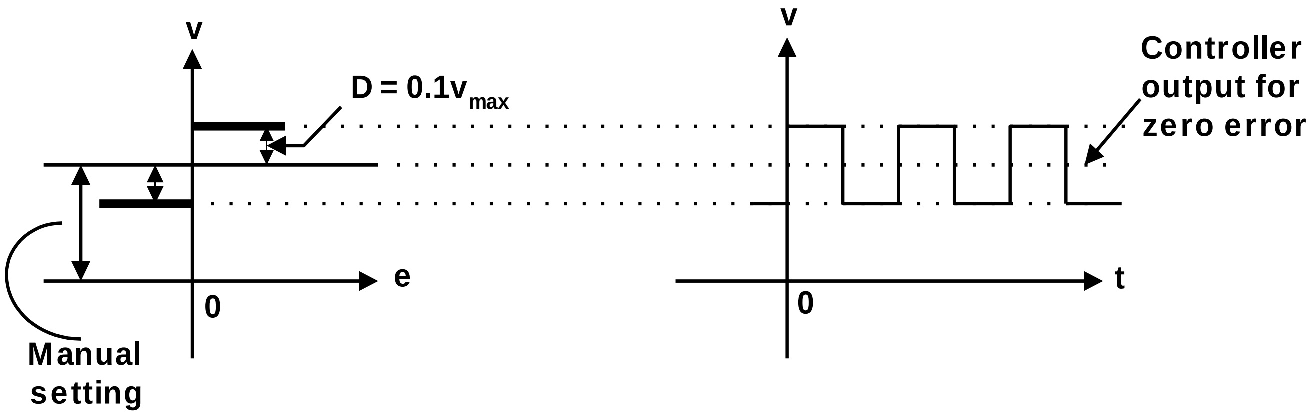


➢The relay amplitude 'D' is initially set to 10% of the controller output-range.

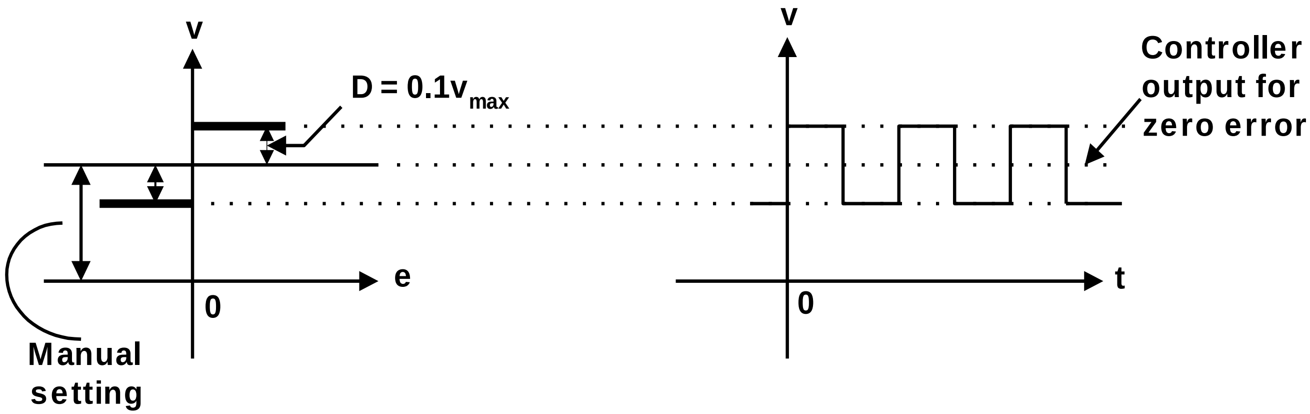
➢This amplitude is adjusted after one and a half period to give oscillation of 2% of the mean output.

Modified relay characteristic for a non-zero set-point



- ➢The relay amplitude ‘D’ is initially set to 10% of the controller output-range.
- ➢This amplitude is adjusted after one and a half period to give oscillation of 2% of the mean output.
- ➢This ensures minimum disturbance at the process output due to tuning.

Modified relay characteristic for a non-zero set-point



- The relay amplitude 'D' is initially set to 10% of the controller output-range.
- This amplitude is adjusted after one and a half period to give oscillation of 2% of the mean output.
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- This adjustment is done by measuring the change in output during the first one and a half period.

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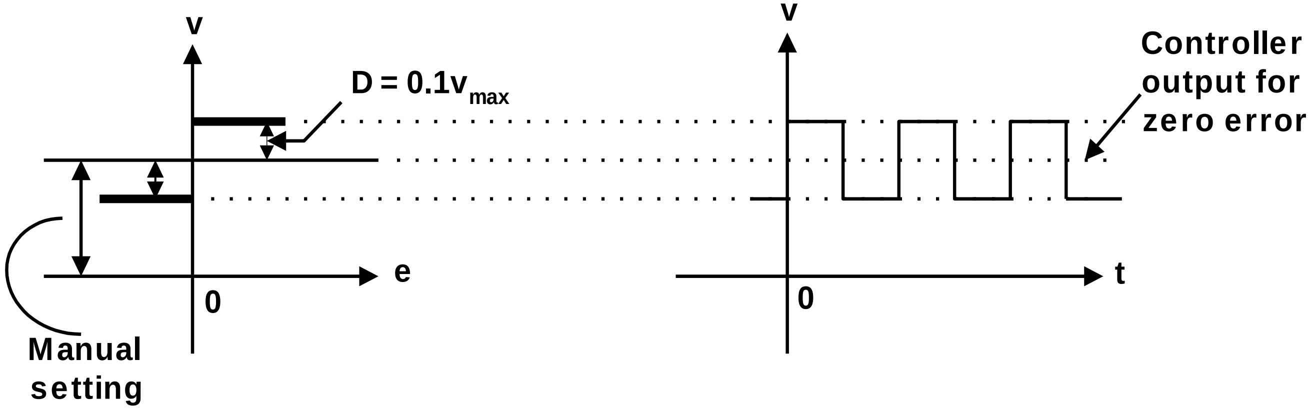

