An Introduction to Digital Control

by

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Design of Digital Controllers

Traditional Approach:

- ✓ **Design the Analog Controller to meet a particular design specification.**

- ✓ **This can be carried out using Root Locus or Bode Plots or using Process Reaction Curve methods of Ziegler-Nichols or Cohen and Coon.**

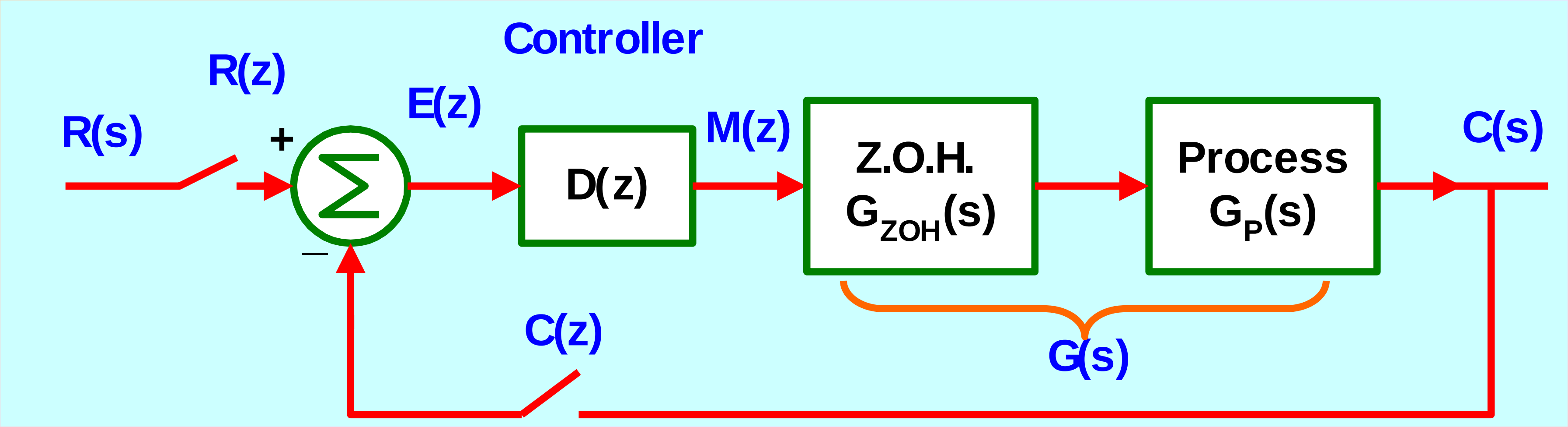
- ✓ **Transform this Analog Controller to its corresponding Digital Version by employing Z-transform or using difference equations.**

Design of Digital Controllers

Alternate Approach:

- ✓ **Design the Digital Controller directly in the discrete domain, based on the time domain specification of a closed-loop system response.**
- ✓ **The controlled plant is represented by a discretized model (a continuous system observed, analyzed and controlled at discrete intervals of time).**
- ✓ **This method of designing controllers, on the basis of desired closed-loop response, is called Direct Synthesis Method.**

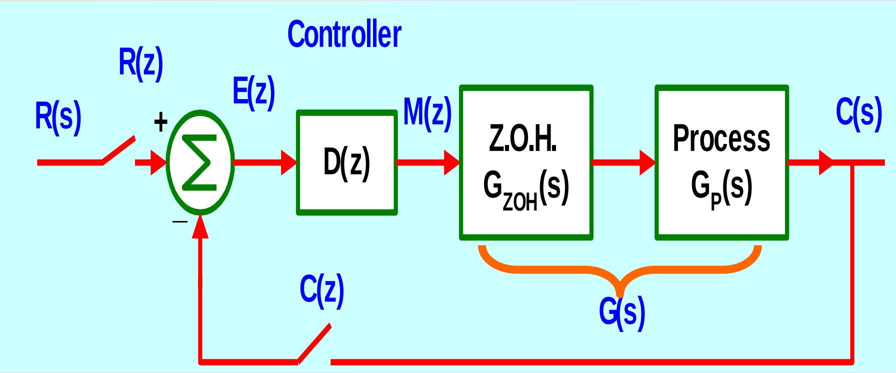
Direct Synthesis Method



A conventional Sampled Data Control System

✓ ***Assumption:*** **The process can be represented by low-order models.**

Direct Synthesis Method



A conventional Sampled Data Control System

Laplace Transform of Z.O.H:

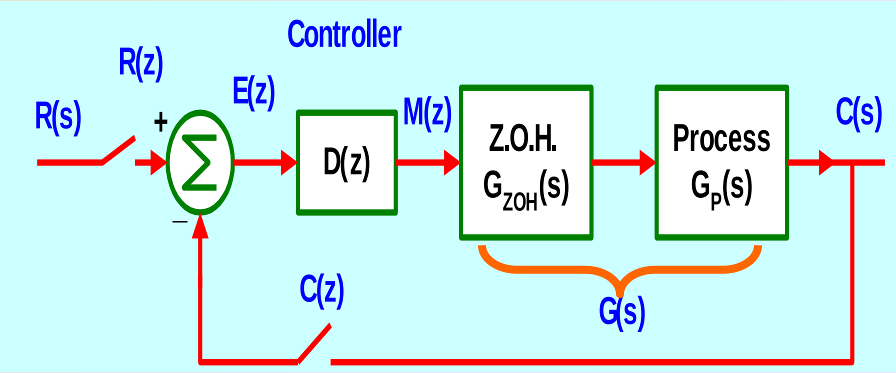
$$= \frac{1 - e^{-sT_s}}{s}$$

Pulse Transfer function of the process with the sample-and-hold operation of the DAC:

$$G(z) = Z[G(s)] = Z\left[\left(\frac{1 - e^{-sT_s}}{s}\right) G_p(s)\right]$$

- ✓ **The process can be represented by a first-order lag (τ) with delay (L) or a second-order lag (τ_1, τ_2) with delay (L).**

Direct Synthesis Method



A conventional Sampled Data Control System

The closed loop transfer function

$$= H(z) = z\left[\frac{C(s)}{R(s)}\right] = \frac{D(z)G(z)}{1+D(z)G(z)}$$

$$D(z) = \frac{1}{G(z)}.\frac{H(z)}{\left[1-H(z)\right]}$$

- ✓ ***Our Objective:*** **the closed loop should respond in a desired manner, following a desired dynamics specified in *H*(*z*).**

conclusion

- ✓ **The controller *D*(*z*) can be designed from the knowledge of the process model and the desired *H*(*z*) specified. Depending on designer specified *H*(*z*), different controllers may arise.**

Direct Synthesis Method

A Process described by First-Order Lag (τ) and Time Delay (L):

$$G(z) = Z\left[\left(\frac{1-e^{-sT_s}}{s}\right)\left(\frac{Ke^{-Ls}}{1+s\tau}\right)\right]$$

$$G(z) = K * Z\left[\left(1-e^{-sT_s}\right)e^{-Ls}\left(\frac{1}{s}-\frac{\tau}{1+s\tau}\right)\right]$$

$$G(z) = K * Z\left[\left(\frac{e^{-Ls}}{s}-\frac{e^{-Ls}}{s+\frac{1}{\tau}}\right)-\left\{\frac{e^{-s(L+T_s)}}{s}-\frac{e^{-s(L+T_s)}}{s+\frac{1}{\tau}}\right\}\right]$$

✓ ***Assumption:*** **The delay is an integral multiple of sampling time i.e. $L = NT_s$.**

Direct Synthesis Method

A Process described by First-Order Lag (τ) and Time Delay (L) (contd …):

$$G(z) = K * Z\left[\left(\frac{e^{-Ls}}{s} - \frac{e^{-Ls}}{s+\frac{1}{\tau}}\right) - \left\{\frac{e^{-s(L+T_s)}}{s} - \frac{e^{-s(L+T_s)}}{s+\frac{1}{\tau}}\right\}\right]$$

$$G(z) = K\left[\frac{z^{-N}}{1-z^{-1}} - \frac{z^{-N}}{\left(1-e^{-\frac{T_s}{\tau}}z^{-1}\right)} - \left\{\frac{z^{-(N+1)}}{1-z^{-1}} - \frac{z^{-(N+1)}}{\left(1-e^{-\frac{T_s}{\tau}}z^{-1}\right)}\right\}\right]$$

$$= K\left[z^{-N} - z^{-(N+1)}\right]\left[\frac{1}{1-z^{-1}} - \frac{1}{\left(1-e^{-\frac{T_s}{\tau}}z^{-1}\right)}\right]$$

$$G(z) = \frac{K\left(1-e^{-\frac{T_s}{\tau}}\right)z^{-(N+1)}}{\left(1-e^{-\frac{T_s}{\tau}}z^{-1}\right)}$$

Direct Synthesis Method

A Process described by Second-Order Lag (τ_1, τ_2) and Time Delay (L):

$$G(z) = Z\left[\frac{1-e^{-sT_s}}{s}\frac{Ke^{-Ls}}{(1+s\tau_1)(1+s\tau_2)}\right] = K * Z\left[\frac{1-e^{-sT_s}}{s}\frac{e^{-sNT_s}}{(1+s\tau_1)(1+s\tau_2)}\right] \quad (\text{Substituting } L \approx NT_s)$$

$$\vdots$$

$$= \frac{K\left(b_1 + b_2 z^{-1}\right)z^{-(N+1)}}{\left(1-e^{-\frac{T_s}{\tau_1}}z^{-1}\right)\left(1-e^{-\frac{T_s}{\tau_2}}z^{-1}\right)}$$

$$b_1 = \frac{\tau_1 e^{-T_s/\tau_1} - \tau_2 e^{-T_s/\tau_2}}{\tau_2 - \tau_1} + 1$$

$$b_2 = e^{-T_s\left(\frac{1}{\tau_2}+\frac{1}{\tau_1}\right)} + \frac{\tau_1 e^{-\frac{T_s}{\tau_2}} - \tau_2 e^{-\frac{T_s}{\tau_1}}}{\tau_2 - \tau_1}$$

Direct Synthesis Method

Constraint of Causality

- ✓ **For the digital controller to be physically realizable, causality should be ensured.**

What does it mean ??

