Digital Communication

by

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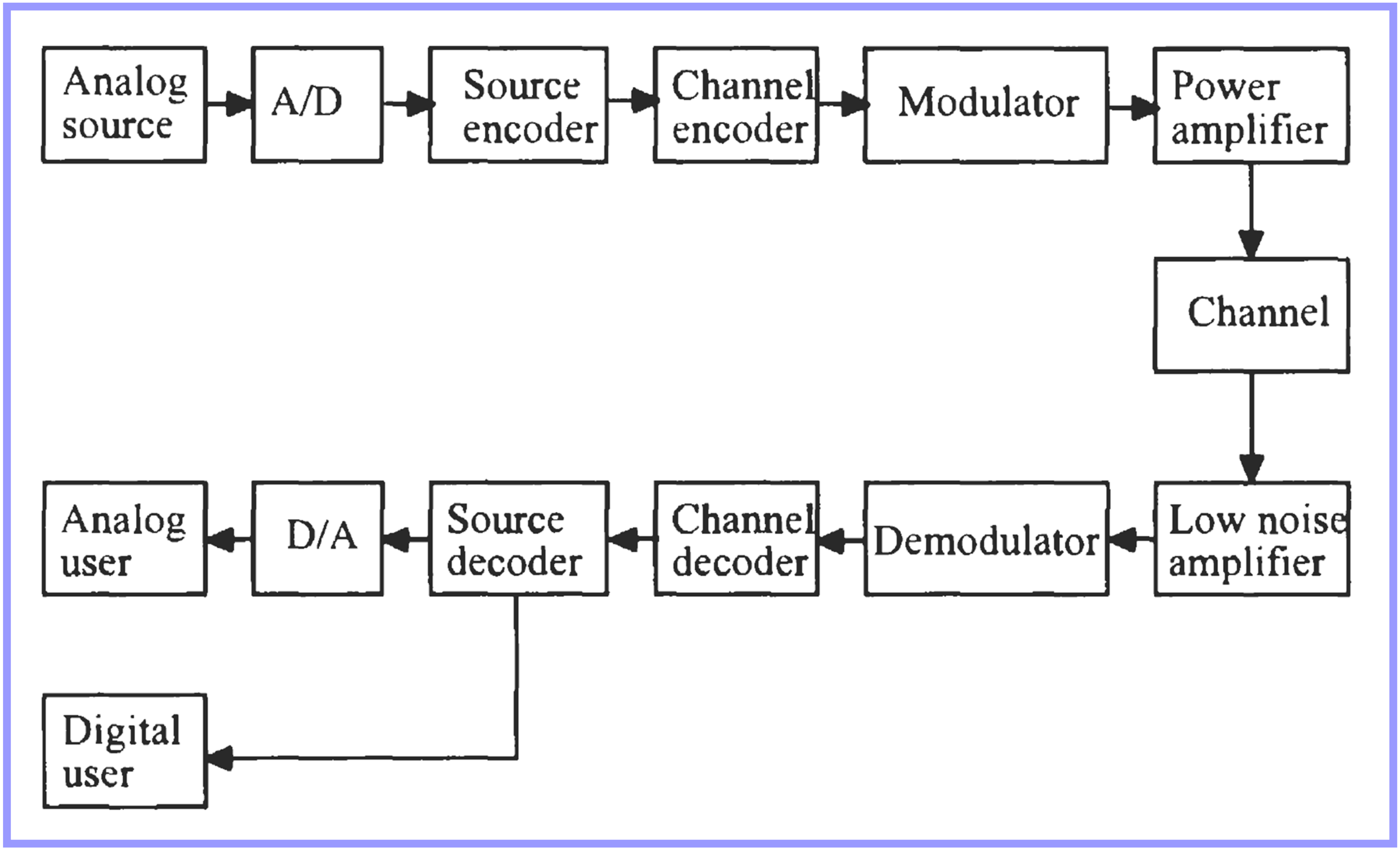
DIGITAL COMMUNICATION SYSTEMS: AN OVERVIEW

Digital Communication Systems



Digital Communication Systems

Features

- **The message to be sent can be from an analog source (e.g. voice) or from a digital source (e.g. computer data).**

- **The ADC samples and quantizes the analog signal in digital form (bit 1 or 0).**

- **The Source encoder accepts the digital signal and encodes it into a shorter digital signal. This is called source encoding.**

- **Source encoding reduces the redundancy, hence the transmission speed. This in turn reduces the bandwidth requirement of the system.**

- **Channel encoder accepts the output digital signal of the source encoder and encodes it into a longer digital signal. This is done to correct errors at the receiver, caused by noise or interference during transmission through channel.**

Digital Communication Systems

Features

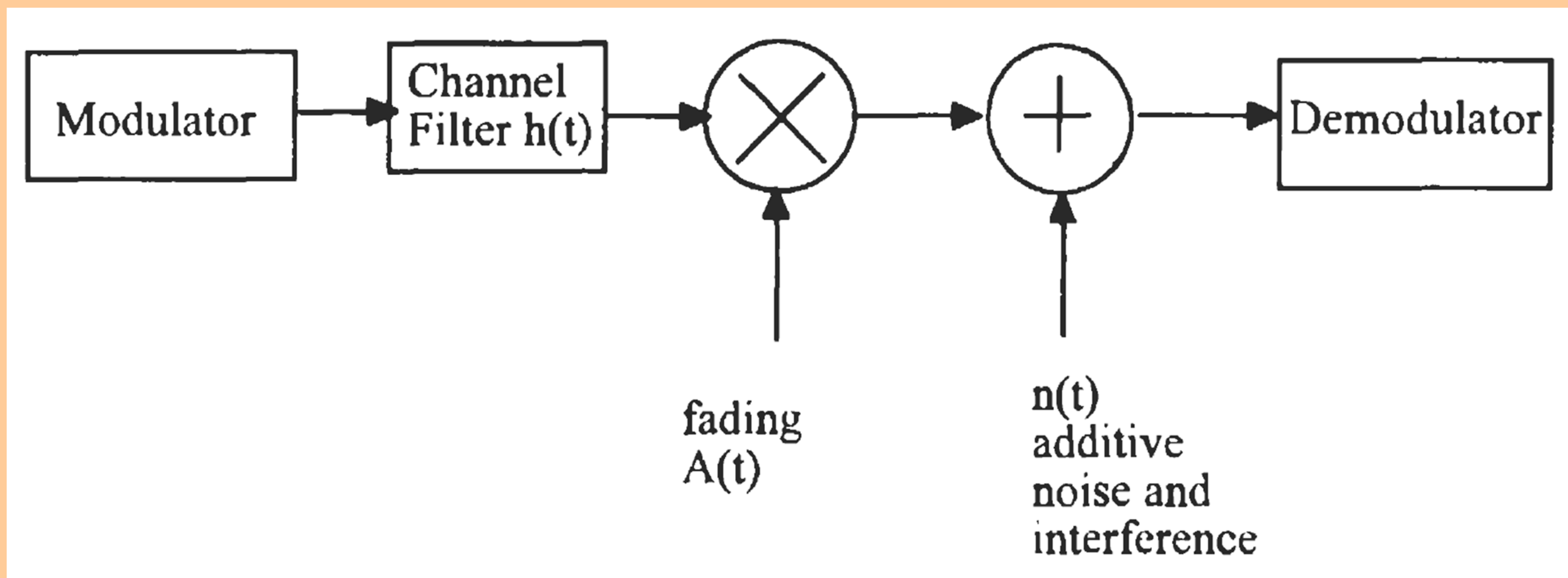
- **Most often the transmission is bandpass transmission in a high frequency passband. Here the modulator impresses the encoded digital symbols onto a carrier.**

- **In baseband transmission, the modulator is a baseband modulator (also called formator), which formats the encoded digital symbols into a waveform suitable for transmission.**

- **The channel is the transmission medium, where noise adds to the signal and fading and attenuation effects appear as a complex multiplicative factor on the symbol. The channel can also be viewed as a filter because of its limited frequency bandwidth.**

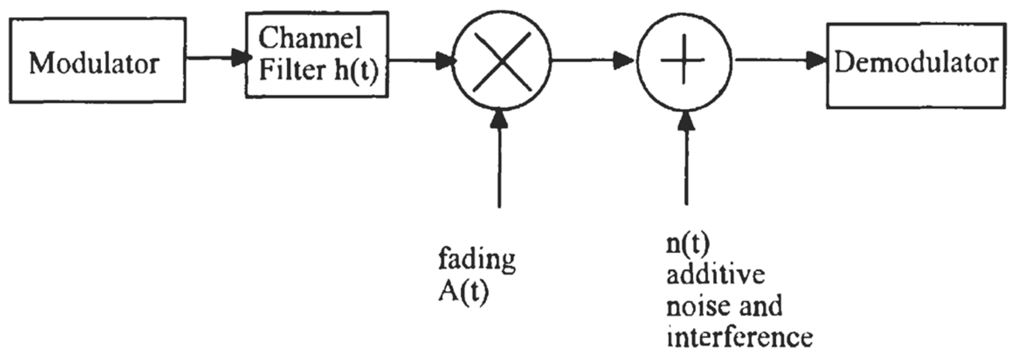
- **In the receiver, virtually the reverse signal processing happens.**

Digital Communication Systems



Digital Communication System Model for Modulation and Demodulation

Digital Communication Systems



The received signal at the input of the demodulator

$$r(t) = A(t)\left[s(t) * h(t)\right] + n(t)$$

The channel filter is a composite filter with T.F.

$$H(f) = H_T(f)H_C(f)H_R(f)$$

$H_T(f)$, $H_C(f)$, and $H_R(f)$: Transfer functions of transmitter, channel, and receiver, respectively.

impulse response of the channel filter

$$h(t) = h_T(t) * h_C(t) * h_R(t)$$

$h_T(t)$, $h_C(t)$, and $h_R(t)$: impulse responses of transmitter, channel, and receiver, respectively.

GEOMETRIC REPRESENTATION OF SIGNALS

Geometric Representation of Signals

It represents any set of M energy signals $\{s_i(t)\}$ as linear combinations of N orthonormal basis functions, where $N \leq M$.

Given a set of real-valued energy signals $s_1(t)$, $s_2(t)$, …, $s_M(t)$, each of duration T seconds, one can write:

$$s_i(t) = \sum_{j=1}^{N} s_{ij}\phi_j(t), \qquad \begin{cases} 0 \leq t \leq T \\ i = 1,2,\cdots,M \end{cases}$$

where the coefficients of expansion are defined as:

$$s_{ij} = \int_0^T s_i(t)\phi_j(t)\,dt, \qquad \begin{cases} i = 1,2,\cdots,M \\ j = 1,2,\cdots,N \end{cases}$$

Geometric Representation of Signals

The real-valued basis functions $\phi_1(t), \phi_2(t), \ldots, \phi_N(t)$ are *orthonormal* i.e.

$$\int_0^T \phi_i(t)\phi_j(t)dt = \delta_{ij} = \begin{cases} 1 & \text{if } i = j \\ 0 & \text{if } i \neq j \end{cases}$$

where δ_{ij} = Kronecker delta.

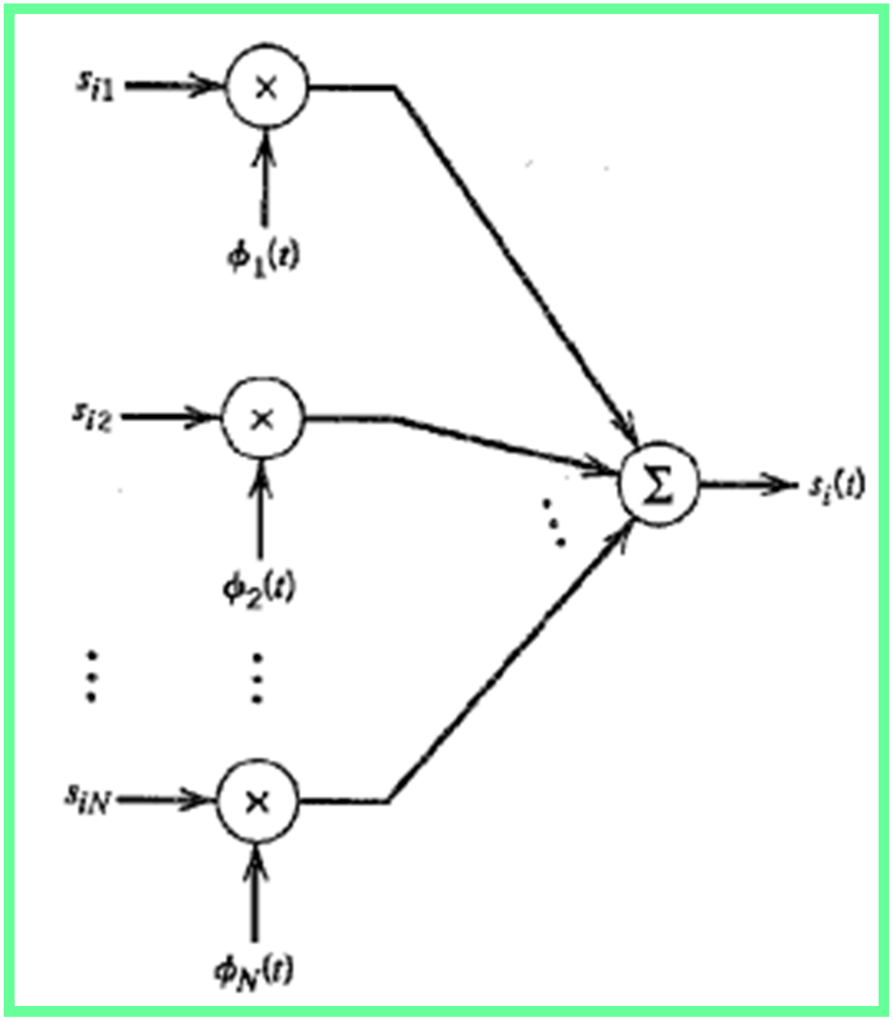
implication

i) Each basis function is normalized to have unit energy.

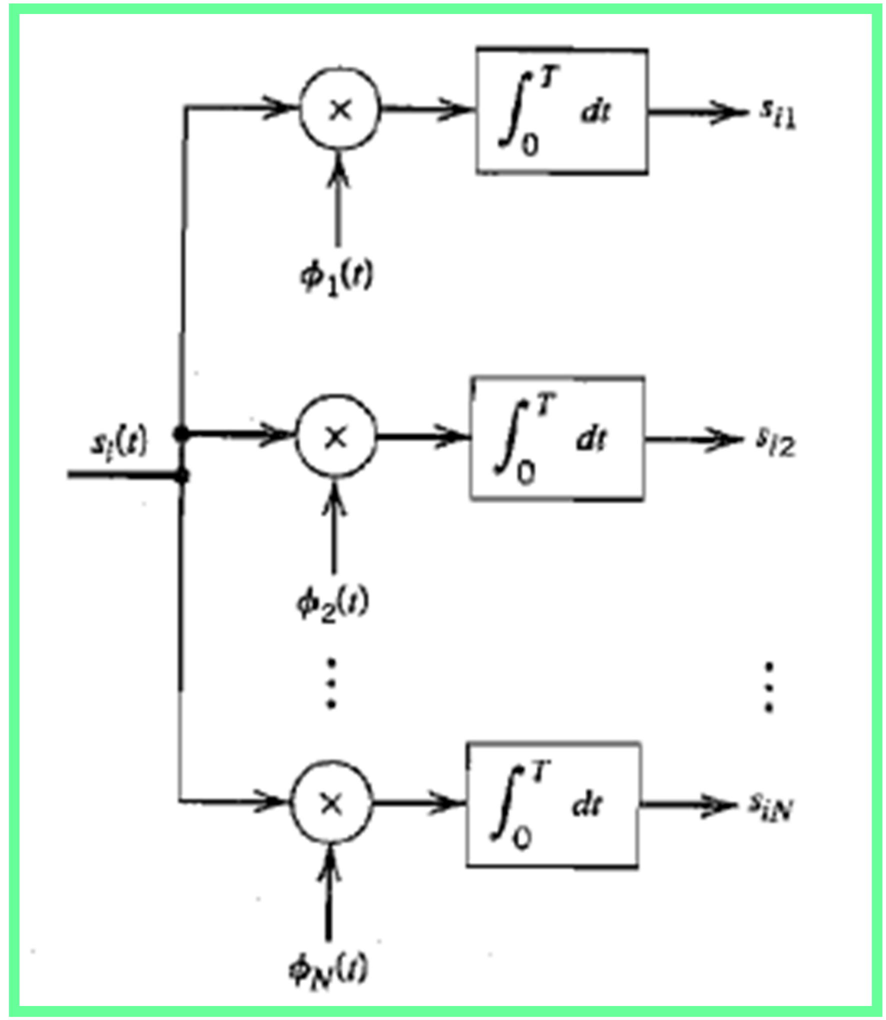
ii) The basis functions $\phi_1(t), \phi_2(t), \ldots, \phi_N(t)$ are orthogonal with respect to each other over the interval $0 \le t \le T$.

The set of coefficients $\{s_{ij}\}_{j=1}^{N}$ forms an N-dimensional vector $\mathbf{s}_i$. The vector $\mathbf{s}_i$ bears a one-to-one relationship with the transmitted signal.

Geometric Representation of Signals



(a)



(b)

(a) Synthesizer for generating signal $s_i(t)$. (b) Analyzer for generating the set of signal vectors $\{s_i\}$.

Geometric Representation of Signals

The synthesizer consists of a bank of *N* multipliers, with each multiplier having its own basis function, followed by a summer.
The analyzer consists of a bank of *N* product-integrators or correlators with a common input, and with each one of them supplied with its own basis function.

Each signal in the set $\{s_i(t)\}$ is completely determined by the vector of its coefficients:

$$\mathbf{s}_i = \begin{bmatrix} s_{i1} \\ s_{i2} \\ \vdots \\ s_{iN} \end{bmatrix}, \quad i = 1, 2, \cdots, M$$

The vector $\mathbf{s}_i$ is called a signal vector.

Geometric Representation of Signals

The set of signal vectors $\{\mathbf{s}_i \mid i = 1, 2, \ldots, M\}$ is a corresponding set of M points in an N-dimensional Euclidean space, with N mutually perpendicular axes labeled $\phi_1, \phi_2, \ldots, \phi_N$. This N-dimensional Euclidean space is called the signal space.

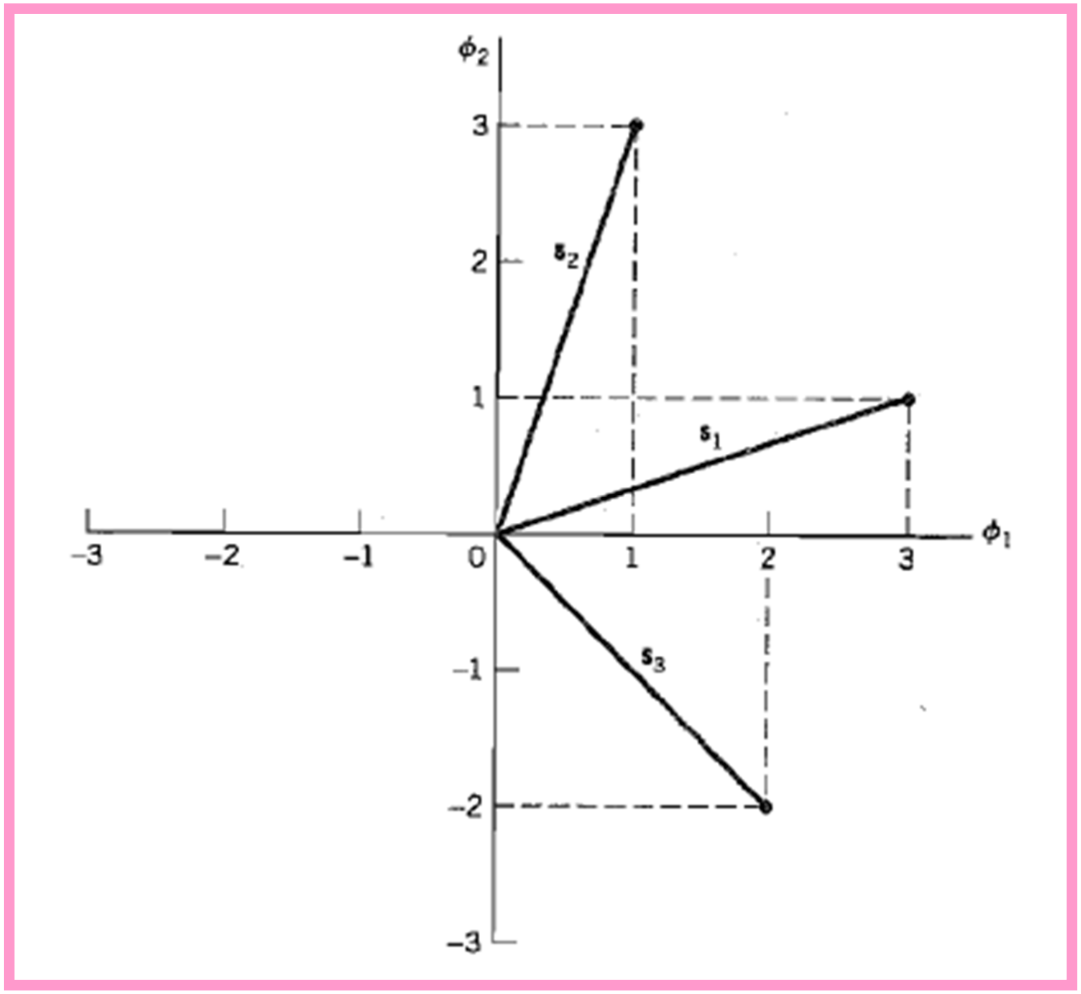
The length of a signal vector $\mathbf{s}_i$ (also called the absolute value or norm) in an N-dimensional Euclidean space is given by the symbol $\|\mathbf{s}_i\|$. The squared-length of any signal vector $\mathbf{s}_i$ is defined to be the inner product or dot product of $\mathbf{s}_i$ with itself, given as:

$$\|\mathbf{s}_i\|^2 = \mathbf{s}_i^T \mathbf{s}_i = \sum_{j=1}^{N} s_{ij}^2, \qquad i = 1, 2, \cdots, M$$

where s_{ij} is the jth element of $\mathbf{s}_i$.

Geometric Representation of Signals

An Example



The geometric representation of signals for a two-dimensional signal space with three signals i.e. when $N = 2$ and $M = 3$.

Geometric Representation of Signals

The energy of a signal $s_i(t)$ of duration T seconds is equal to the squared length of the signal vector $\mathbf{s}_i$ representing it.

$$E_i = \int_0^T s_i^2(t)\,dt \quad \Rightarrow \quad E_i = \sum_{j=1}^{N} s_{ij}^2 = \|\mathbf{s}_i\|^2$$

In the case of a pair of signals $s_i(t)$ and $s_k(t)$, represented by the signal vectors $\mathbf{s}_i$ and $\mathbf{s}_k$, respectively, it can be shown that:

$$\int_0^T s_i(t)\,s_k(t)\,dt = \mathbf{s}_i^T \mathbf{s}_k$$

The inner product of the signals $s_i(t)$ and $s_k(t)$ over the interval $[0, T]$, using their time-domain representations, is equal to the inner product of their respective vector representations $\mathbf{s}_i$ and $\mathbf{s}_k$.

The two vectors $\mathbf{s}_i$ and $\mathbf{s}_k$ are orthogonal or perpendicular to each other if their inner product $\mathbf{s}_i^T \mathbf{s}_k$ is zero.

COMMUNICATION CHANNELS

Communication Channels

Channel Characteristic **plays an important role in studying, choosing and designing modulation schemes.**

Important channel models in communication

- **Additive White Gaussian Noise (AWGN) channel.**
- **Bandlimited channel.**
- **Fading Channel.**

Communication Channels

Additive White Gaussian Noise (AWGN) Channels

AWGN Channel **is a universal channel model for analyzing modulation schemes. The channel does nothing but add a white Gaussian noise to the signal passing through it.**

Implication

The amplitude frequency response of the channel is flat (with infinite bandwidth) and phase frequency response is linear for all frequencies. ***Hence modulated signals pass through it without any amplitude loss and phase distortion of frequency components.***

Fading **does not exist. The only distortion is introduced by the AWGN.**

Communication Channels

Additive White Gaussian Noise (AWGN) Channels

The received signal: $r(t) = s(t) + n(t)$ **$n(t)$: AWGN**

AWGN Channel **The whiteness of $n(t)$ implies that it is a *stationary random process with a flat PSD* for all frequencies. It is convention to assume PSD as:**

$$N(f) = N_o/2, \quad -\infty < f < \infty$$

A white process has infinite power.

This is a mathematical idealization.

Communication Channels

Additive White Gaussian Noise (AWGN) Channels

According to Wiener-Khintchine theorem, the autocorrelation function of the AWGN is:

$$
\begin{aligned}
R(\tau) &\triangleq E\{n(t)n(t-\tau)\} = \int_{-\infty}^{\infty} N(f)e^{j2\pi f\tau}\,df \\
&= \int_{-\infty}^{\infty} \frac{N_o}{2} e^{j2\pi f\tau}\,df = \frac{N_o}{2}\delta(\tau)
\end{aligned}
$$

$\delta(\tau)$: Dirac delta function

This shows that the noise samples are uncorrelated no matter how close they are in time.

Communication Channels

Additive White Gaussian Noise (AWGN) Channels

At any time instance, the amplitude of *n*(*t*) obeys a Gaussian probability density function:

$$p(\eta) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\{-\frac{\eta^2}{2\sigma^2}\}$$

η: used to represent the values of the random process *n*(*t*)

σ^2: variance of the random process

$\sigma^2 = \infty$, for the AWGN process (as σ^2 is the power of the noise)

Communication Channels

Additive White Gaussian Noise (AWGN) Channels

However, when $r(t)$ is correlated with a orthonormal function $\phi(t)$, the noise in the output has a finite variance.

$$r = \int_{-\infty}^{\infty} r(t)\phi(t)dt = s + n$$

$$s = \int_{-\infty}^{\infty} s(t)\phi(t)dt$$

$$n = \int_{-\infty}^{\infty} n(t)\phi(t)dt$$

Communication Channels

Additive White Gaussian Noise (AWGN) Channels

The variance of *n* is:

$$
\begin{aligned}
E\{n^2\} &= E\left\{\left[\int_{-\infty}^{\infty} n(t)\phi(t)dt\right]^2\right\} \\
&= E\left\{\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} n(t)\phi(t)n(\tau)\phi(\tau)dtd\tau\right\} \\
&= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} E\{n(t)n(\tau)\}\phi(t)\phi(\tau)dtd\tau \\
&= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \frac{N_o}{2}\delta(t-\tau)\phi(t)\phi(\tau)dtd\tau \\
&= \frac{N_o}{2}\int_{-\infty}^{\infty} \phi^2(t)dt = \frac{N_o}{2}
\end{aligned}
$$

Communication Channels

Additive White Gaussian Noise (AWGN) Channels

Then the probability density function (PDF) of *n* is:

$$p(n) = \frac{1}{\sqrt{\pi N_o}} \exp\{-\frac{n^2}{N_o}\}$$

Conclusion: **Strictly speaking, the AWGN channel does not exist, since *no channel can have an infinite bandwidth*. However, when the signal bandwidth is smaller than the channel bandwidth, many practical channels are approximately AWGN channels.**

Communication Channels

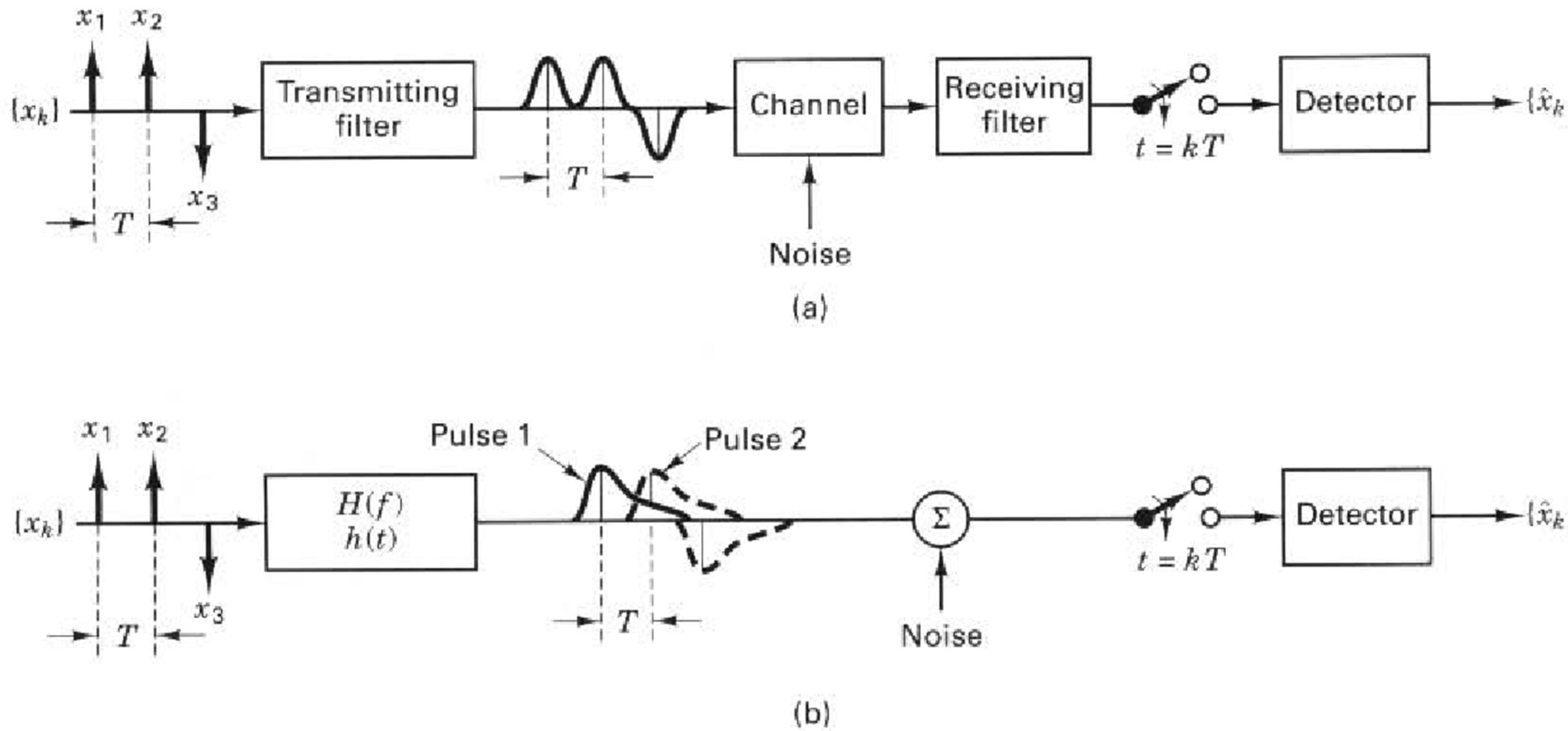
Bandlimited Channel

When the channel bandwidth is smaller than the signal bandwidth, the channel is bandlimited.

Severe bandwidth limitation causes intersymbol interference (*ISI*) (i.e. digital pulses will extend beyond their transmission duration (symbol period T_s)) and interfere with the next symbol or even more symbols.

The *ISI* causes an increase in the bit error probability (P_b) or bit error rate (*BER*). When increasing the channel bandwidth is impossible or not cost-efficient, channel equalization techniques are used for combating *ISI*.

