Capacitive Transducers

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Capacitive Transducers

- ✓**Capacitive transducers operate on the principle of variation of capacitance with the variation of the physical variable under measurement.**

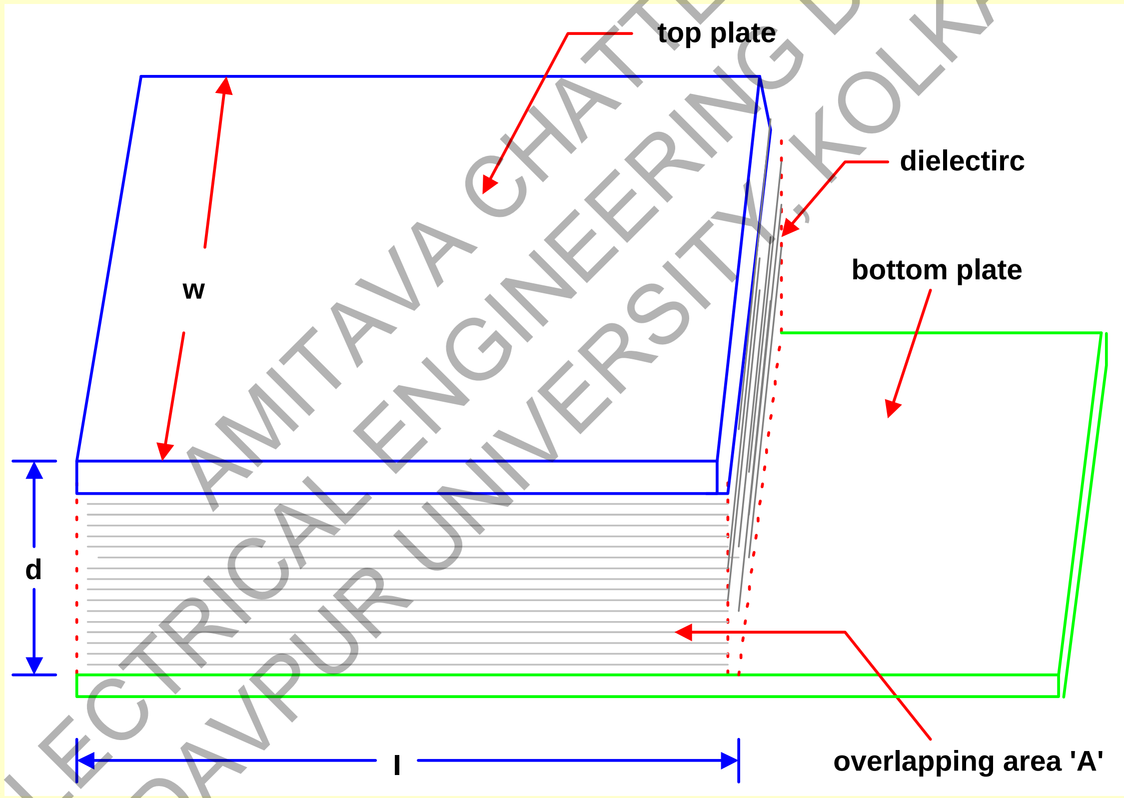
Parallel Plate Capacitors

- ✓**For parallel plate capacitors, employing rectangular plates, capacitance is given by** $C = \frac{\varepsilon A}{d}$. **Here, change in capacitance is caused by:**
  - **change in overlapping area *A*,**
  - **change in the distance between the plates *d*,**
  - **change in dielectric constant.**

Transducers using Change in Overlapping Area

Measurement of Linear Displacement

Capacitors employing rectangular plates



Transducers using Change in Overlapping Area (contd…)

Capacitors employing rectangular plates

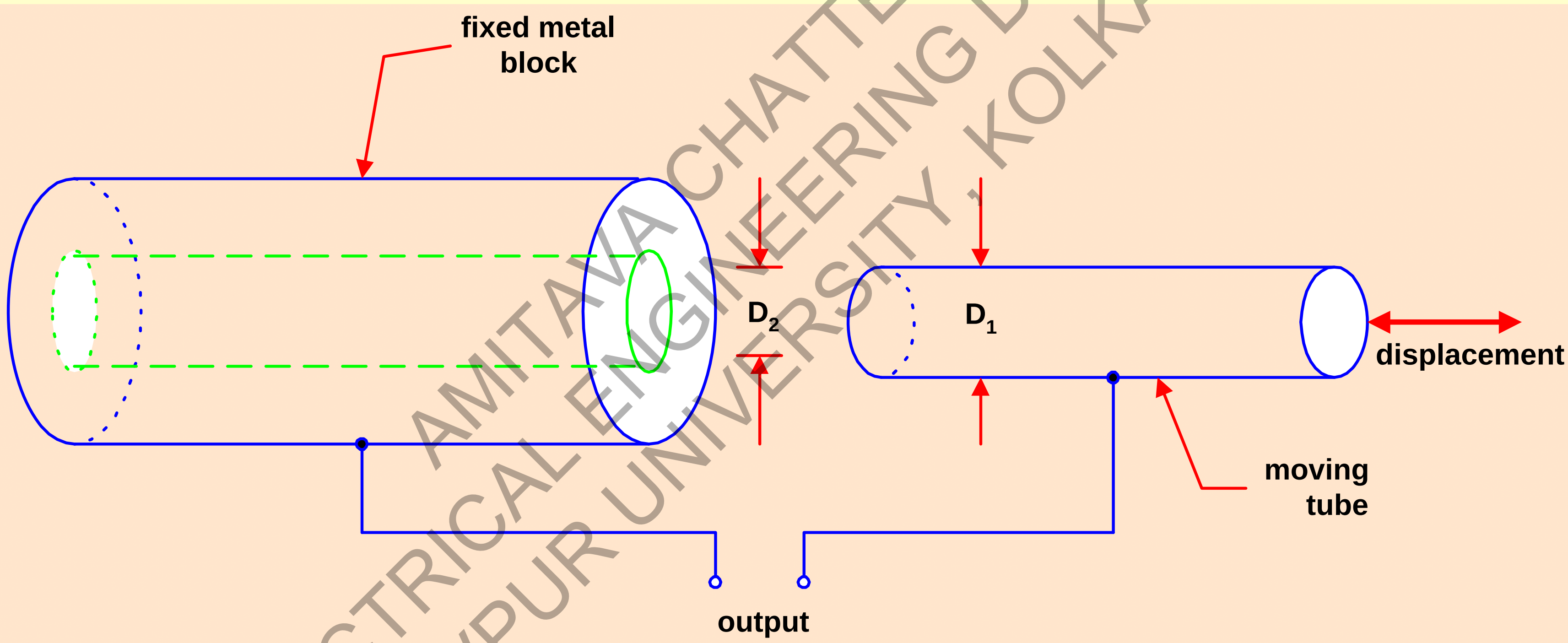
$$C = \frac{\varepsilon A}{d} = \frac{\varepsilon l w}{d}$$

w **= width of the overlapping part of the plates,**
l **= length of the overlapping part of the plates.**

Sensitivity of the transducer, $S = \dfrac{\Delta C}{\Delta l} = \dfrac{\varepsilon w}{d}$.

Transducers using Change in Overlapping Area (contd…)

Capacitors employing cylindrical electrodes



Transducers using Change in Overlapping Area (contd…)