- ✓ **The controller output m_n at any instant n should depend only on the present and the past values of the input error sequence i.e., e_n, e_{n-1}, e_{n-2} etc. and not on future values of input i.e., e_{n+1}, e_{n+2} etc.**

Direct Synthesis Method

Constraint of Causality (contd…)

Consider the *z*-T.F. of a controller of the form:

$$D(z)=\frac{M(z)}{E(z)}=\frac{b_o+b_1z+b_2z^2+\dots+b_kz^k}{a_o+a_1z+a_2z^2+\dots+a_jz^j}\qquad (\text{j and k are positive integers})$$

For causality, the necessary condition is $j \geq k$.

a logical extension

- ✓ **For a process which has a dead time, represented by z^{-N}, the desired C.L.T.F., $H(z)$, must also include the same dead time.**

Direct Synthesis Method

Dead-Beat Controller

- ✓ The dead-beat controller aims for the best response possible to a set point change.

What does it mean ??

- ✓ Following a set-point change, and after a time period equal to the system time-delay, the output should be at set-point and remain there.

↓

$$H(z) = z^{-N} \quad (N \geq 1)$$

***N*: the system delay, where $L = NT_s$.**

- ✓ ***Note:*** In ***Digital Control Loops***, there is always a minimum delay of one sampling interval.

Direct Synthesis Method

Dead-Beat Controller (contd…)

The *z*-T.F. of the dead-beat controller:

$$D(z) = \frac{1}{G(z)} \frac{z^{-N}}{1 - z^{-N}}$$

Design of a Dead-Beat Controller: An Example

Let the *plant* under control be:

$$G_p(s) = \frac{2\,\mathbf{exp}(-4s)}{(1+20s)}$$

$$G(z) = Z\left\{\frac{1-\mathbf{exp}(-sT_s)}{s} G_p(s)\right\} = (1\text{-}z^{-1}) Z\left\{\frac{2\,\mathbf{exp}(-4s)}{s(1+20s)}\right\}$$

- ✓ ***Next:*** **Select the *Sampling Interval* (T_s), here chosen as 10% of the time constant, i.e. T_s = 2. Thus, process delay N = (4/2) = 2.**

Direct Synthesis Method

Dead-Beat Controller (contd…)

Design of a Dead-Beat Controller: Example contd…

$$G(z) = (1\text{-}z^{-1})Z\left\{\frac{2\,\mathbf{exp}(-4s)}{s(1+20s)}\right\} = 2z^{-2}(1\text{-}z^{-1})Z\left\{\frac{1}{s(1+20s)}\right\}$$

$$Z\left\{\frac{1}{s(1+20s)}\right\} = Z\left\{\frac{1/20}{s\left(1/20+s\right)}\right\} = \frac{z\left[1-\mathbf{exp}\left(\frac{-T_s}{20}\right)\right]}{[z-1]\left[z-\mathbf{exp}\left(\frac{-T_s}{20}\right)\right]}$$

$$= \left(\frac{1}{1-z^{-1}}\right)\frac{\{1-\mathbf{exp}(-0.1)\}z^{-1}}{1-\mathbf{exp}(-0.1)z^{-1}}$$

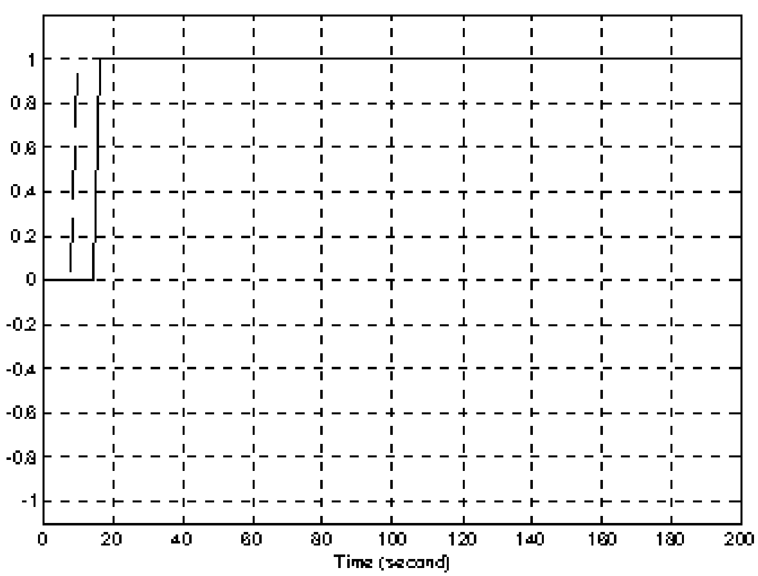
$$G(z) = 2z^{-2}(1-z^{-1})\left(\frac{1}{1-z^{-1}}\right)\left[\frac{\{1-\mathbf{exp}(-0.1)\}z^{-1}}{1-\mathbf{exp}(-0.1)z^{-1}}\right] = \frac{0.190z^{-3}}{1\text{-}0.905z^{-1}}$$

Direct Synthesis Method

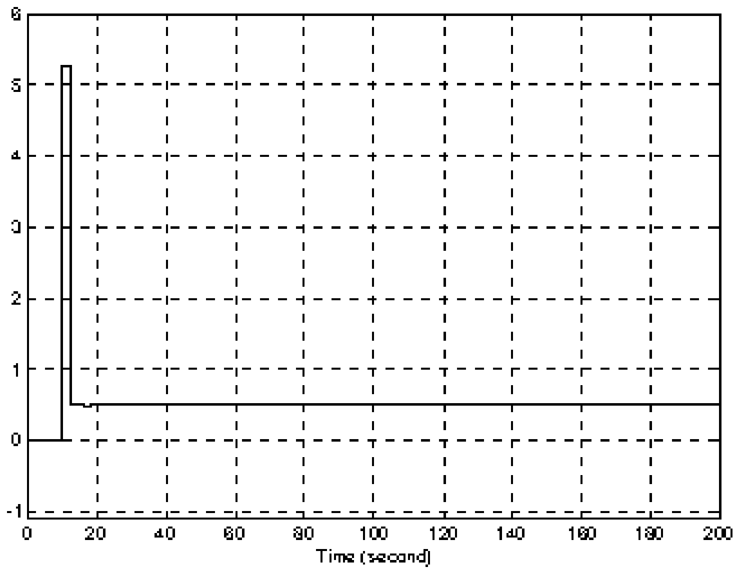
Dead-Beat Controller (contd…)

Design of a Dead-Beat Controller: Example contd…

$$G(z) = \frac{0.190z^{-3}}{1-0.905z^{-1}} \Rightarrow D(z) = \left(\frac{1-0.905z^{-1}}{0.190z^{-3}}\right)\left(\frac{z^{-3}}{1-z^{-3}}\right) = 5.263\left(\frac{1-0.905z^{-1}}{1-z^{-3}}\right)$$



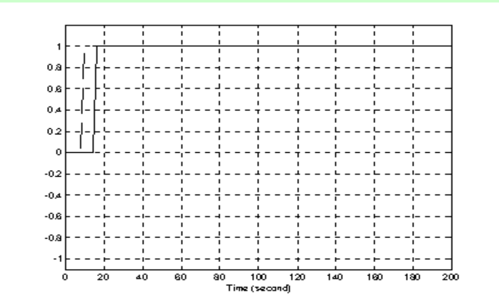
Closed-loop Response using Dead-Beat Controller



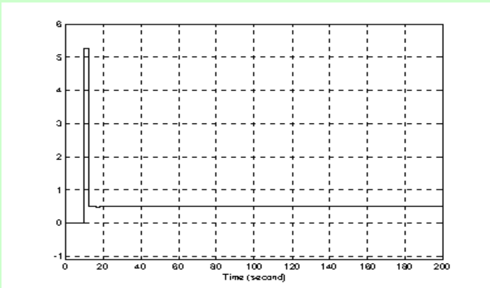
Control signal for the system using Dead-Beat Controller

Direct Synthesis Method

Dead-Beat Controller (contd…)



Closed-loop Response using **Dead-Beat Controller**



Control signal for the system using **Dead-Beat Controller**

The *Dahlin Controller* can satisfy these requirements.

Any Solution ??

Strength of this method…

- ✓ **The dead-beat controller provided an excellent closed loop response.**

Weakness of this method…

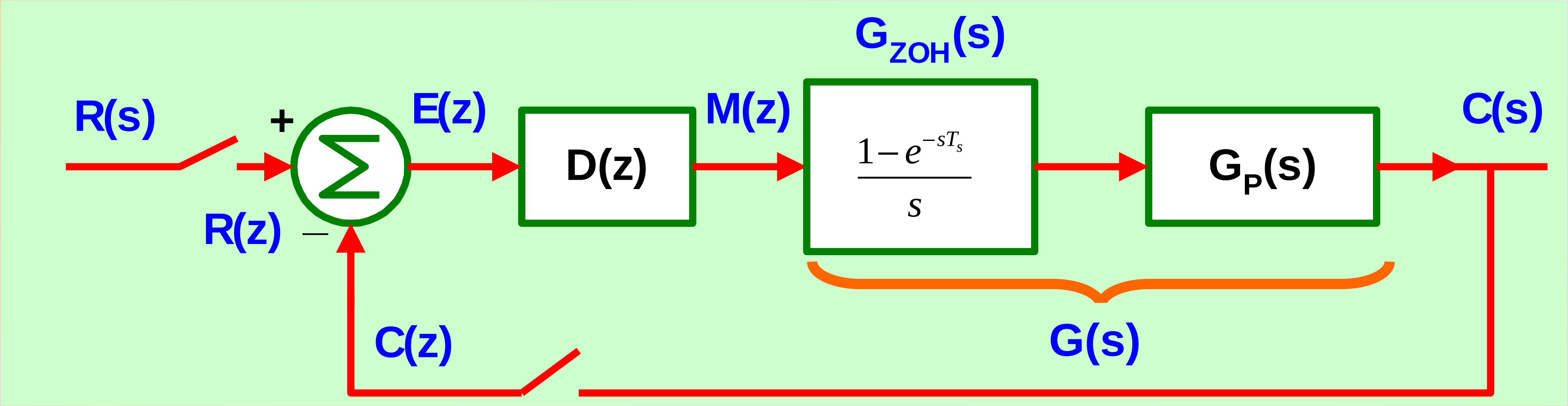
- ✓ **Large excursions of control signal may cause excessive wear and tear of the final control elements.**

Conclusion…

May be it will be better to specify a less exacting closed loop performance.