Intersymbol Interference (ISI)

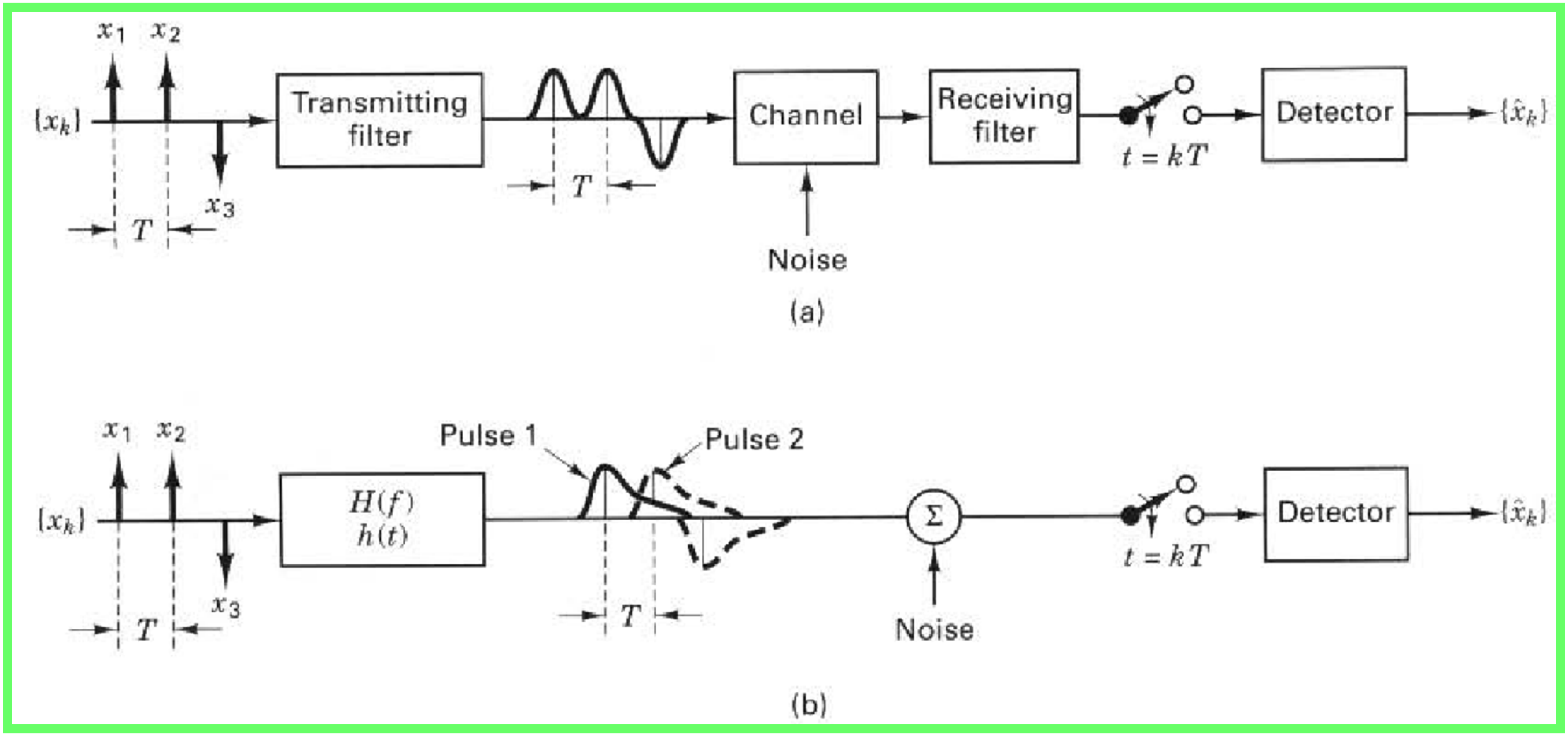


Intersymbol interference in the detection process.
(a) Typical baseband digital system. (b) Equivalent model.

Overall equivalent system filter T.F.:

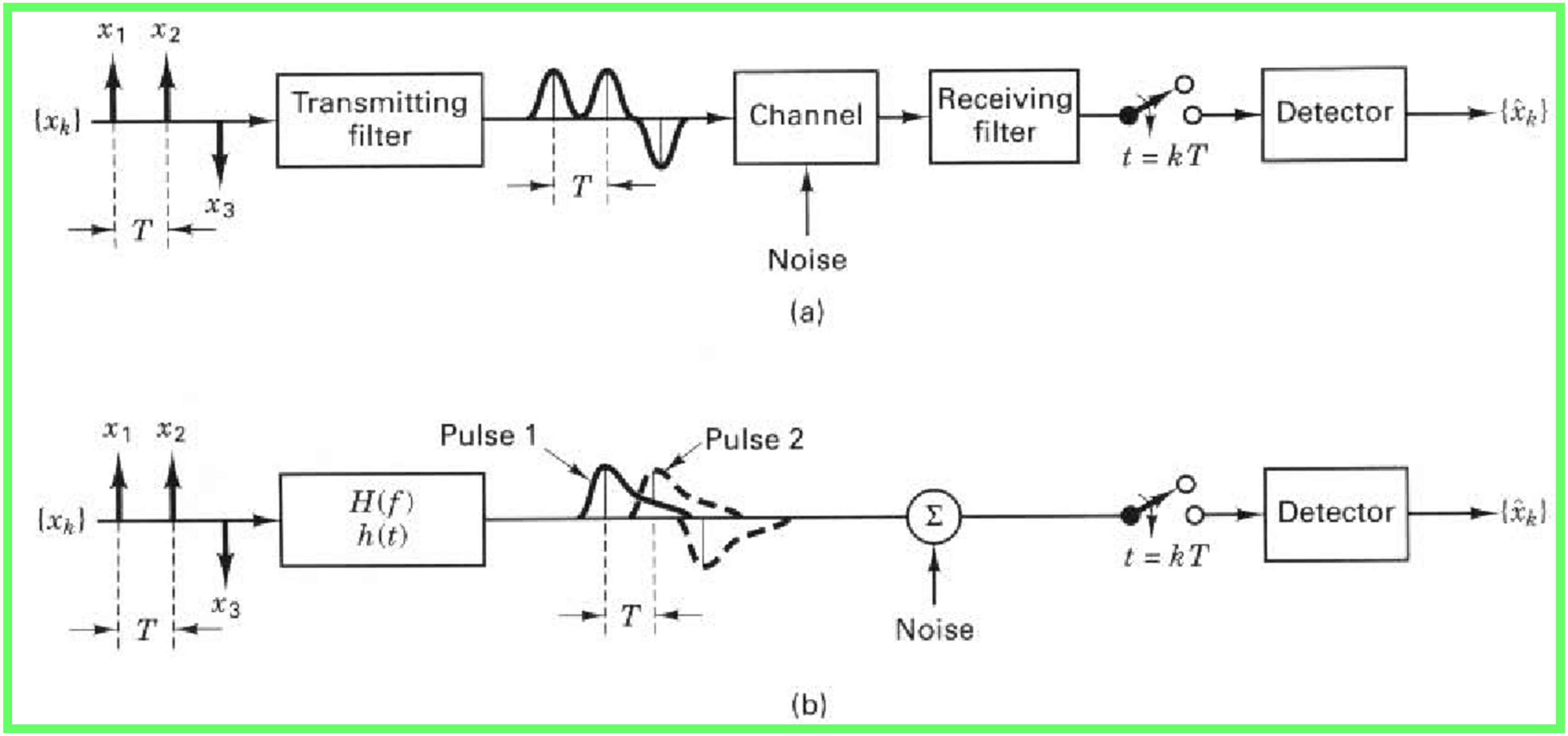
$$H(f) = H_t(f)H_c(f)H_r(f)$$

Intersymbol Interference (ISI)



When the receiving filter is configured to *compensate for the distortion caused by both the transmitter and the channel*, it is called an equalizing filter or receiving/equalizing filter.

Intersymbol Interference (ISI)



Due to the effects of system filtering, the received pulses can overlap one another. The tail of a pulse can "smear" into adjacent symbol interval(s). This can give rise to *interference with the detection process and degradation in error performance*. Even in absence of noise, the effects of filtering and channel-induced distortion can give rise to ISI.

Intersymbol Interference (ISI)

A common design problem is that, *given a specified $H_c(f)$, determine $H_t(f)$ and $H_r(f)$, such that the ISI is minimized at the output of $H_r(f)$*.

Nyquist showed that the theoretical minimum system bandwidth required to detect R_s symbols/sec., without ISI, is ($R_s/2$) Hz.

For most communication systems the goal is to reduce the required system bandwidth as much as possible (to generate more revenue). Nyquist provided a basic limitation to such bandwidth reduction.

If one attempts to force a system to operate at smaller bandwidths than the constraint permits, this will lead to degradation in system performance due to increased ISI.

Communication Channels

Fading Channel

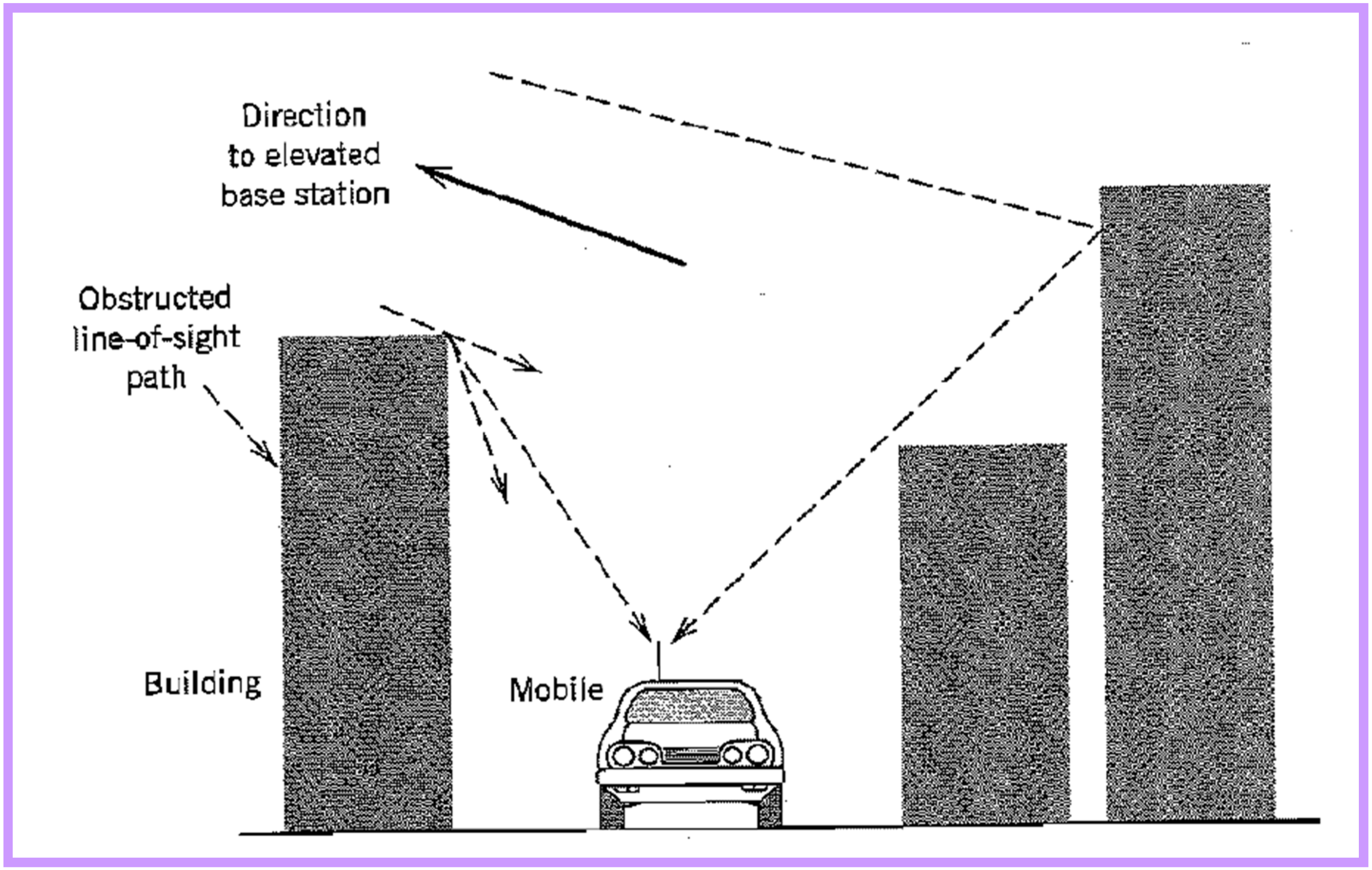
Fading is a phenomenon occurring when the amplitude and phase of a radio signal change rapidly over a short period of time or travel distance.

Fading is caused by interference between two or more versions of the transmitted signal which arrive at the receiver at slightly different times.

These waves, called multipath waves, combine at the receiver antenna to give a resultant signal which can vary widely in magnitude and phase.

Communication Channels

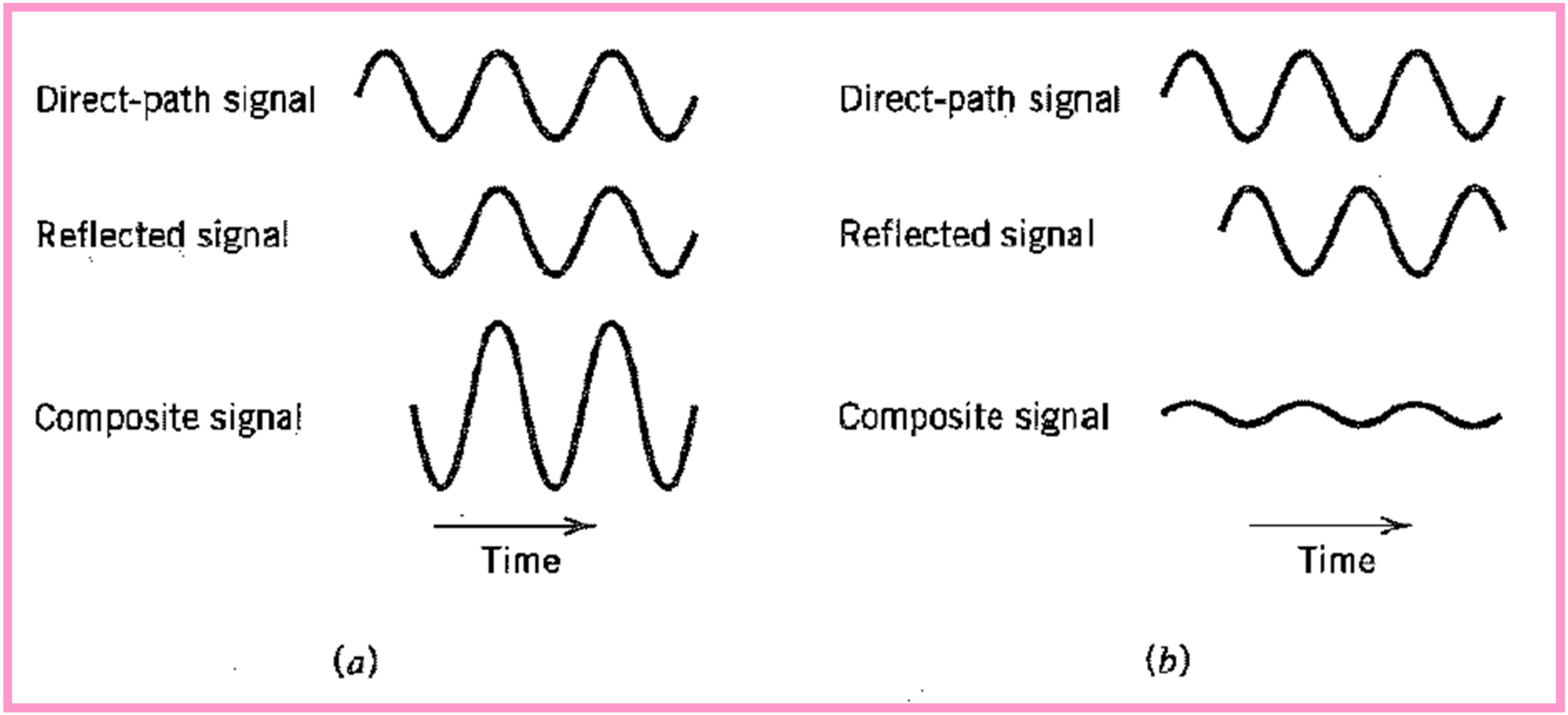
Propagation effects



The mechanism of radio propagation in urban areas.

Communication Channels

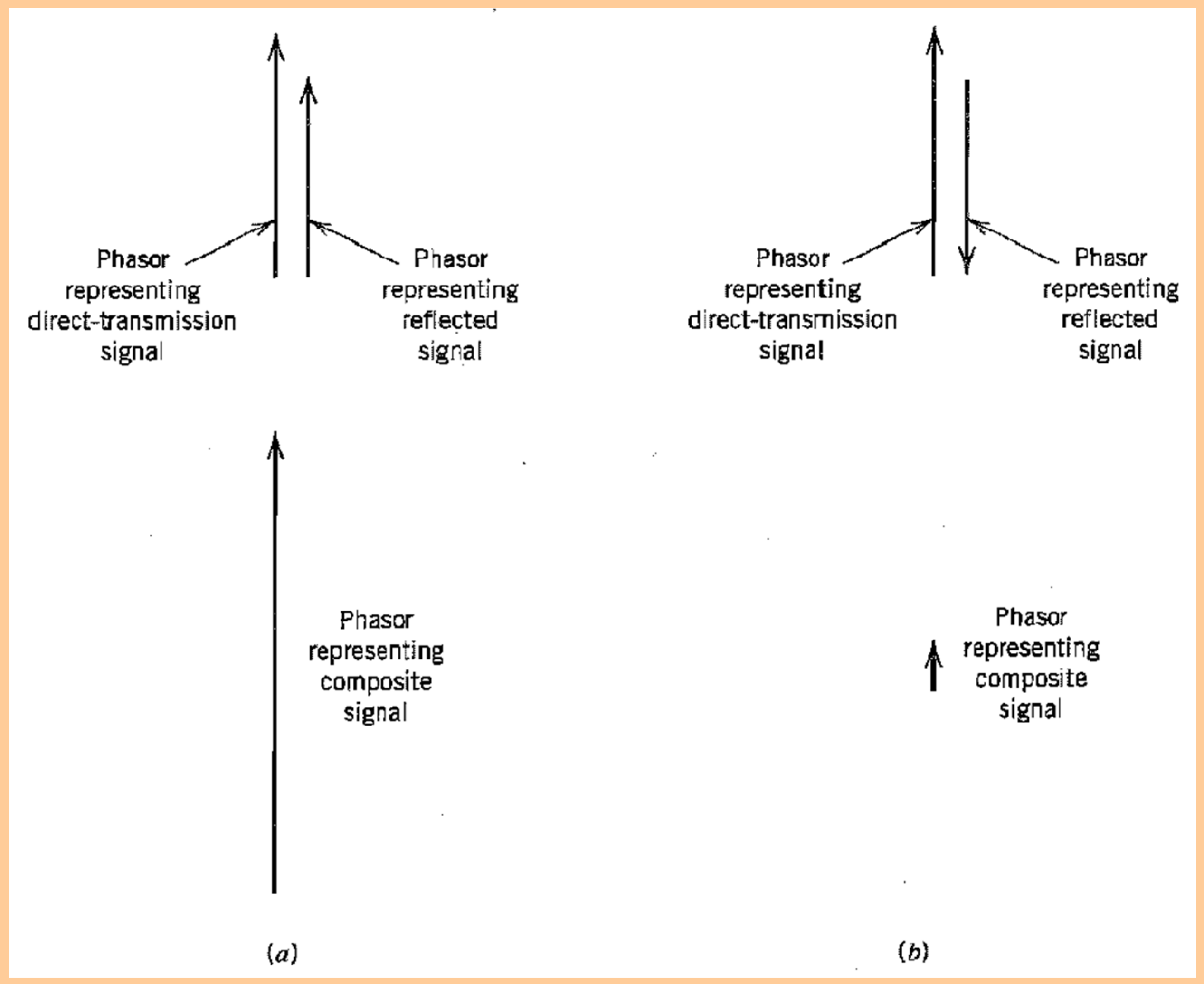
Propagation effects



(a) Constructive and (b) destructive forms of the multi path phenomenon for sinusoidal signals (static multi path environment).

Communication Channels

Propagation effects



Phase representations of (a) constructive and (b) destructive forms of the multi path (static multi path environment).

Communication Channels

Propagation effects

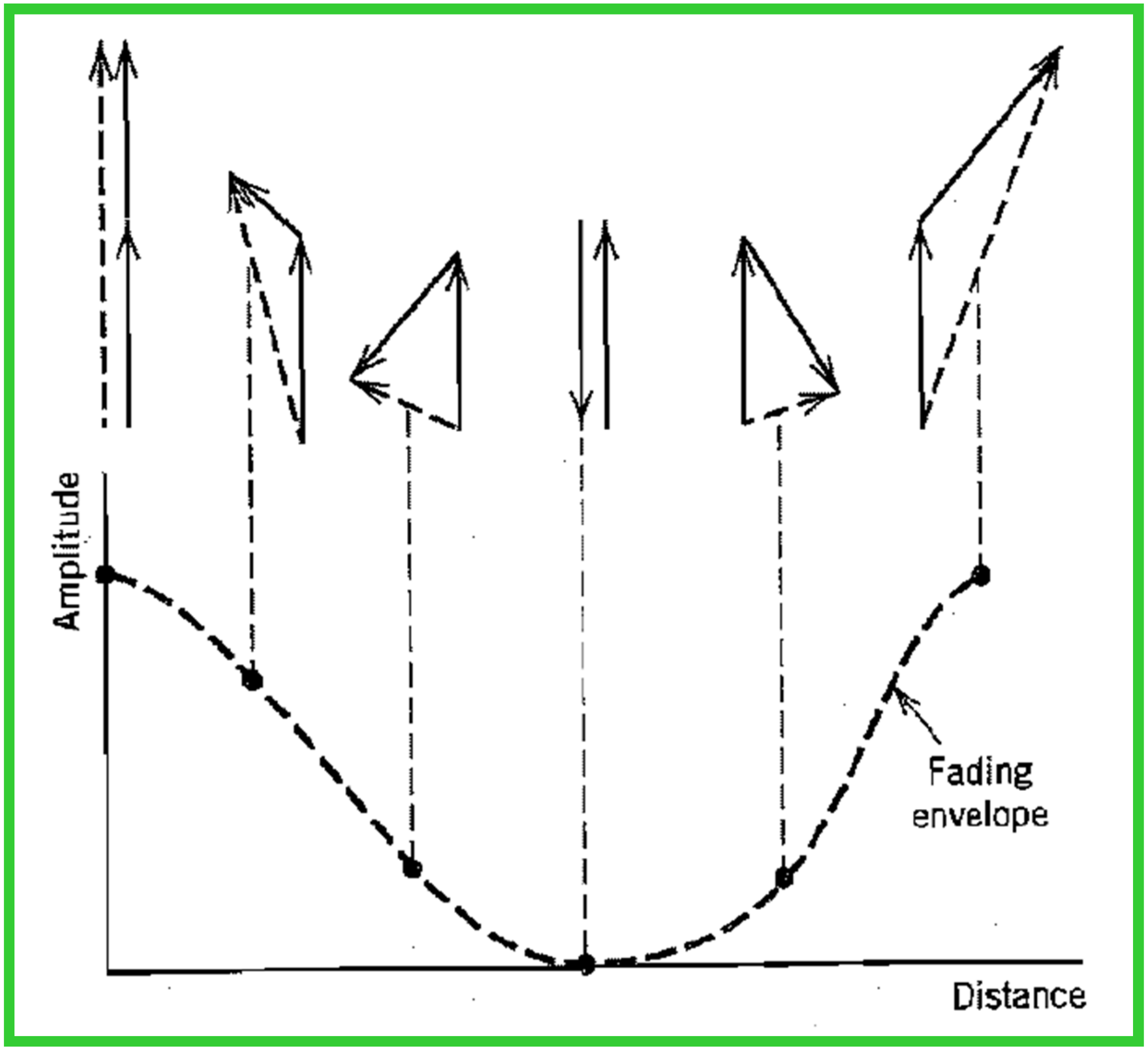


Illustration of how the envelope fades as two incoming signals combine with different phases (dynamic multi path environment).

MODULATION TECHNIQUES

Basic Modulation Methods

Digital Modulation is a process that impresses a digital symbol onto a signal suitable for transmission.

Modulation Techniques

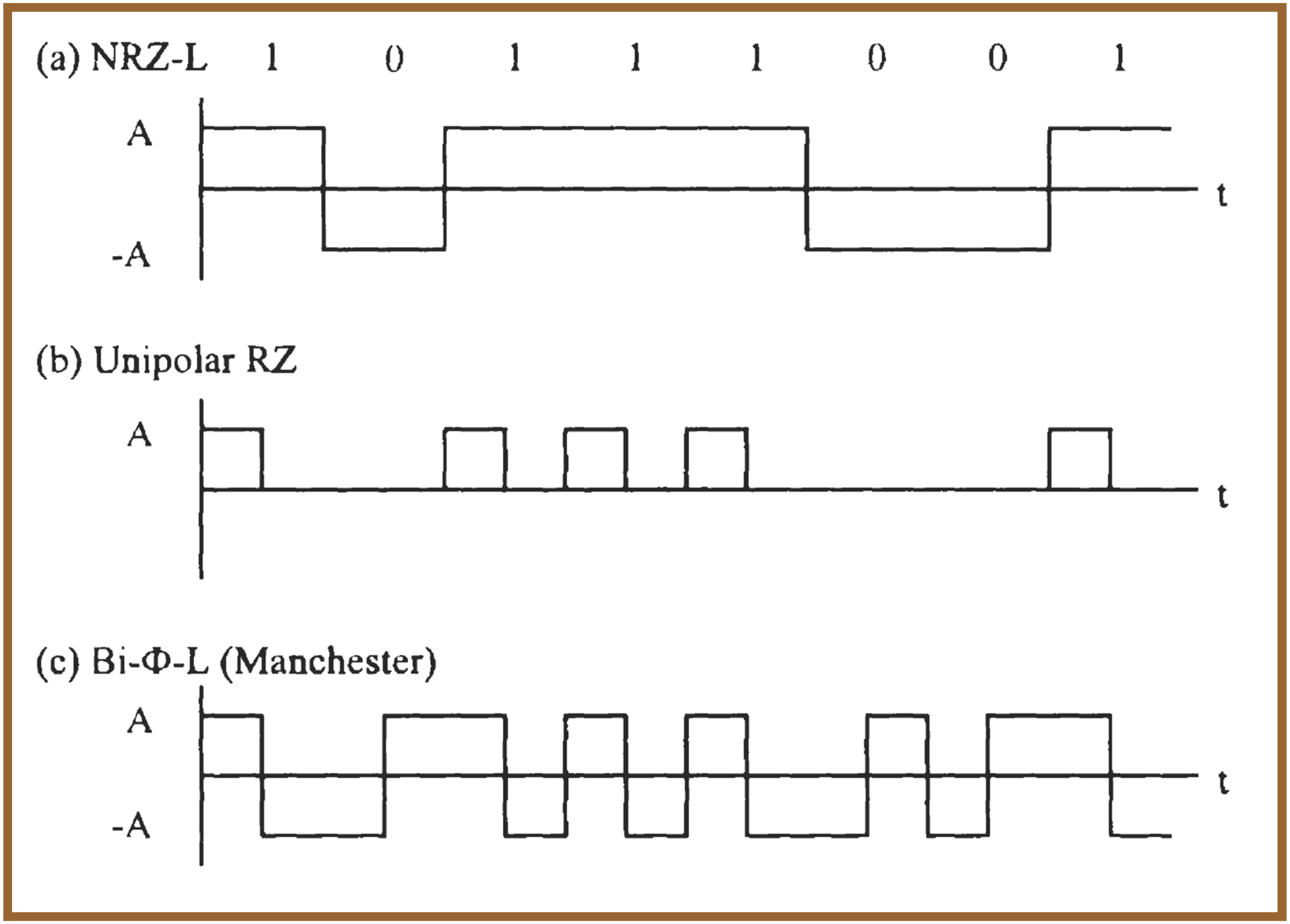
Baseband Modulation

Used for short distance transmissions and often called Line Coding.

Bandpass Modulation

Used for long distance and wireless transmissions. It is also called Carrier Modulation.

Baseband Digital Modulation



Baseband Modulation Example Waveforms.

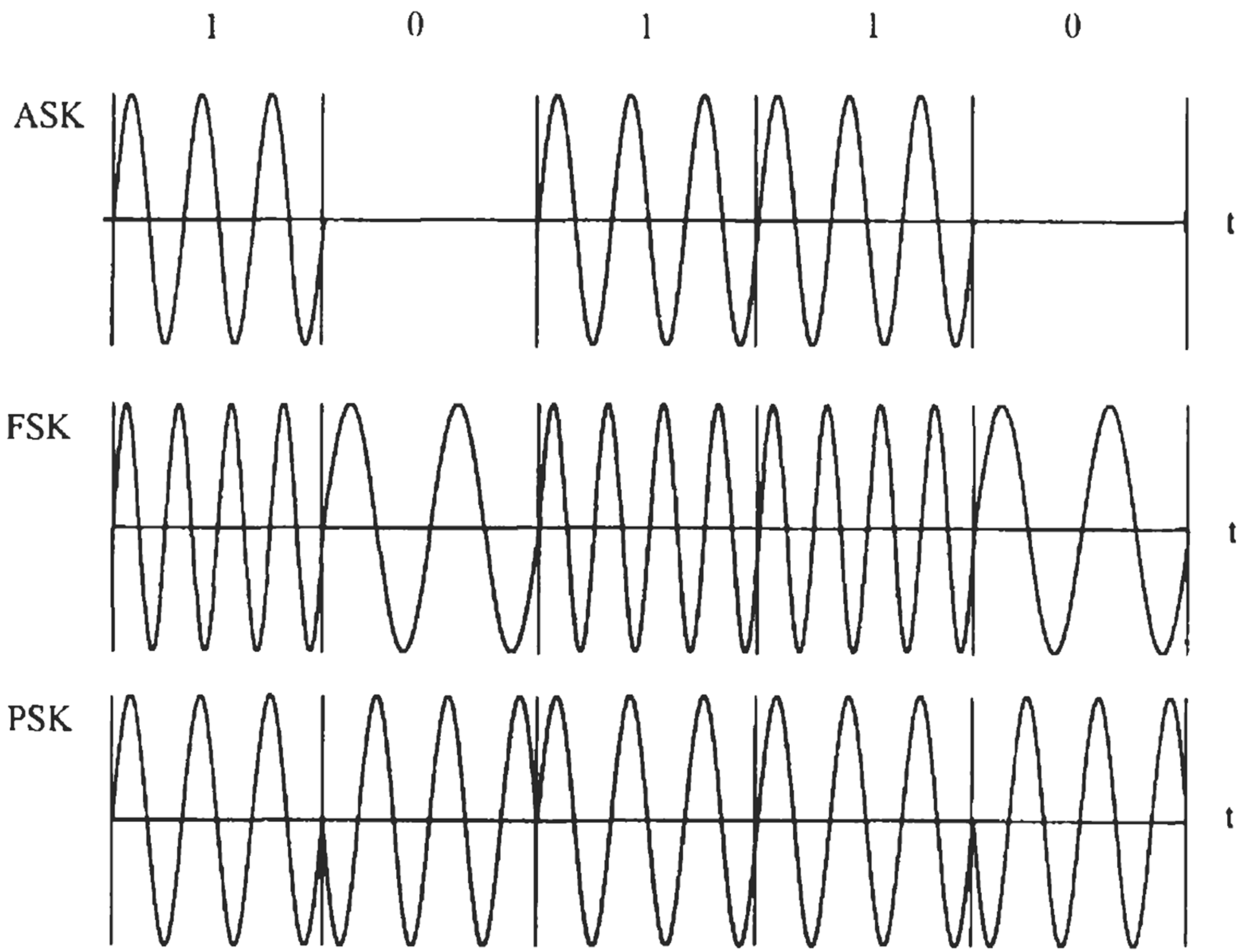
Bandpass Digital Modulation

In Bandpass Modulation, a sequence of digital symbols is used to alter the parameters of a high frequency sinusoidal signal i.e. amplitude, frequency, and phase.

Basic Bandpass Digital Modulation Schemes

- **Amplitude Shift Keying (ASK).**
- **Frequency Shift Keying (FSK).**
- **Phase Shift Keying (PSK).**

Bandpass Digital Modulation



Three basic Bandpass Modulation Schemes.

Criteria of Choosing Modulation Schemes

- ✓ **The essence of digital modem design is to efficiently transmit digital bits and recover them from corruptions from the noise and other channel impairments.**

Three primary criteria of choosing modulation schemes:

- Power Efficiency.
- Bandwidth Efficiency.
- System Complexity.

Criteria of Choosing Modulation Schemes

Power Efficiency

- ✓ The bit error rate or bit error probability of a modulation scheme is inversely related to (E_b/N_o), the bit energy to noise spectral density ratio.

- ✓ For example, P_b of ASK in the AWGN channel is:

$$P_b = Q\left(\sqrt{\frac{2E_b}{N_o}}\right)$$

E_b: Average bit energy, N_o: noise PSD,

$Q(x)$: The Gaussian integral.