Capacitors employing cylindrical electrodes

$$C = \frac{2\pi\varepsilon l}{\log_e\left(\frac{D_2}{D_1}\right)}$$

l = length of the overlapping part of the cylinders,

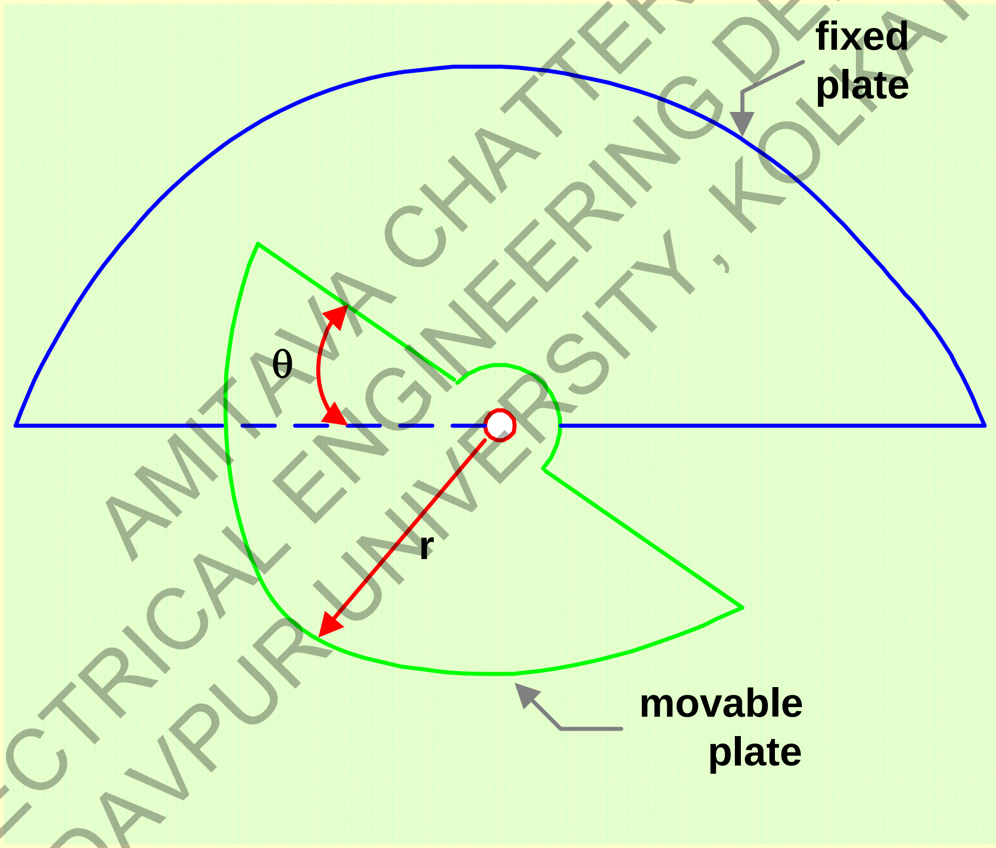
D_2 = inner diameter of the outer cylindrical electrode,

D_1 = outer diameter of the inner cylindrical electrode.

Sensitivity of the transducer, $S = \frac{\partial C}{\partial l} = \frac{2\pi\varepsilon}{\log_e\left(\frac{D_2}{D_1}\right)}$.

Transducers using Change in Overlapping Area (contd…)

Measurement of Angular Displacement



Transducers using Change in Overlapping Area (contd…)

Measurement of Angular Displacement

$$C = \frac{\varepsilon r^2}{2d}\theta$$

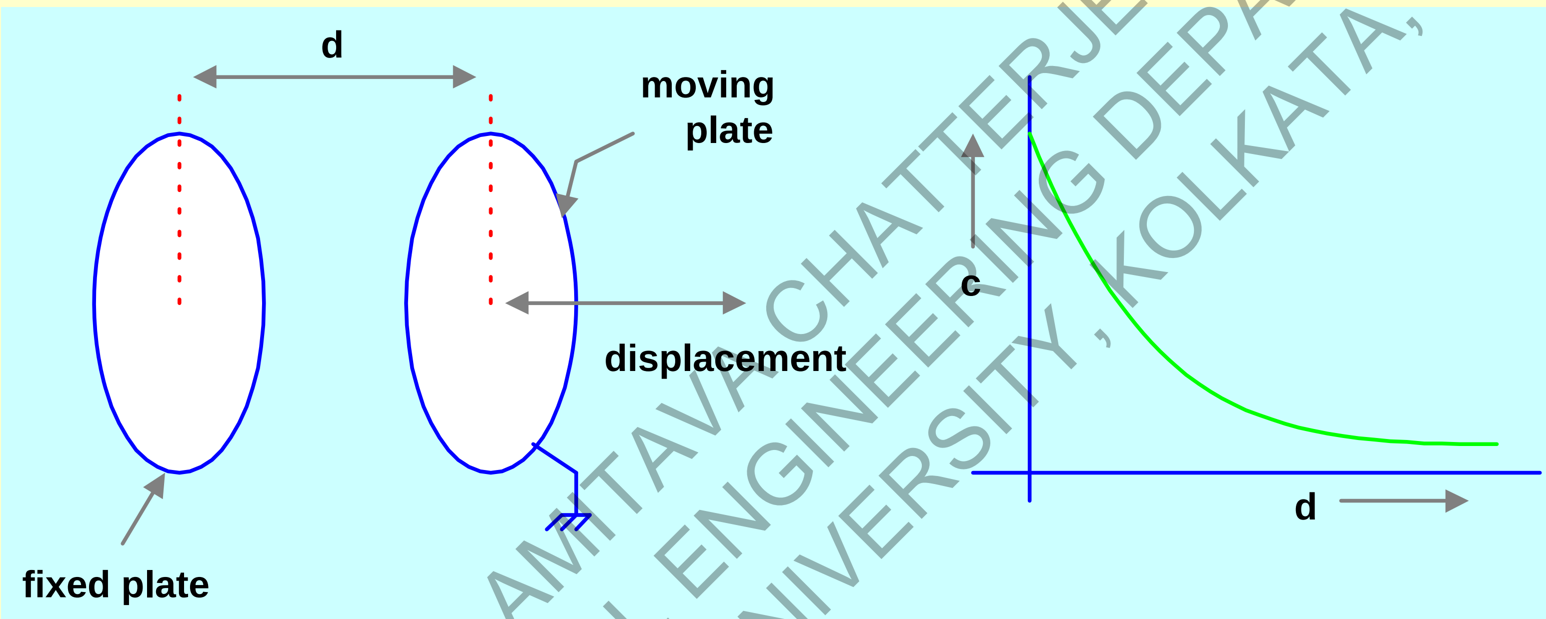
θ = angular displacement of the movable plate (in radian),
r = radius of the smaller plate,
d = distance between the plates.

⇩

Sensitivity of the transducer, $S = \dfrac{\partial C}{\partial \theta} = \dfrac{\varepsilon r^2}{2d}.$

Transducers using Change in Distance between the Plates

Measurement by moving one of the parallel plates

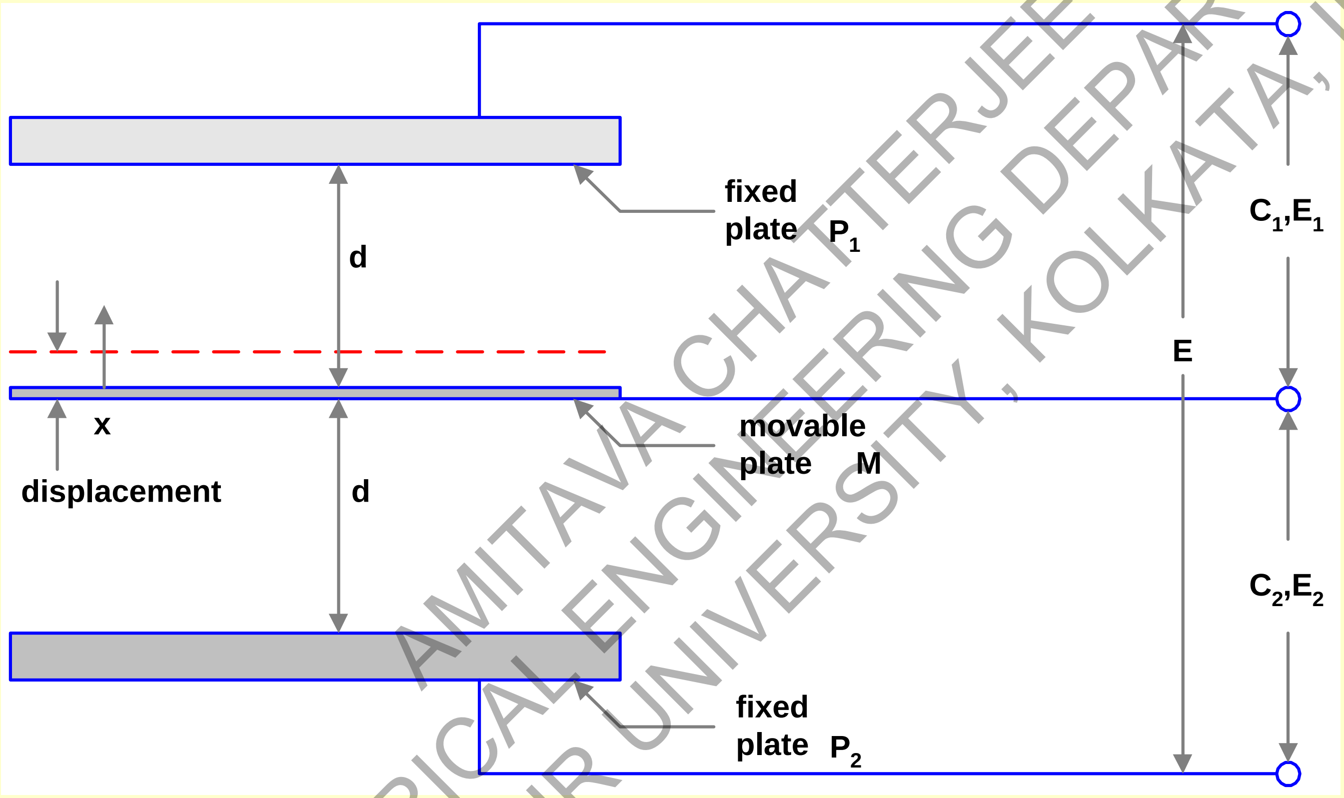


$$C = \frac{\varepsilon A}{d}$$

$$S = \frac{\partial C}{\partial d} = -\frac{\varepsilon A}{d^2}$$

Transducers using Change in Distance between the Plates (contd…)