Direct Synthesis Method

Dahlin Controller



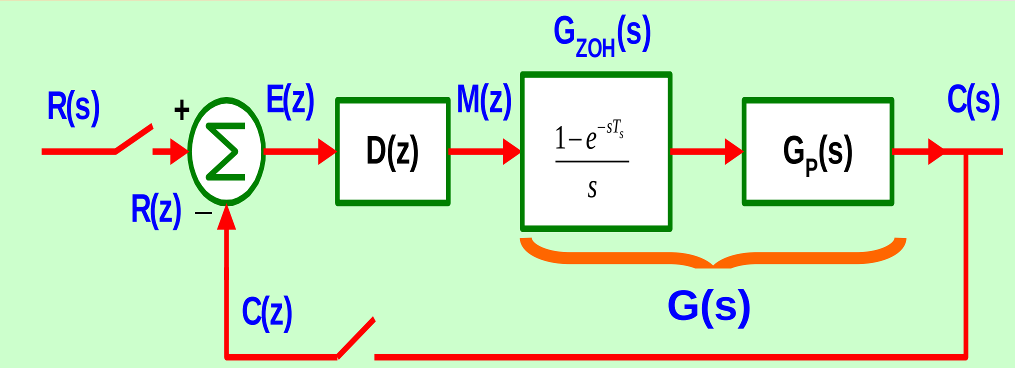
✓ In ***Dahlin's Method***, **the plant is assumed to be modeled by a first-order or second-order transfer function, and the desired C.L.T.F., *H*(*s*), is considered to be a first order lag with dead time, with unity gain.**

$$H(s)=\frac{\lambda e^{-Ls}}{s+\lambda}=\frac{e^{-Ls}}{1+s/\lambda}$$

$$\left(\begin{array}{l}\text{dead time} = \text{L} \\ \text{time const} = \frac{1}{\lambda}\end{array}\right)$$

Direct Synthesis Method

Dahlin Controller (contd…)



The *z*-domain C.L.T.F. (with the Z.O.H. included for physical realizability of the system):

$$H(z) = \frac{\left(1 - e^{-\lambda T_s}\right) z^{-N-1}}{1 - e^{-\lambda T_s} z^{-1}}$$

- ✓ Here λ is called the ***Tuning Parameter***, and it is the reciprocal of the closed loop time constant (τ).

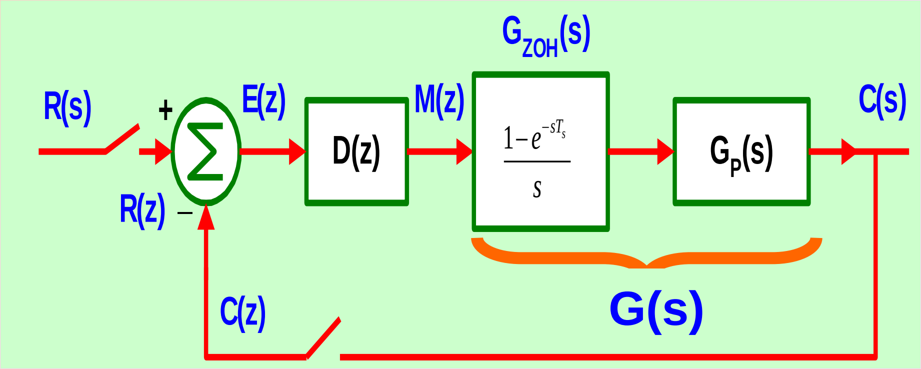
Large λ… → Closed-loop system response will be ***faster***.

Small λ… → Closed-loop system response will be ***sluggish***.

Note: λ is normally chosen as 2 to 3 times as fast as the process time constant.

Direct Synthesis Method

Dahlin Controller (contd…)



The *z*-T.F. of the controller:

$$D(z)=\frac{1}{G(z)}\frac{H(z)}{1-H(z)}\left[G(z)=Z\left[G(s)\right]\right]$$

$$\therefore D(z)=\frac{1}{G(z)}\frac{\left(1-e^{-\lambda T_s}\right)z^{-N-1}}{\left[1-e^{-\lambda T_s}z^{-1}-\left(1-e^{-\lambda T_s}\right)z^{-N-1}\right]}=\frac{M(z)}{E(z)}$$

(putting $L=NT_s$)

Direct Synthesis Method

Dahlin Controller (contd...)

Design of a Dahlin Controller: An Example

Let the *plant* under control be:

$$G_p(s) = \frac{2\,\mathbf{exp}(-4s)}{(1+20s)}$$

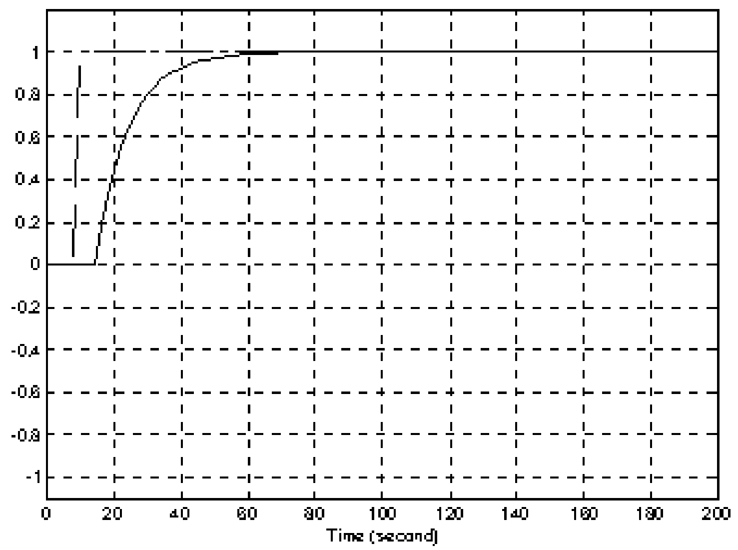
- ***Next:*** **We choose *Closed Loop Time Constant* (τ) = 10. For *Sampling Interval* T_s = 2,** $e^{-\lambda T_s} = e^{-\left(T_s/\tau\right)} = 0.819$.

$$D(z) = \left(\frac{1-0.905z^{-1}}{0.190z^{-3}}\right)\left(\frac{0.181z^{-3}}{1-0.819z^{-1}-0.181z^{-3}}\right) = 0.953\left(\frac{1-0.905z^{-1}}{1-0.819z^{-1}-0.181z^{-3}}\right)$$

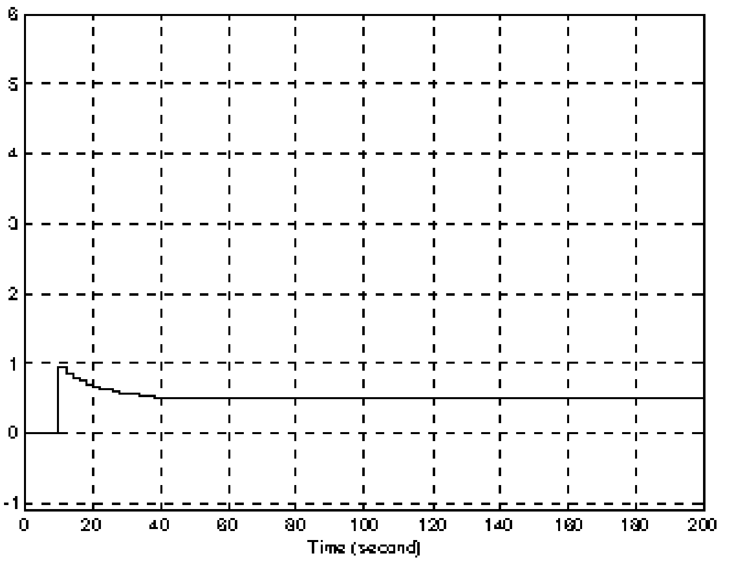
Direct Synthesis Method

Dahlin Controller (contd…)

Design of a Dahlin Controller: Example contd…



Closed-loop Response using Dahlin Controller



Control signal for the system using Dahlin Controller

Conclusion…

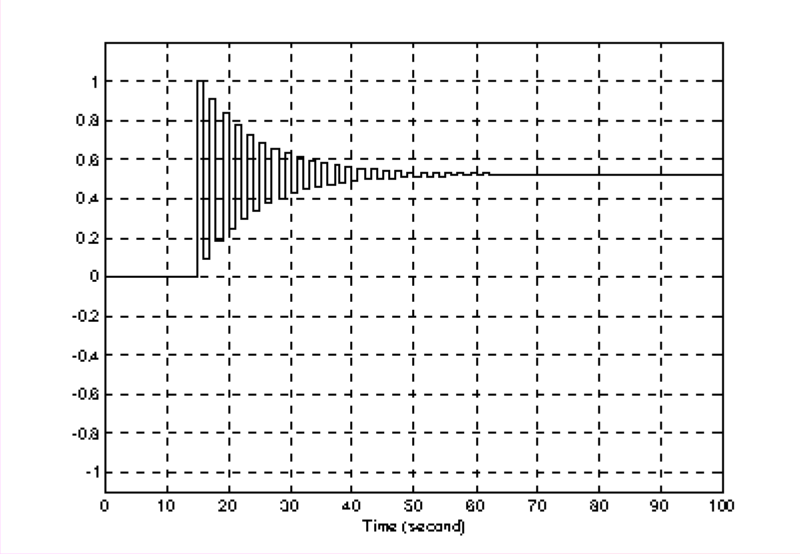
✓ **The controlled output approaches the set point in a slower manner. But the control signal is much more acceptable in practical situations.**

Ringing of Digital Controller

What is Ringing ??