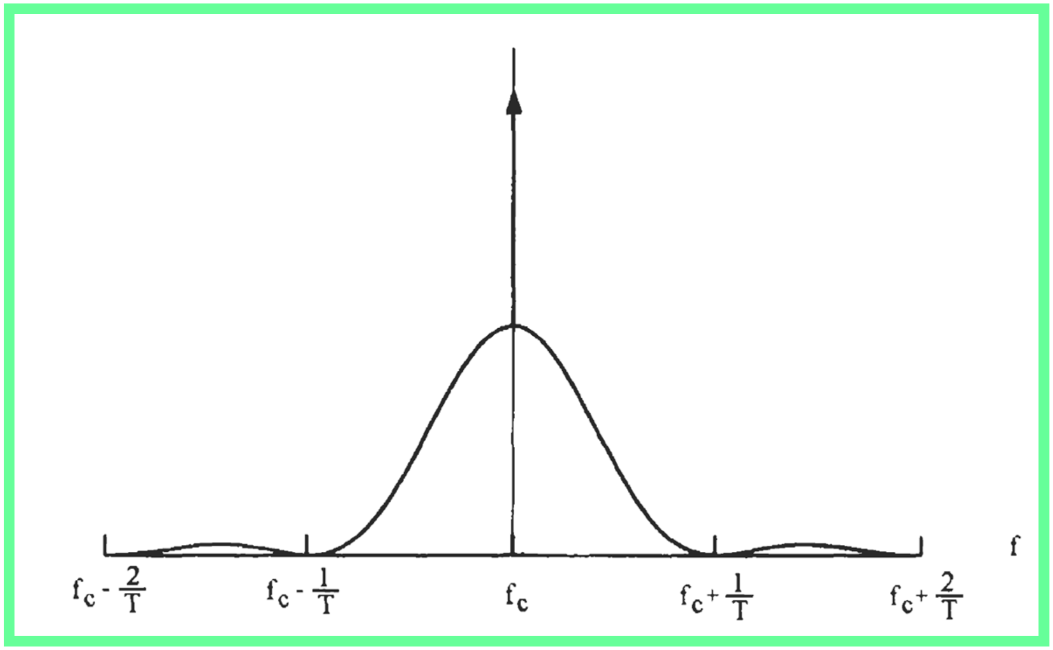
$$Q(x) = \int_x^{\infty} \frac{1}{\sqrt{2\pi}} e^{-u^2} du$$

$Q(x)$ is a monotonically decreasing function of x. Therefore the power efficiency of a modulation scheme is defined as the required E_b/N_o for a certain bit error probability P_b over an AWGN channel.

Criteria of Choosing Modulation Schemes

Bandwidth Efficiency

✓ **The bandwidth efficiency is defined as the number of bits per second that can be transmitted in one Hertz of system bandwidth. It depends on the requirement of system bandwidth for a certain modulated signal.**



PSD of ASK.

To perfectly transmit signal, infinite system bandwidth required, which is impractical. Practical system bandwidth requirement is finite and varies depending on different criteria.

Criteria of Choosing Modulation Schemes

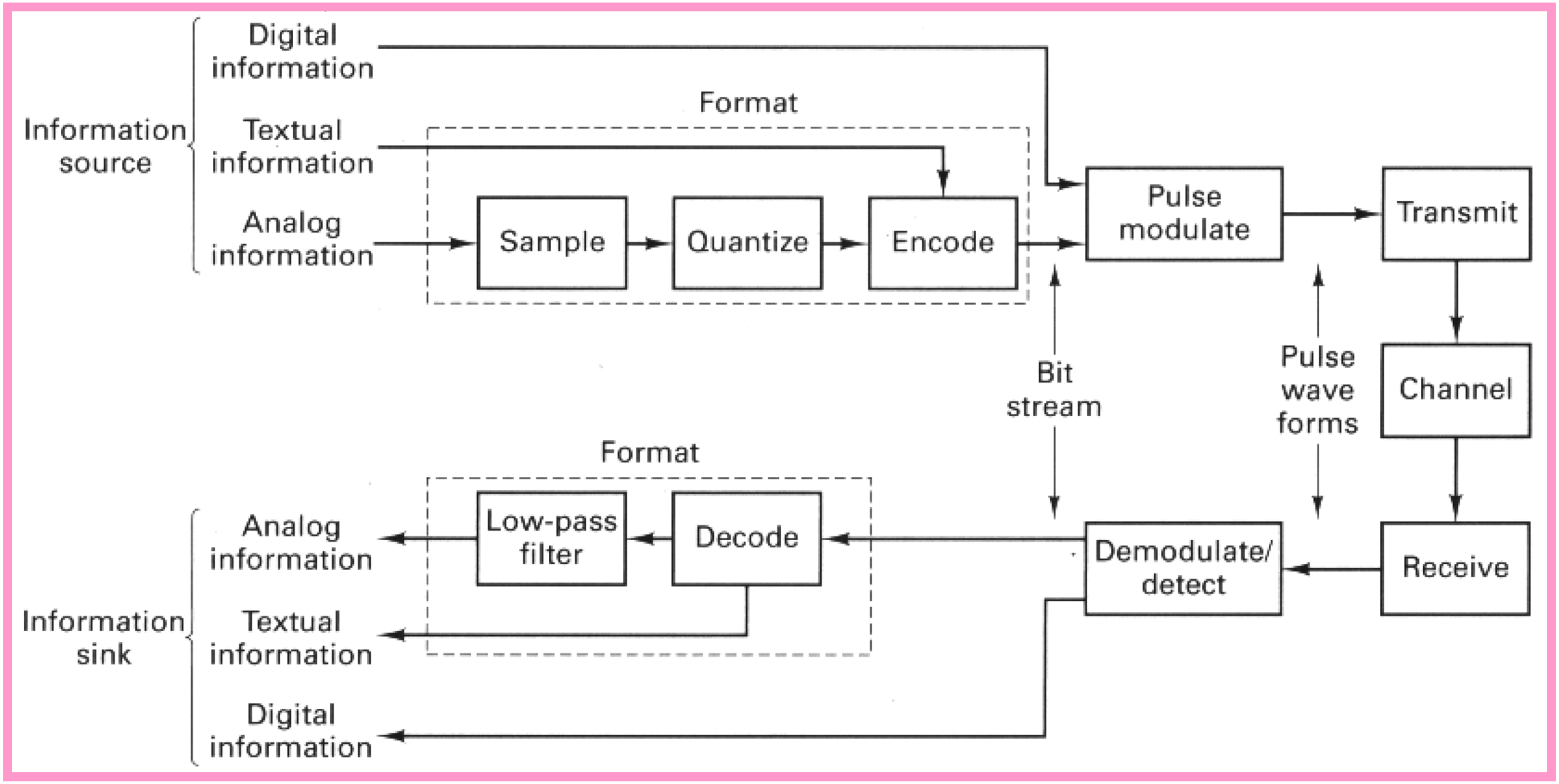
Bandwidth Efficiency

Three bandwidth efficiency criteria are quite popular:

- **Nyquist Bandwidth Efficiency.**
- **Null-to-null Bandwidth Efficiency.**
- **Percentage Bandwidth Efficiency.**

FORMATTING AND BASEBAND MODULATION

Baseband Systems



Formatting and transmission of baseband signals.

Messages, Characters, and Symbols

- ✓ **Textual messages comprise a sequence of alphanumeric characters. When digitally transmitted, the characters are first encoded into a sequence of bits, called a bit stream or baseband signal.**

↓

- ✓ **Groups of k bits can then be combined to form new digits or symbols, from a finite symbol set or alphabet of $M = 2^k$ such symbols.**

↓

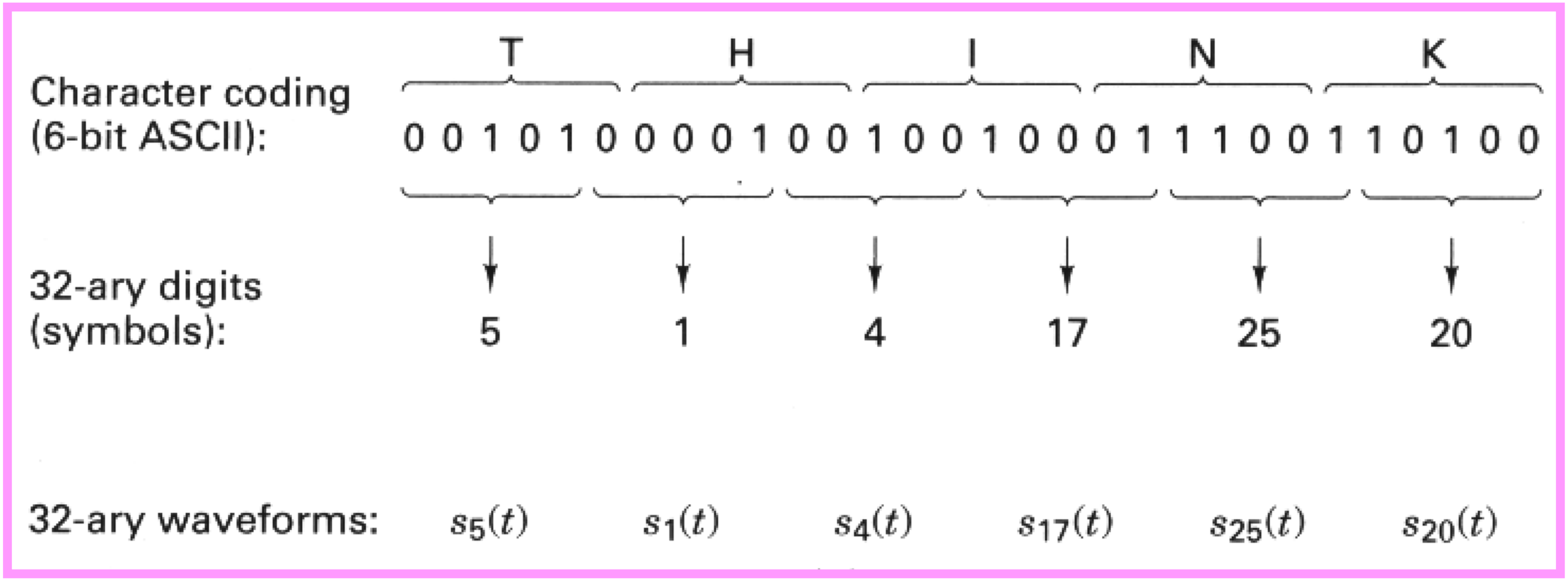
A system using a symbol set size of M is referred to as an M-ary system. The value of k or M represents an important initial choice. If $k = 1$, system is termed *binary* where $M = 2$. For $k = 2$, the system is termed *quaternary* or *4-ary* ($M = 4$).

Messages, Characters, and Symbols

Message (text):	"THINK"									
Character coding (6-bit ASCII):	T		H		I		N		K	
	0 0 1	0 1 0	0 0 0	1 0 0	1 0 0	1 0 0	0 1 1	1 0 0	1 1 0	1 0 0
8-ary digits (symbols):	1	2	0	4	4	4	3	4	6	4
8-ary waveforms:	$s_1(t)$	$s_2(t)$	$s_0(t)$	$s_4(t)$	$s_4(t)$	$s_4(t)$	$s_3(t)$	$s_4(t)$	$s_6(t)$	$s_4(t)$

An 8-ary example.

Messages, Characters, and Symbols



A 32-ary example.

Formatting Analog Information

- ✓ **If the information is analog, it can not be character encoded as in the case of textual data. The information must first be transformed into a digital format. This process starts with sampling the waveform to produce a discrete pulse-amplitude-modulated waveform.**

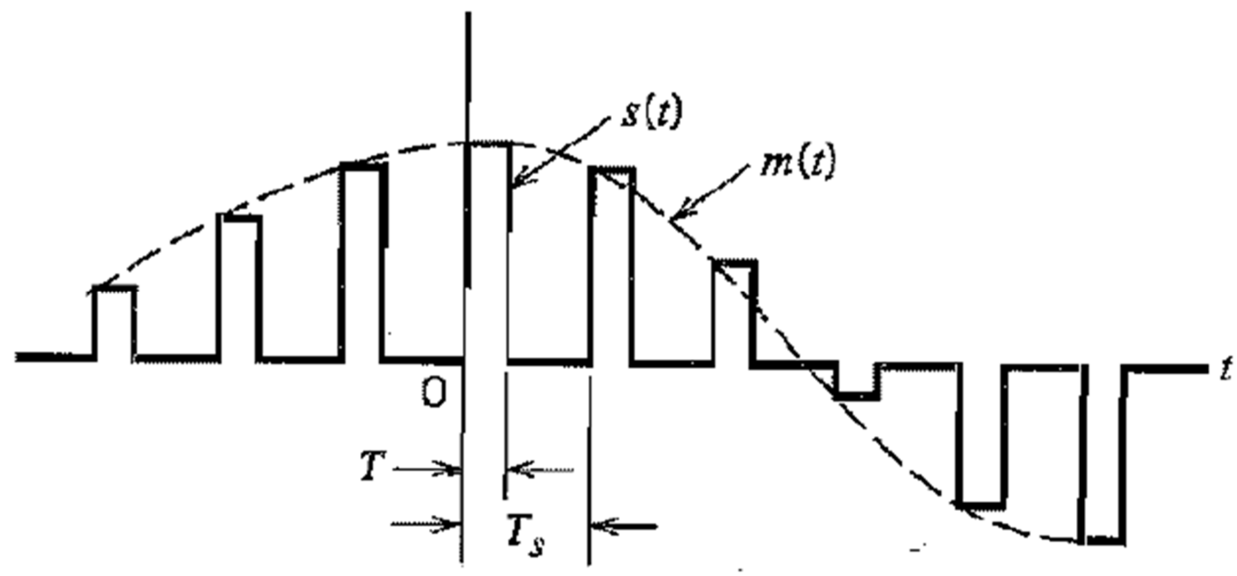
The Sampling Theorem:

A band limited signal of finite energy, which has no frequency components higher than f_m hertz, can be uniquely determined by values sampled at uniform intervals of $(T_s \leq (1/2f_m))$ sec.

The sampling rate $f_s = 2f_m$ is also called the *Nyquist rate*.

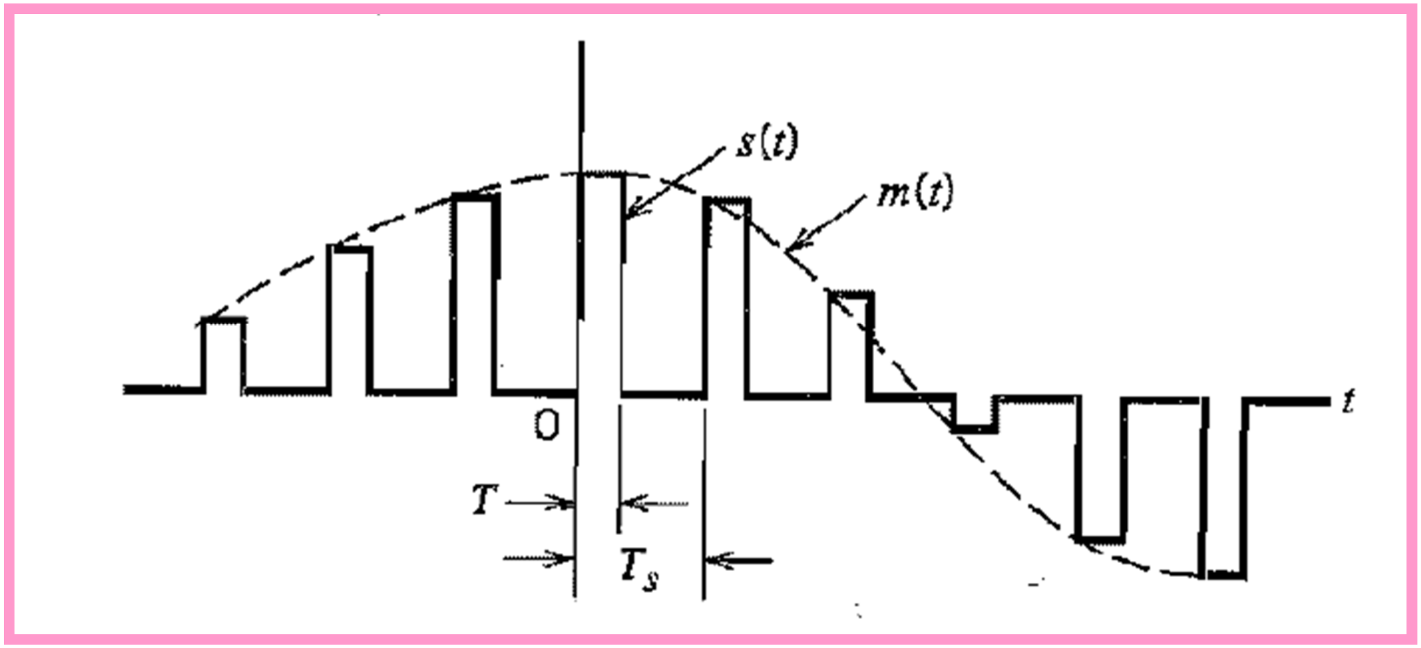
Pulse-Amplitude Modulation (PAM)

In PAM, the amplitudes of regularly spaced pulses are varied in proportion to the corresponding sample values of a continuous message signal. The pulses can be of a rectangular form or some other appropriate shape.



Flat-top samples, representing an analog signal. *m(t)*: message signal and *s(t)*: the corresponding PAM signal.

Pulse-Amplitude Modulation (PAM)



There are two operations involved in the generation of the **PAM** signal:

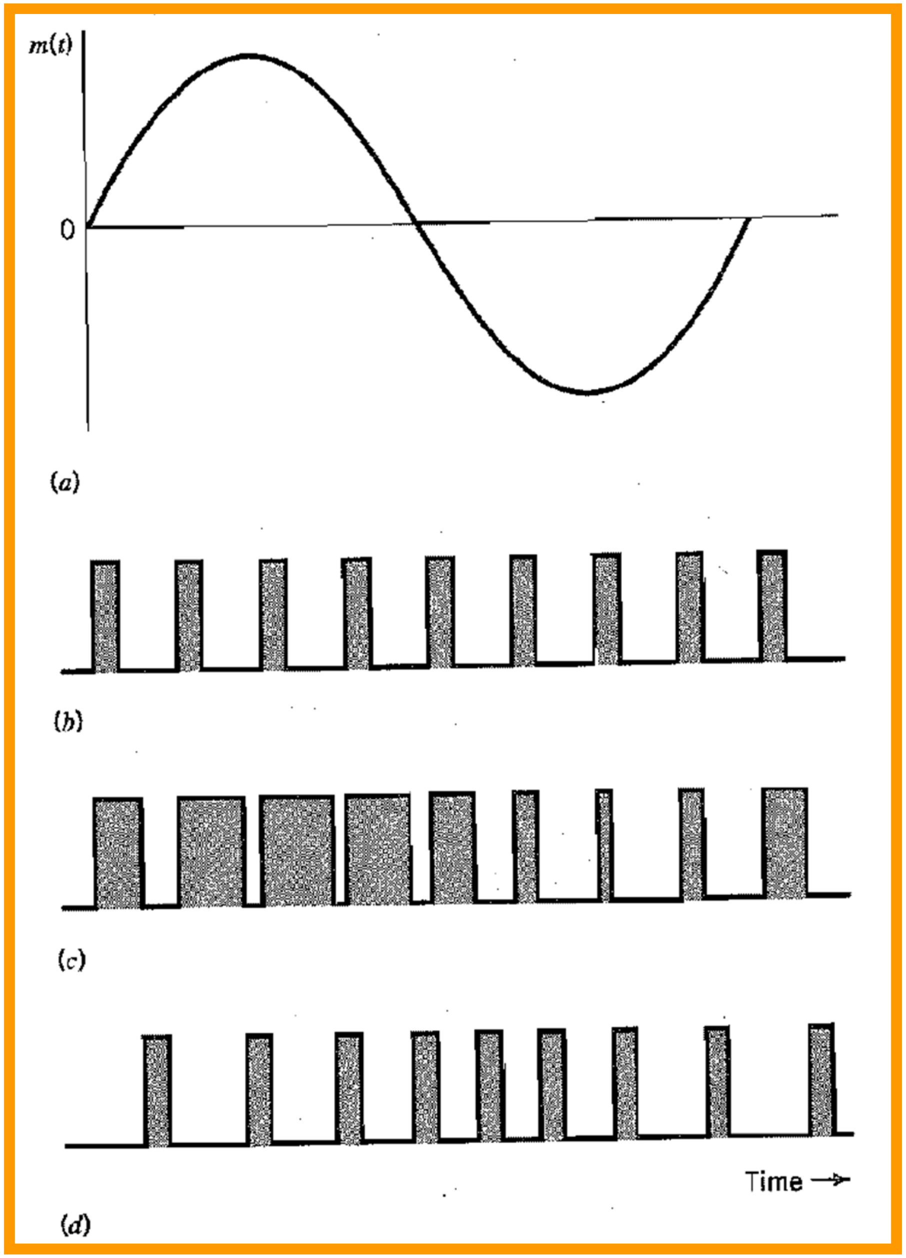
- **Instantaneous sampling** of the message signal $m(t)$ every T_s seconds and
- **Lengthening** the duration of each sample so obtained to some constant value T.

Other Forms of Pulse Modulation

- **Pulse-duration modulation (PDM), also known as pulse width modulation (PWM) – here samples of the message signal are used to vary the duration of the individual pulses in the carrier.**

- **Pulse-position modulation (PPM) – here the position of a pulse relative to its unmodulated time of occurrence is varied in accordance with the message signal.**

Other Forms of Pulse Modulation



(a) Modulating wave.

(b) Pulse carrier.

(c) PDM wave.

(d) PPM wave.

PPM is more efficient form of pulse modulation than PDM.

Sources of Corruption

The analog signal recovered from the sampled, quantized, and transmitted pulses will contain corruption from several sources.

Sources of Corruption

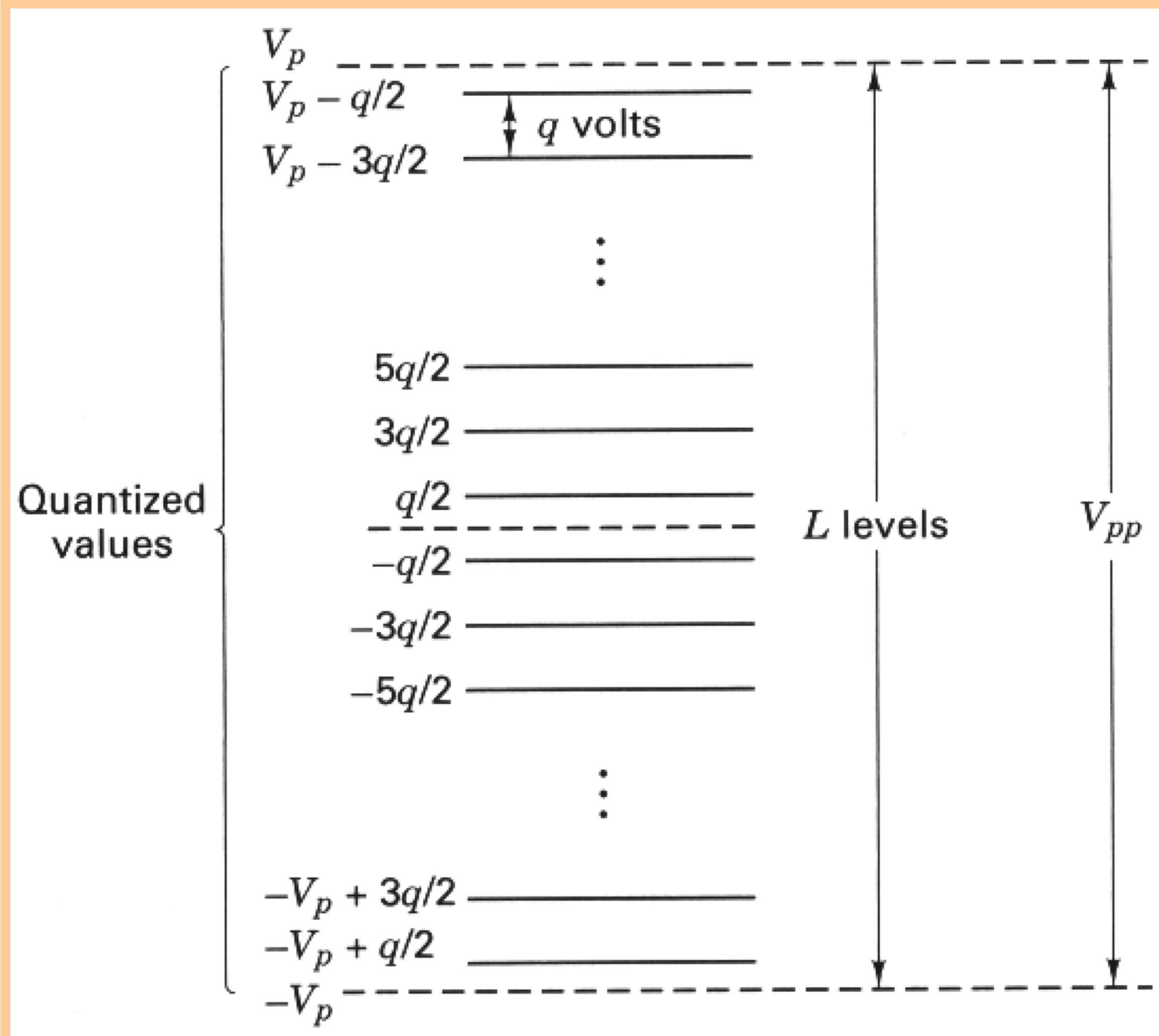
Sampling and Quantization effects

- Qunatization Noise
- Quantizer Saturation
- Timing Jitter

Channel effects

- Channel Noise
- Intersymbol Interference

Signal-to-Noise Ratio for Quantized Pulses



Quantization levels for an L-level linear quantizer for an analog signal with a peak-to-peak voltage range of $V_{pp} = V_p - (-V_p) = 2V_p$ volts.

Signal-to-Noise Ratio for Quantized Pulses

Salient Points

The step size between quantization levels is called the quantile interval. In the previous figure, quantile interval = *q* volts.

When the quantization levels are uniformly distributed over the full range, the quantizer is called a uniform or linear quantizer.

The degradation of the signal due to quantization is limited to half a quantile interval, ±(q/2) volts.

Can we use any useful Figure of Merit ??

***YES*. A useful figure of merit for the uniform quantizer is the quantizer variance (mean-square error assuming zero mean).**

Signal-to-Noise Ratio for Quantized Pulses

Quantizer Variance as a Figure of Merit for the Uniform Quantizer…

- ✓ **Let us assume that the quantization error, *e*, is uniformly distributed over a single quantile interval *q*-wide (i.e. the analog input takes on all values with equal probability). Then the quantizer error variance:**

$$\sigma^2 = \int_{-q/2}^{+q/2} e^2 p(e)\, de$$

$$= \int_{-q/2}^{+q/2} e^2 \frac{1}{q}\, de = \frac{q^2}{12}$$

$p(e) = (1/q) =$ (uniform) probability density function of the quantization error and

$\sigma^2 =$ variance, corresponds to the average quantization noise power.

Signal-to-Noise Ratio for Quantized Pulses

Quantizer Variance as a Figure of Merit for the Uniform Quantizer…

The peak power of the analog signal (normalized to 1Ω):

$$V_p^2 = \left(\frac{V_{pp}}{2}\right)^2 = \left(\frac{Lq}{2}\right)^2 = \frac{L^2q^2}{4}$$

L **= number of quantization levels.**

The ration of peak signal power to average quantization noise power $(S/N)_q$, assuming that there are no errors due to ISI or channel noise):

$$\left(\frac{S}{N}\right)_q = \frac{L^2q^2/4}{q^2/12} = 3L^2$$

Conclusion: **$(S/N)_q$ improves as a function of the number of quantization levels required. With an infinite number of quantization levels, there is zero quantization noise.**

Pulse Code Modulation (PCM)

The PCM is the name given to the class of baseband signals obtained from the quantized PAM signals by encoding each quantized sample into a *digital word*.

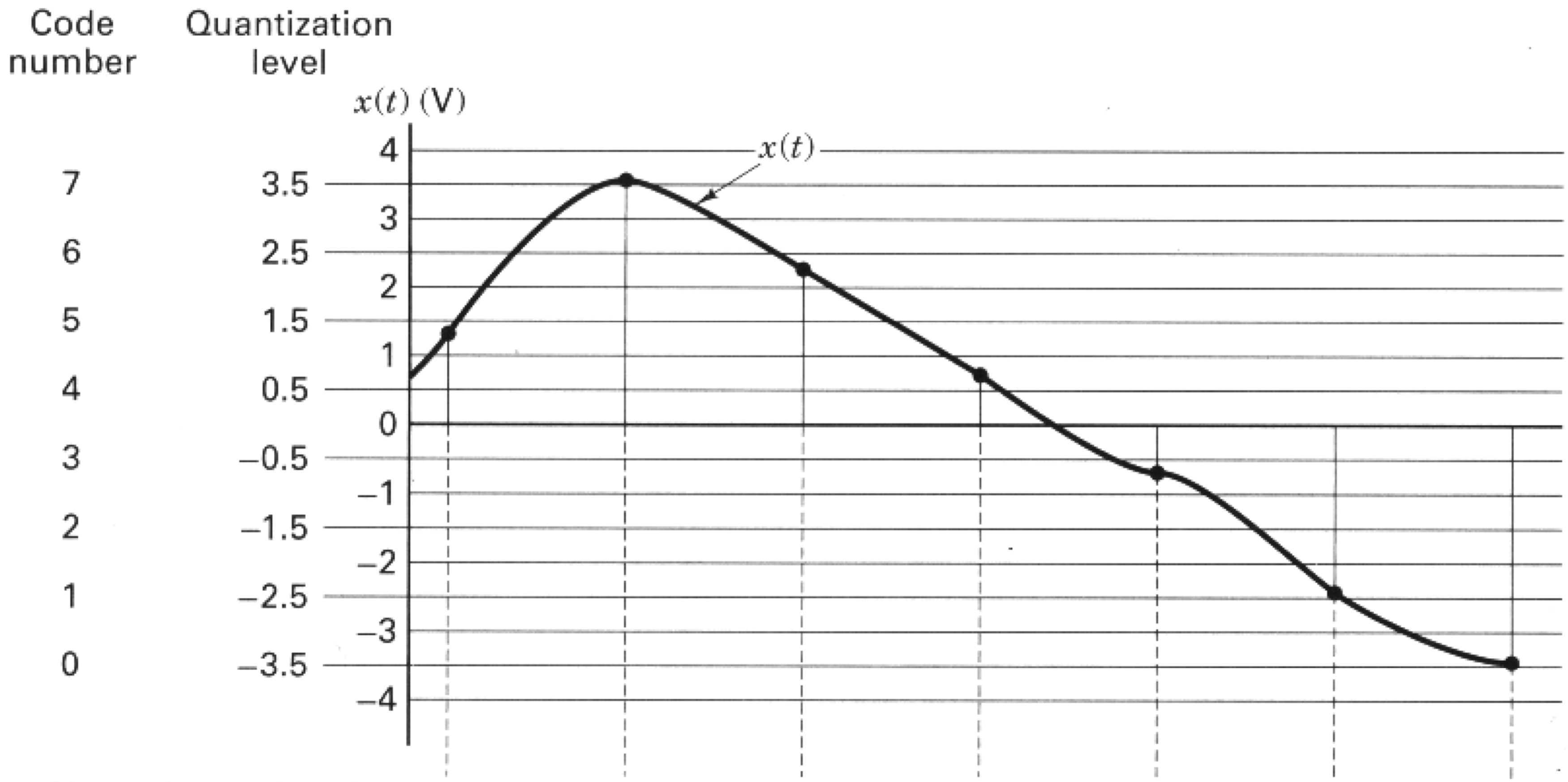
↓

The source information is sampled and quantized to one of *L* levels. Then each quantized sample is digitally encoded into an *l*-bit ($l = \log_2 L$) codeword.

↓

For baseband transmission, the codeword bits will then be transformed to pulse waveforms.