Measurement by differential arrangement



$$C_1 = \frac{\varepsilon A}{d - x}$$

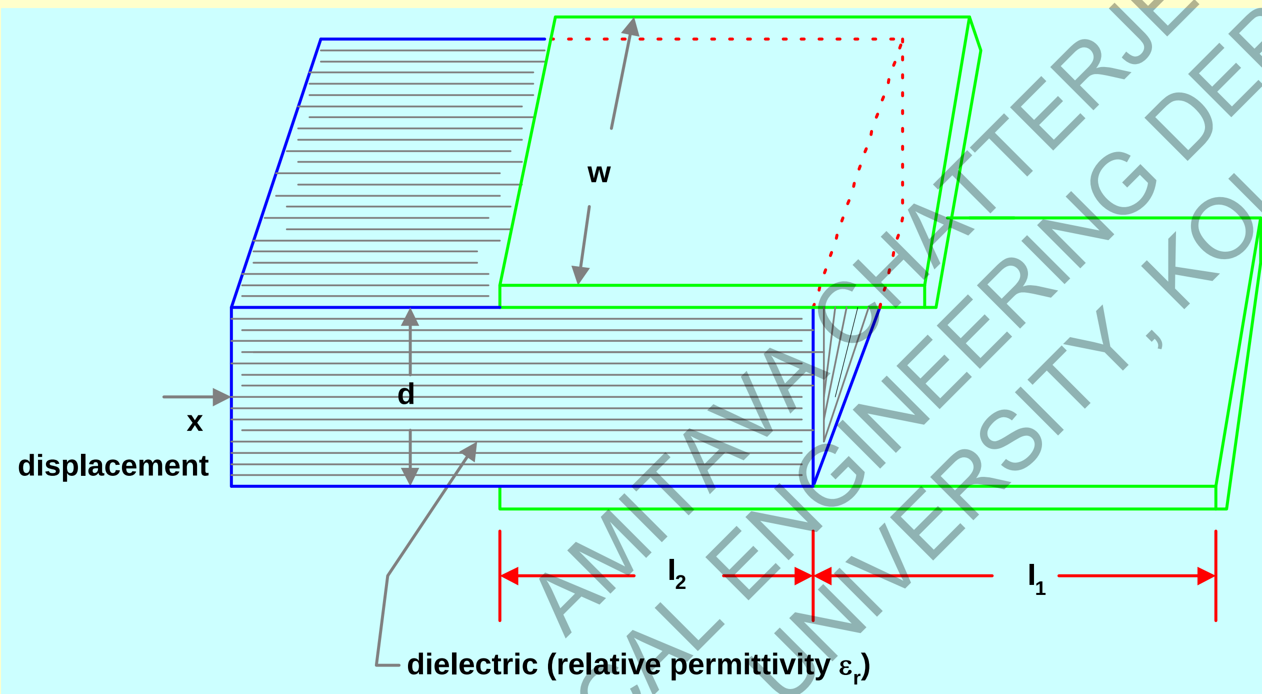
$$C_2 = \frac{\varepsilon A}{d + x}$$

$$\Delta E = E_2 - E_1 = \frac{x}{d} E$$

$$S = \frac{\Delta E}{x} = \frac{E}{d}$$

Transducers using Variation of Dielectric constant

Measurement of Displacement



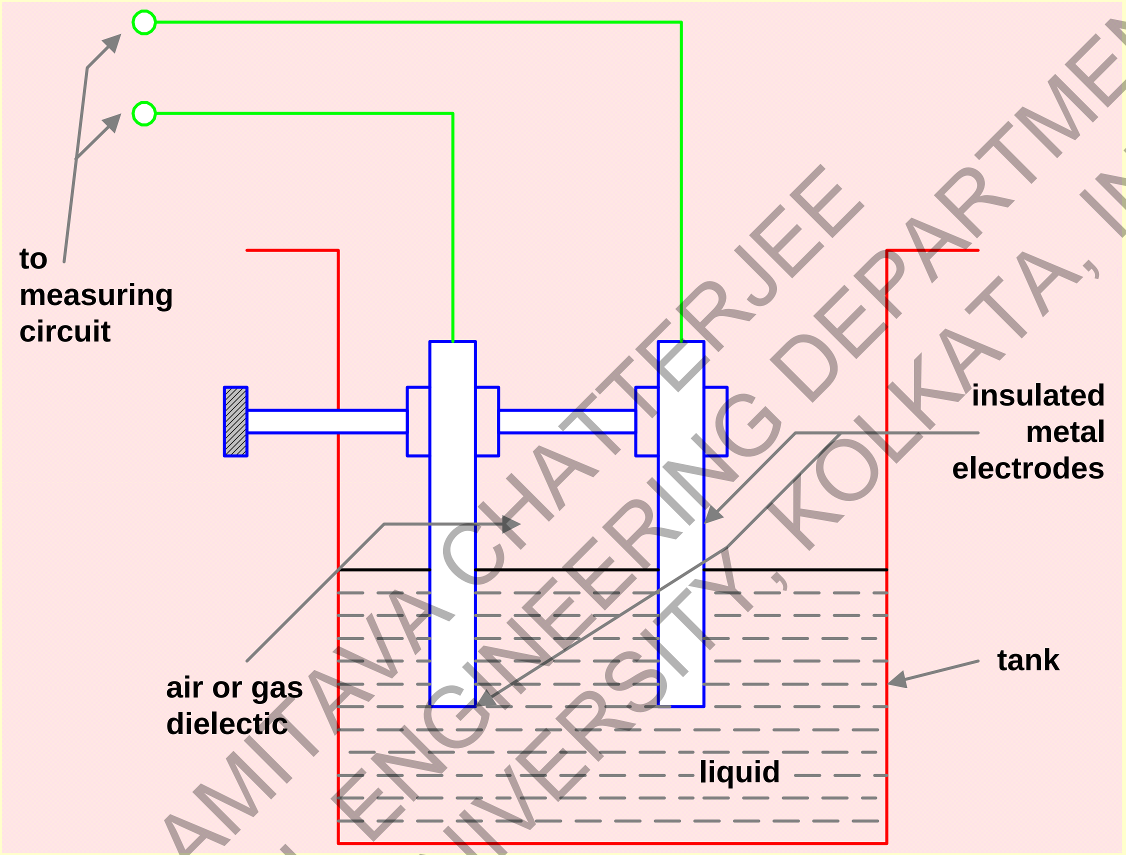
Initial Capce.

$$C_0 = \varepsilon_0 \frac{wl_1}{d} + \varepsilon_0 \varepsilon_r \frac{wl_2}{d}$$

$$= \varepsilon_0 \frac{w}{d}\left(l_1 + \varepsilon_r l_2\right)$$

New Capce. $$C = C_0 + \Delta C = C_0 + \varepsilon_0 \frac{wx}{d}\left(\varepsilon_r - 1\right) \quad \therefore \Delta C = \varepsilon_0 \frac{wx}{d}\left(\varepsilon_r - 1\right)$$

Capacitive Level Gauge



- ✓ Works on the principle of variation of capacitance with the variation of dielectric constant, caused by a corresponding change in liquid level.
- ✓ The capacitance can be measured by a suitable capacitive Wheatstone bridge.