- ✓ **Sometimes, a digital controller produces a control signal that keeps oscillating with decreasing amplitude about the final equilibrium value. This phenomenon is called *Ringing*.**

a typical example



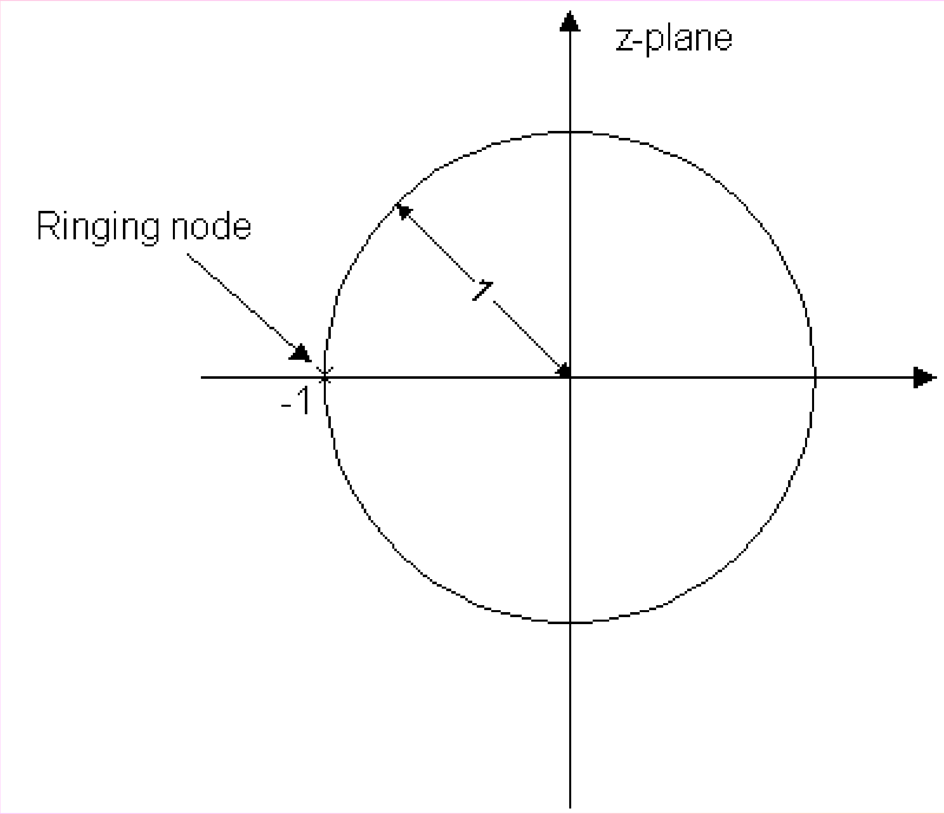
What is the Cause of Ringing ??

✓ **It is caused by negative real controller poles. For example, this can happen due to a poor choice of τ in the Dahlin Controller.**

Ringing of Digital Controller (contd…)

Effect of Controller Poles on Ringing …

✓ **The closer the controller pole to the –1 point in the *z*-plane, the more severe the ringing. The –1 point is called the *Ringing node*.**

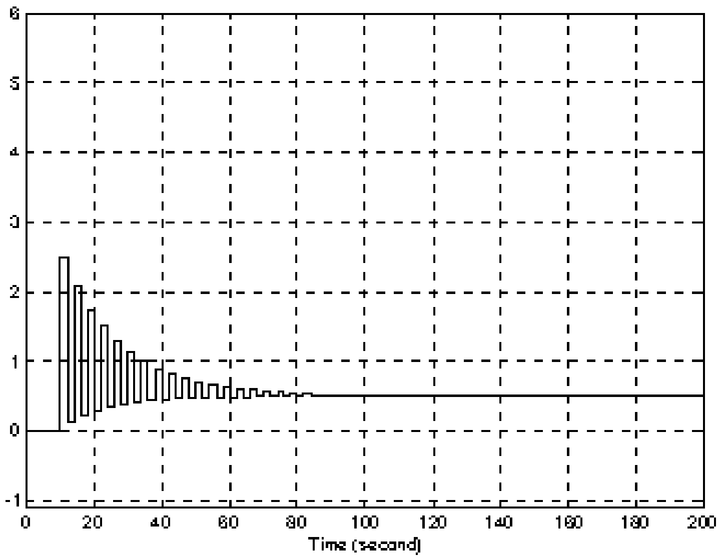


Ringing of Digital Controller (contd…)

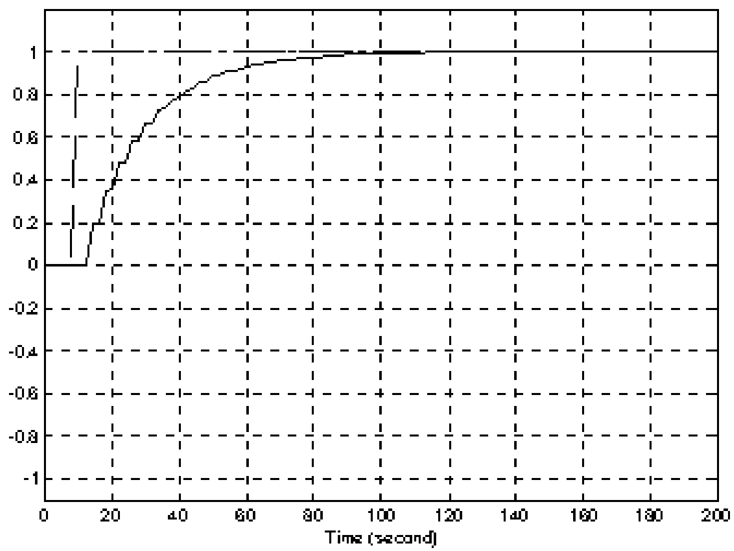
An Important Point …

✓ **Even if the control signals are oscillating, they may not have any reflections in the controlled output.**

an example



Control Signal



Closed-loop Response

Ringing of Digital Controller (contd…)

We consider a digital controller with T.F.:

$$D(z)=\frac{1}{1-bz^{-1}}=\frac{M(z)}{E(z)}$$

(*b* is a *real pole* of the T.F.)

$$\therefore E(z)=M(z)-bz^{-1}M(z)$$

$$or, M(z)=E(z)+bz^{-1}M(z)$$

$$\therefore m_n=e_n+bm_{n-1}$$

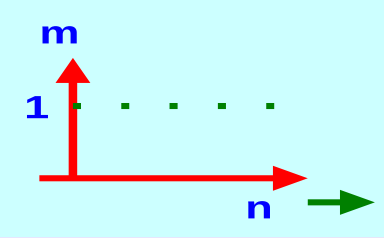
We consider that an error equal to unit impulse enters the controller at *t* = 0.

$$e_n=1, \quad \text{for} \quad n=0,$$
$$e_n=0, \quad \text{for} \quad n=1,2,3,\cdots$$

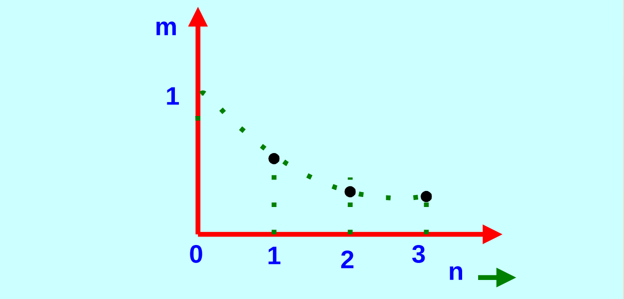
$$m_n=1, \quad \text{for} \quad n=0,$$
$$m_n=b, \quad \text{for} \quad n=1,$$
$$m_n=b^2, \quad \text{for} \quad n=2,$$
$$\ldots$$

$$m_n=b^n$$

If *b* = 1, *m* is a discrete-time step



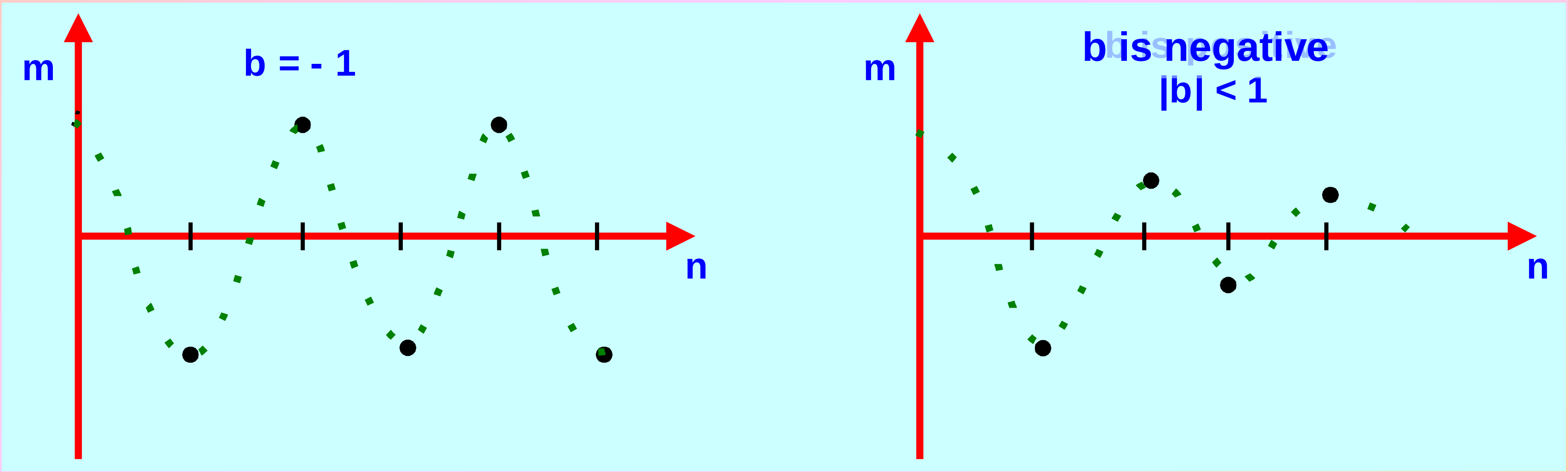
If *b* is a +ve real number < 1, *m* will gradually decay with time



Ringing of Digital Controller (contd…)

If *b* is a negative real number: $m_n = (-1)^n |b|^n$

Then there will be successive changes in the sign of the controller output.