Pulse Code Modulation (PCM)

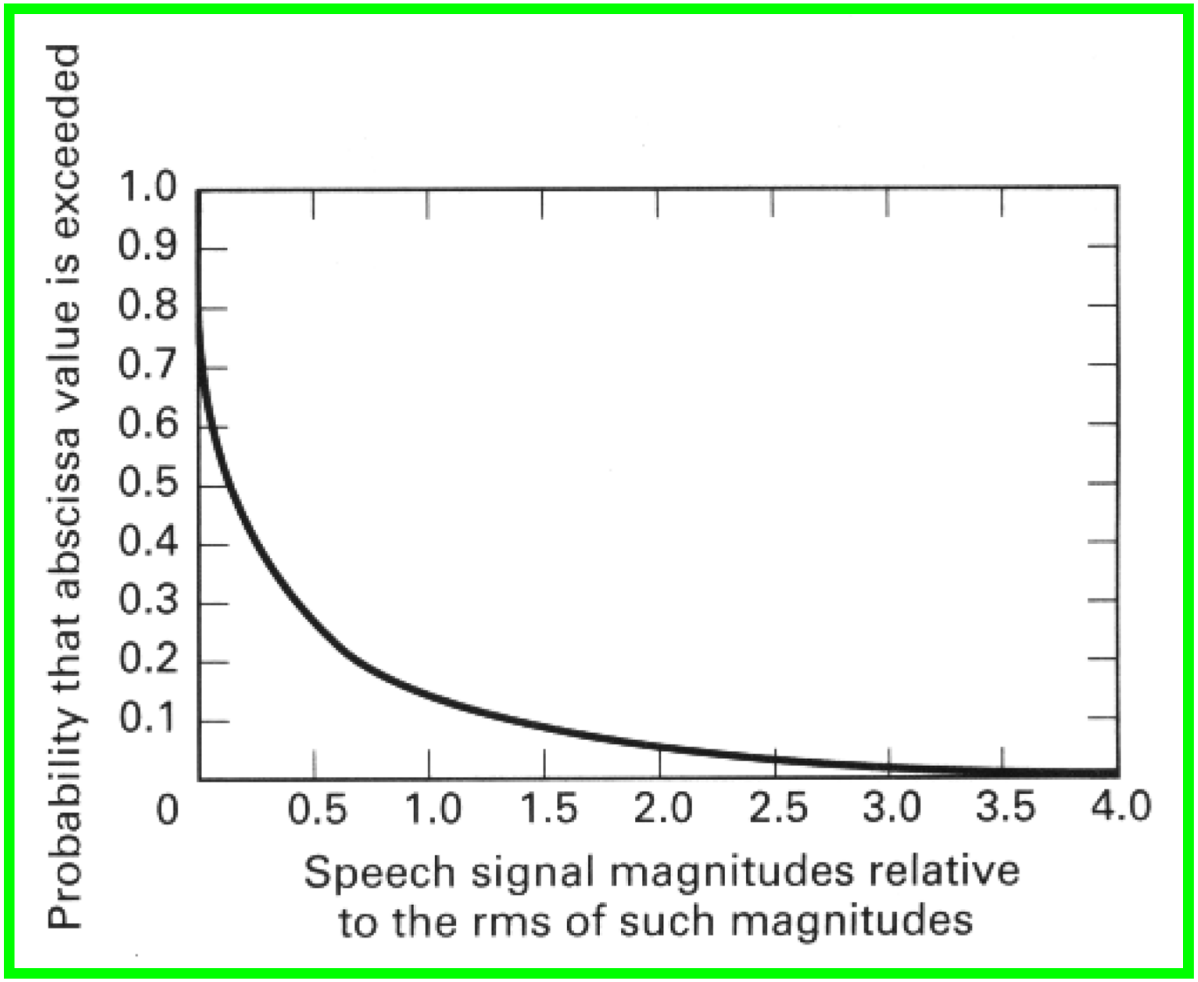


Natural sample value	1.3	3.6	2.3	0.7	–0.7	–2.4	–3.4
Quantized sample value	1.5	3.5	2.5	0.5	–0.5	–2.5	–3.5
Code number	5	7	6	4	3	1	0
PCM sequence	101	111	110	100	011	001	000

Natural samples, quantized samples, and pulse code modulation: An example.

Uniform and Nonuniform Quantization

Statistics of Speech Amplitudes



Statistical distribution of single-talker speech signal magnitudes.

Uniform and Nonuniform Quantization

Statistics of Speech Amplitudes

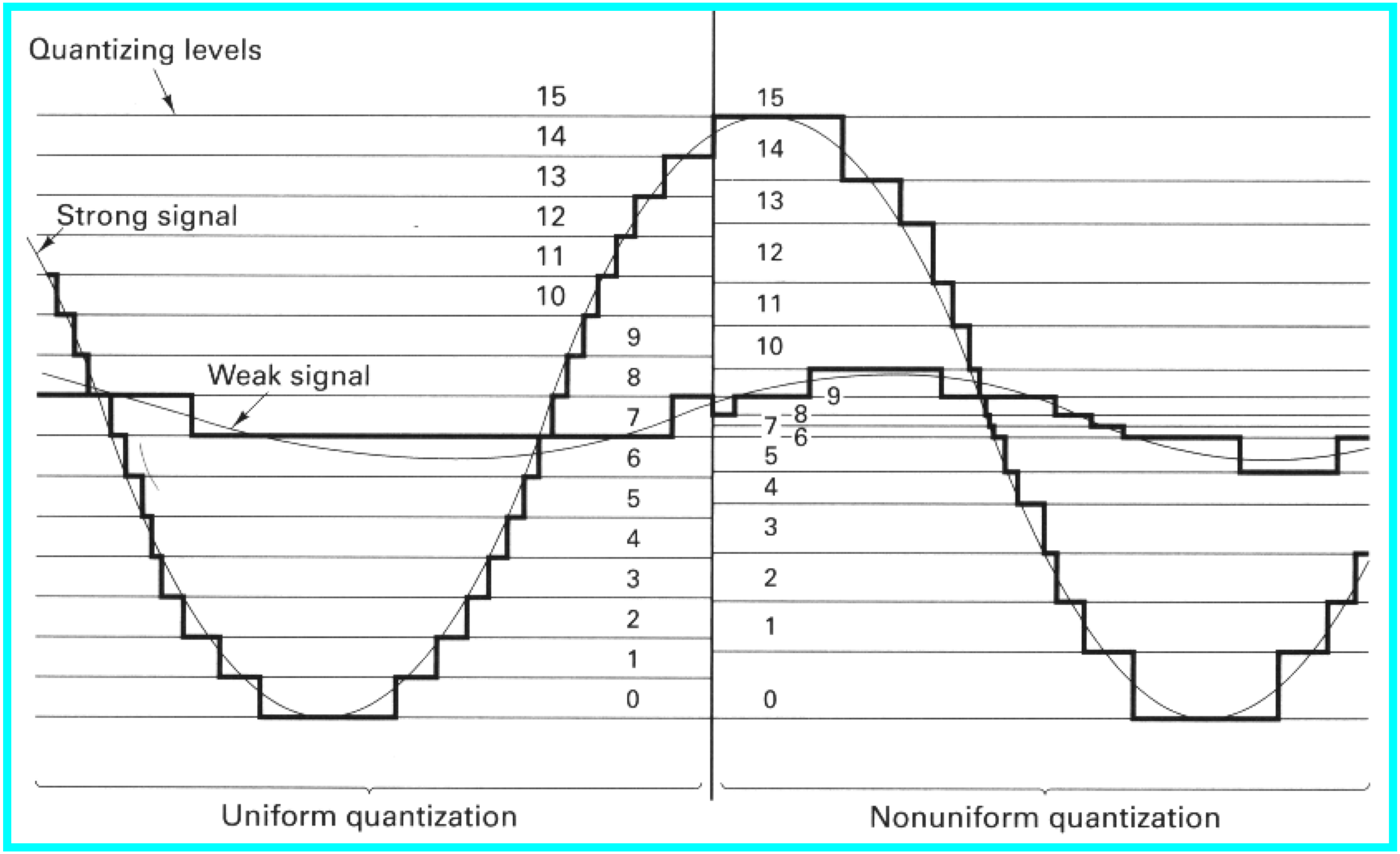
For most voice communication channels, very low speech volumes predominate. 50% of the time, the voltage characterizing speech energy is less than one-fourth of the rms value.

Large amplitude values are relatively rare. Only 15% of the time does the voltage exceed the rms value.

Implication …

A system with uniform quantization would be wasteful for speech signals as many of the quantizing steps would rarely be used. Therefore, with uniform quantization, the SNR is worse for low-level signals than for high-level signals.

Uniform and Nonuniform Quantization



Uniform and nonuniform quantization of signals.

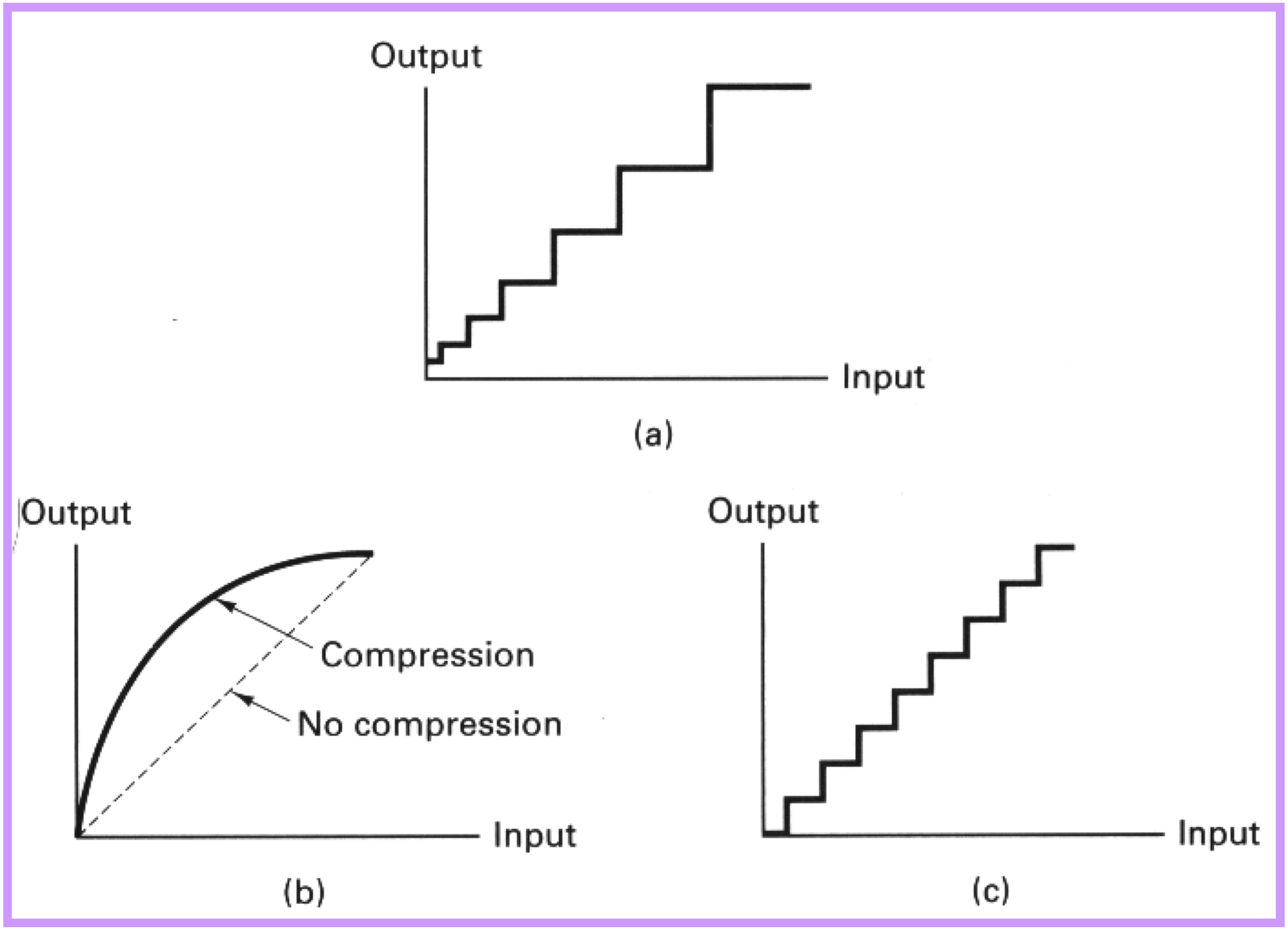
Nonuniform Quantization

One way of achieving nonuniform quantization is to utilize a nonuniform quantizer characteristic.

More often, nonuniform quantization is achieved by first distorting the original signal with a logarithmic compression characteristic, and then using a uniform quantizer.

For small magnitude signals, the compression characteristic has a much steeper slope than for large magnitude signals. Thus, a given signal change at small magnitudes will carry the uniform quantizer through more steps than the same change at large magnitudes.

Nonuniform Quantization



(a) Nonuniform quantizer characteristic. (b) Compression characteristic. (c) Uniform quantizer characteristic.

Nonuniform Quantization

Companding Characteristics

μ-law characteristic

$$y = y_{\max} \frac{\log_e[1 + \mu(|x| / x_{\max})]}{\log_e(1 + \mu)} \operatorname{sgn} x$$

$$\operatorname{sgn} x = \begin{cases} +1 & \text{for } x \geq 0 \\ -1 & \text{for } x < 0 \end{cases}$$

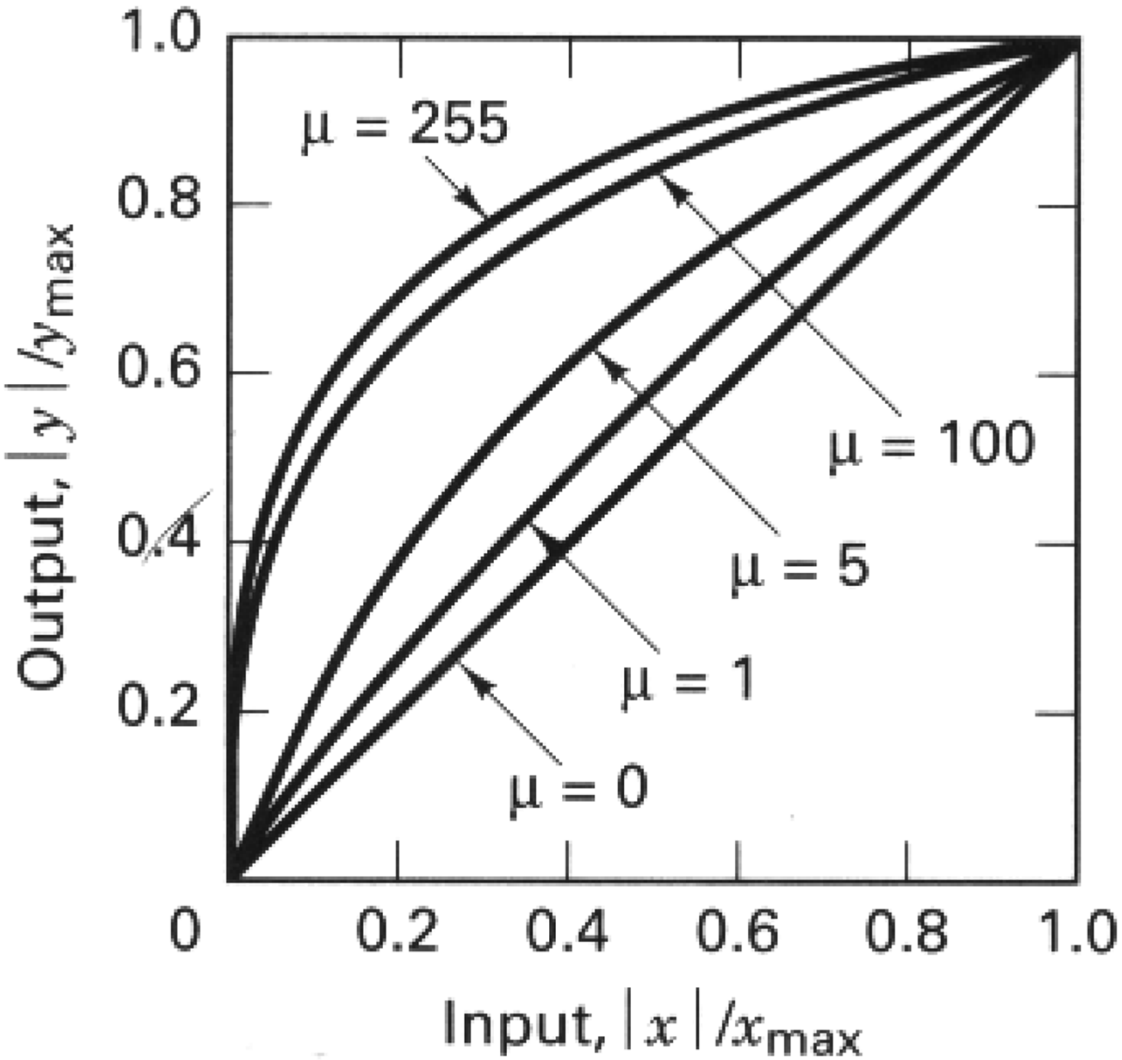
μ: a positive constant
x and y: input and output voltages

A-law characteristic

$$y = \begin{cases} y_{\max} \dfrac{A(|x| / x_{\max})}{1 + \log_e A} \operatorname{sgn} x & 0 < \dfrac{|x|}{x_{\max}} \leq \dfrac{1}{A} \\ y_{\max} \dfrac{1 + \log_e[A(|x| / x_{\max})]}{1 + \log_e A} \operatorname{sgn} x & \dfrac{1}{A} < \dfrac{|x|}{x_{\max}} < 1 \end{cases}$$

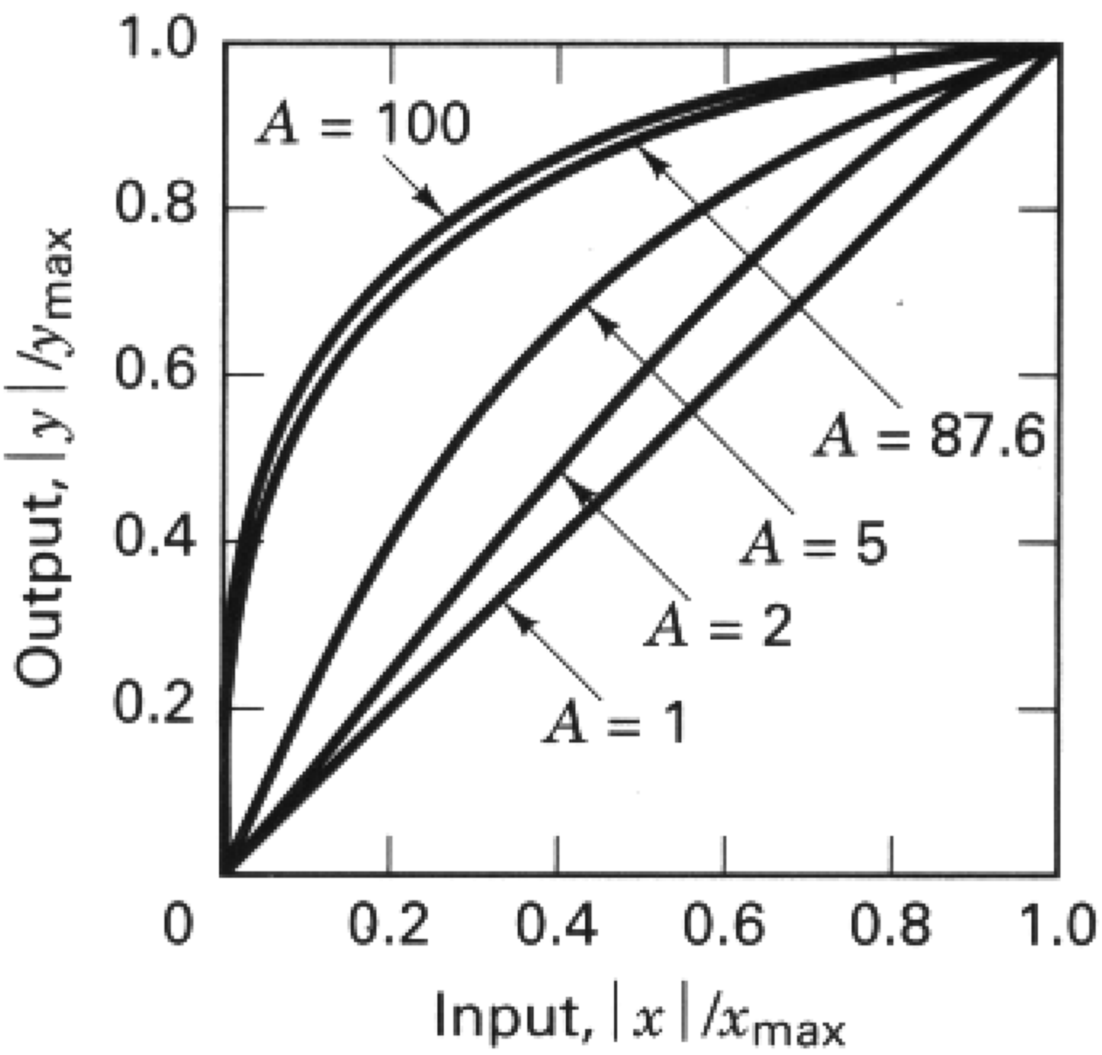
A: a positive constant
x and y: input and output voltages

Nonuniform Quantization



μ-law compression characteristic.

Nonuniform Quantization



A-law compression characteristic.

Baseband Transmission

Waveform Representation of Binary Digits

Digits are just abstractions – a way to describe the message information. Hence a physical quantity is required that will represent or "carry" the digits.

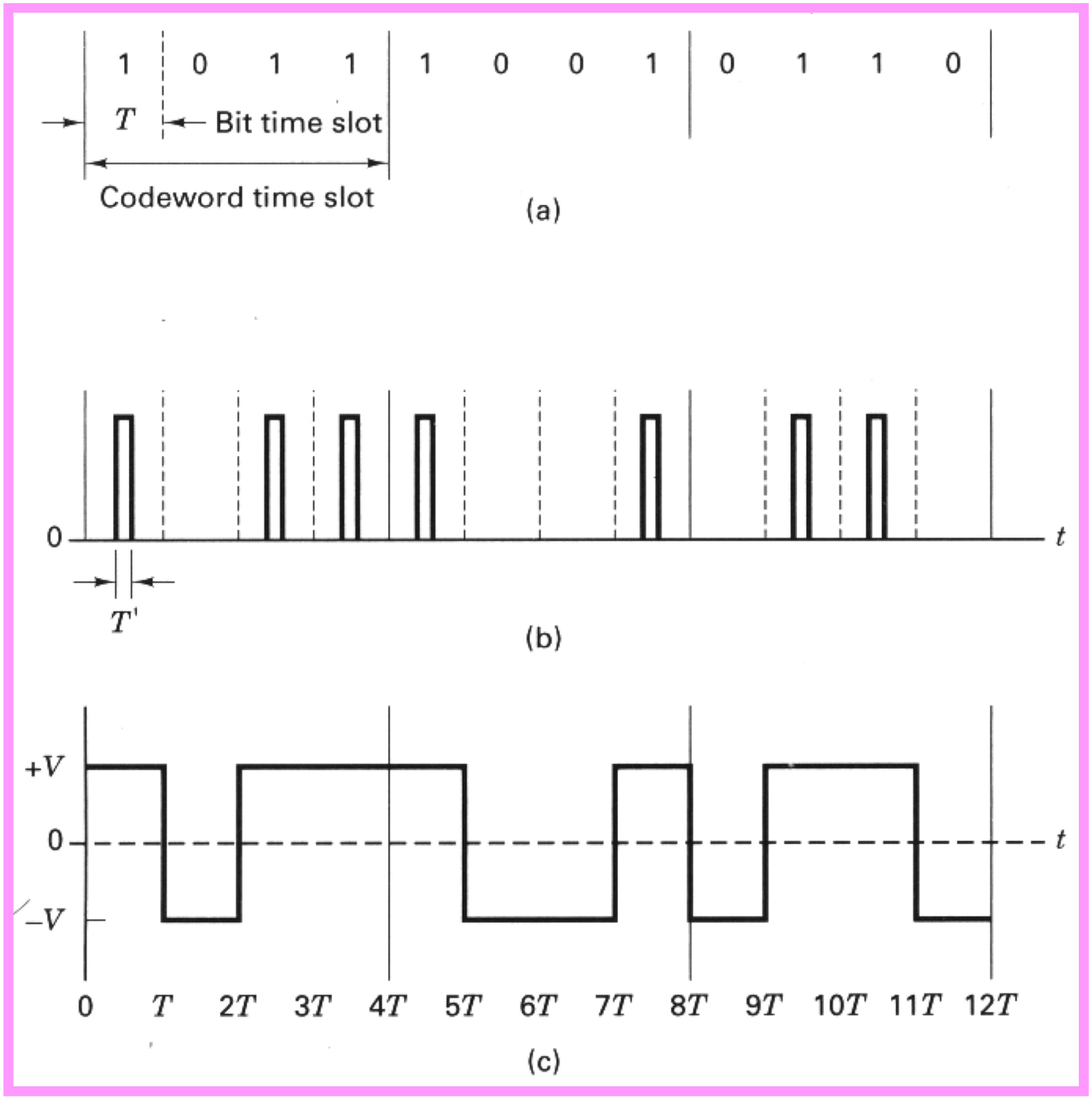
How??

Binary Digits are represented by electrical pulses in order to transmit them through a baseband channel. A sequence of electrical pulses, conforming to a pattern, can be used to transmit the information in the PCM bit stream, and hence the information in the quantized samples of message.

At the receiver, a determination must be made as to the presence or absence of a pulse in each bit time slot.

Baseband Transmission

Waveform Representation of Binary Digits: An example



(a) PCM sequence.

(b) Pulse representation of PCM.

(c) Pulse waveform (transition between two levels).

Baseband Transmission

PCM Waveform Types

When pulse modulation is applied to a binary symbol, the resulting binary waveform is called pulse-code nodulation (PCM) waveform.

In telephony applications, these waveforms are often called line codes.

When pulse modulation is applied to a nonbinary symbol, the resulting waveform is called an M-ary pulse-modulation waveform.

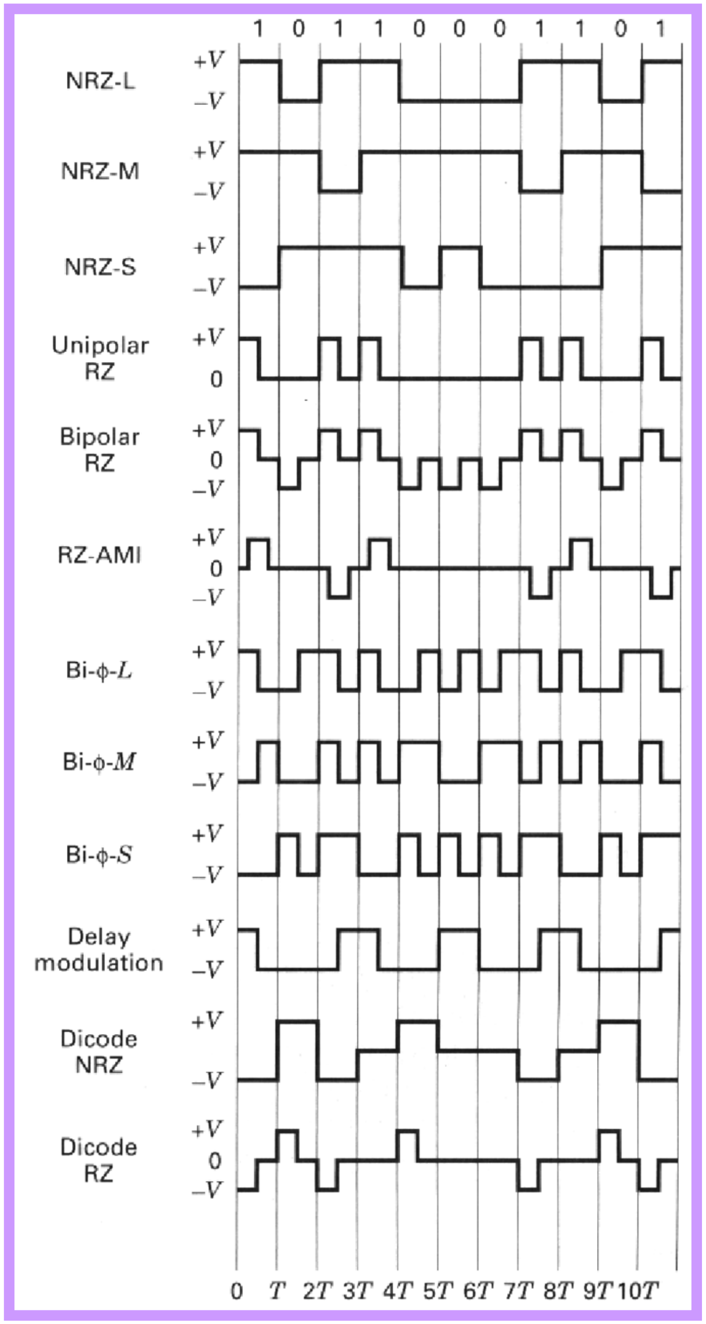
Baseband Transmission

PCM Waveform Types

PCM waveforms fall into the following four groups:

- Nonreturn-to-zero (NRZ).
- Return-to-zero (RZ).
- Phase Encoded.
- Multilevel Binary.

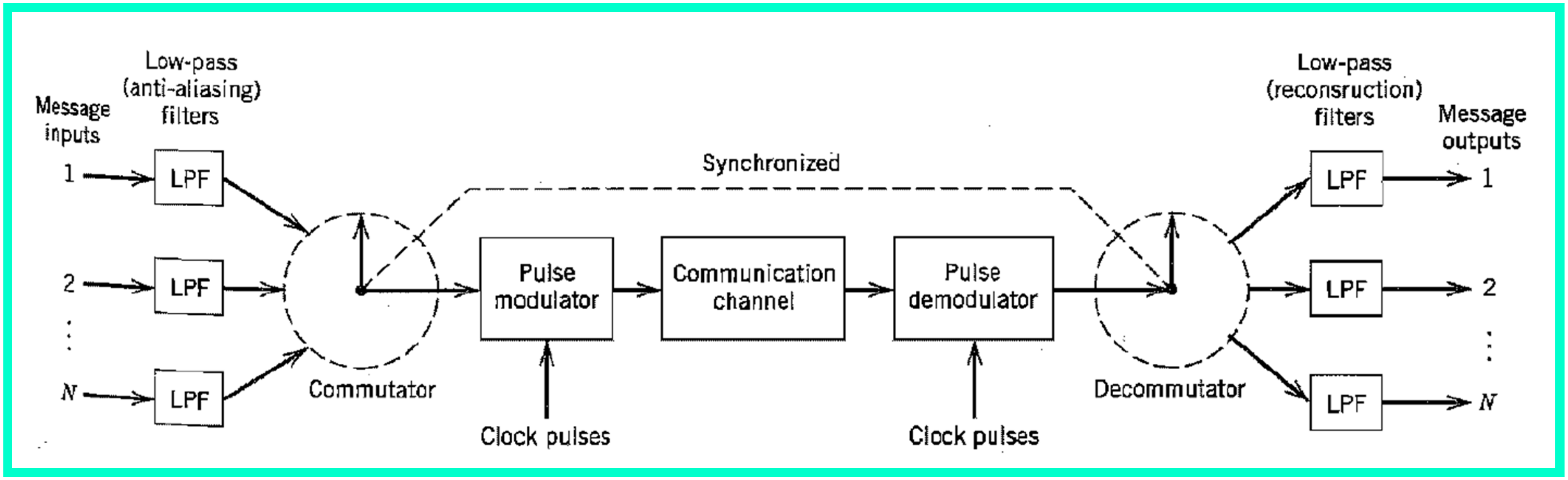
Baseband Transmission



Various PCM waveforms.

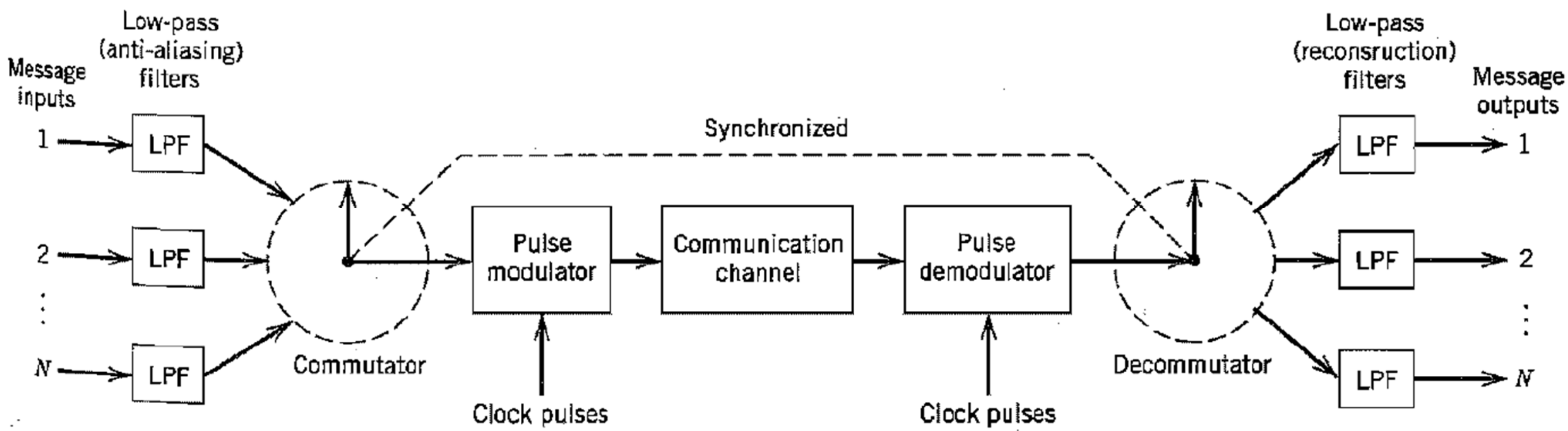
Time-Division Multiplexing (TDM)

A TDM system enables the joint utilization of a common communication channel by a plurality of independent message sources, without mutual interference among them.