Capacitive Level Gauge (contd…)

Features

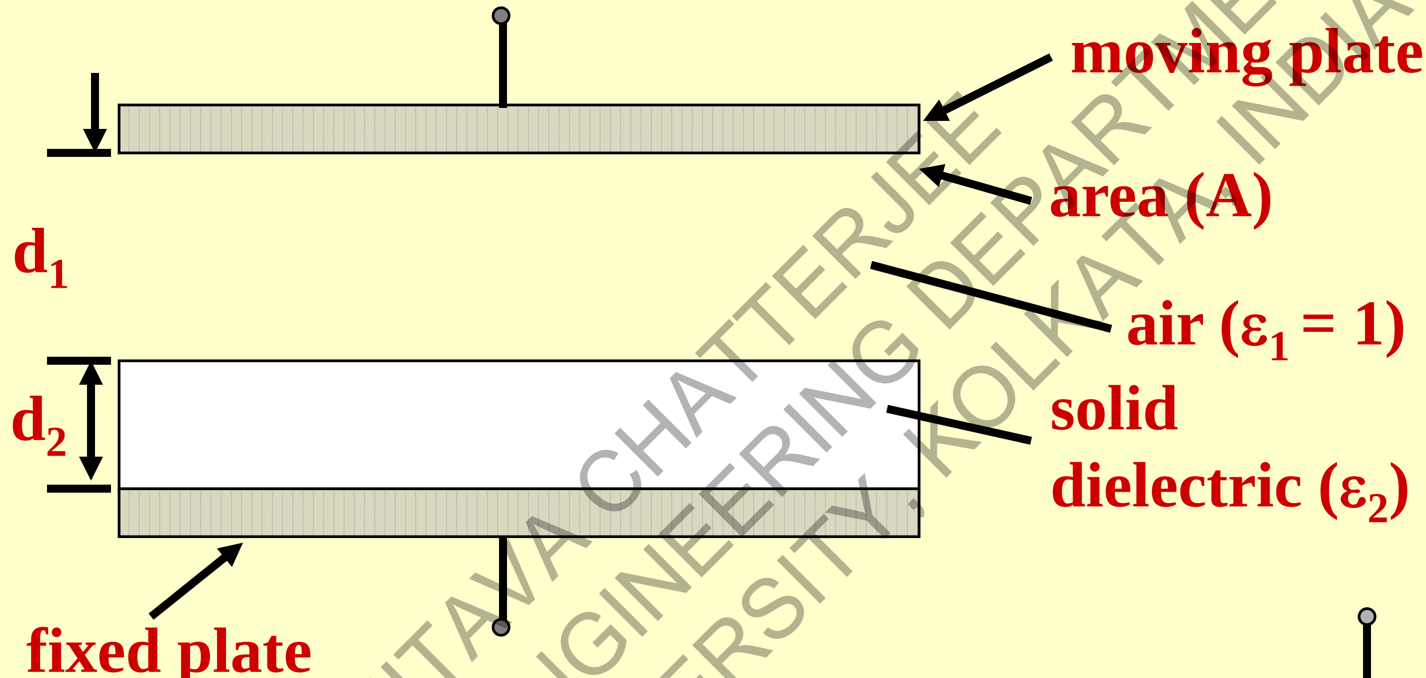
- ✓ **For non-conducting liquids, a bare probe arrangement may be satisfactory.**

- ✓ **For conducting liquids, probe plates are insulated.**

- ✓ **Capacitive level gauges are popular because they are relatively inexpensive, versatile, reliable, and require minimum maintenance.**

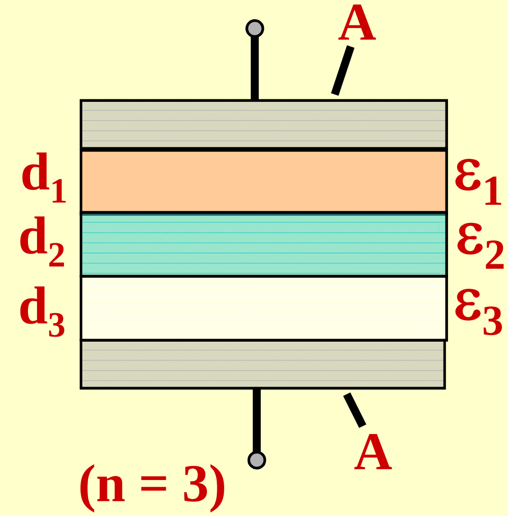
- ✓ **Can be used for measuring levels from a few cm. to more than 100 m.**

Capacitive Transducer with Solid Dielectric and Variable Air Gap between Parallel Plates

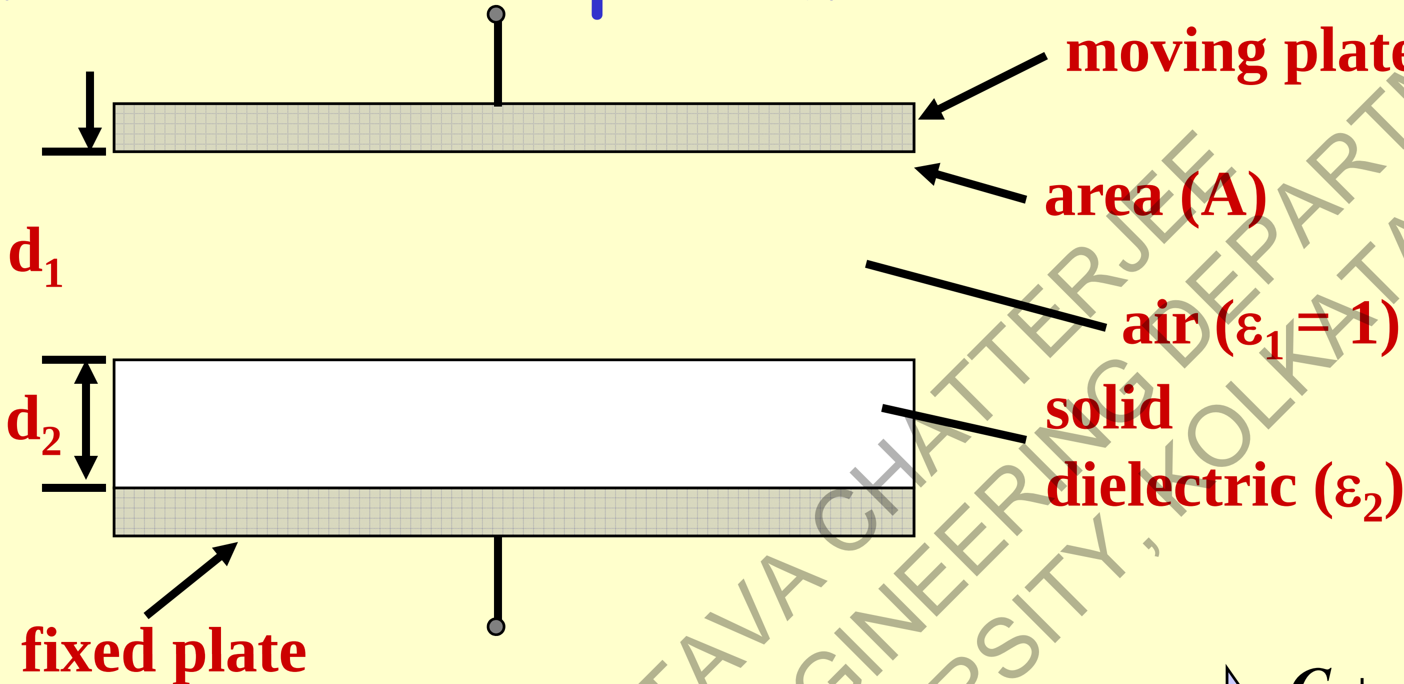


Capacitance C with parallel plates, each of an area A, and n layers of various dielectric materials:

$$C = \frac{\varepsilon_0 A}{\left[\frac{d_1}{\varepsilon_1} + \frac{d_2}{\varepsilon_2} + \cdots + \frac{d_n}{\varepsilon_n}\right]}$$



Capacitive Transducer with Solid Dielectric and Variable Air Gap between Parallel Plates (contd…)