✓ **Smaller the value of |*b*|, more the damping of oscillations of the controller output.**

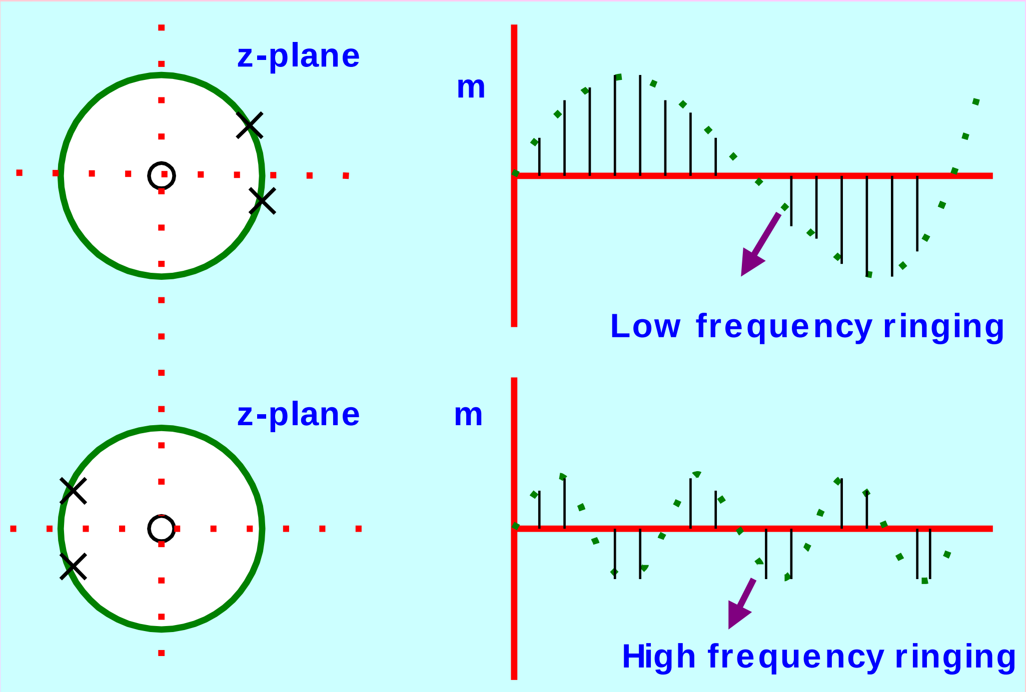
Ringing of Digital Controller (contd…)

We consider a digital controller with T.F.:

$$D(z)=\frac{M(z)}{E(z)}=\frac{ze^{-a\tau}\sin\omega\tau}{z^2-2e^{-a\tau}z\sin\omega\tau+e^{-2a\tau}}=Z\left(e^{-an\tau}\sin\omega n\tau\right)$$

***M*(*z*)** → ***z*-transform of the impulse response of the controller**

***D*(*z*)** → **It has complex conjugate pole pair**

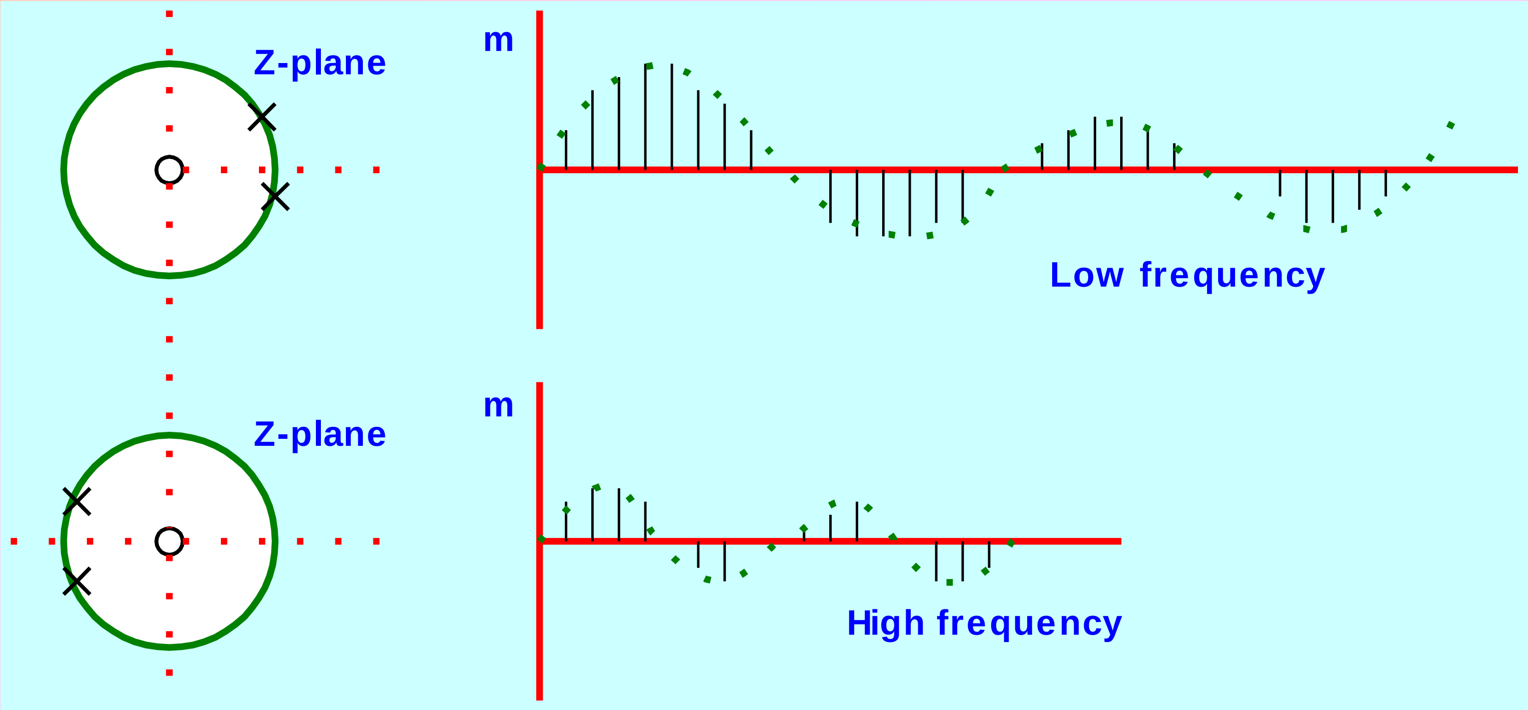


Conclusion …

If the pole pair is in the left half of the *s – plane*, the frequency of oscillation is higher.

Ringing of Digital Controller (contd…)

If the complex conjugate pole pair lie within the unit circle:



Conclusion …

This gives rise to damped oscillation of the controller output. Hence, complex conjugate pole pairs of the z-T.F. of a digital controller also gives rise to controller ringing.

Ringing of Digital Controller (contd…)

Final Conclusions …

- ✓ **Negative real poles and complex conjugate pole pairs of the controller's *z*-T.F. give rise to *ringing*. *Ringing* is more pronounced for complex conjugate pole pairs in left half of z-plane than that in right half of z-plane.**

- ✓ **The closer a ringing pole is to the unit circle, the higher will be the ringing of the controller.**

What are the Effects of Ringing ??

- ✓ **Ringing increases the wear and tear of final control elements, and can cause system instability within a multi-loop environment. This is *unacceptable* in industrial practice.**

Ringing of Digital Controller (contd…)

How to Overcome Ringing ??

- **By using more accurate process models. But developing accurate process models can be quite complicated.**

Is There any Easy Way-out ??

- ***Yes*, Dahlin suggested an easier and quicker solution for this problem.**

- ***Step 1*: Locate the errant controller pole.**
- ***Step 2*: Replace that factor by its steady-state equivalent.**

Ringing of Digital Controller (contd…)

Removal of Ringing Poles in Dahlin Controller: An Example

We consider a Dahlin controller with T.F.:

$$D(z)=\left(\frac{1}{G(z)}\right)\left(\frac{\left(1-e^{-\frac{T_s}{\tau}}\right)z^{-N}}{1-e^{-\frac{T_s}{\tau}}z^{-1}-\left(1-e^{-\frac{T_s}{\tau}}\right)z^{-N}}\right)$$

- ✓ **Let N = 2 and we choose,** $e^{-\lambda T_s}=e^{-\left(T_s/\tau\right)}=0.01$.

$$D(z)=\left(\frac{1}{G(z)}\right)\left(\frac{0.99z^{-2}}{1-0.01z^{-1}-0.99z^{-2}}\right)$$

$$D(z)=\left(\frac{1}{G(z)}\right)\left(\frac{0.99z^{-2}}{(1-z^{-1})(1+0.99z^{-1})}\right)$$

- ✓ **There is a controller pole at z = -0.99 that can cause ringing problems. This should be removed, without causing any change in the gain of the controller.**

Ringing of Digital Controller (contd…)

Removal of Ringing Poles in Dahlin Controller: Example contd…

$$D(z)=\left(\frac{1}{G(z)}\right)\left(\frac{0.99z^{-2}}{1-0.01z^{-1}-0.99z^{-2}}\right) \Rightarrow D(z)=\left(\frac{1}{G(z)}\right)\left(\frac{0.99z^{-2}}{(1-z^{-1})(1+0.99z^{-1})}\right)$$

How to Remove this Ringing Pole ??