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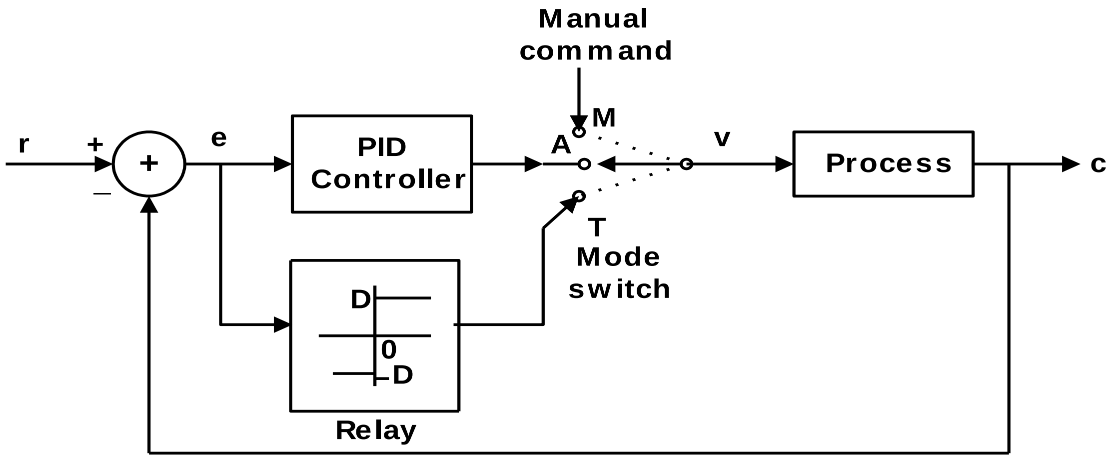
- This amplitude is adjusted after one and a half period to give oscillation of 2% of the mean output.
- This ensures minimum disturbance at the process output due to tuning.
- This adjustment is done by measuring the change in output during the first one and a half period.
- The modified relay amplitude is stored for the next tuning operation.

Satt Control Autotuner



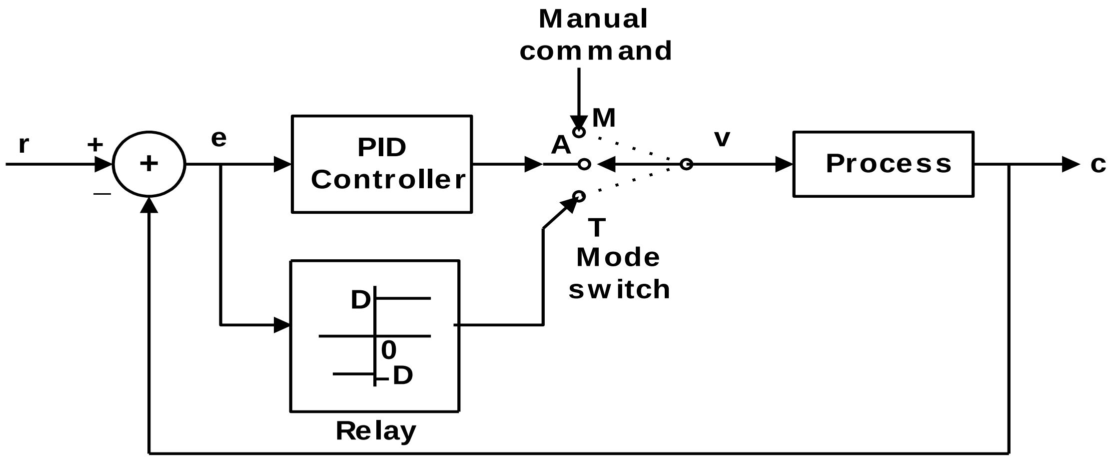
M → Manual position

A → Auto position

T → Tune position

The system is automatically switched to Auto mode after estimating the critical gain K_c and critical time period T_c during first 5½ period of oscillation.

Satt Control Autotuner



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The parameters of PID controller (viz. K_p, T_i and T_d) are determined from K_c and T_c according to Z-N rule.

References

1. **Process Control Systems** *by* Shinskey
2. **Automatic Process Control** *by* Eckman
3. **Principles of Process Control** *by* Patranabis
4. **Process Control** *by* Harriott
5. **Process Systems Analysis and Control** *by* Coughanowr and Koppel
6. **Process Control** *by* Pollard
7. **Chemical Process Control** *by* Stephanopoulos
8. **Modern Control Engineering** *by* Ogata
9. **Applied Process Control** *by* Chidambaram

Thank You