Here d_1 is variable, but d_2 is constant

$$C = \frac{\varepsilon_0 A}{\left[d_1 + \frac{d_2}{\varepsilon_2}\right]}$$

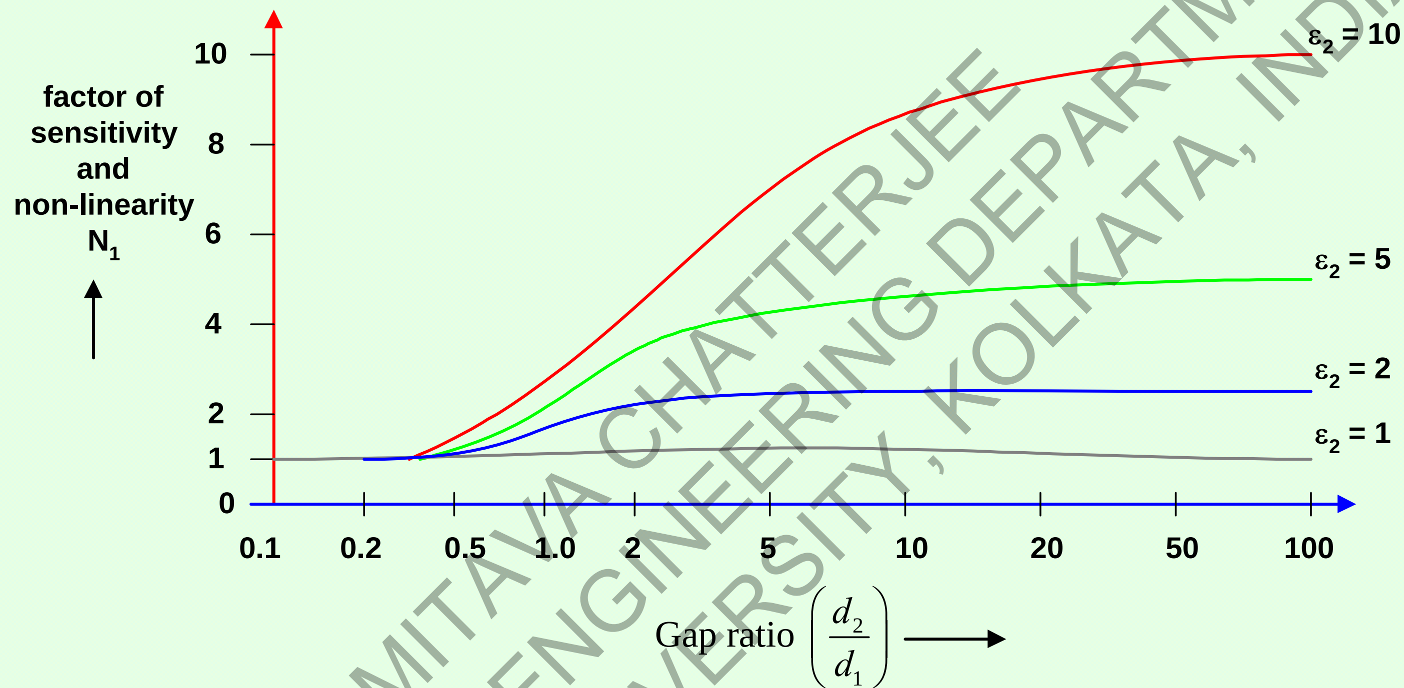
If the air gap is decreased by x, new capce.:

$$C + \Delta C = \frac{\varepsilon_0 A}{\left[(d_1 - x) + \frac{d_2}{\varepsilon_2}\right]}$$

$$\frac{\Delta C}{C} \approx \frac{x}{d_1 + d_2} N_1$$

$$N_1 = \frac{d_1 + d_2}{d_1 + \frac{d_2}{\varepsilon_2}} = \frac{1 + \frac{d_2}{d_1}}{1 + \frac{d_2}{d_1 \varepsilon_2}}$$

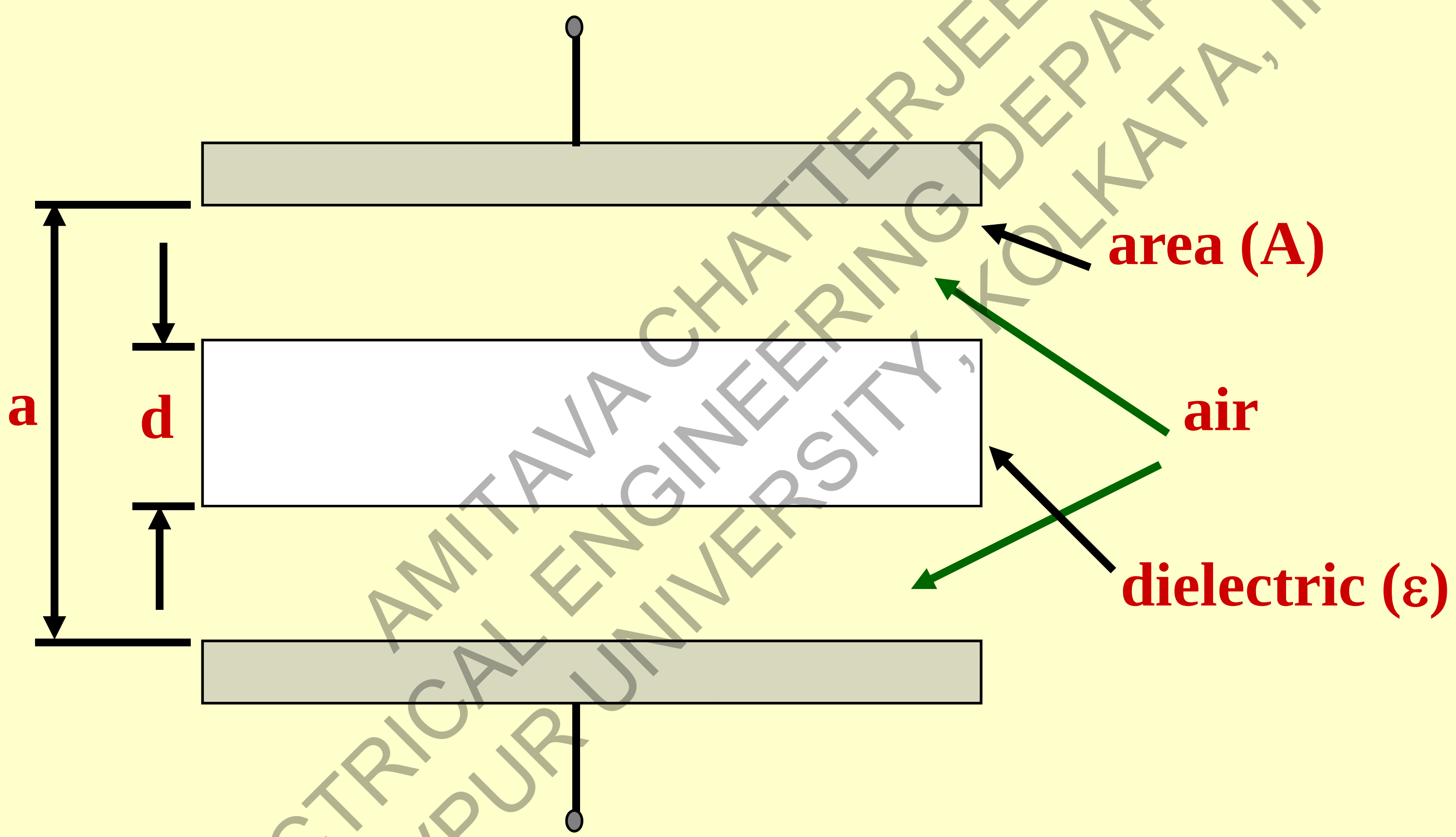
Capacitive Transducer with Solid Dielectric and Variable Air Gap between Parallel Plates (contd…)



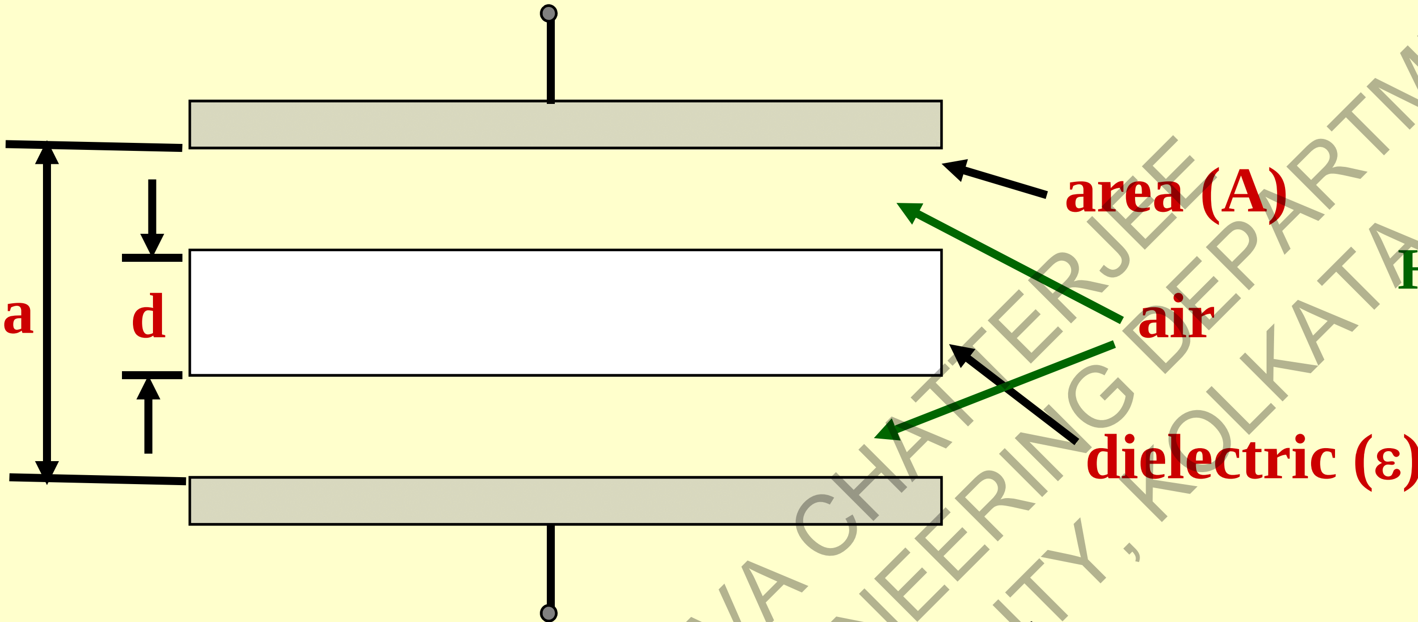
N_1 : both sensitivity and non-linearity factor