- ✓ **evaluate** $(1+0.99z^{-1})$ **at** $z = 1$.

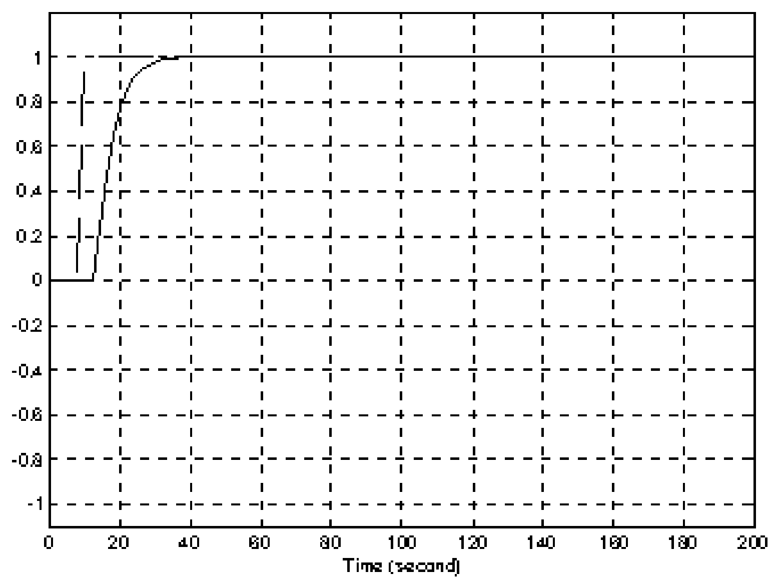
$$D(z)=\left(\frac{1}{G(z)}\right)\left(\frac{0.99z^{-2}}{1.99(1-z^{-1})}\right)$$

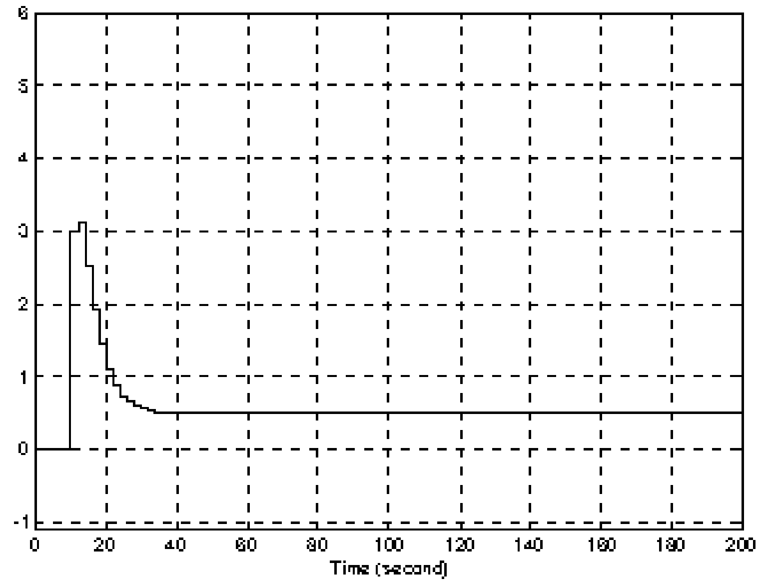
- ✓ **This removal of ringing pole results in a better behaved response.**

Ringing of Digital Controller (contd…)

Removal of Ringing Poles in Dahlin Controller:

The Results obtained in the Example …





Closed-loop Response

Control Signal

(After removal of the ringing pole)

Design of Predictive Controllers

The Key Essence:

- ✓ **The simplest predictive controller requires an estimate of the process output at the next sampling instant and based on this information generates an actuation signal that strives to make the output equal to the desired value.**

Why is it called Predictive Control ??

- ✓ **Because it depends upon a process model to *predict* the future value of the controlled variable and uses this value as a controller input.**

Source of its Richness…

- ✓ **This method is simple and can be readily extended to handle measurable disturbances using feed forward.**

Design of Predictive Controllers

Direct Single-Step Design

T.F. of a second- order plant without time delay:

$$G(z)=\frac{C(z)}{M(z)}=\frac{K\left(b_1+b_2z^{-1}\right)z^{-1}}{1+a_1z^{-1}+a_2z^{-2}} \quad \dots (1)$$

where

$$a_1=-\left(e^{-T_s/\tau_1}+e^{-T_s/\tau_2}\right);\, a_2=e^{-T_s\left(\frac{1}{\tau_1}+\frac{1}{\tau_2}\right)};$$

$$b_1=1+\frac{\tau_1e^{-T_s/\tau_1}-\tau_2e^{-T_s/\tau_2}}{\tau_2-\tau_1};\, b_2=e^{-T_s\left(\frac{1}{\tau_2}+\frac{1}{\tau_1}\right)}+\frac{\tau_1e^{-T_s/\tau_2}-\tau_2e^{-T_s/\tau_1}}{\tau_2-\tau_1};$$

Assumption: **the constant K is absorbed in the b_1 and b_2 coefficients.**

Design of Predictive Controllers

Direct Single-Step Design (contd…)

From eq. (1), we get: $C(z)\left(1+a_1z^{-1}+a_2z^{-2}\right)=M(z)\left(b_1+b_2z^{-1}\right)z^{-1}$

In time-domain difference equation form: $c_{n+1}=-a_1c_n-a_2c_{n-1}+b_1m_n+b_2m_{n-1}$

… (2)

c_{n+1}: predicted system output at sampling instant $(n+1)$ under the present control action m_n.

This prediction is one-sample-ahead prediction.

Design of Predictive Controllers
Direct Single-Step Design (contd…)

In simplest predictive control strategy, we choose current actuation m_n in such a way that the next system output c_{n+1} is equal to the desired set point r_n.

From eq. (2), we get:

$$b_1 m_n + b_2 m_{n-1} = r_n + a_1 c_n + a_2 c_{n-1}$$

… (3)

$$m_n = \frac{1}{b_1}\left[r_n + a_1 c_n + a_2 c_{n-1} - b_2 m_{n-1}\right]$$

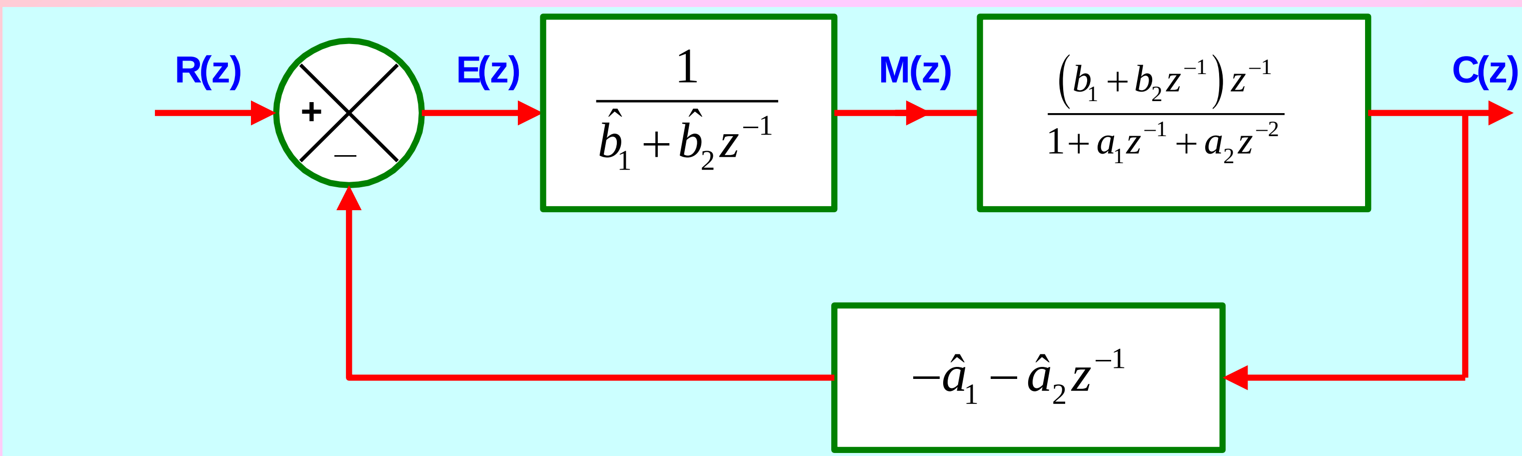
This gives the control action required at the current sampling instant, in our strive to make c_{n+1} equal r_n.

Design of Predictive Controllers

Direct Single-Step Design (contd…)

z-transform of eq. (3) gives:

$$M(z)\left(b_1 + b_2 z^{-1}\right) = R(z) + C(z)\left(a_1 + a_2 z^{-1}\right)$$



Direct Single Step Design (Deadbeat).

The controller designed based on this philosophy is called a *Deadbeat Controller*.

Design of Predictive Controllers
Direct Single-Step Design (contd…)

The Salient Feature of Dead-Beat Controllers:

- ✓ **This design procedure requires that the closed-loop response has a finite settling time, minimum rise time, and zero steady-state error.**

Any Problem with this Design Methodology ??

- ✓ ***Yes*, this is an *unrealistic design*, because it is attempting to drive the plant to the desired value in a single sampling instant and this assumes that there is no limit on the actuation signal.**

- ✓ ***Also*, this controller assumes that the plant model accurately represents the process, with plant parameters being exact, which again is an *unrealistic assumption*.**

Design of Predictive Controllers

Then, do we have a Better Solution than Dead-Beat Controllers ??

- ✓ **The answer is *YES*. We can design for the output to follow a set point with a pre-determined transient response profile. This is called the Model Following Design.**

What is its Design Specification ??