Block diagram of TDM system.

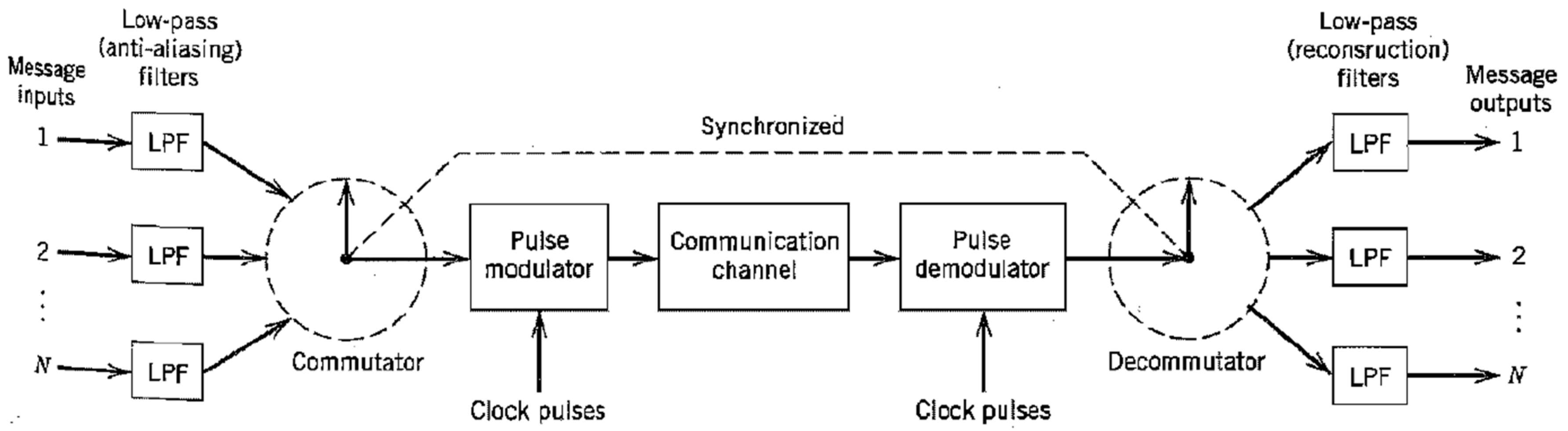
Time-Division Multiplexing (TDM)



Commutator: it takes a narrow sample of each of the N input messages at a rate f_s that is slightly higher than $2W$ (W = cut-off frequency of the anti-alias filter) and sequentially interleaves these N samples inside the sampling interval T_s.

Pulse Modulator: it transforms the multiplexed signal into a form suitable for transmission over the comnon channel.

Time-Division Multiplexing (TDM)



Pulse Demodulator: it receives the signal at the receiving end and performs the reverse operation of the pulse modulator.

Decommutator: it distributes the narrow samples produced at the pulse demodulator output to the appropriate low-pass reconstruction filters. The decommutator operates in synchronism with the commutator in the transmitter.

Delta Modulation (DM)

In DM, an incoming message signal is oversampled to purposely increase the correlation between adjacent samples of the signal. This is done to permit the use of a simple quantization strategy for constructing the encoded signal.

In its basic form, DM provides a staircase approximation to the oversampled version of the message signal. The difference between the input and the approximation is quantized into two levels, namely, $\pm\Delta$, corresponding to positive and negative differences.

Let $m(t)$ denote the input (message) signal and $m_q(t)$ denote its staircase approximation. We shall use $m[n] = m(nT_s)$, $n = 0, \pm 1, \pm 2, \ldots$. Here T_s = sampling period and $m(nT_s)$ is a sample of the signal $m(t)$ taken at time $t = nT_s$.

Delta Modulation (DM)

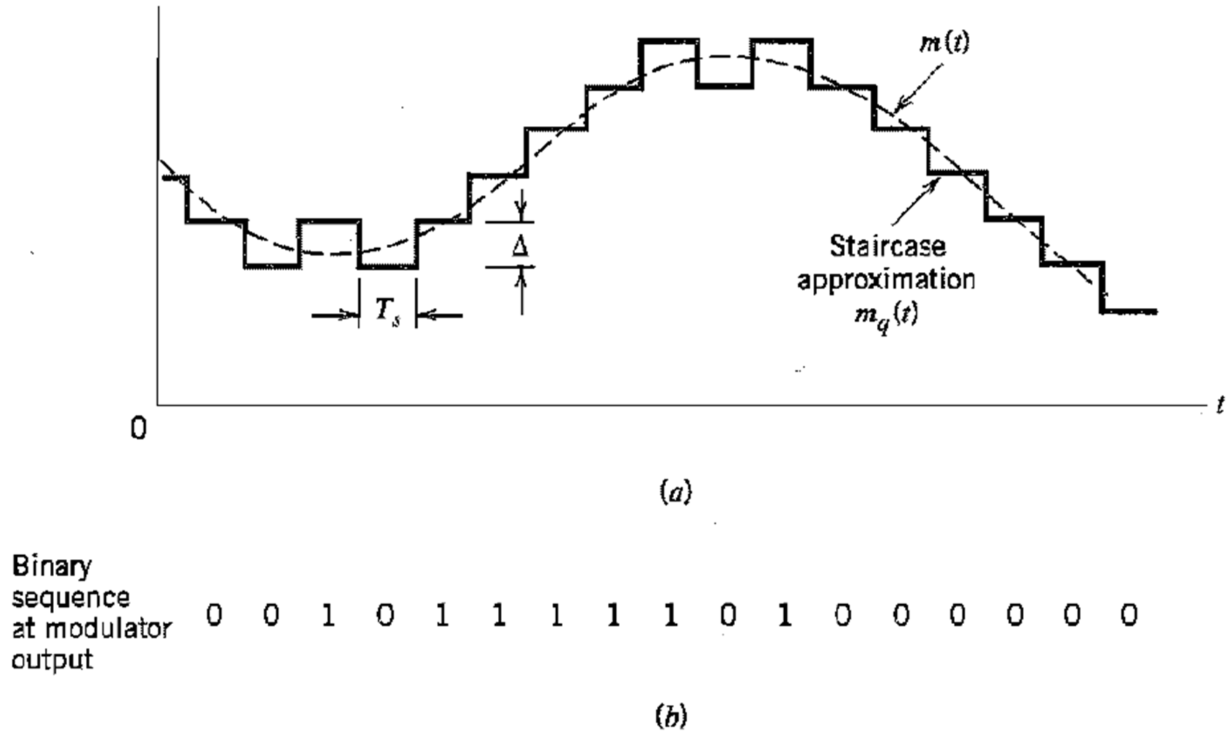


Illustration of DM.

From basic principles of DM:

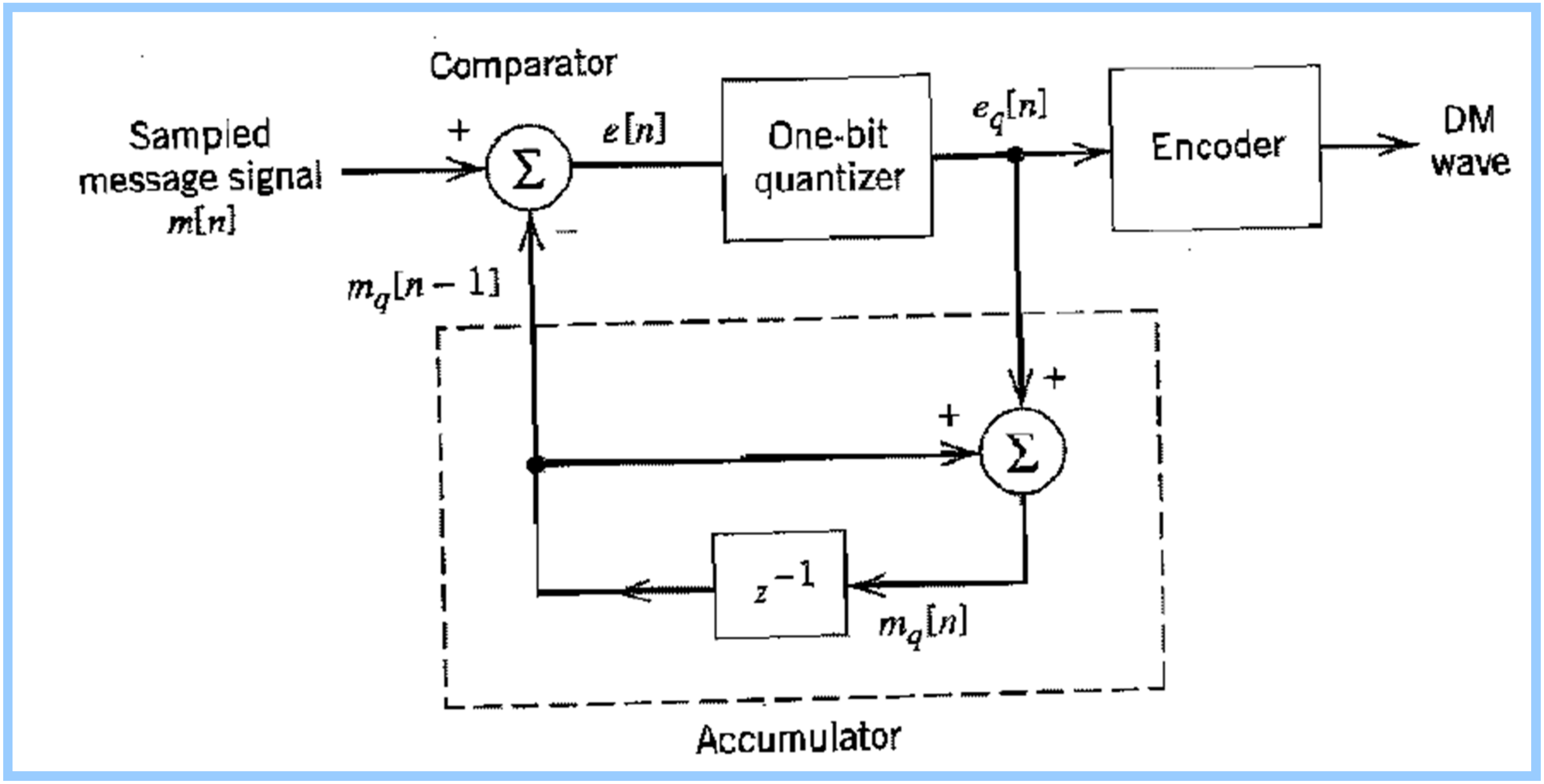
$$e[n] = m[n] - m_q[n-1]$$

$$e_q = \Delta\,\mathbf{sgn}(e[n])$$

$$m_q[n] = m_q[n-1] + e_q[n]$$

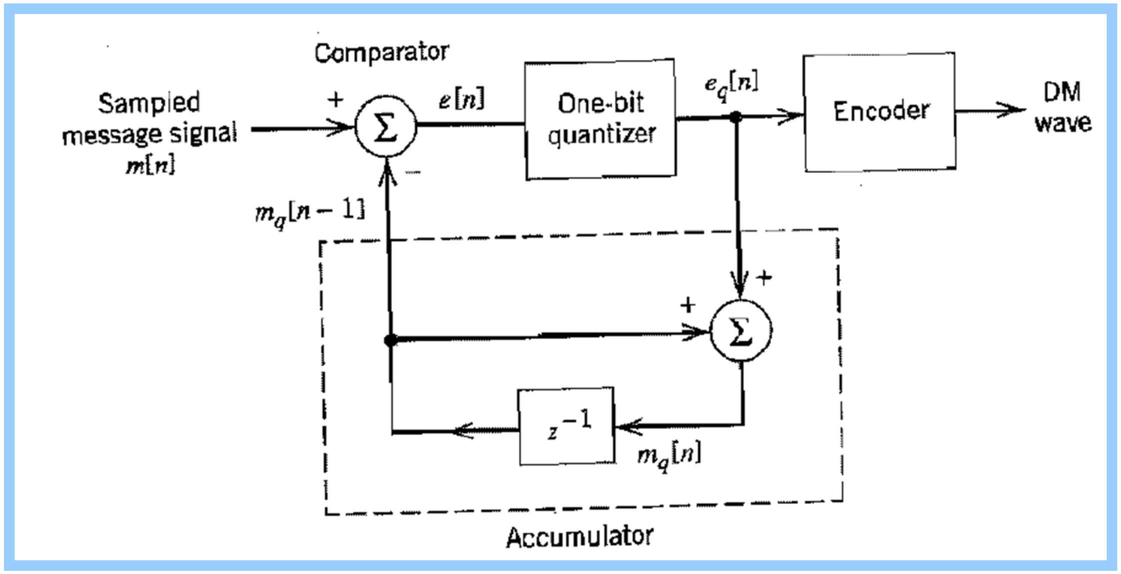
$e[n]$: error signal, signifying difference between present sample input and the latest approximation.

Delta Modulation (DM)



Transmitter of the DM system.

Delta Modulation (DM)

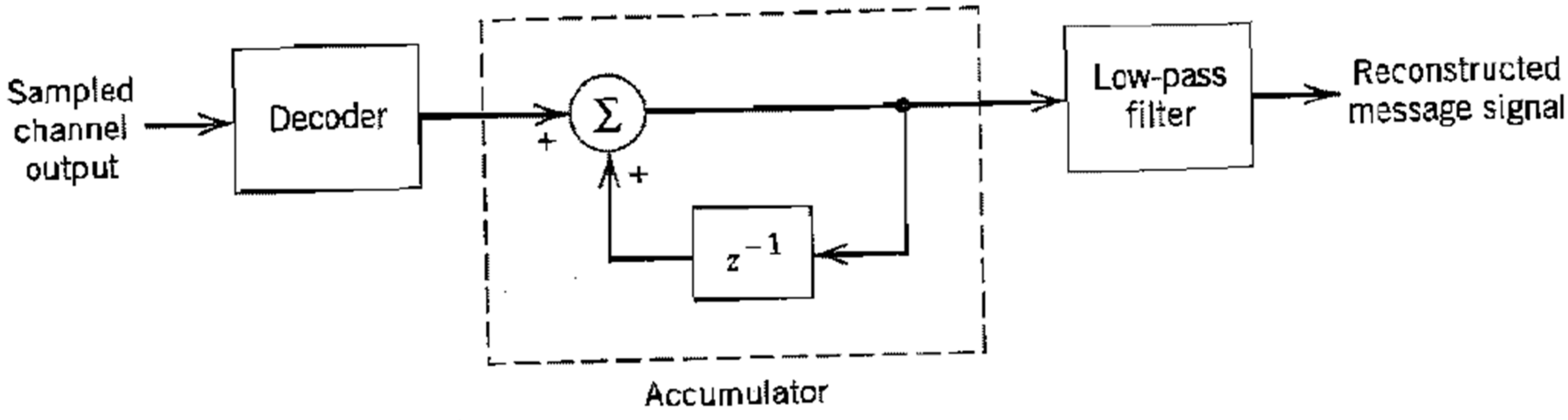


Transmitter of the DM system.

The modulator comprises a comparator, quantizer and accumulator. The quantizer comprises a hard limiter whose input-output relation is a scaled version of the signum function. The quantizer output is applied to an accumulator, producing the result:

$$m_q[n] = \Delta \sum_{i=1}^{n} \mathbf{sgn}(e[i]) = \sum_{i=1}^{n} e_q[i]$$

Delta Modulation (DM)



Receiver of the DM system.

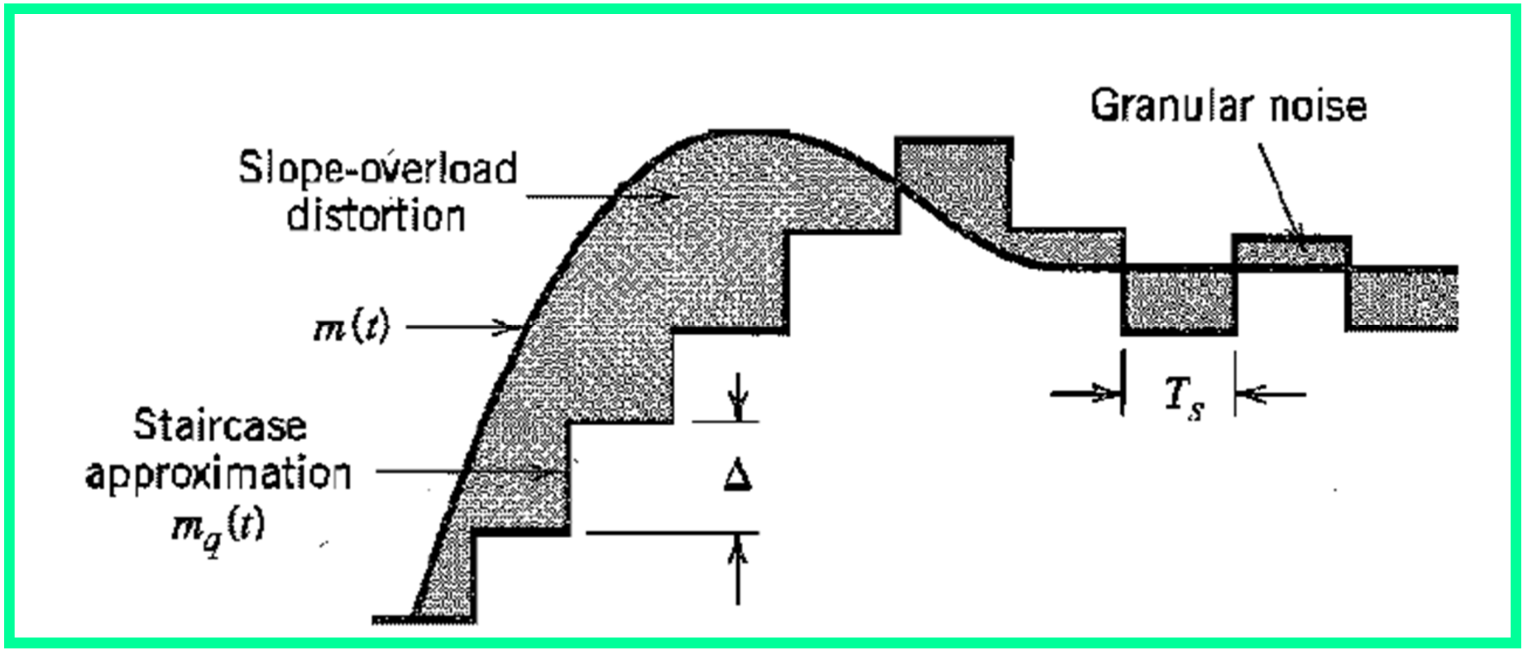
In the receiver, the staircase approximation $m_q(t)$ is reconstructed by passing the sequence of positive and negative pulses, produced at the decoder output, through an accumulator in a manner similar to that used in the transmitter.

Delta Modulation (DM)

Quantization errors

Slope overload distortion

Granular noise



Baseband Demodulation/Detection

For any binary channel, the transmitted signal over a symbol interval (0, T) is represented by:

$$s_i(t) = \begin{cases} s_1(t) & 0 \le t \le T \quad \text{for a binary 1} \\ s_2(t) & 0 \le t \le T \quad \text{for a binary 0} \end{cases}$$

The received signal $r(t)$, degraded by noise $n(t)$, and possibly degraded by the impulse response of the channel $h_c(t)$ is:

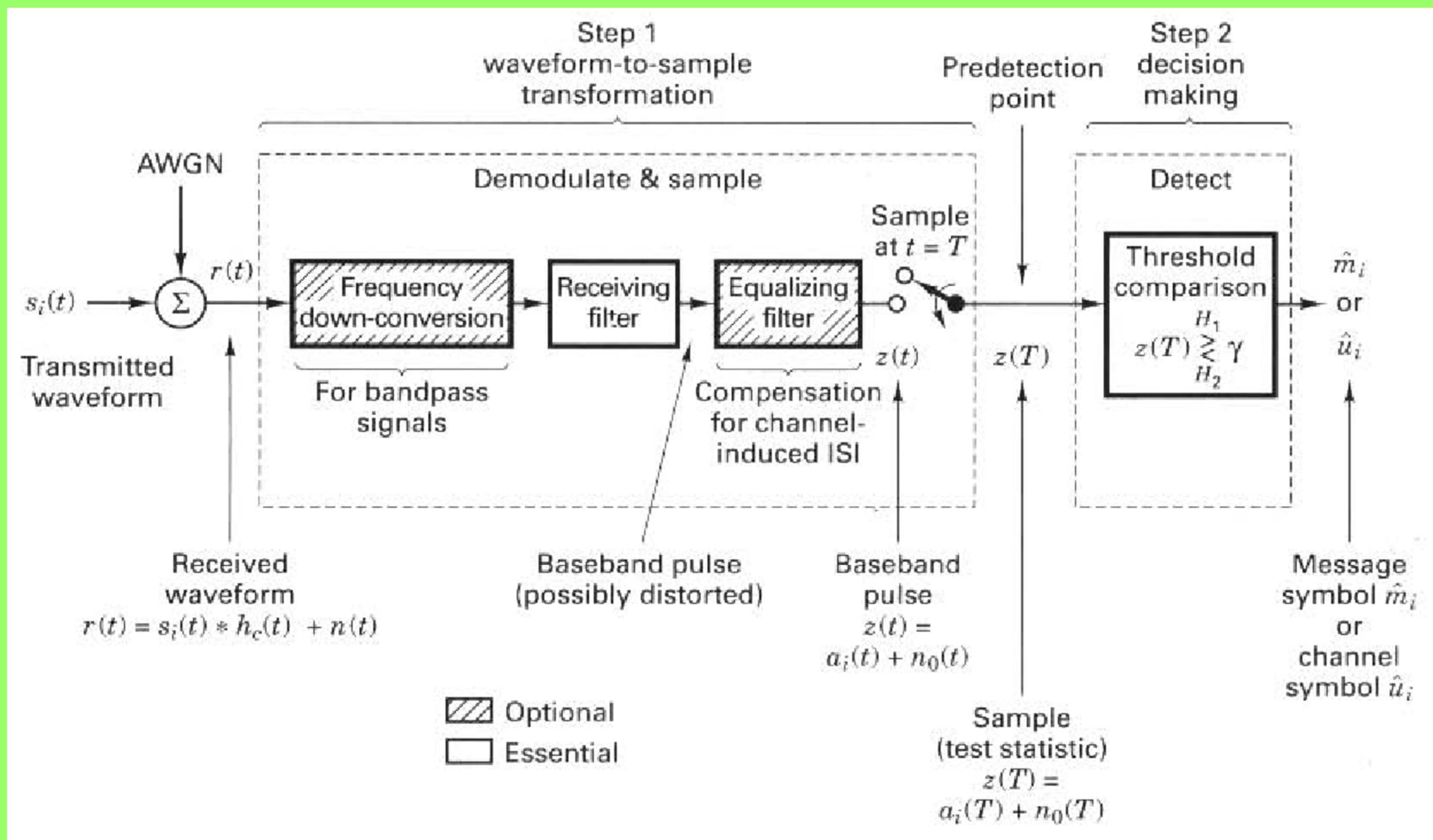
$$r(t) = s_i(t) * h_c(t) + n(t) \qquad i = 1, \ldots, M$$

$n(t)$ is assumed to be a zero mean AWGN process.

For binary transmission over an ideal distortionless channel:

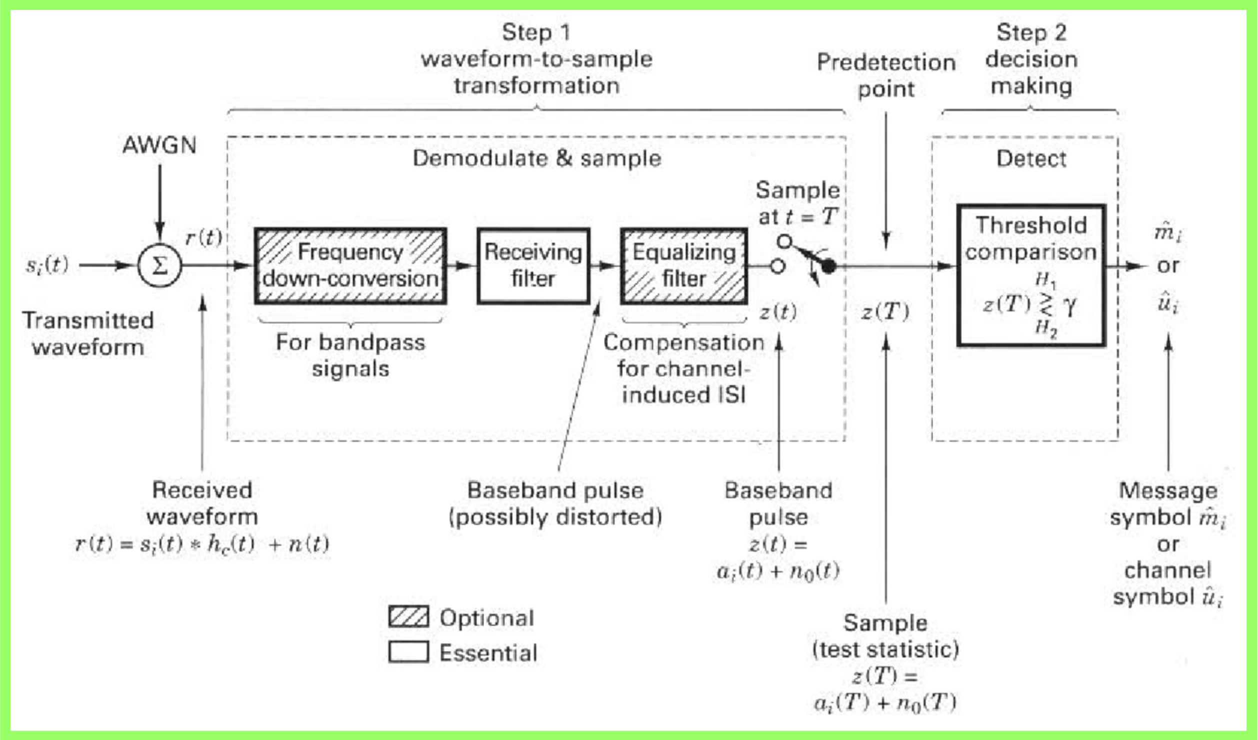
$$r(t) = s_i(t) + n(t) \qquad i = 1, 2, \qquad 0 \le t \le T$$

Baseband Demodulation/Detection



Two basic steps in the demodulation/detection of digital signals.

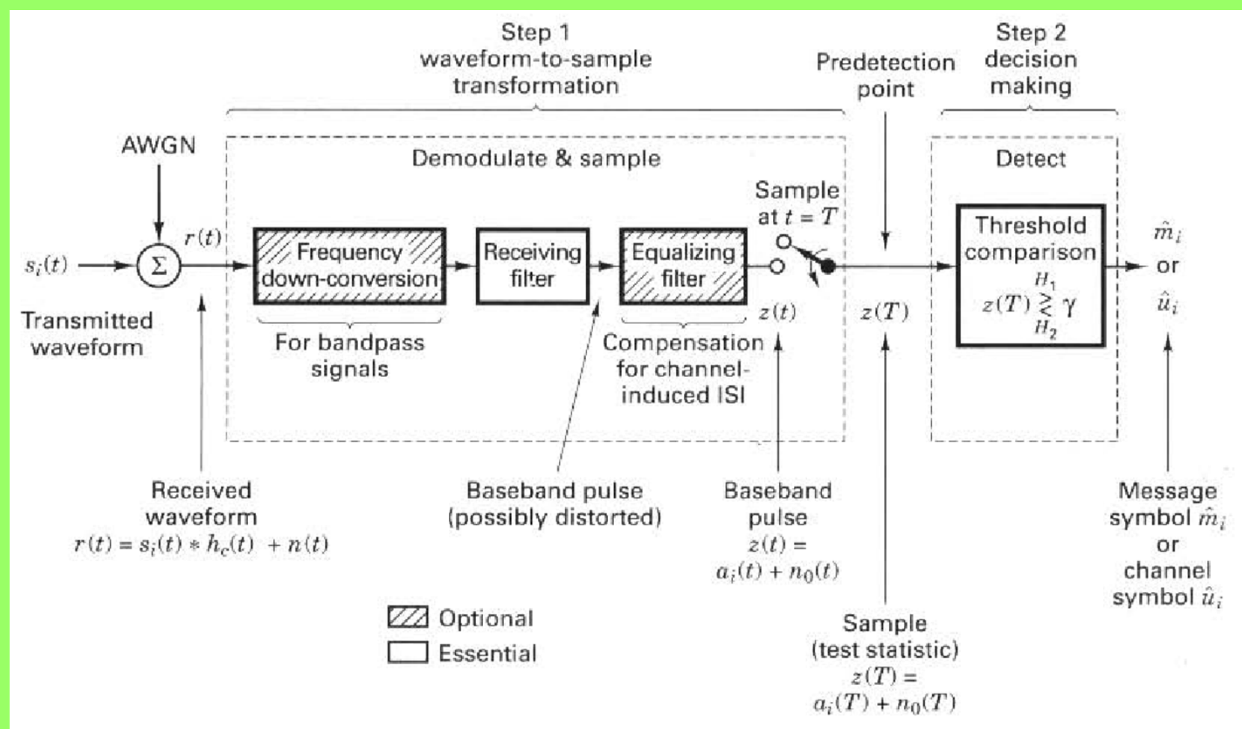
Baseband Demodulation/Detection



The objective of the receiving filter is to recover a baseband pulse with the best possible signal-to-noise ratio (SNR), free of any ISI.

The optimum receiving filter to accomplish this is called a matched filter or correlator.

Baseband Demodulation/Detection



At the end of each sample duration *T*, the output of the sampler yields a sample *z*(*T*), sometimes called the *test statistic*. Assuming the input noise as a Gaussian random process and the demodulator is linear:

$$z(T) = a_i(T) + n_0(T) \qquad i = 1, 2$$

$a_i(T)$: desired signal component

$n_0(T)$: noise component

Baseband Demodulation/Detection

Now n_0 is a zero mean Gaussian random variable, and thus $z(T)$ is a Gaussian random variable with a mean of either a_1 or a_2, depending on whether a binary one or binary zero was sent.

The probability density function (pdf) of the Gaussian random noise n_0 is:

$$p(n_0) = \frac{1}{\sigma_0 \sqrt{2\pi}} \exp\left[-\frac{1}{2}\left(\frac{n_0}{\sigma_0}\right)^2\right]$$

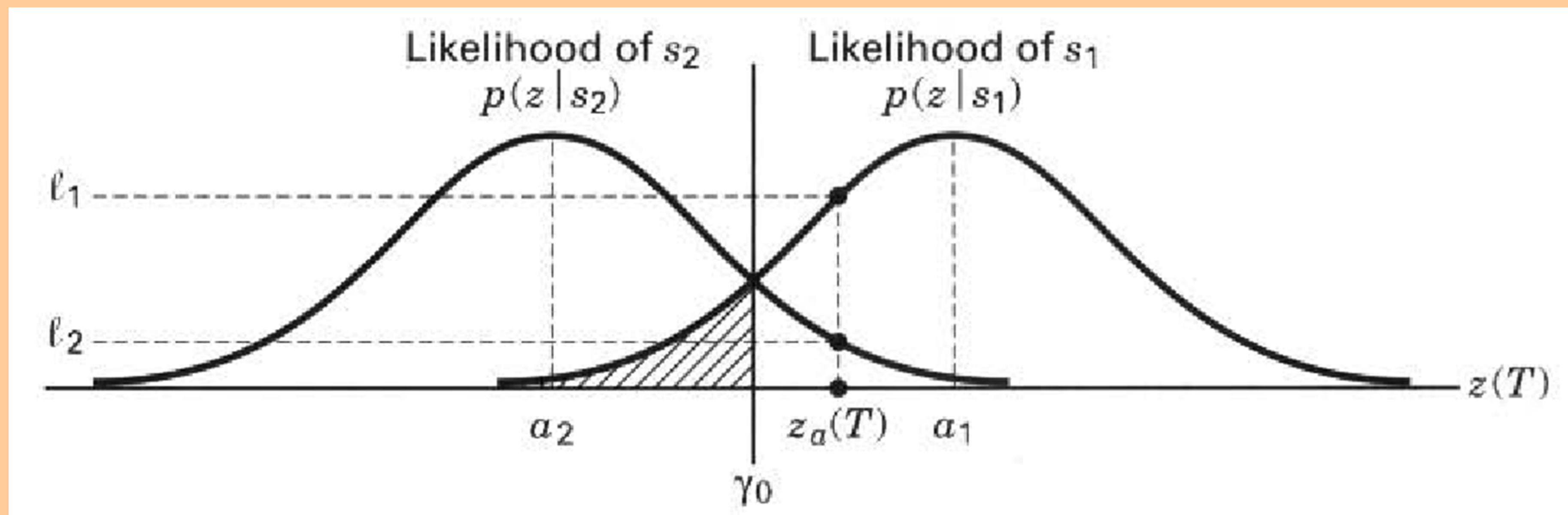
σ_0^2: noise variance

The conditional pdfs are:

$$p(z|s_1) = \frac{1}{\sigma_0 \sqrt{2\pi}} \exp\left[-\frac{1}{2}\left(\frac{z - a_1}{\sigma_0}\right)^2\right]$$

$$p(z|s_2) = \frac{1}{\sigma_0 \sqrt{2\pi}} \exp\left[-\frac{1}{2}\left(\frac{z - a_2}{\sigma_0}\right)^2\right]$$

Baseband Demodulation/Detection



Conditional pdfs: $p(z|s_1)$ and $p(z|s_2)$.