$$N_1 = \frac{d_1 + d_2}{d_1 + \frac{d_2}{\varepsilon_2}} = \frac{1 + \frac{d_2}{d_1}}{1 + \frac{d_2}{d_1 \varepsilon_2}}$$

Capacitive Transducer with Solid Dielectric of Variable Permittivity or Thickness and Air Gap between Parallel Plates



Case I: Solid Dielectric with Variable Permittivity



Here d is constant, but ε is variable

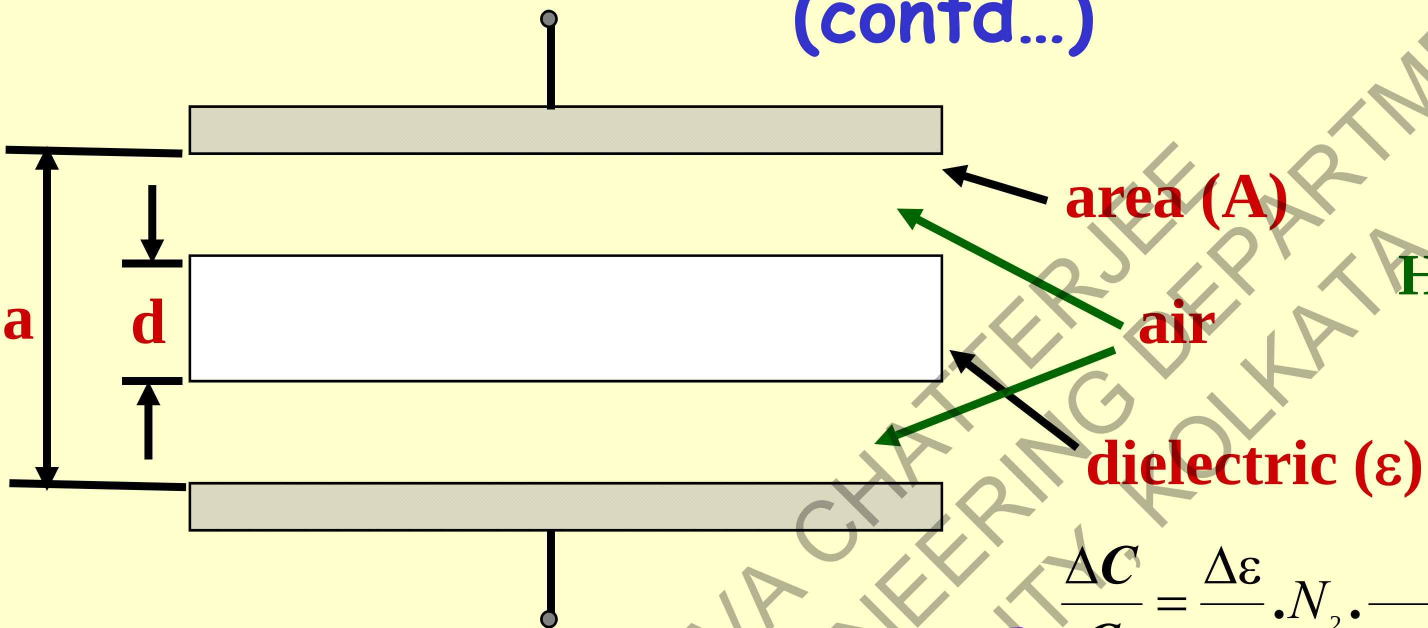
Initial capacitance:

$$C = \frac{\varepsilon_0 A}{\left[(a - d) + \frac{d}{\varepsilon}\right]}$$

If the permittivity increases by $\Delta\varepsilon$, new capacitance:

$$C + \Delta C = \frac{\varepsilon_0 A}{\left[(a - d) + \frac{d}{(\varepsilon + \Delta\varepsilon)}\right]}$$

Case I: Solid Dielectric with Variable Permittivity (contd…)



Here d is constant, but ε is variable

$$\frac{\Delta C}{C}=\frac{\dfrac{\varepsilon_0 A}{\left[(a-d)+\dfrac{d}{(\varepsilon+\Delta\varepsilon)}\right]}-\dfrac{\varepsilon_0 A}{\left[(a-d)+\dfrac{d}{\varepsilon}\right]}}{\dfrac{\varepsilon_0 A}{\left[(a-d)+\dfrac{d}{\varepsilon}\right]}}$$

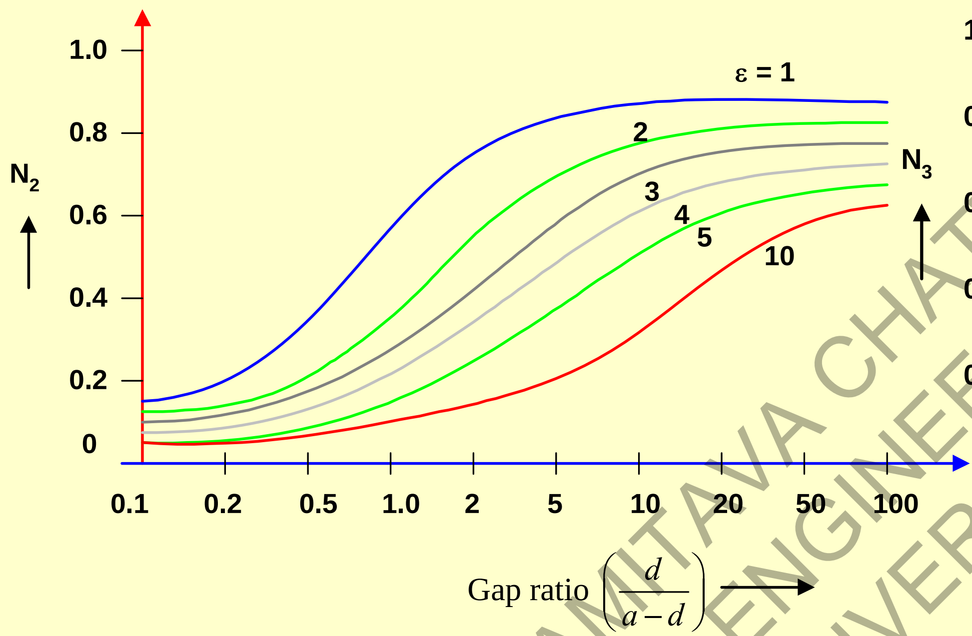
$$\frac{\Delta C}{C}=\frac{\Delta\varepsilon}{\varepsilon}.N_2.\frac{1}{1+\left(\dfrac{\Delta\varepsilon}{\varepsilon}\right)N_3}$$