- ✓ **Instead of reaching the set point r_n in one sample time, the plant should only reach a fraction (α) of it. $0 < \alpha \leq 1$.**

⇩

- ✓ **By choosing a lower and lower value for weighting factor α, the response can be made a slower and slower one. But finally the response approaches infinitesimally close to r_n.**

Design of Predictive Controllers

Model Following Design

Let: $$c_{n+1} = c_n + \alpha\left(r_n - c_n\right) \quad \ldots (4)$$

Taking z-transform: $$\frac{C(z)}{R(z)} = \frac{\alpha z^{-1}}{1-(1-\alpha)z^{-1}} = \frac{(1-\beta)z^{-1}}{1-\beta z^{-1}}$$

$\beta = 1 - \alpha$

Any Similarity with Dahlin's Design ??

- ✓ ***Yes***, **this transfer function is very similar to that in Dahlin's design with** $\beta = e^{-\lambda T_s}$ **.**

- ✓ **Hence the design philosophy is similar to the Dahlin's design method, although it results in a different structure with both feedback and feedforward components to the controller.**

Design of Predictive Controllers
Model Following Design (contd...)

From eq. (4):

$$c_{n+1} = c_n + (1-\beta)(r_n - c_n)$$

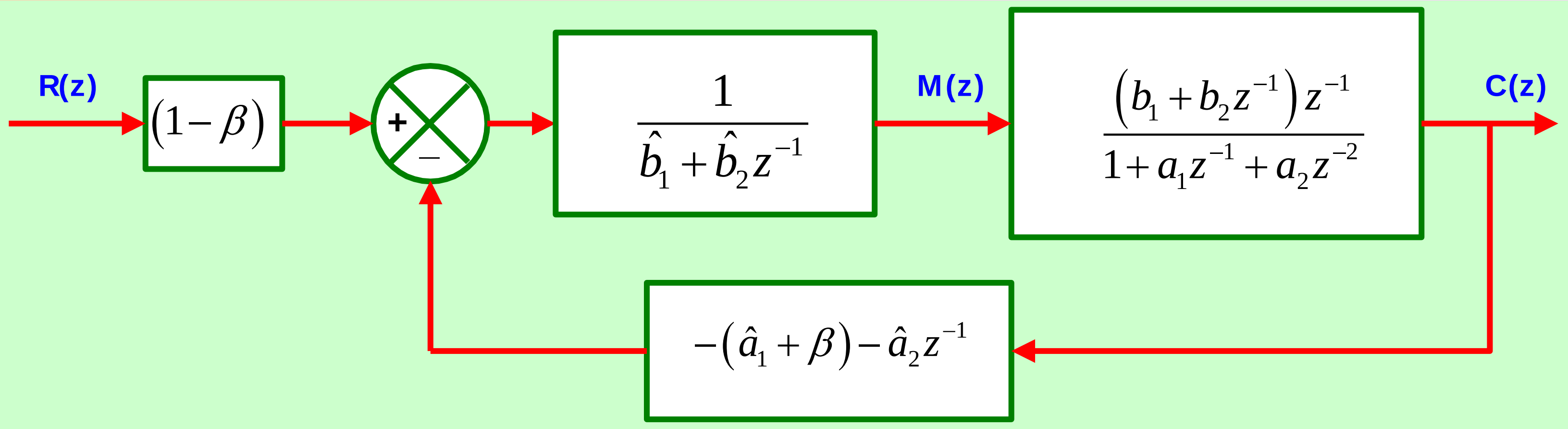
$$c_n + (1-\beta)(r_n - c_n) = -a_1 c_n - a_2 c_{n-1} + b_1 m_n + b_2 m_{n-1}$$

$$m_n = \frac{1}{b_1}\left[(a_1+\beta)c_n + a_2 c_{n-1} - b_2 m_{n-1} + (1-\beta) r_n\right]$$

$$M(z) = \frac{1}{b_1 + b_2 z^{-1}}\left[(1-\beta)R(z) + \left\{(a_1+\beta) + a_2 z^{-1}\right\}C(z)\right]$$

Design of Predictive Controllers

Model Following Design (contd...)



Model Following Design.

To accommodate a more general closed loop characteristic, it is usual to form an auxiliary output $\Phi(z)$ from the system output $C(z)$.

$$\phi(z)=P(z)C(z)$$

$P(z)$: **the inverse of the desired closed loop response**

Design of Predictive Controllers
Model Following Design (contd…)

If a dead-beat controller is designed now to set $\phi_{n+1} = r_n$, the output:

$$C(z) = \frac{\phi(z)}{P(z)} = \frac{R(z)}{P(z)}$$

It has a response $\dfrac{1}{P(z)}$ as required.

Any Important Consideration for this 1/P(z) ??

- ✓ ***Yes*, it should have a time constant that is reasonable for the plant characteristics under consideration and have a steady-state gain of unity to ensure the steady-state system output is equal to the set point value.**

Design of Predictive Controllers

Comparison Between Dahlin's Design and Predictive Designs

- ✓ **As in Dahlin's Design, in Predictive Designs also, the plant zero gets cancelled by the controller, and consequently, the *ringing pole* problem still occurs.**

Any Solution ??

- ✓ **This problem can be removed by the same procedure as that adopted with the Dahlin's Algorithm.**

Design of Predictive Controllers

Control Weighting Design

- ✓ **The predictive control design philosophies described so far are all based on a simple minimization of a *Performance Criterion*.**

How ??

- ✓ **For *dead-beat controllers*, the squared error between c_{n+1} and r_n is minimized by setting $c_{n+1} = r_n$.**

- ✓ **For *model following control*, we minimize the squared error between prediction c_{n+1} and some function of the set point.**

Design of Predictive Controllers

Control Weighting Design (contd…)

But, We are not Considering an Important Point …

- ✓ **In addition with errors in model following, another important consideration is actuating signal.**

How to Take This into Account ??

- ✓ **By defining a *more general performance criterion* that will take both factors into consideration.**

What will be the Effect ??

- ✓ **The *accuracy of model following can be traded* for large excursions in plant actuations, and a suitable compromise can be reached.**

Design of Predictive Controllers

Control Weighting Design (contd…)

The generalized performance index (*J*) to be minimized for controller design:

$$J = (c_{n+1} - r_n)^2 + \gamma m_n^2$$

γ : a weighting factor

Minimizing *J* with respect to m_n:

$$\frac{dJ}{dm_n} = 2[c_{n+1} - r_n]\frac{\partial c_{n+1}}{\partial m_n} + 2\gamma m_n = 0$$

From the prediction equation:

$$c_{n+1} = -a_1 c_n - a_2 c_{n-1} + b_1 m_n + b_2 m_{n-1}$$

Therefore:

$$\frac{\partial c_{n+1}}{\partial m_n} = b_1$$

$$\therefore (c_{n+1} - r_n) + \frac{\gamma}{b_1} m_n = 0$$

$$-a_1 c_n - a_2 c_{n-1} + b_1 m_n + b_2 m_{n-1} - r_n + \frac{\gamma}{b_1} m_n = 0$$

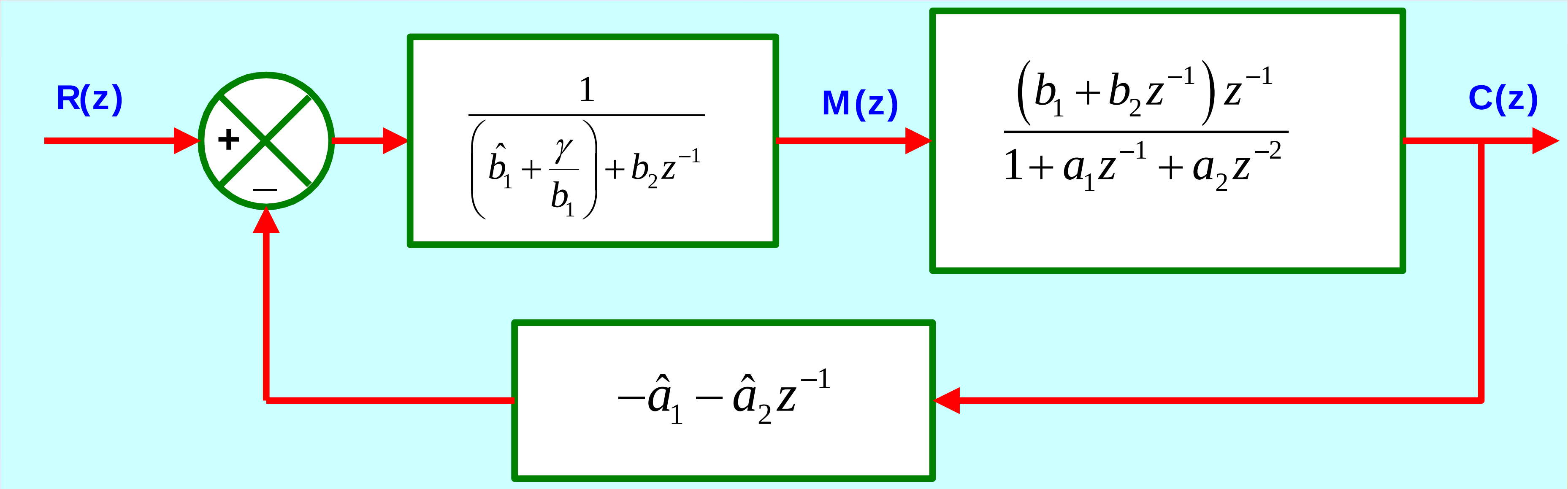
$$m_n = \frac{1}{\left(b_1 + \frac{\gamma}{b_1}\right)}[r_n + a_1 c_n + a_2 c_{n-1} - b_2 m_{n-1}]$$

Design of Predictive Controllers

Control Weighting Design (contd…)

Taking *z*-transform:

$$M(z)=\frac{1}{\left[\left(b_1+\frac{\gamma}{b_1}\right)+b_2z^{-1}\right]}\left[R(z)+C(z)\left(a_1+a_2z^{-1}\right)\right]$$



Design using Control Weighting.