The rightmost conditional pdf $p(z|s_1)$ is called the likelihood of s_1 and it illustrates the pdf of the random variable $z(T)$, given that symbol s_1 was transmitted.

Similarly, the leftmost conditional pdf $p(z|s_2)$ is called the likelihood of s_2 and it illustrates the pdf of the random variable $z(T)$, given that symbol s_2 was transmitted.

Baseband Demodulation/Detection

The received signal energy (not the shape) is the important parameter in the detection process. Hence the detection analysis for baseband signals is the same as that for bandpass signals.

Now $z(T)$ is a voltage signal proportional to the energy of the received symbol. The larger the magnitude of $z(T)$, the more error free will be the detection process.

The detection is performed as:

$$z(T) \underset{H_2}{\overset{H_1}{\gtrless}} \gamma$$

H_1 and H_2: two possible (binary) hypotheses.

Choosing H_1 is equivalent to deciding that signal $s_1(t)$ was sent and hence a binary 1 is detected. Similarly, choosing H_2 is equivalent to deciding that signal $s_2(t)$ was sent and hence a binary 0 is detected.

Basic SNR Parameter for Digital Communication Systems

In analog communication, average signal power to average noise power ratio (S/N or SNR) is used as a figure of merit. In digital communication, E_b/N_o, a normalized version of SNR, is used as a natural figure of merit.

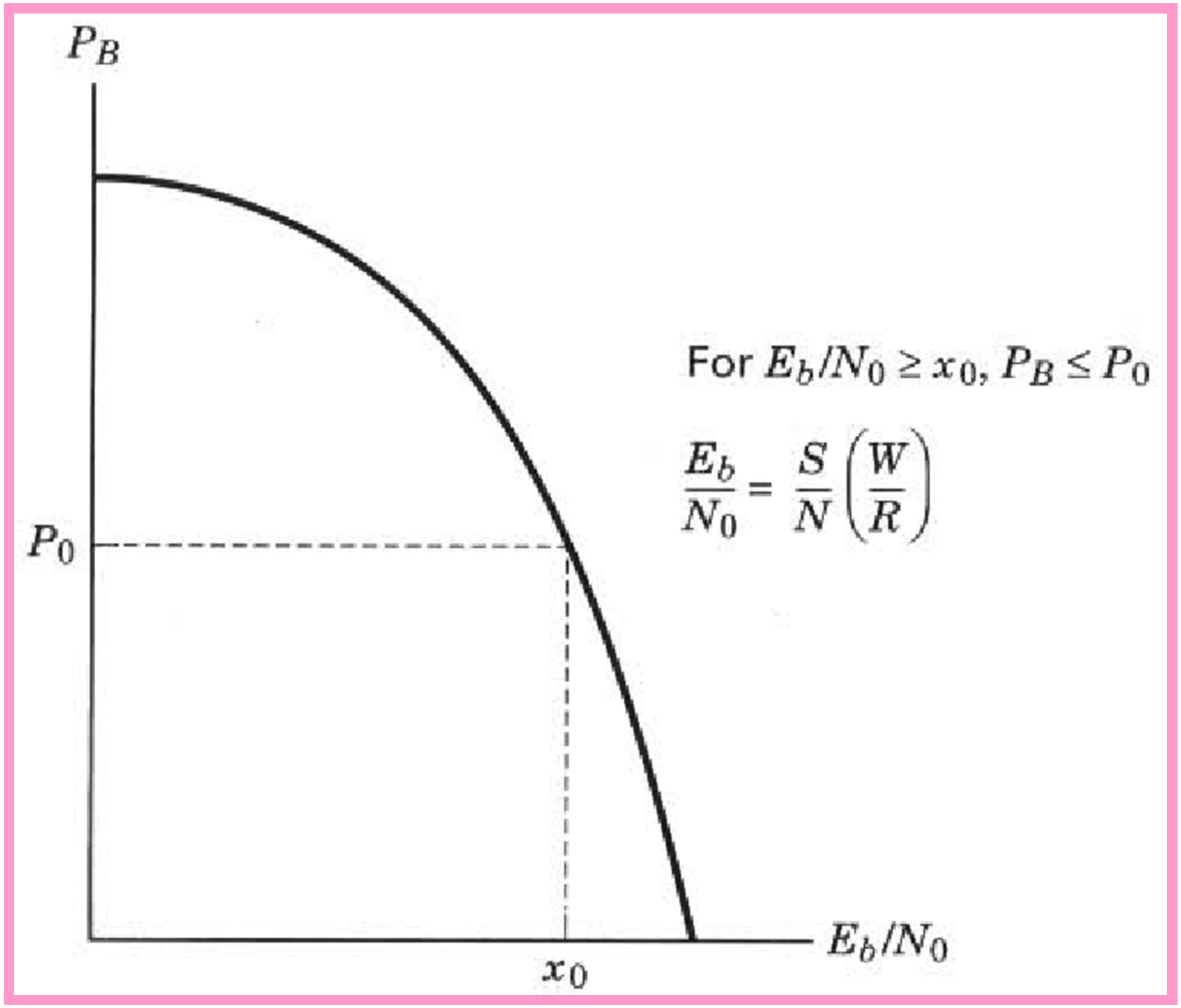
$$\frac{E_b}{N_0} = \frac{S\ T_b}{N/W} = \frac{S/R_b}{N/W}$$

$$\frac{E_b}{N_0} = \frac{S}{N}\left(\frac{W}{R}\right)$$

E_b: bit energy; S: signal power; T_b: bit time; N_o: noise power spectral density; W: bandwidth; $R_b = R =$ bit rate (bits/s).

One of the most important metrics of performance in digital communication systems is a plot of bit error probability (P_b) vs. E_b/N_o.

Basic SNR Parameter for Digital Communication Systems



General shape of the P_b vs. E_b/N_o curve.

Detection of Binary Signals in Gaussian Noise

Maximum Likelihood Receiver Structure

The detection making criterion is:

$$z(T) \underset{H_2}{\overset{H_1}{\gtrless}} \gamma$$

- ✓ **A popular choice for choosing the threshold level γ for the binary decision is based on minimizing the probability of error.**

- ✓ **For the computation of this minimum error value of $\gamma = \gamma_0$, one has to compute an inequality expression between the ratio of conditional pdfs and the signal *a priori* probabilities. This is called the likelihood ratio test and is given as:**

$$\frac{p(z|s_1)}{p(z|s_2)} \underset{H_2}{\overset{H_1}{\gtrless}} \frac{P(s_2)}{P(s_1)}$$

$P(s_1)$ and $P(s_2)$: *a priori* probabilities that $s_1(t)$ and $s_1(t)$, respectively, are transmitted.
H_1 and H_2: two possible hypotheses.

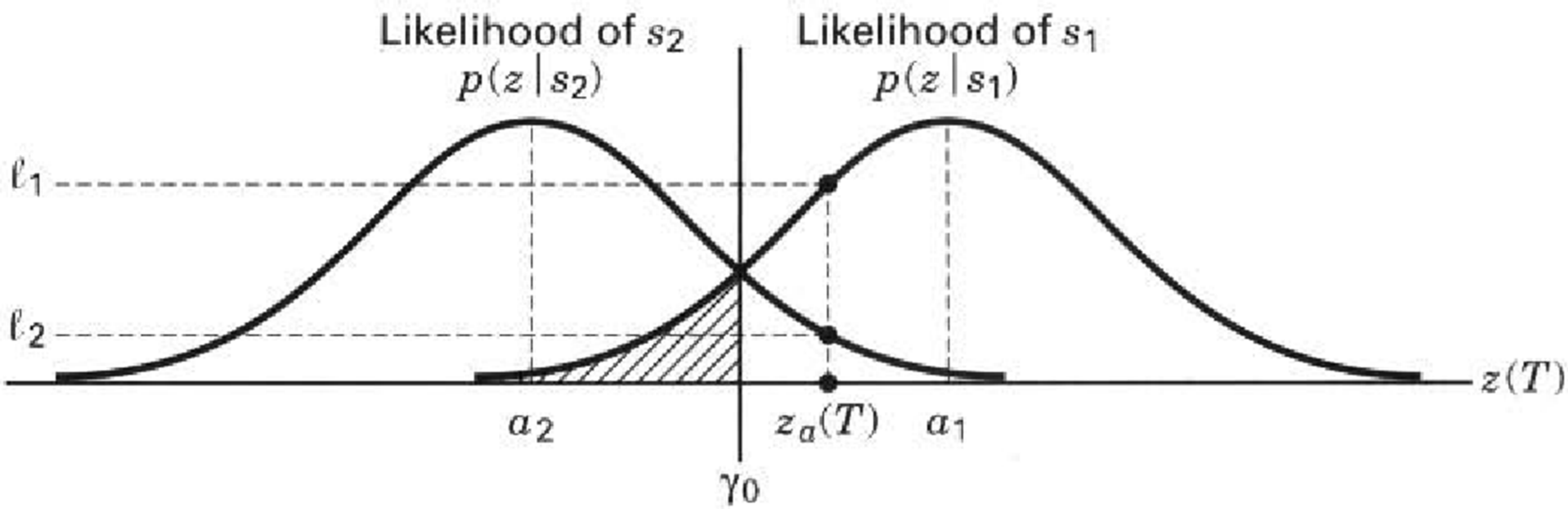
Detection of Binary Signals in Gaussian Noise

Maximum Likelihood Receiver Structure

✓ **If $P(s_1) = P(s_2)$ and if the likelihoods $p(z|s_i)$ $(i = 1, 2)$, are symmetrical then:**

$$z(T) \underset{H_2}{\overset{H_1}{\gtrless}} \frac{a_1 + a_2}{2} = \gamma_0$$

γ_0: *optimum threshold* for minimizing the probability of making an incorrect decision for this case. (This is also called Minimum Error Criterion).



Detection of Binary Signals in Gaussian Noise

The Matched Filter

A matched filter is a linear filter designed to provide the maximum signal-to-noise power ratio at its output for a given transmitted symbol waveform.

Let a known signal $s(t)$ plus AWGN $n(t)$ is the input to an LTI receiving filter followed by the sampler, shown earlier. The ratio of the instantaneous signal power to average noise power at time $t = T$, out of the sampler in step 1:

$$\left(\frac{S}{N}\right)_T = \frac{a_i^2}{\sigma_0^2}$$

a_i: signal component; σ_0^2: output noise variance.

***Objective*: Find the filter transfer function $H_0(f)$ that maximizes $(S/N)_T$.**

Detection of Binary Signals in Gaussian Noise

The Matched Filter

The signal $a_i(t)$ at the filter output:

$$a_i(t) = \int_{-\infty}^{\infty} H(f)S(f)e^{j2\pi ft}\,df$$

S(*f*): Fourier transform of the input signal. *H*(*f*): Filter transfer function.

The output noise power:

$$\sigma_0^2 = \frac{N_0}{2}\int_{-\infty}^{\infty} |H(f)|^2\,df$$

($N_0/2$): Two sided PSD of the input noise.

$$\left(\frac{S}{N}\right)_T = \frac{\left|\int_{-\infty}^{\infty} H(f)S(f)e^{j2\pi fT}\,df\right|^2}{N_0/2\int_{-\infty}^{\infty} |H(f)|^2\,df}$$

Detection of Binary Signals in Gaussian Noise

The Matched Filter

To find that $H(f) = H_0(f)$ that maximizes $(S/N)_T$, Schwarz's inequality can be used:

$$\left|\int_{-\infty}^{\infty} f_1(x) f_2(x)\, dx\right|^2 \leq \int_{-\infty}^{\infty} |f_1(x)|^2\, dx \int_{-\infty}^{\infty} |f_2(x)|^2\, dx$$

The equality holds if:

$$f_1(x) = k f_2^*(x)$$

k: an arbitrary constant;
*: complex conjugate operator.

$$\left|\int_{-\infty}^{\infty} H(f) S(f) e^{j2\pi fT}\, df\right|^2 \leq \int_{-\infty}^{\infty} |H(f)|^2\, df \int_{-\infty}^{\infty} |S(f)|^2\, df$$

Detection of Binary Signals in Gaussian Noise

The Matched Filter

$$\left|\int_{-\infty}^{\infty} H(f)S(f)e^{j2\pi fT}\,df\right|^2 \le \int_{-\infty}^{\infty} |H(f)|^2\,df \int_{-\infty}^{\infty} |S(f)|^2\,df$$

$$\left(\frac{S}{N}\right)_T \le \frac{2}{N_0}\int_{-\infty}^{\infty} |S(f)|^2\,df$$

$$\max\left(\frac{S}{N}\right)_T = \frac{2E}{N_0} \quad \ldots(1)$$

The energy E of the input signal $s(t)$:

$$E = \int_{-\infty}^{\infty} |S(f)|^2\,df$$

***Conclusion*: The maximum output $(S/N)_T$ depends on the input signal energy and the power spectral density of the noise not on the particular shape of the waveform used.**

Detection of Binary Signals in Gaussian Noise

The Matched Filter

The equality in eqn. (1) holds only if the optimum filter transfer function $H_0(f)$ is employed:

$$H(f) = H_0(f) = kS^*(f)e^{-j2\pi fT}$$

$$\Downarrow$$

$$h(t) = \mathcal{F}^{-1}\{kS^*(f)e^{-j2\pi fT}\}$$

Since $s(t)$ is a real-valued signal:

$$h(t) = \begin{cases} ks(T-t) & 0 \le t \le T \\ 0 & \text{elsewhere} \end{cases}$$

Detection of Binary Signals in Gaussian Noise

The Correlation Realization of the Matched Filter

The output $z(t)$ of a causal filter can be described in the time domain as the convolution of a received input waveform $r(t)$ with the impulse response of the filter.

$$z(t) = r(t) * h(t) = \int_0^t r(\tau)h(t-\tau)\,d\tau$$

Substituting $h(t)$ of the matched filter into $h(t - \tau)$ and arbitrarily setting $k = 1$:

$$\begin{aligned} z(t) &= \int_0^t r(\tau)s[T-(t-\tau)]\,d\tau \\ &= \int_0^t r(\tau)s(T-t+\tau)\,d\tau \end{aligned}$$

Detection of Binary Signals in Gaussian Noise

The Correlation Realization of the Matched Filter

When $t = T$, →

$$z(T) = \int_{0}^{T} r(\tau)s(\tau)\, d\tau$$

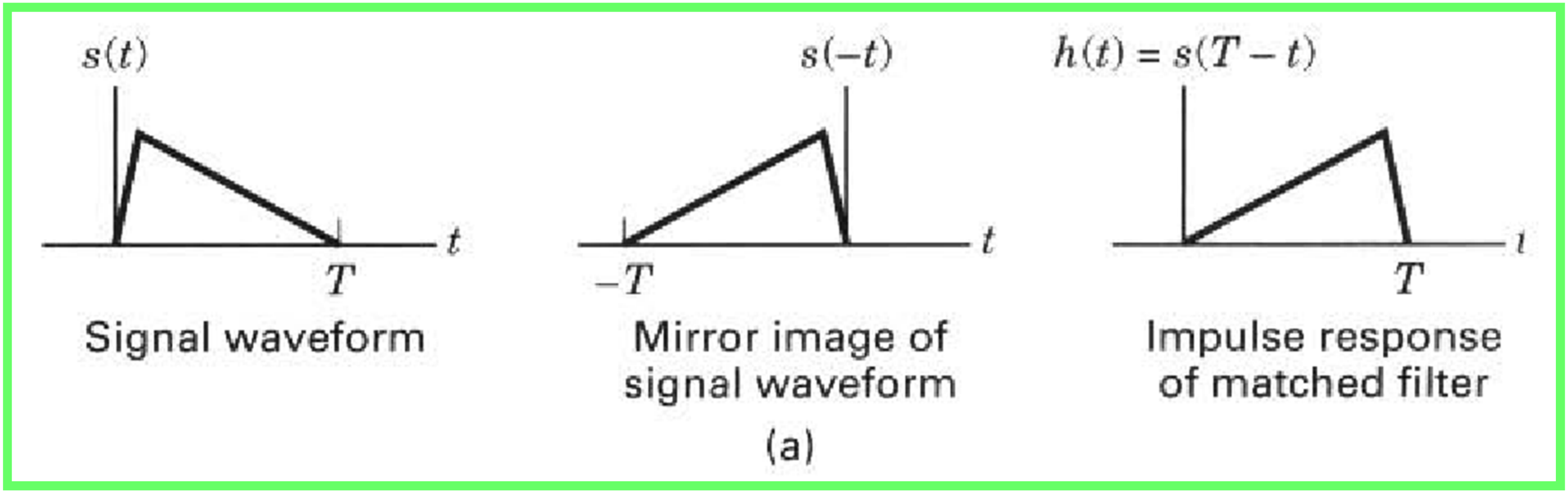
The product integration of the received signal $r(t)$ with a replica of the transmitted waveform $s(t)$ over one symbol interval is known as the correlation of $r(t)$ with $s(t)$.

Let us consider that a received signal $r(t)$ is correlated with each prototype signal $s_i(t)$ ($i = 1, \ldots, M$) using a bank of M correlators. The signal $s_i(t)$ whose product integration or correlation with $r(t)$ yields the maximum output $z_i(T)$ is the signal that matches $r(t)$ better than all the other $s_j(t)$, $j \neq i$.

The term matched filter is often used synonymously with correlator.

Detection of Binary Signals in Gaussian Noise

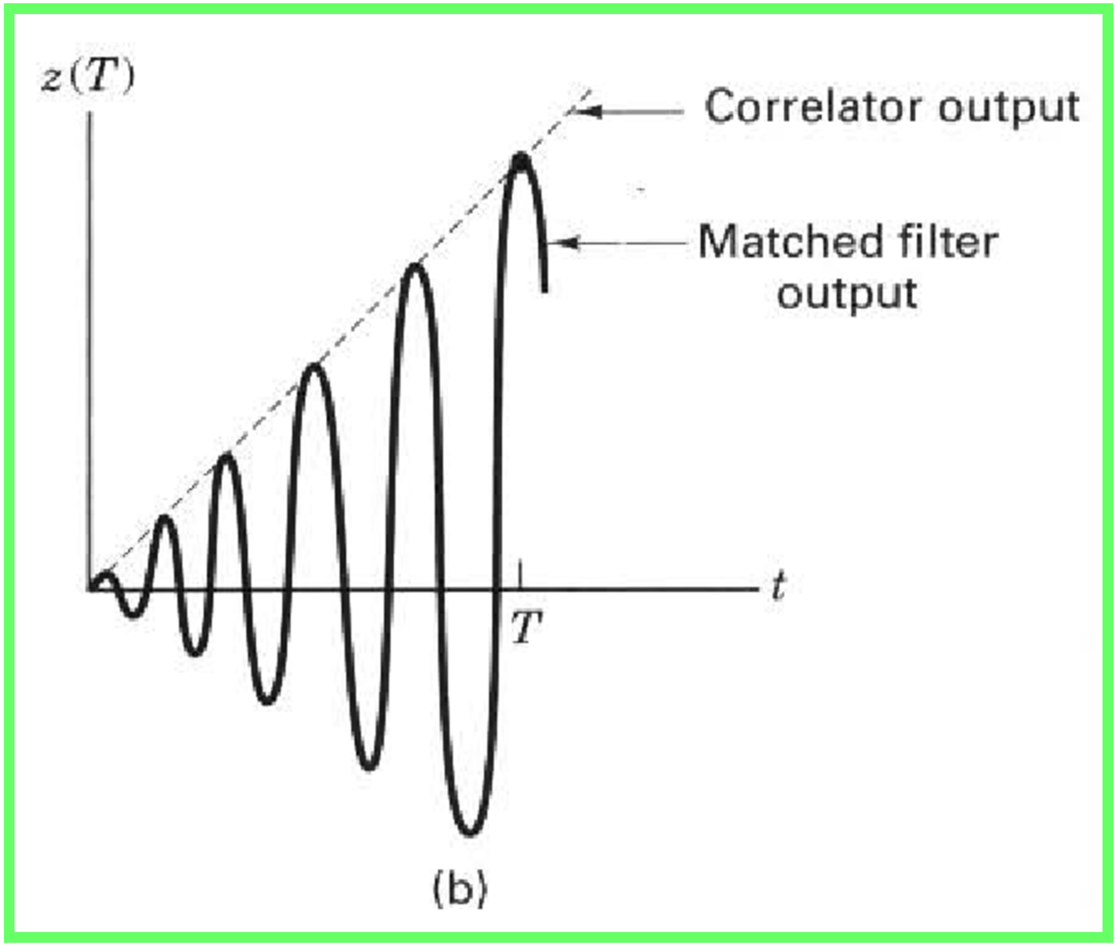
The Correlation Realization of the Matched Filter



Correlator and matched filter. (a) Matched filter characteristic.

Detection of Binary Signals in Gaussian Noise

The Correlation Realization of the Matched Filter

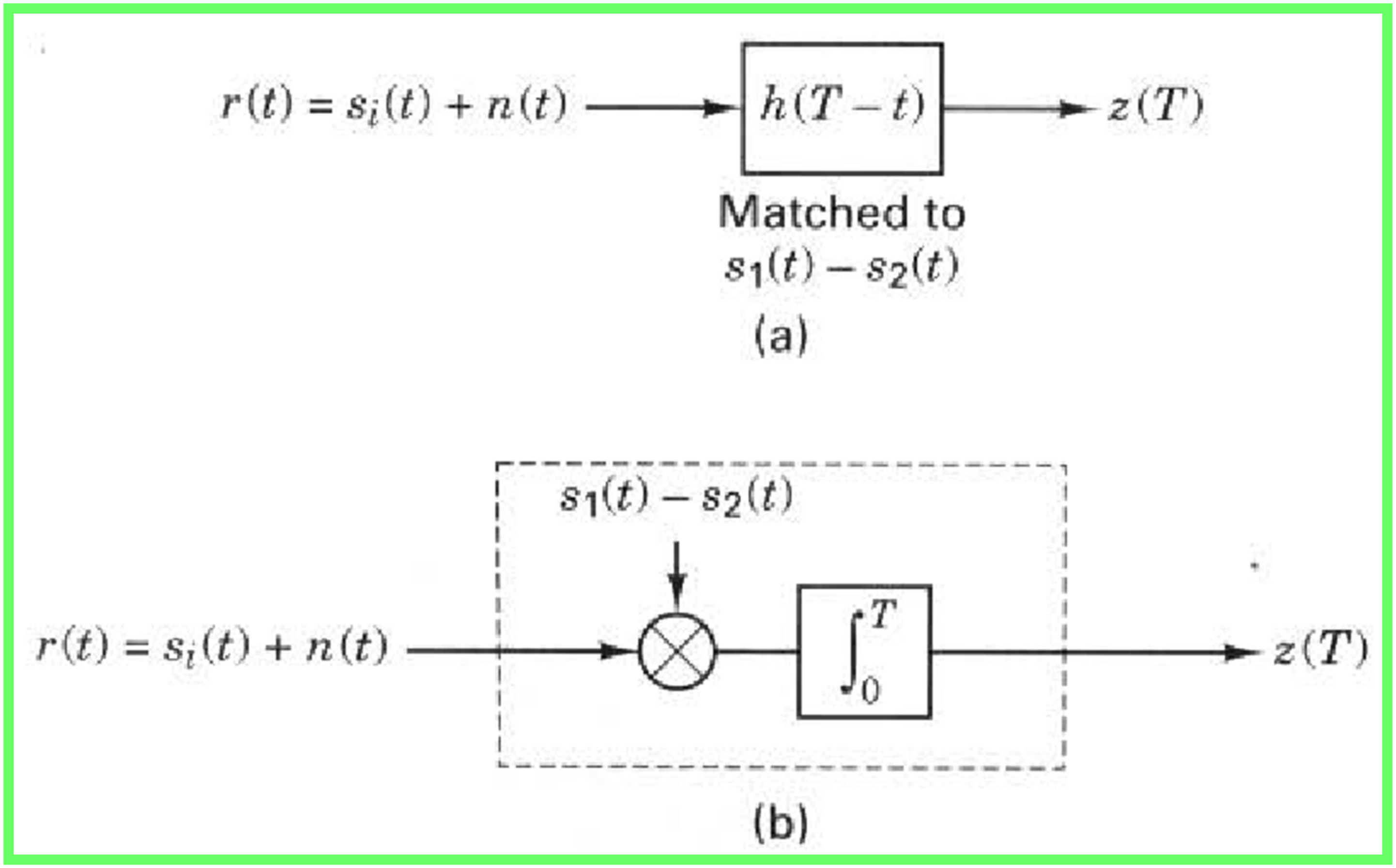


Correlator and matched filter. (b) Comparison of correlator and matched filter outputs, for a sine-wave input.

Detection of Binary Signals in Gaussian Noise

The Correlation Realization of the Matched Filter

Conclusion: The matched filter output and the correlator output are identical at the sampling instant $t = T$.



Equivalence of matched filter and correlator. (a) Matched filter. (b) Correlator.

DIGITAL BANDPASS MODULATION AND DEMODULATION

Bandpass Modulation

Why Modulate??

Digital Modulation is the process by which digital symbols are transformed into waveforms that are compatible with the characteristics of the channel.

In the case of Baseband Modulation, these waveforms usually take the form of shaped pulses. In Bandpass Modulation, the shaped pulses modulate a sinusoid called a carrier wave, or simply a carrier.

Bandpass Modulation

Advantages…

Bandpass Modulation can provide other important benefits in signal transmission. If more than one signal utilizes a single channel, modulation may be used to separate the different channels. This technique is called Frequency-division Multiplexing.

Modulation can also be used to minimize the effects of interference. A class of such modulation schemes, known as spread-spectrum modulation, requires a system bandwidth much larger than the minimum bandwidth that would be required by the message.

Modulation can also be used to place a signal in a frequency band where design requirements, such as filtering and amplification, can be easily met.

FREQUENCY SHIFT KEYING

(FSK)

Binary FSK Signal and Modulator

- ✓ **In its most general form, the binary FSK scheme uses two signals with different frequencies to represent binary 1 and 0.**

FSK Techniques

Noncoherent FSK | **Coherent FSK**

Binary FSK Signal and Modulator

Noncoherent FSK

$$s_1(t) = A\cos(2\pi f_1 t + \Phi_1), \quad kT \le t \le (k+1)T, \text{ for } 1$$
$$s_2(t) = A\cos(2\pi f_2 t + \Phi_2), \quad kT \le t \le (k+1)T, \text{ for } 0$$

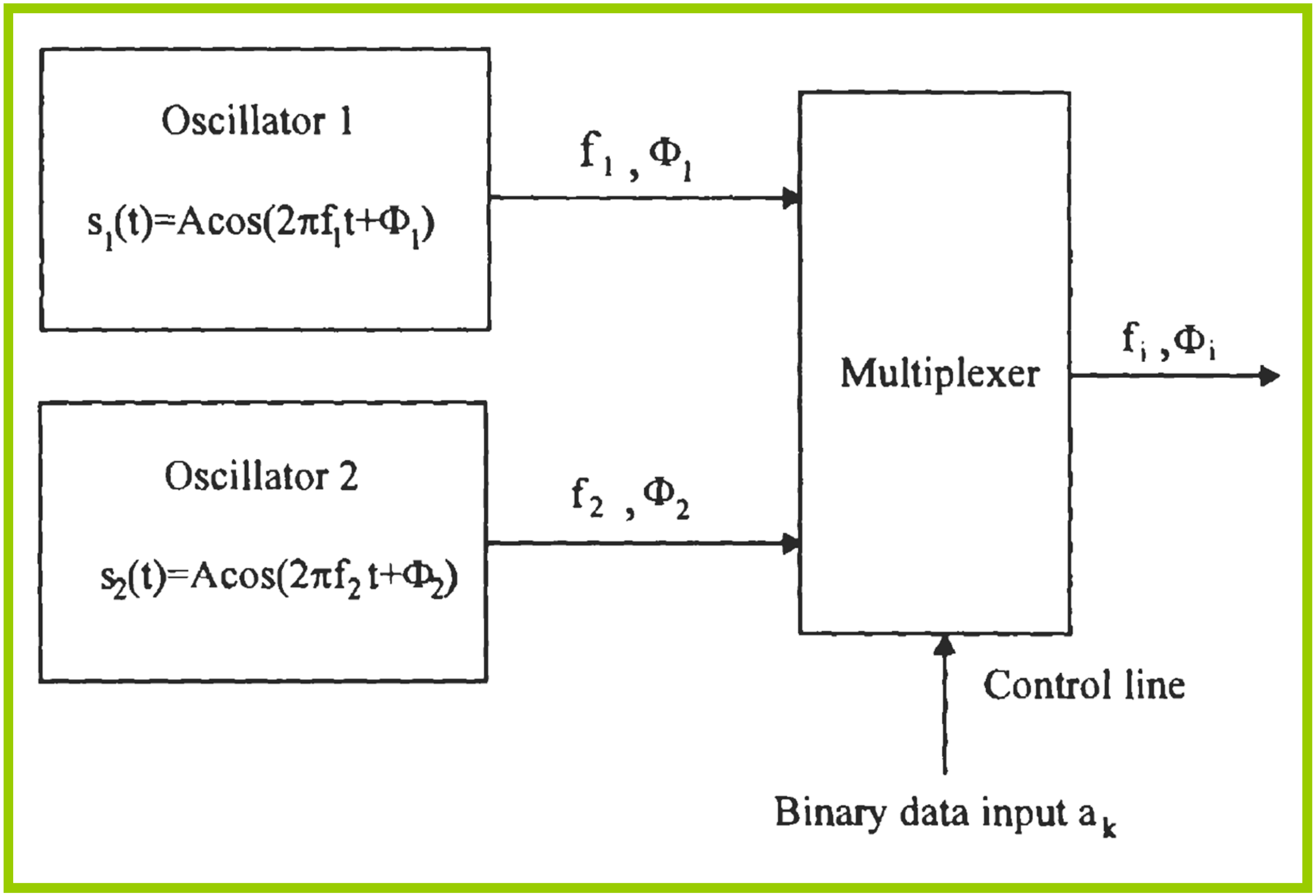
Φ_1 and Φ_2 : Initial phases at $t = 0$

T: bit period of the binary data

- ✓ ***Note*: These two signals are not coherent since Φ_1 *and* Φ_2 are not the same in general. Hence, the waveform is not continuous at bit transitions.**

Binary FSK Signal and Modulator

Noncoherent FSK Modulator



Binary FSK Signal and Modulator

Coherent FSK

$$s_1(t) = A\cos(2\pi f_1 t + \Phi), \quad kT \le t \le (k+1)T, \text{ for } 1$$
$$s_2(t) = A\cos(2\pi f_2 t + \Phi), \quad kT \le t \le (k+1)T, \text{ for } 0$$