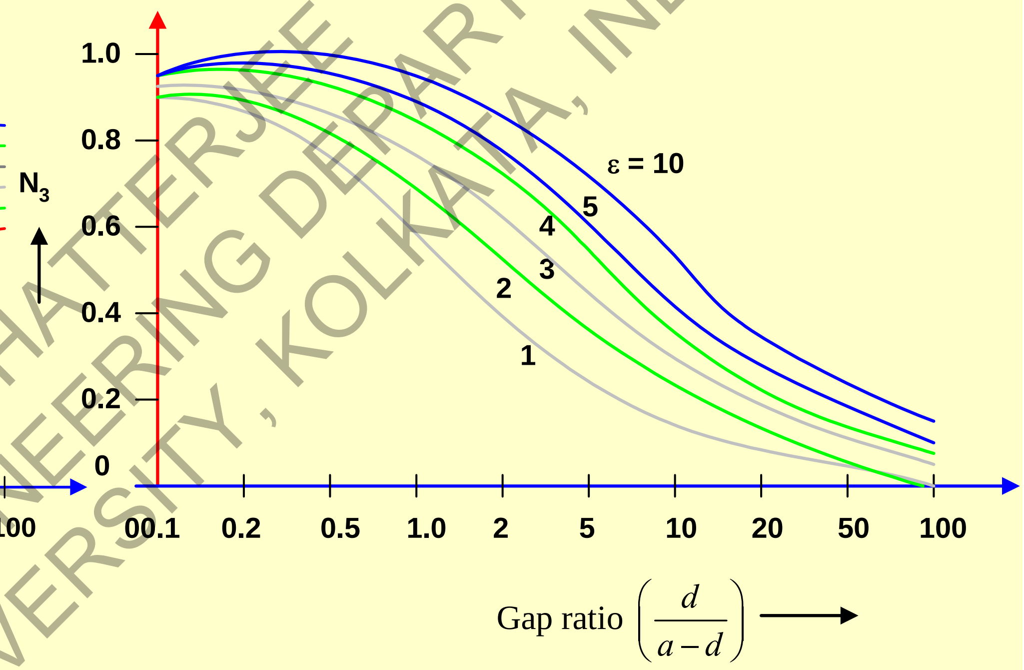
$$N_2=\frac{1}{1+\varepsilon\left(\dfrac{a-d}{d}\right)}$$

$$N_3=\frac{1}{1+\dfrac{1}{\varepsilon}\left(\dfrac{d}{a-d}\right)}$$

$$\frac{\Delta C}{C}\approx\frac{\Delta\varepsilon}{\varepsilon}N_2$$

Case I: Solid Dielectric with Variable Permittivity (contd…)





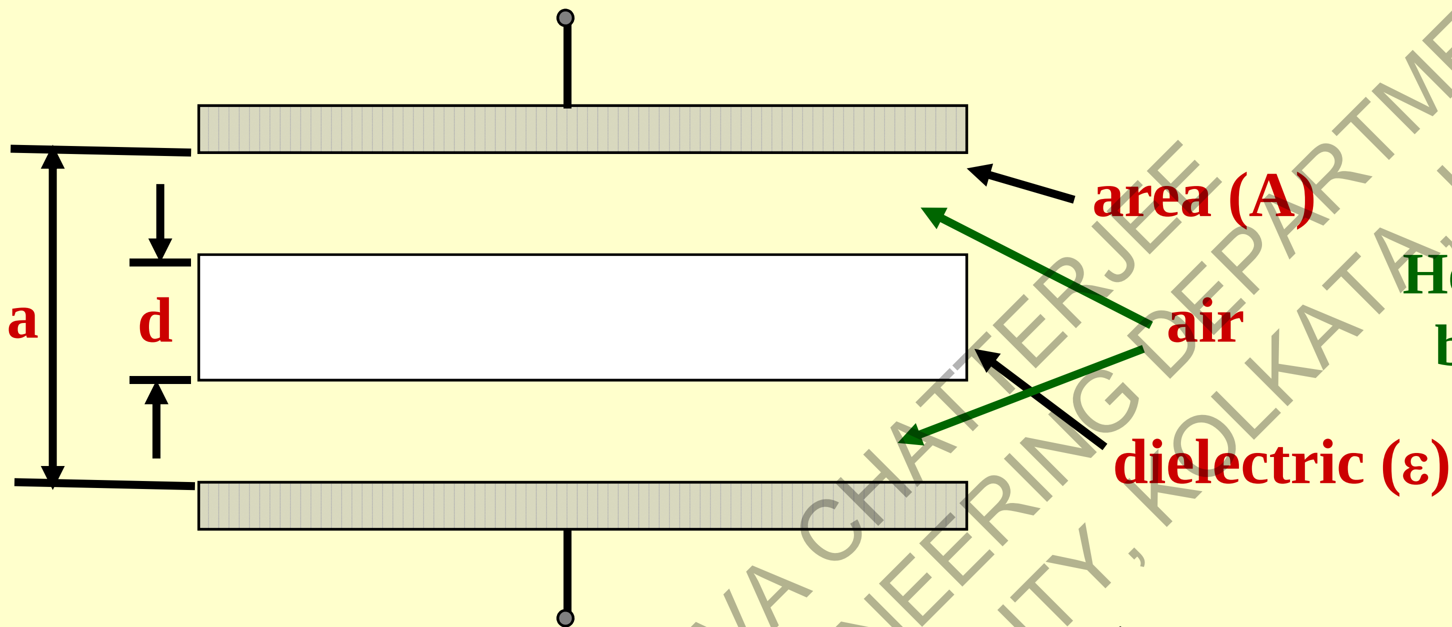
N_2 : sensitivity factor

$$N_2 = \frac{1}{1+\varepsilon\left(\frac{a-d}{d}\right)}$$

N_3 : non-linearity factor

$$N_3 = \frac{1}{1+\frac{1}{\varepsilon}\left(\frac{d}{a-d}\right)}$$

Case II: Solid Dielectric with Variable Thickness

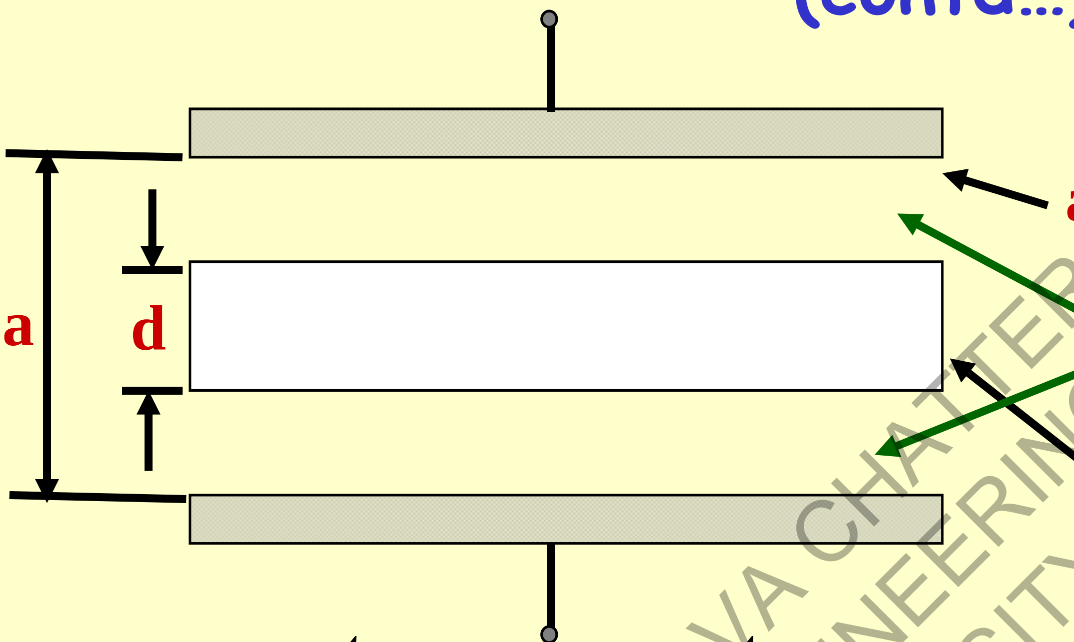


Here d is variable, but ε is constant

Initial capacitance: $$C = \frac{\varepsilon_0 A}{\left[(a - d) + \frac{d}{\varepsilon}\right]}$$

If the dielectric thickness increases by x, new capacitance: $$C + \Delta C = \frac{\varepsilon_0 A}{\left[(a - d - x) + \frac{d + x}{\varepsilon}\right]}$$

Case II: Solid Dielectric with Variable Thickness (contd…)