Design of Predictive Controllers

Control Weighting Design (contd…)

Special Features of this Design …

- ✓ **The pole position of the controller is now dependent on the value of γ.**

⇓

- ✓ **As γ varies, this pole moves from a position where it cancels the zero exactly (when $\gamma = 0$) to a point closer to the origin, with a consequent reduction in the forward path gain.**

Design of Predictive Controllers

Control Weighting Design (contd…)

How does it Compare with the Dahlin's Design ??

- **In Dahlin's Design, in the case of the plant having a zero near the unit circle no cancellation occurs, but the steady state gain of the controller is maintained.**

- **In Control Weighting Design, a compromise can be obtained by moving the ringing pole closer to the origin.**

Any Penalty Paid ??

- ***Yes*, the penalty paid for this is that the model following is less exact but it does give the advantage of significant reductions in the actuation signal.**

Design of Predictive Controllers

Incremental Form of the Predictor

Constraints of the Predictive Control Approaches, discussed so far

- ✓ **Unfortunately, zero steady state error is only guaranteed in the absence of disturbances, and if the model coefficients are exact.**

- ✓ **In Deadbeat Design and Model Following Design, the steady state conditions of the loop for the steady state value of the output to be equal to the set point, results in an actuating signal:**

$$m_{ss} = r_{ss} \frac{\left(1 + \hat{a}_1 + \hat{a}_2\right)}{\hat{b}_1 + \hat{b}_2}$$

But, There is a Catch …

This value of actuation is only produced if the estimated plant parameters are exact and if there is no load disturbance.

Design of Predictive Controllers

Incremental Form of the Predictor (contd…)

How to Overcome this Problem ??

- ✓ **By writing the *Predictor Equation* in *Incremental Form* so that it may predict the absolute output c_{n+1} from previous outputs and changes in actuations.**

$$c_{n+1}-c_n=-a_1\left(c_n-c_{n-1}\right)-a_2\left(c_{n-1}-c_{n-2}\right)+b_1\Delta m_n+b_2\Delta m_{n-1}$$

where

$$\Delta m_n=m_n-m_{n-1}$$

$$c_{n+1}=\left(1-a_1\right)c_n+\left(a_1-a_2\right)c_{n-1}+a_2c_{n-2}+b_1\Delta m_n+b_2\Delta m_{n-1}$$

$$m_n=\frac{1}{b_1}\left[\left(1-\beta\right)r_n+\left(a_1+\beta-1\right)c_n+\left(a_2-a_1\right)c_{n-1}-a_2c_{n-2}+b_1m_{n-1}-b_2\left\{m_{n-1}-m_{n-2}\right\}\right]$$

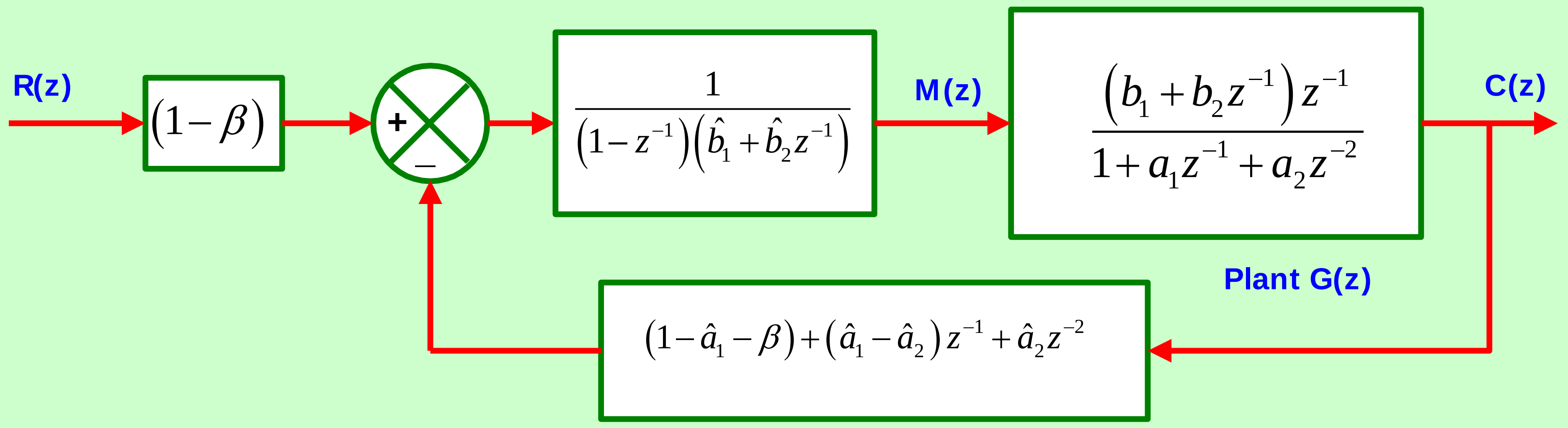
Design of Predictive Controllers

Incremental Form of the Predictor (contd...)

$$m_n = \frac{1}{b_1}\left[(1-\beta)r_n + (a_1+\beta-1)c_n + (a_2-a_1)c_{n-1} - a_2c_{n-2} + b_1m_{n-1} - b_2\{m_{n-1}-m_{n-2}\}\right]$$

z-transform

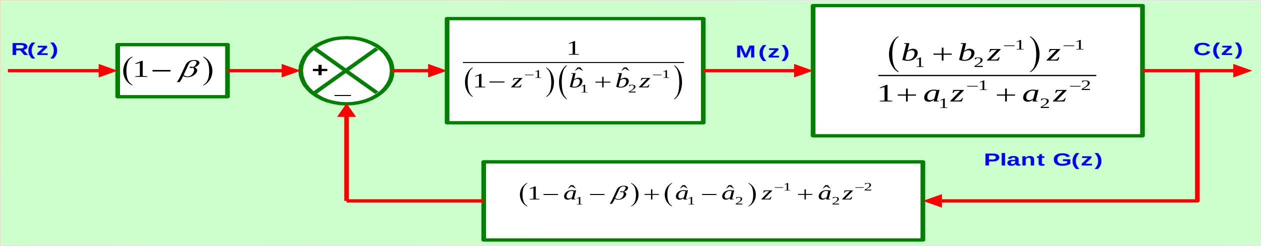
$$M(z) = \frac{1}{(1-z^{-1})(b_1+b_2z^{-1})}\left[(1-\beta)R(z) + \{(a_1+\beta-1)+(a_2-a_1)z^{-1} - a_2z^{-2}\}C(z)\right]$$



Model Following Design using Incremental Form of Controller.

Design of Predictive Controllers

Incremental Form of the Predictor (contd…)



Model Following Design using Incremental Form of Controller.

Characteristic Features …

- ✓ **An integrator term $\left(1-z^{-1}\right)^{-1}$ has been introduced in the forward path.**
- ✓ **In the *Steady State*, the feedback path gets reduced to:**

$$\left(1-\hat{a}_1-\beta\right)+\left(\hat{a}_1-\hat{a}_2\right)+\hat{a}_2=1-\beta$$

It is independent of the accuracy of the estimated plant parameters

This controller can reduce steady-state errors caused either by load variations or by parameter inaccuracies, to zero

Design of Predictive Controllers

Feedforward Compensation

- ✓ One important feature of predictive control is that measurable disturbances can be readily incorporated in the algorithms.

- ✓ Let us consider a disturbance $v(t)$ which affects the plant.

Plant Output

$$C(z)A(z)=B(z)M(z)+L(z)V(z)$$

where

$$L(z)=l_1z^{-1}+l_2z^{-2}+l_3z^{-3}+........$$

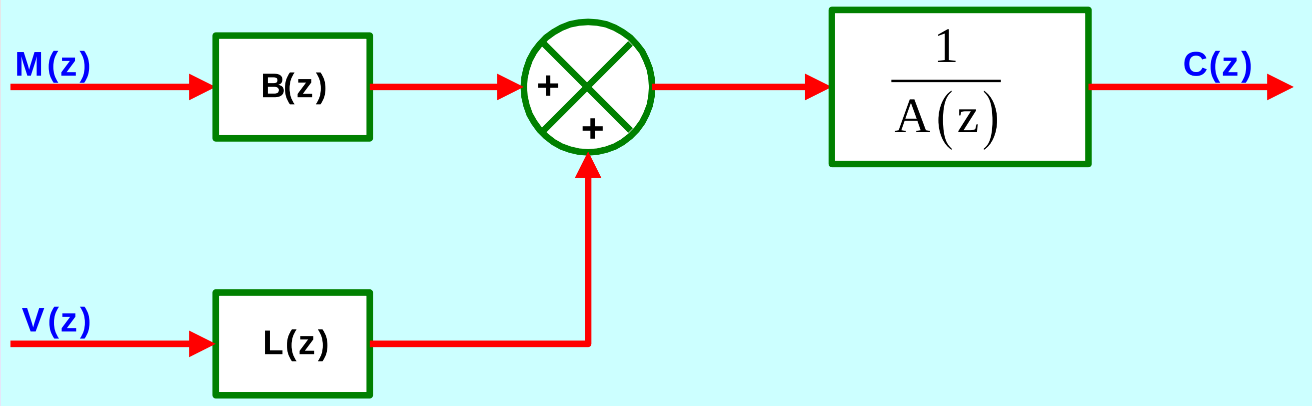
In Difference Equation Form

$$c_{n+1}=-a_1c_n-a_2c_{n-1}+b_1m_n+b_2m_{n-1}+l_1v_n+l_2v_{n-1}$$

Design of Predictive Controllers

Feedforward Compensation (contd...)

$$C(z)A(z) = B(z)M(z) + L(z)V(z)$$



Disturbance Structure for Feedforward Compensation.

For the plant output expressed in the upper form, the denominator dynamics are common to both the actuations and the disturbances.

Effectively this means that both actuations and disturbances act on the plant at the same point, as shown in the above figure.

Digital Control

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Thank You…