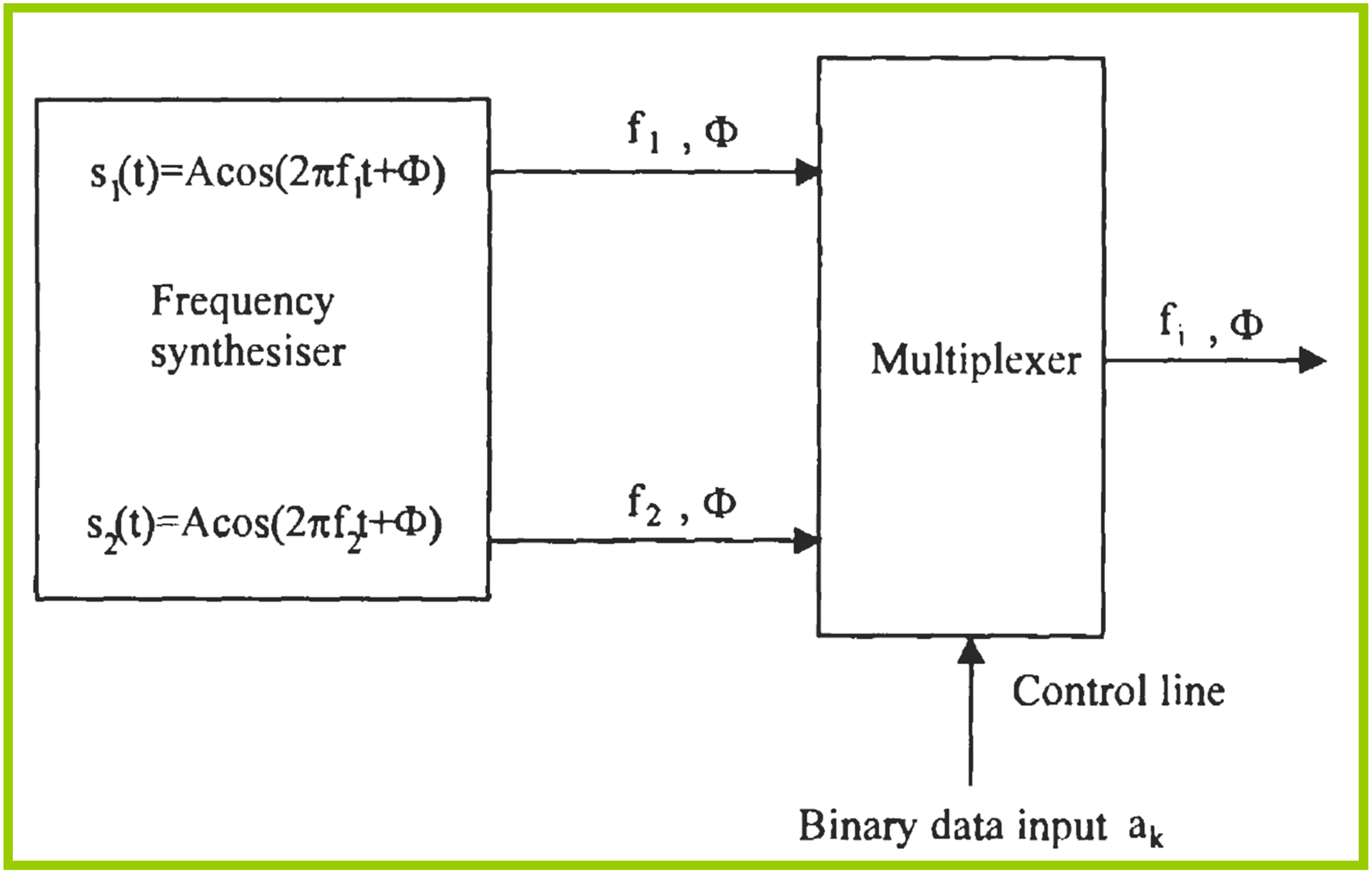
Φ : **Initial phase at $t = 0$**

T: **bit period of the binary data**

✓ ***Note*: These two signals have the same initial phase Φ at $t = 0$.**

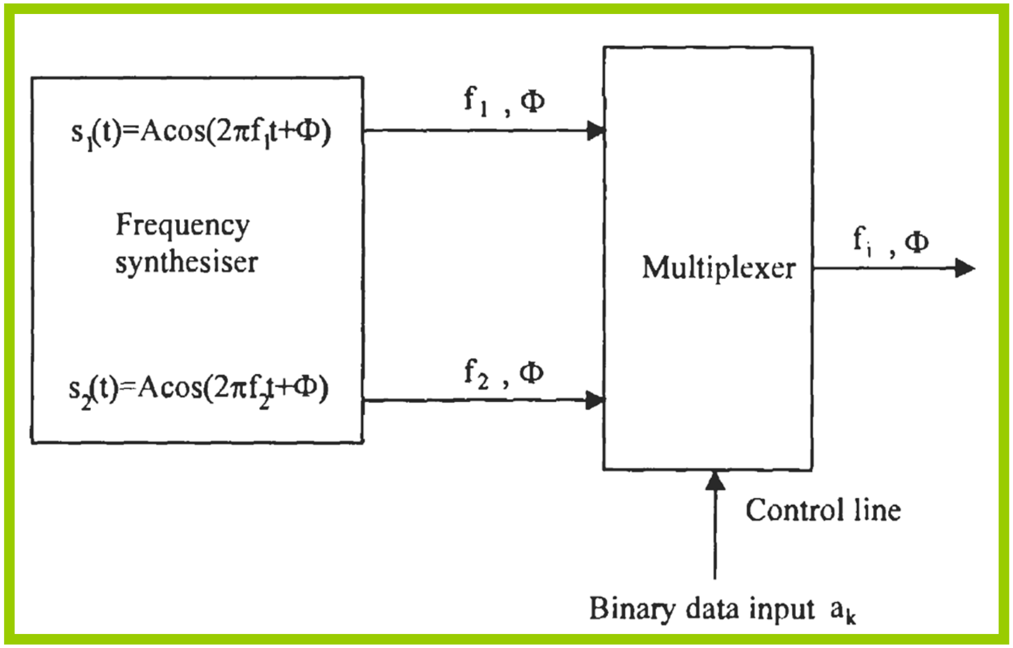
Binary FSK Signal and Modulator

Coherent FSK Modulator



Binary FSK Signal and Modulator

Coherent FSK Modulator



- ✓ **The frequency synthesizer generates two frequencies, f_1 and f_2, which are synchronized.**
- ✓ **The binary input data controls the multiplexer. The bit timing must be synchronized with the carrier frequencies.**
- ✓ **If $a_k = 1$, $s_1(t)$ will pass. If $a_k = 0$, $s_2(t)$ will pass.**

Binary FSK Signal and Modulator

Coherent FSK Modulator

- ✓ **For coherent demodulation of the coherent FSK signal, the two frequencies are so chosen that the two signals are orthogonal.**

$$\int_{kT}^{(k+1)T} s_1(t)s_2(t)dt = 0$$

$$\begin{aligned}
&\int_{kT}^{(k+1)T} \cos(2\pi f_1 t + \Phi)\cos(2\pi f_2 t + \Phi)dt \\
= \quad & \frac{1}{2}\int_{kT}^{(k+1)T} [\cos[2\pi(f_1 + f_2)t + 2\Phi] + \cos 2\pi(f_1 - f_2)t]dt \\
= \quad & \frac{1}{4\pi(f_1 + f_2)}[\cos 2\Phi \sin 2\pi(f_1 + f_2)t + \sin 2\Phi \cos 2\pi(f_1 + f_2)t]\Big|_{kT}^{(k+1)T} \\
& + \frac{1}{4\pi(f_1 - f_2)} \sin 2\pi(f_1 - f_2)t]\Big|_{kT}^{(k+1)T} \\
= \quad & 0
\end{aligned}$$

Binary FSK Signal and Modulator

Coherent FSK Modulator

- ✓ **This requires that $2\pi(f_1 + f_2)T = 2n\pi$ and $2\pi(f_1 - f_2)T = m\pi$, where n and m are integers.**

$$f_1 = \frac{2n + m}{4T}$$

$$f_2 = \frac{2n - m}{4T}$$

Let the two frequencies be rewritten as:

$$f_1 = f_c + \Delta f$$

$$f_2 = f_c - \Delta f$$

$$f_c = \frac{f_1 + f_2}{2} = \frac{n}{2T}$$

$$2\Delta f = f_1 - f_2 = \frac{m}{2T}$$

f_c : Nominal carrier frequency, must be integer multiple of $1/2T$, for orthogonality

Conclusion: **For orthogonality, f_1 and f_2 must be integer multiple of $(1/4T)$ and their difference must be integer multiple of $(1/2T)$.**

Binary FSK Signal and Modulator

Coherent FSK Modulator

✓ When the separation is chosen as $(1/T)$, then the phase continuity will be maintained at bit transitions. This FSK is called Sunde's FSK.

Generalization: If the separation is (k/T) (k: an integer), the phase of the coherent FSK signal is always continuous.

Proof: at $t = nT$, the phase of $s_1(t)$ is

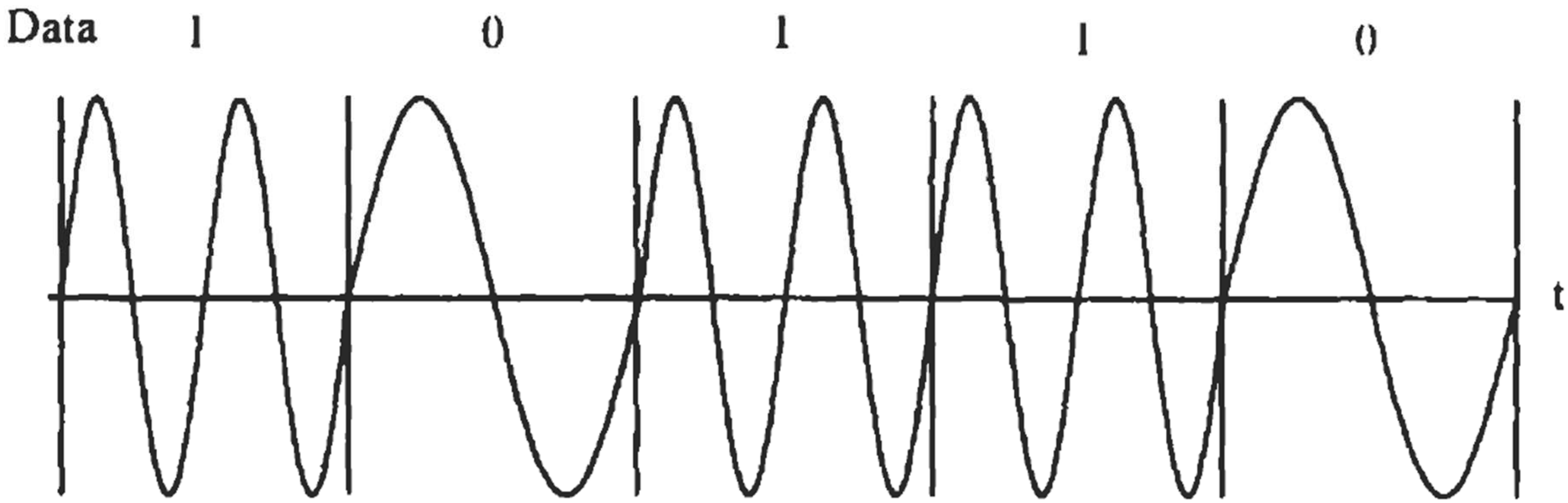
$$
\begin{aligned}
2\pi f_1 nT + \Phi &= 2\pi(f_2 + k/T)nT + \Phi \\
&= 2\pi f_2 nT + 2\pi kn + \Phi \\
&= 2\pi f_2 nT + \Phi \text{ (Modulo-}2\pi\text{)}
\end{aligned}
$$

Exactly same as the phase of $s_2(t)$.

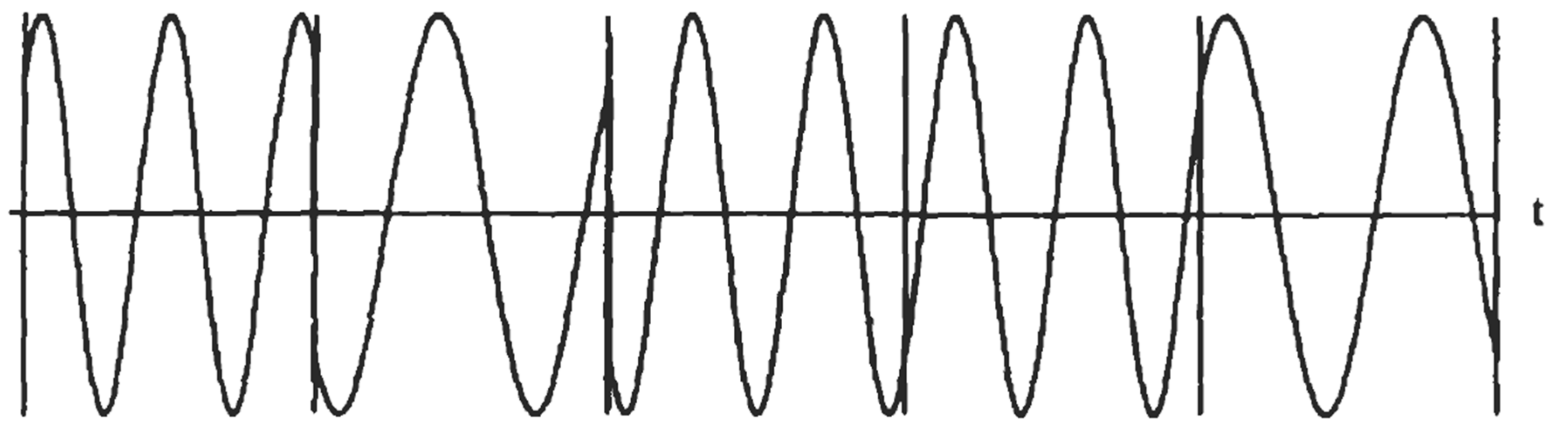
Conclusion: At $t = nT$, if the input bit switches from 1 to 0, the new signal $s_2(t)$ will start at exactly the same amplitude where $s_1(t)$ has ended.

Binary FSK Signal and Modulator

Coherent FSK Modulator



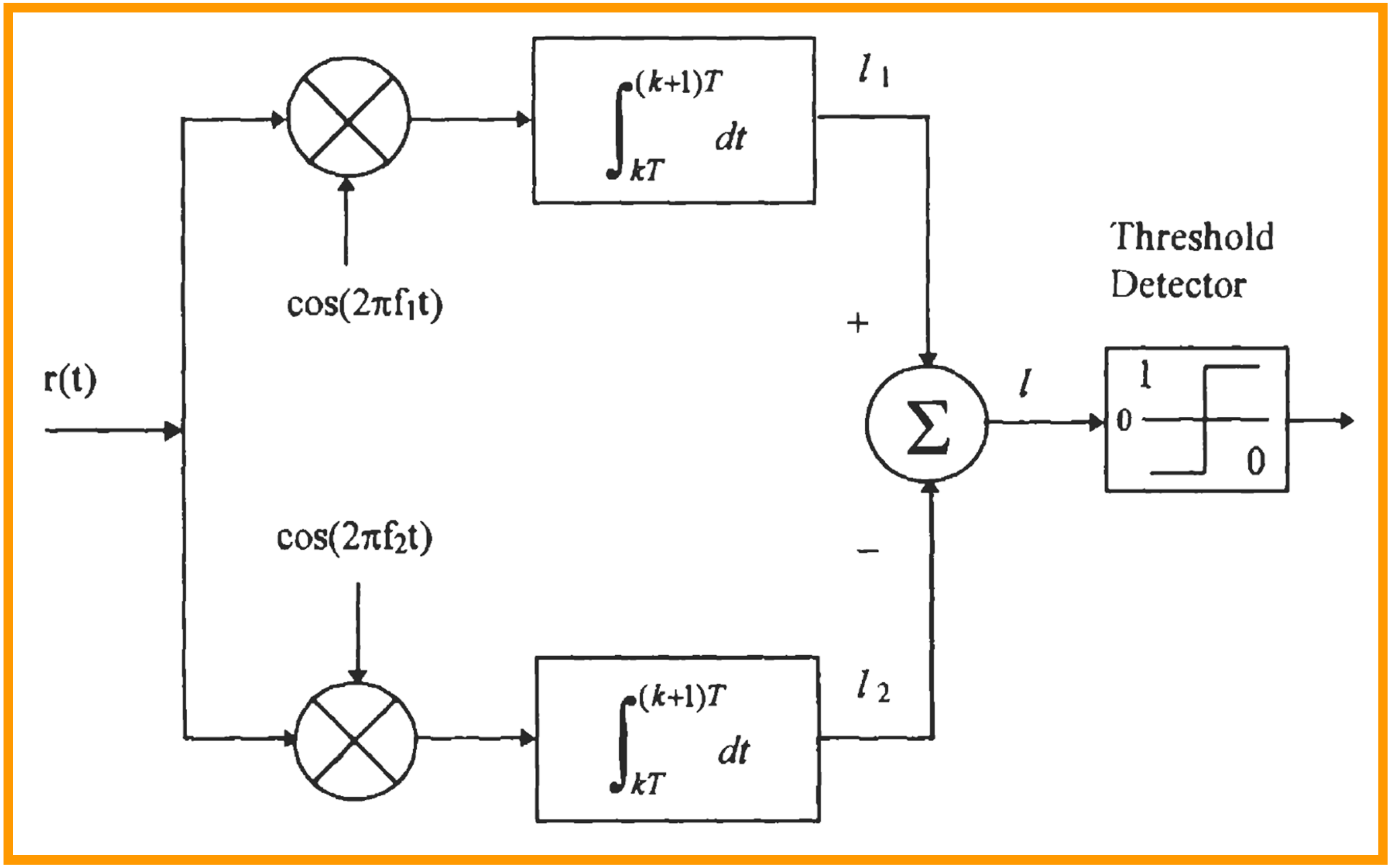
Sunde's FSK: $f_1 = (2/T)$, $f_2 = (1/T)$, $2\Delta f = (1/T)$.



FSK with discontinuous phase: $f_1 = (9/4T)$, $f_2 = (6/4T)$, $2\Delta f = (3/4T)$.

Coherent Demodulation and Error Performance

Coherent FSK Demodulators

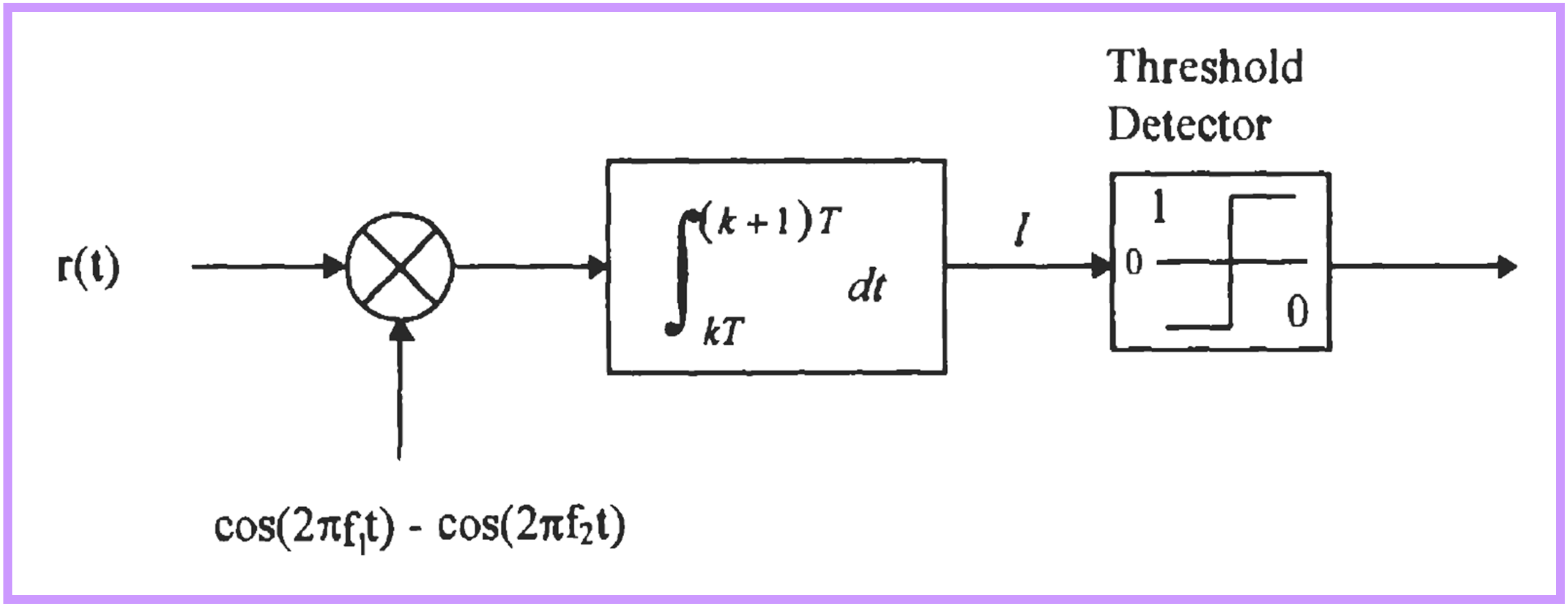


Coherent FSK demodulator: two correlator implementation.

Note: **Two reference signals used are $\cos(2\pi f_1 t)$ and $\cos(2\pi f_2 t)$. They must be synchronized with the received signal.**

Coherent Demodulation and Error Performance

Coherent FSK Demodulators

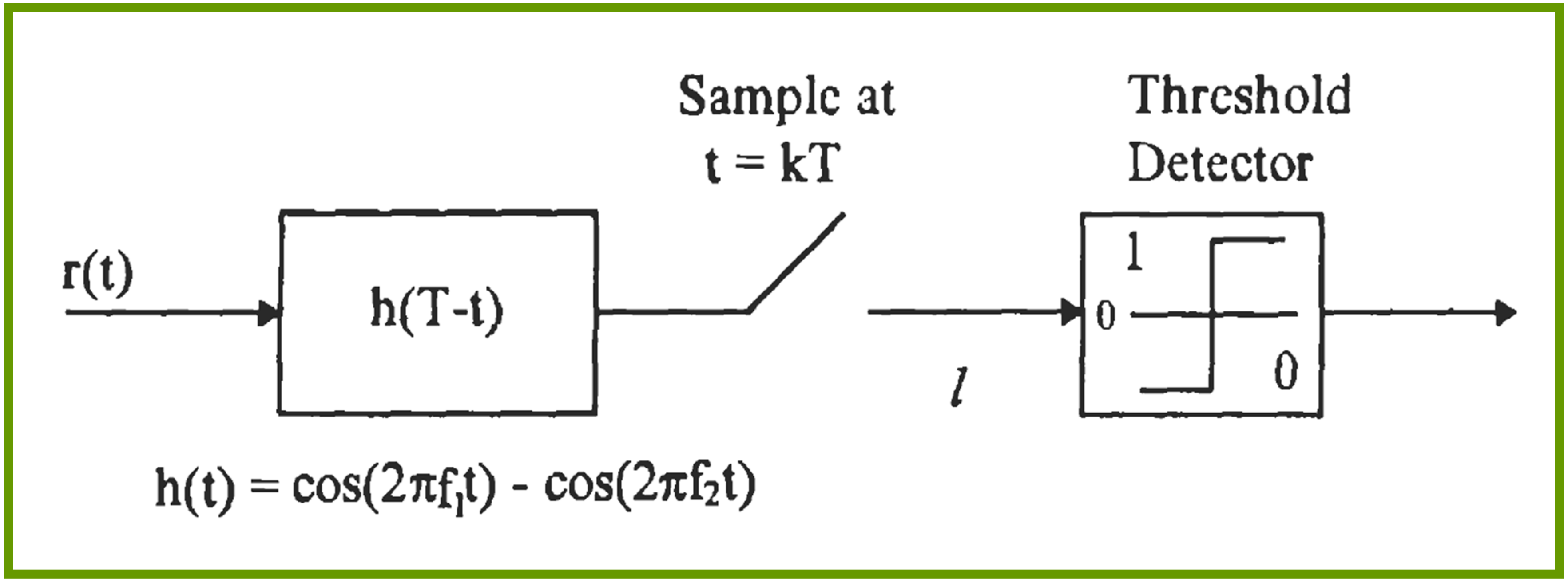


Coherent FSK demodulator: one correlator implementation.

Note: **A single reference signals is used as** $\cos(2\pi f_1 t) - \cos(2\pi f_2 t)$**.**

Coherent Demodulation and Error Performance

Coherent FSK Demodulators



Coherent FSK demodulator: matched filter implementation.

Note: **The correlator in the previous configuration is replaced by a matched filter that matches $\cos(2\pi f_1 t) - \cos(2\pi f_2 t)$.**

Coherent Demodulation and Error Performance

Coherent FSK Demodulators

✓ **All three implementations discussed are equivalent in terms of error performance.**

For an AWGN channel, the received signal is:

$$r(t) = s_i(t) + n(t), \qquad i = 1, 2$$

$n(t)$: additive white Gaussian noise with zero mean and a two-sided power spectral density $(N_o/2)$

The bit error probability, for any equally likely binary signals is:

$$P_b = Q\left(\sqrt{\frac{E_1 + E_2 - 2\rho_{12}\sqrt{E_1 E_2}}{2N_o}}\right)$$

$N_o/2$: two-sided PSD of the AWGN

Coherent Demodulation and Error Performance

Coherent FSK Demodulators

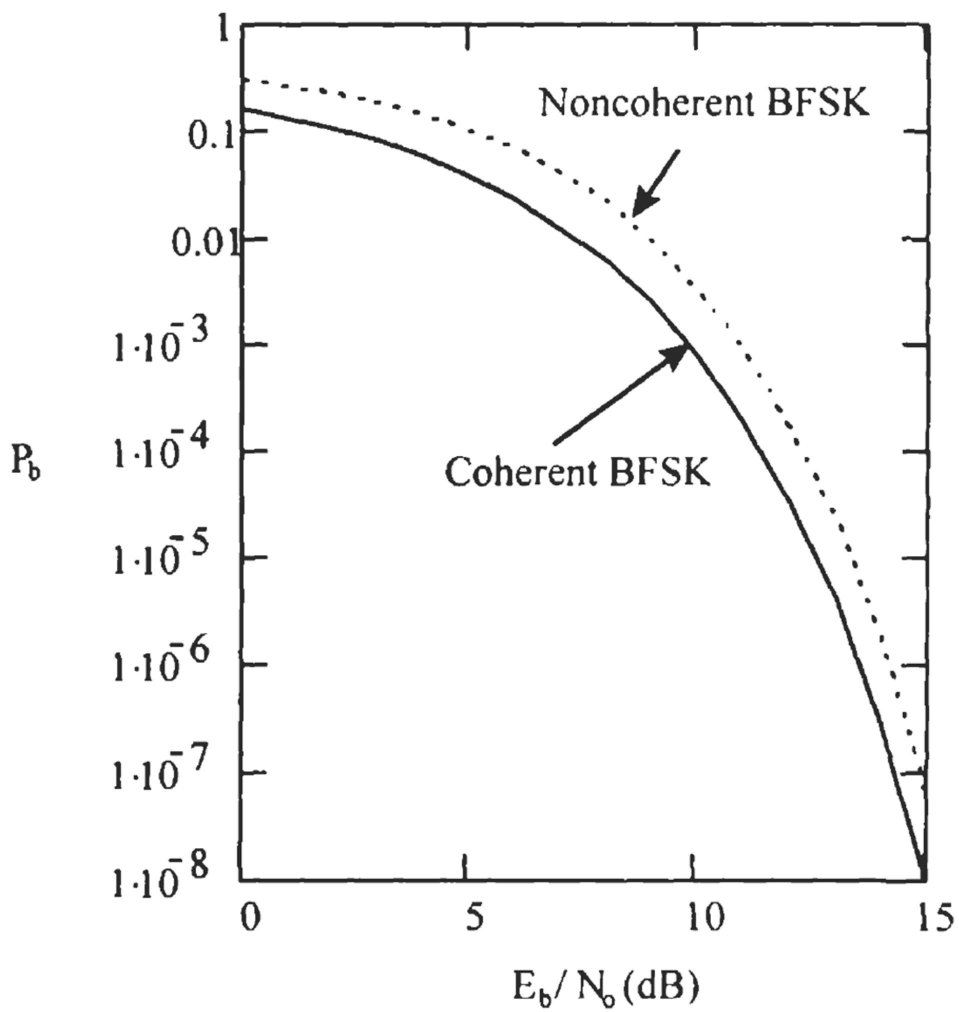
- ✓ For **Sunde's FSK signals**, $E_1 = E_2 = E_b$, $\rho_{12} = 0$ (orthogonal). Thus, **the error probability** is:

$$P_b = Q\left(\sqrt{\frac{E_b}{N_o}}\right)$$

$E_b = (A^2T/2)$: **average bit energy of the FSK signal.**

Coherent Demodulation and Error Performance

Coherent and Noncoherent FSK Demodulators



P_b of coherently and noncoherently demodulated FSK signal

AMPLITUDE SHIFT KEYING

(ASK)

Binary ASK Signal

✓ In its most general form, the **binary ASK** scheme uses **two signal amplitudes of A and 0** to represent **binary 1 and 0**, respectively.

$$s(t) = Am(t)\cos 2\pi f_c t, \qquad 0 \le t \le T$$

A: a constant, $m(t)$: 1 or 0

f_c: carrier frequency, T: bit duration

The signal has a power ($P = A^2/2$) where $A = \text{sqrt}(2P)$.

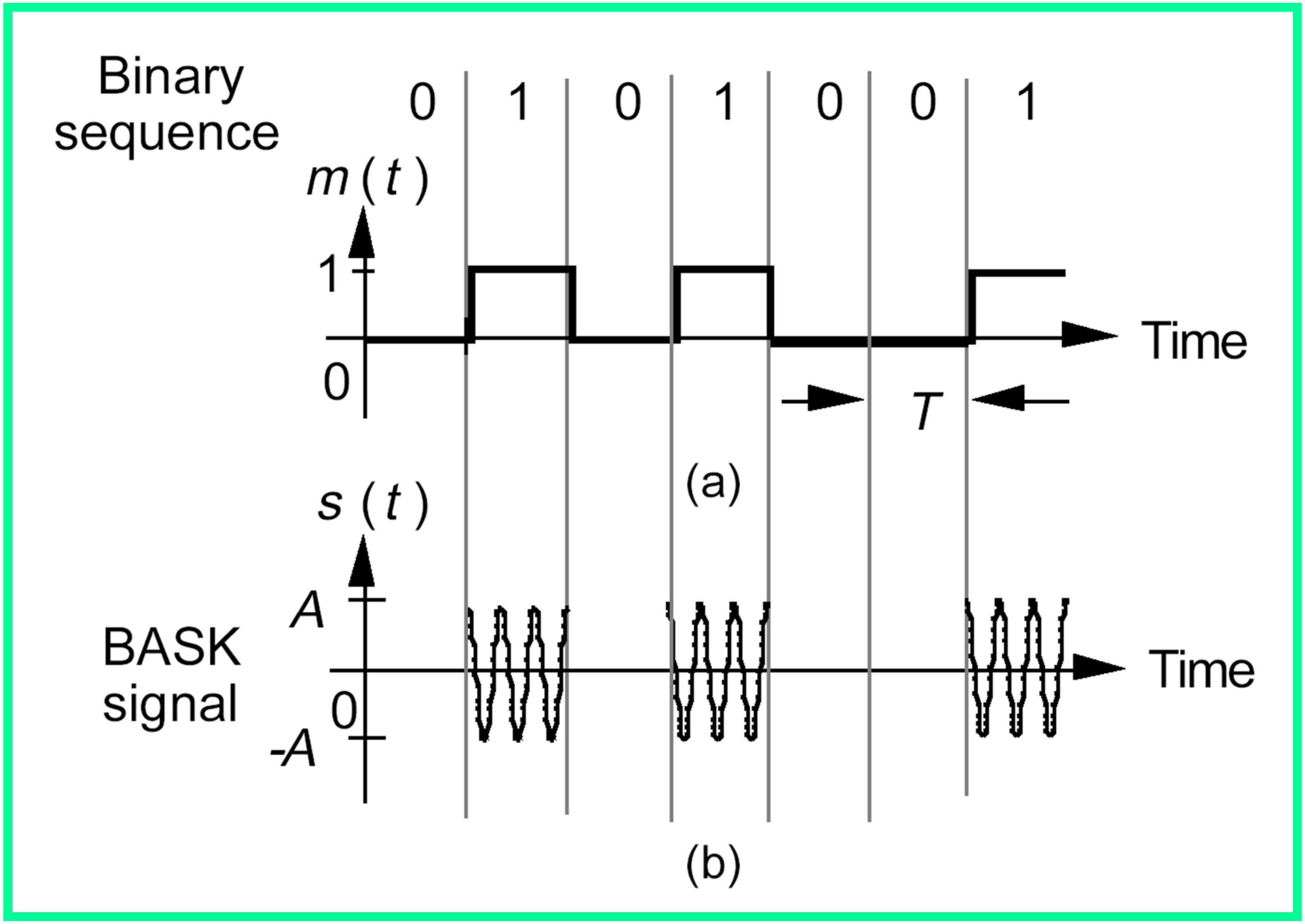
Binary ASK Signal

One can write:

$$\begin{aligned} s(t) &= \sqrt{2P}\ \cos 2\pi f_c t, & 0 \le t \le T \\ &= \sqrt{PT}\sqrt{\frac{2}{T}} \cos 2\pi f_c t, & 0 \le t \le T \\ &= \sqrt{E}\sqrt{\frac{2}{T}} \cos 2\pi f_c t, & 0 \le t \le T \end{aligned}$$

$E = PT$ **: the energy contained in a bit duration**

Binary ASK Signal



(a) Binary modulating signal and (b) BASK signal.

Binary ASK Signal

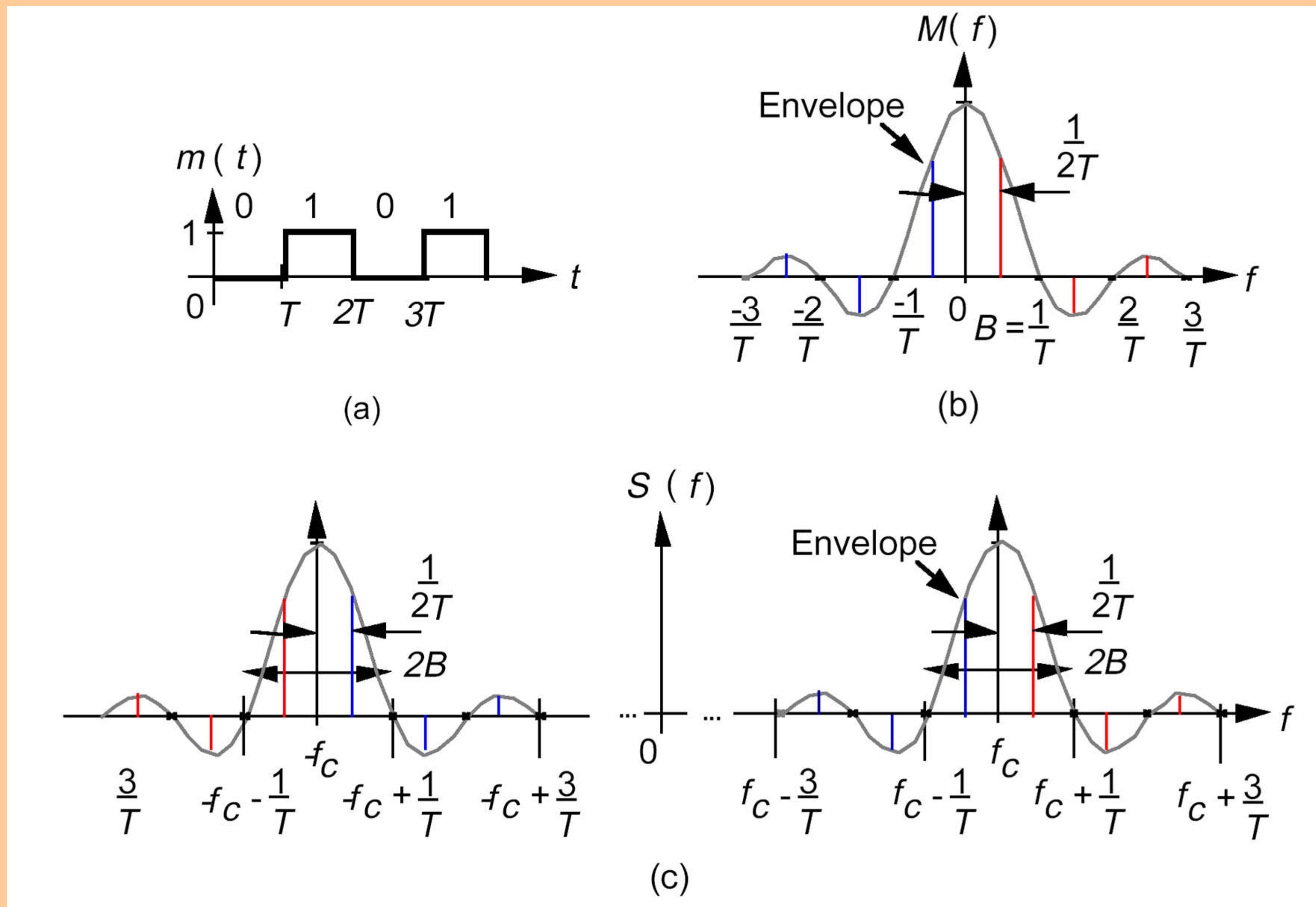
The Fourier transform of BASK signal $s(t)$:

$$S(f) = \frac{A}{2}\int_{-\infty}^{\infty}[m(t)\ e^{j2\pi f_c t}]\, e^{-j2\pi f t}\, dt + \frac{A}{2}\int_{-\infty}^{\infty}[m(t)\ e^{-j2\pi f_c t}]\, e^{-j2\pi f t}\, dt$$

$$S(f) = \frac{A}{2}M(f - f_c) + \frac{A}{2}M(f + f_c)$$

The effect of multiplication by the carrier signal $A\cos 2\pi f_c t$ is to shift the spectrum of the modulating signal $m(t)$ to f_c.

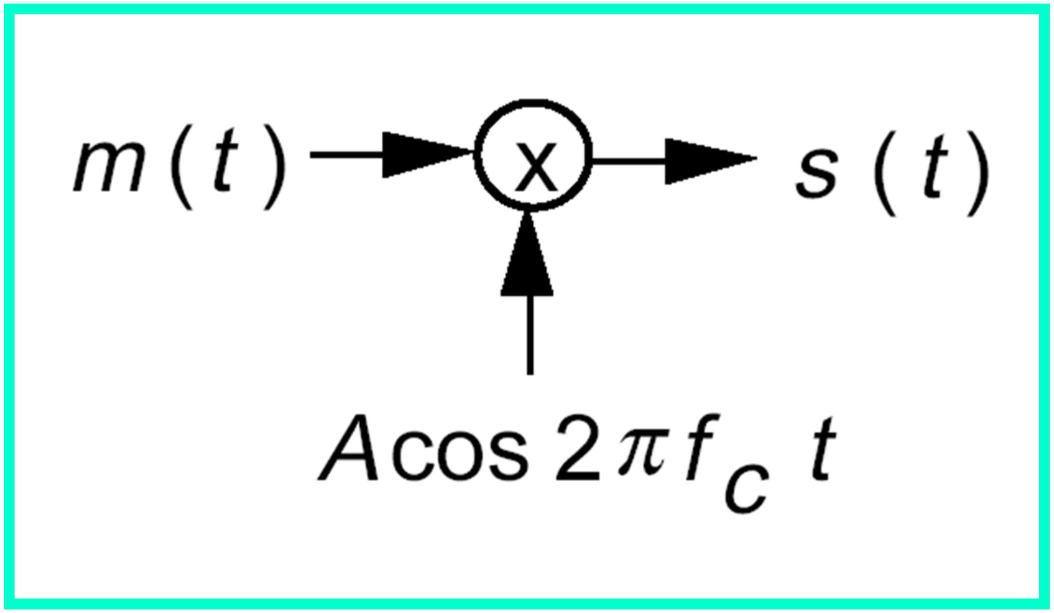
Binary ASK Signal



(a) Modulating signal. (b) Spectrum of (a). (c) Spectrum of **BASK** signals.

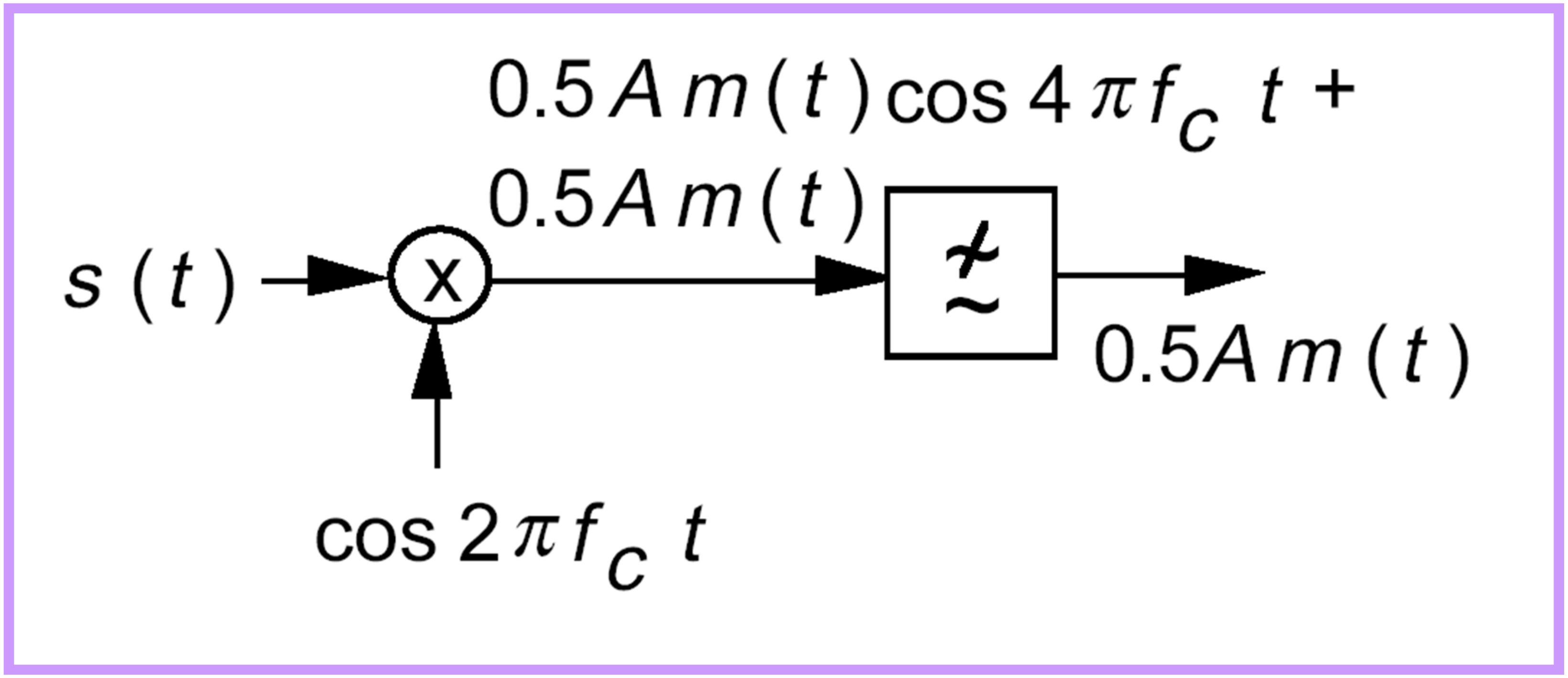
Binary ASK Signal

The bandwidth is defined as the range occupied by the baseband signal $m(t)$ from 0 Hz to the first zero-crossing point. Then the bandwidth for the baseband signal = B Hz and bandwidth for the BASK signal = $2B$ Hz.



BASK modulator.

Binary ASK Signal



A possible implementation of the coherent demodulator for the BASK signals.

PHASE SHIFT KEYING

(PSK)

Binary PSK Signal

✓ In its most general form, the binary PSK scheme uses two signals with different phases of 0 and π to represent binary 1 and 0.

$$
\begin{aligned}
s_1(t) &= A\cos 2\pi f_c t, \quad 0 \le t \le T, \quad \text{for } 1 \\
s_2(t) &= -A\cos 2\pi f_c t, \quad 0 \le t \le T, \quad \text{for } 0
\end{aligned}
$$

➢ **These two signals are called *antipodal*.**

Binary PSK Signal

$$
\begin{aligned}
s_1(t) &= A\cos 2\pi f_c t, \quad 0 \le t \le T, \quad \text{for } 1 \\
s_2(t) &= -A\cos 2\pi f_c t, \quad 0 \le t \le T, \quad \text{for } 0
\end{aligned}
$$

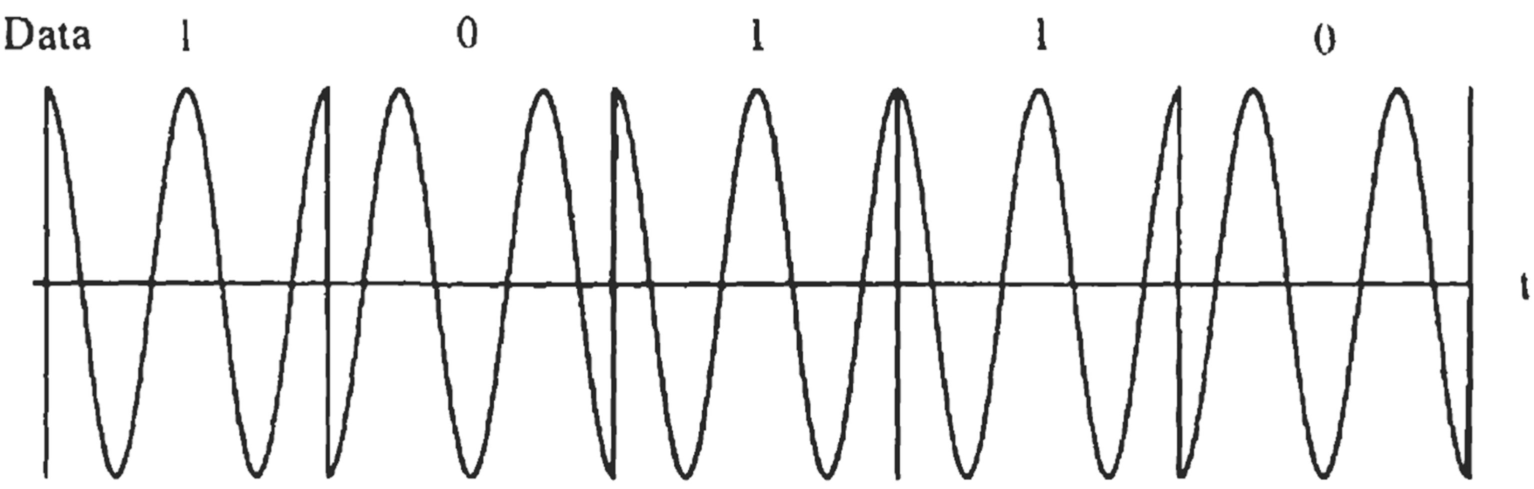
Why are these two signals chosen ?

✓ **Because they have a correlation coefficient of -1. This leads to minimum error probability for same (E_b/N_o). These two signals have the same frequency and energy.**

PSK Techniques

Noncoherent PSK | **Coherent PSK**

Binary PSK Signal

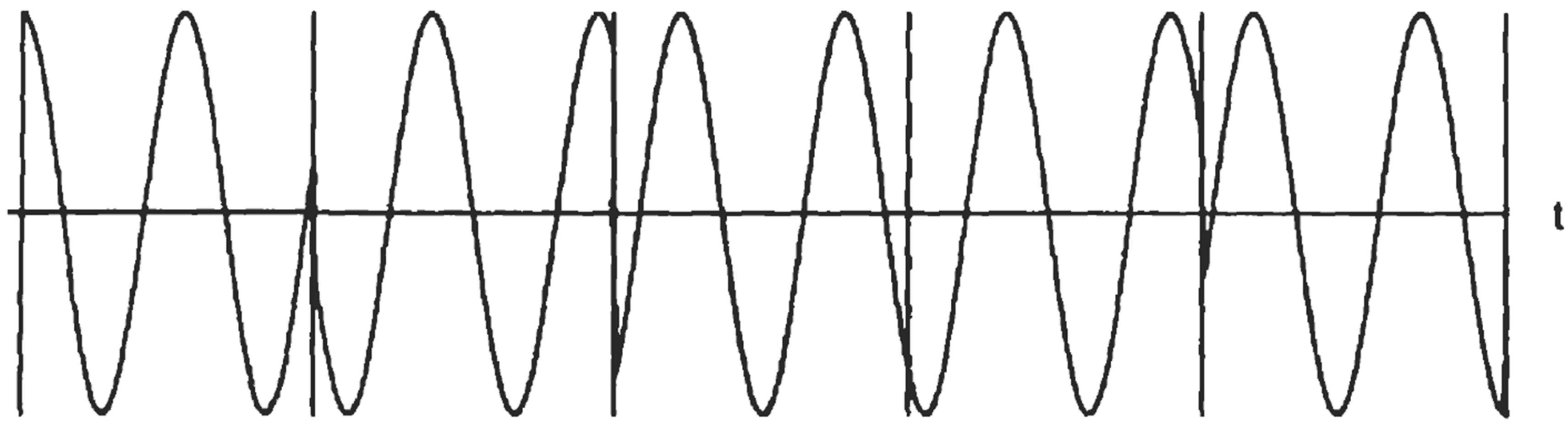


(a) $f_c = 2/T$

The waveform of a BPSK signal, generated by the modulator shown next, for a data stream {10110}. f_c is an integer multiple of R_b.

- ✓ ***Note*: The waveform has a constant envelope like FSK and also constant frequency. If $f_c = mR_b = (m/T)$ (m: an integer and R_b: data bit rate) and the bit timing is synchronous with the carrier, then the initial phase at a bit boundary is either 0 or π , corresponding to data bit 1 or 0.**

Binary PSK Signal



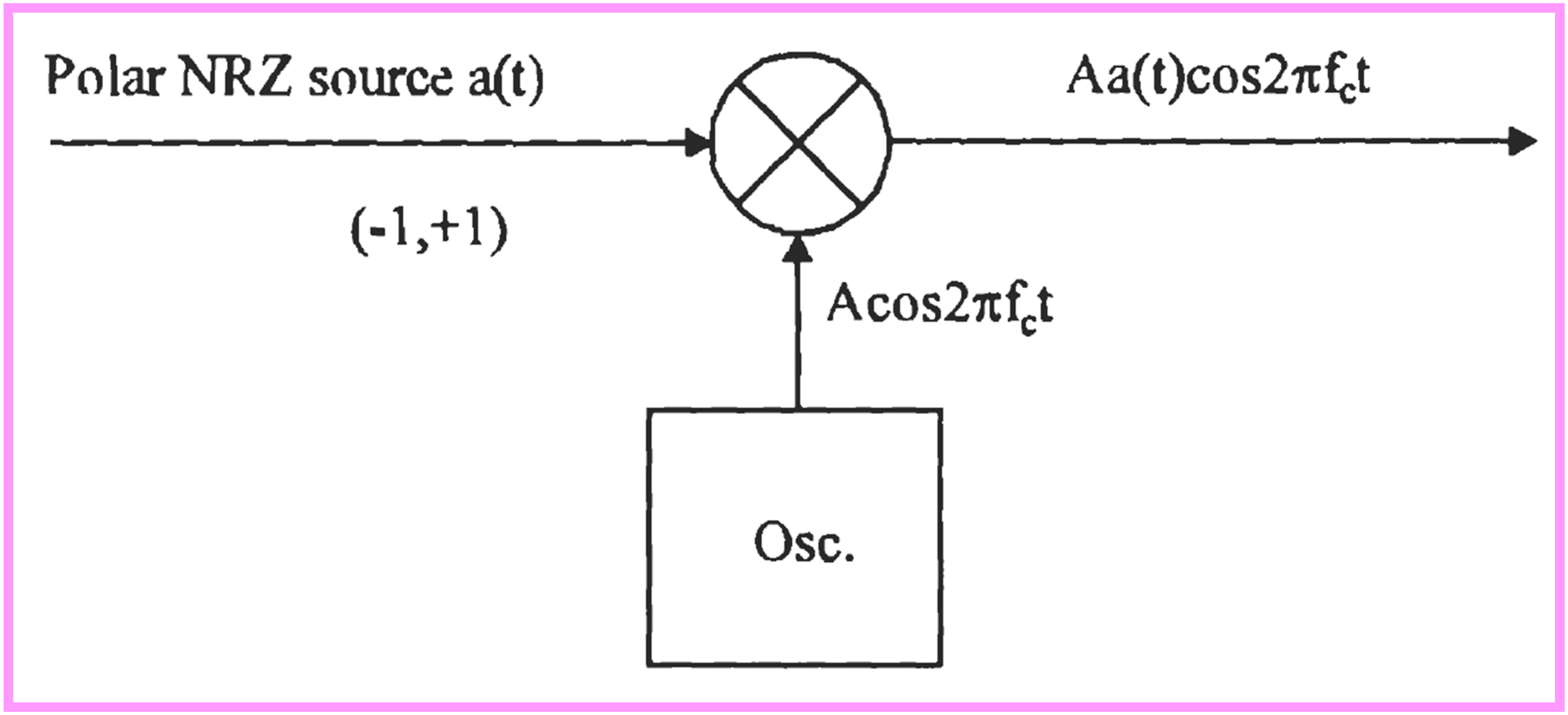
(b) $f_c = 1.8/T$

The waveform of a BPSK signal, generated by the modulator shown next, for a data stream {10110}. f_c is not an integer multiple of R_b.

- ✓ ***Note***: **If f_c is not an integer multiple of R_b, then the initial phase at a bit boundary is neither 0 nor π. The condition $f_c = mR_b$ is necessary to ensure minimum bit error probability.**

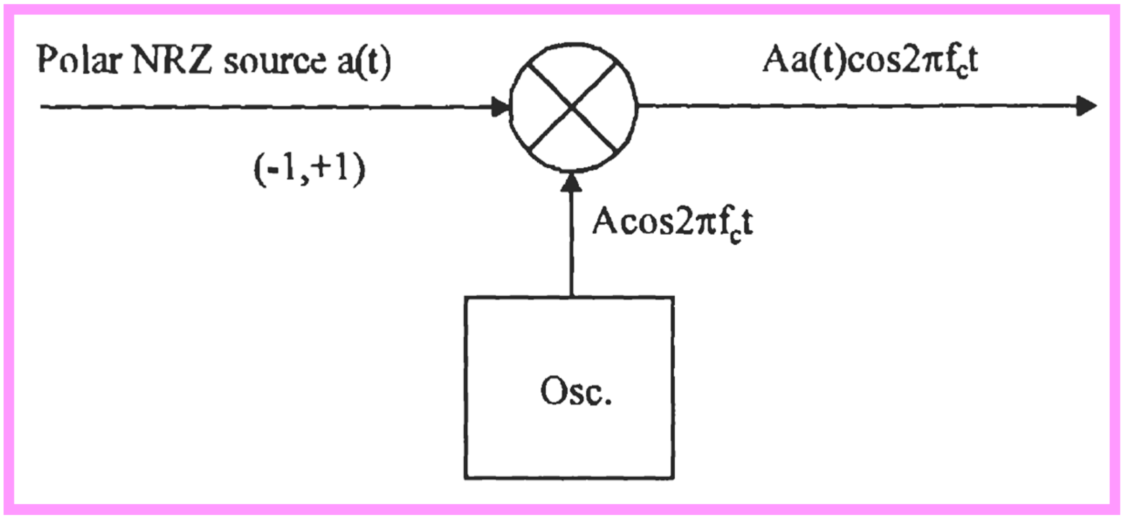
Binary PSK Signal

BPSK Modulator



Binary PSK Signal

BPSK Modulator



First a bipolar data stream *a*(*t*) is formed from the binary data stream:

$$a(t) = \sum_{k=-\infty}^{\infty} a_k p(t - kT)$$

$a_k \in \{+1, -1\}$

***p(t)*: a rectangular pulse with unit amplitude, defined on [0, *T*].**

The BPSK signal

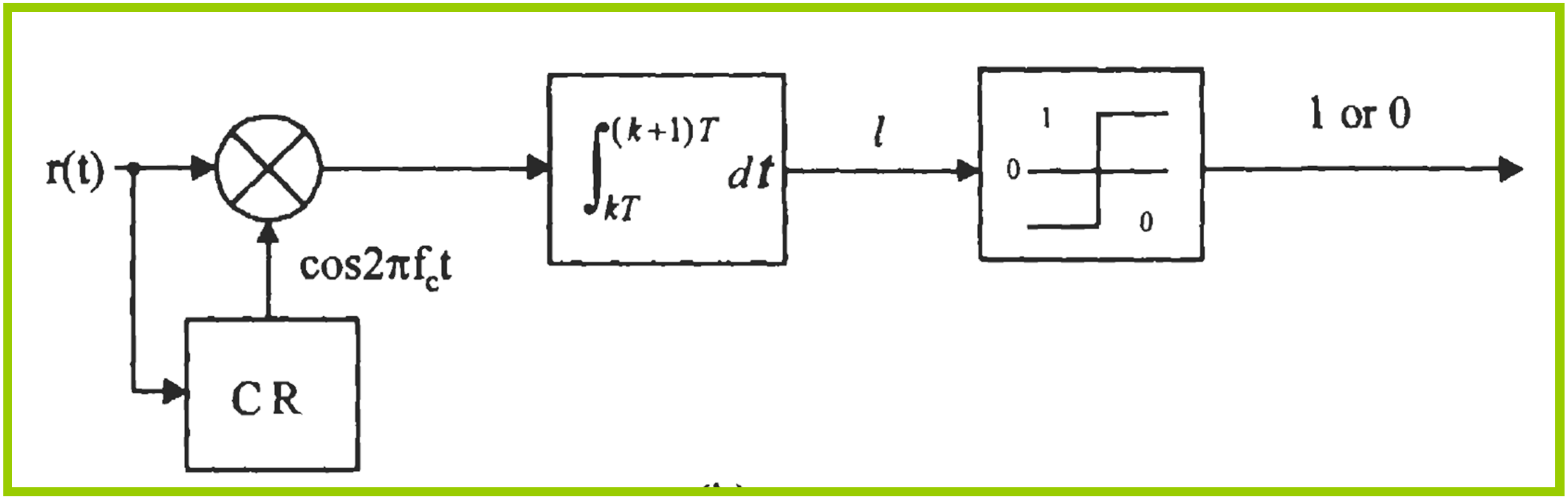
$$s(t) = Aa(t)\cos 2\pi f_c t,$$

$$-\infty < t < \infty$$

multiplication of a(t) with a sinusoidal carrier

Binary PSK Signal

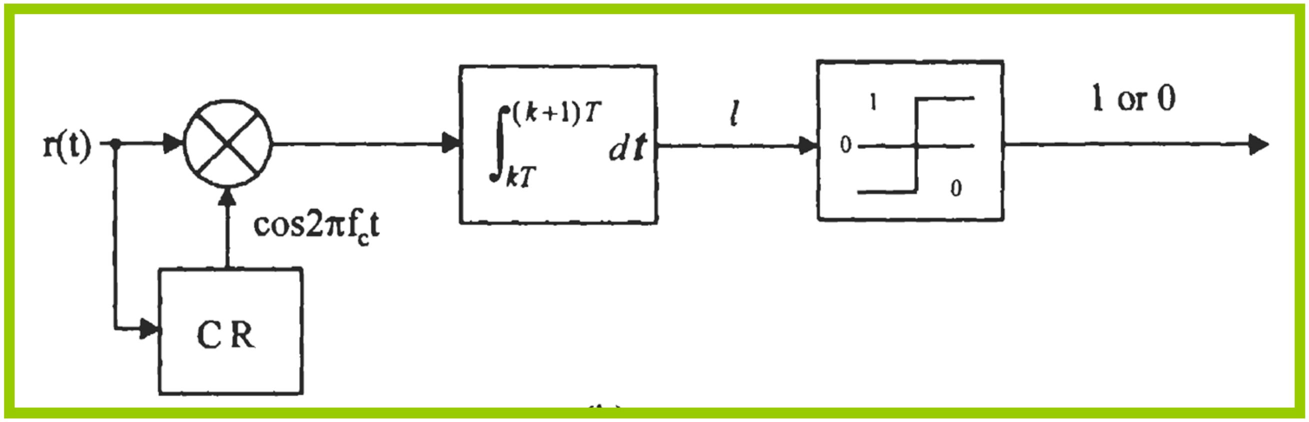
Coherent BPSK Demodulator



Coherent BPSK demodulator: correlator implementation.

Binary PSK Signal

Coherent BPSK Demodulator



Coherent BPSK demodulator: correlator implementation.

- **The correlator's reference signal is a scaled down version of the difference signal $s_d(t) = 2A\cos(2\pi f_c t)$. The reference signal must be synchronous with the received signal in frequency and phase.**
- **It is generated by the carrier recovery (CR) circuit.**
- **Using a matched filter instead of a correlator is not recommended at passband since a filter with $h(t) = \cos(2\pi f_c(T-t))$ is difficult to implement.**

Binary PSK Signal

Coherent BPSK Demodulator

In absence of noise, setting *A*=1, correlator output at *t* = (*k*+1)*T*:

$$\int_{kT}^{(k+1)T} r(t)\cos 2\pi f_c t\,dt$$

$$= \int_{kT}^{(k+1)T} a_k \cos^2 2\pi f_c t\,dt$$

$$= \frac{1}{2}\int_{kT}^{(k+1)T} a_k(1+\cos 4\pi f_c t)\,dt$$

$$= \frac{T}{2}a_k + \frac{a_k}{8\pi f_c}\left[\sin 4\pi f_c(k+1)T - \sin 4\pi f_c kT\right]$$

Conclusion: **If $f_c = mR_b$, the second term is zero. If $f_c \neq mR_b$ and $f_c >> R_b$, then the effect of the second term is negligible.**

Binary PSK Signal

Coherent BPSK Demodulator

The **bit error probability** can be shown as:

$$P_b = Q\left(\sqrt{\frac{E_1 + E_2 - 2\rho_{12}\sqrt{E_2 E_1}}{2N_o}}\right)$$

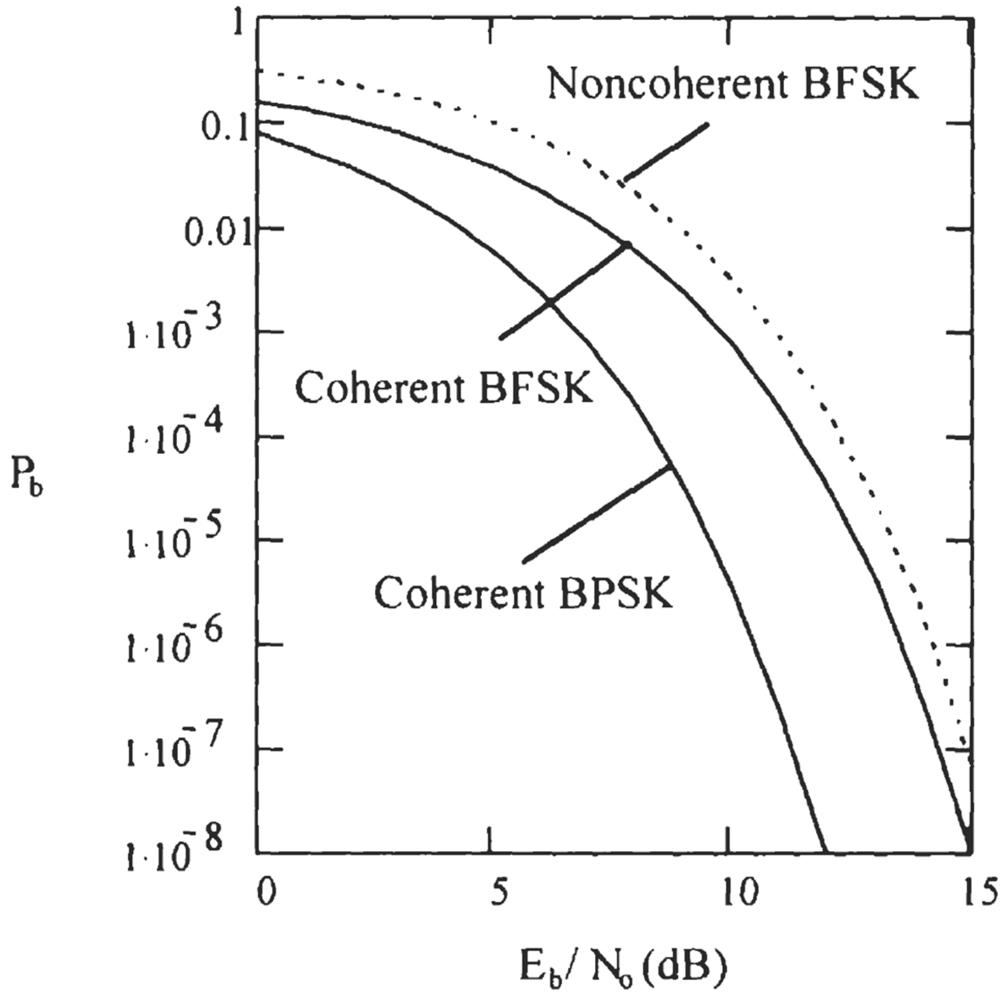
For **BPSK**, $\rho_{12} = -1$, $E_1 = E_2 = E_b$, thus:

$$P_b = Q\left(\sqrt{\frac{2E_b}{N_o}}\right)$$

(for coherent BPSK)

Binary PSK Signal

Coherent BPSK Demodulator



The P_b of coherent BFSK is inferior to coherent BPSK. However coherent BPSK requires that the reference signal at the receiver be synchronized in phase and frequency with the received signal.

P_b of coherent BPSK in comparison with BFSK

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- **Simon Haykin, *Communication Systems*. 4th Edition, Wiley India Edition, 2008.**

Thank You ...