Here *d* is variable, but *ε* is constant

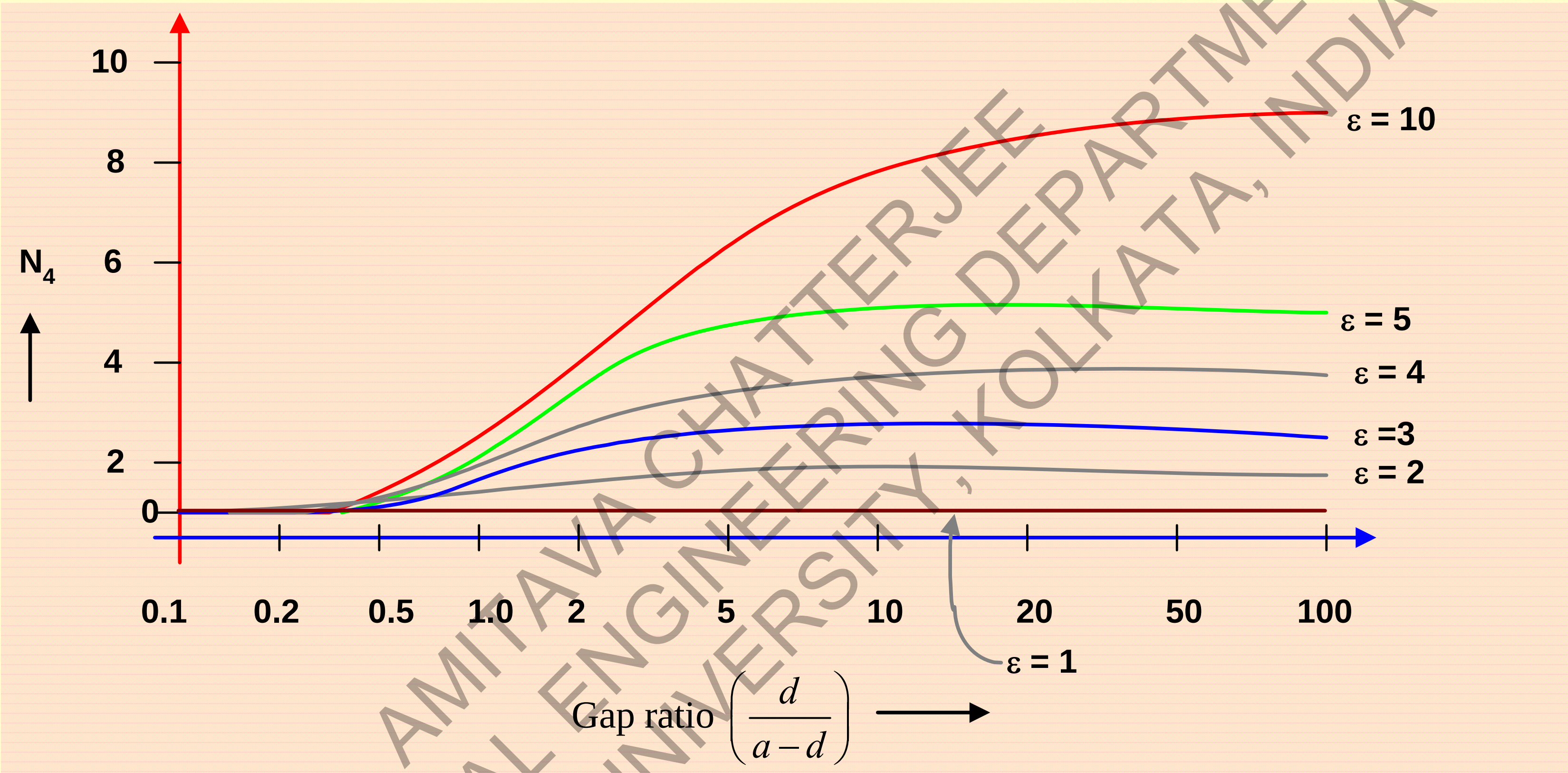
$$\frac{\Delta C}{C}=\frac{\dfrac{\varepsilon_0 A}{\left[(a-d-x)+\dfrac{d+x}{\varepsilon}\right]}-\dfrac{\varepsilon_0 A}{\left[(a-d)+\dfrac{d}{\varepsilon}\right]}}{\dfrac{\varepsilon_0 A}{\left[(a-d)+\dfrac{d}{\varepsilon}\right]}}$$

$$\frac{\Delta C}{C}=\frac{x}{d}\cdot N_4\cdot\frac{1}{1-\left(\dfrac{x}{d}\right)N_4}$$

$$N_4=\frac{\varepsilon-1}{1+\varepsilon\left(\dfrac{a-d}{d}\right)}$$

$$\frac{\Delta C}{C}\approx\frac{x}{d}N_4$$

Case II: Solid Dielectric with Variable Thickness (contd…)



N_4 : both sensitivity and non-linearity factor

$$N_4 = \frac{\varepsilon - 1}{1 + \varepsilon\left(\frac{a-d}{d}\right)}$$

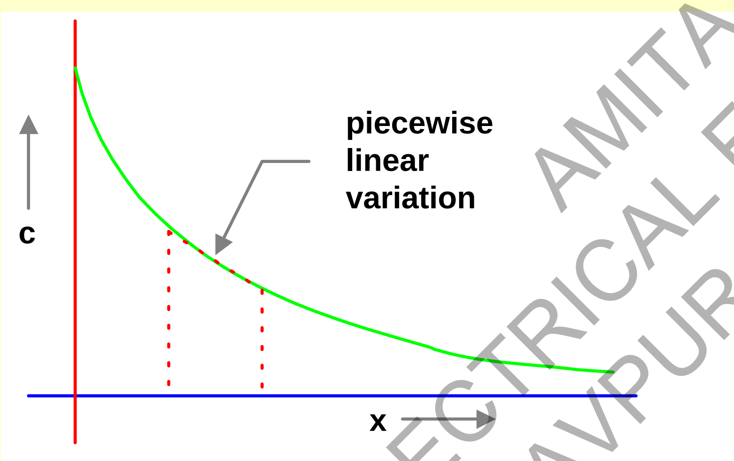
Capacitor Microphones:

an application of a displacement transducer using change in distance between the plates

Basic Theory

$$S = \frac{dC}{dx} = -\frac{\varepsilon A}{x^2}$$

x = **distance between the two plates.**

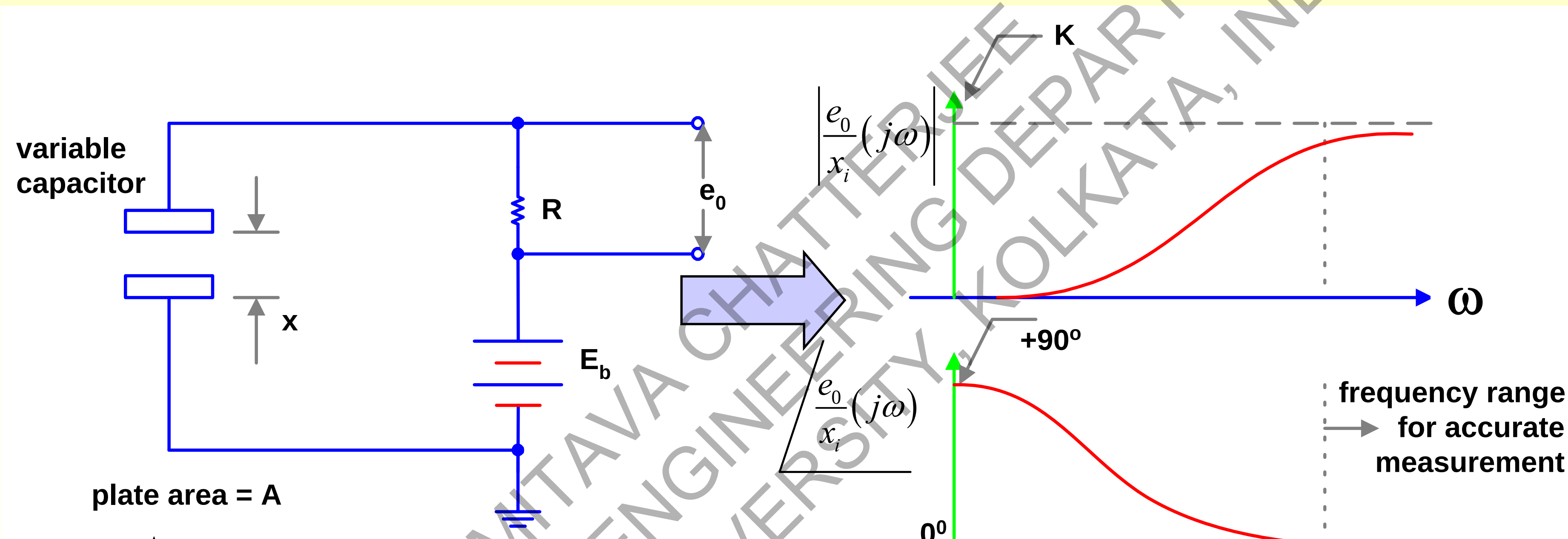


For small displacements about neutral position:

$$\frac{dC}{dx} = -\frac{C}{x} \quad \text{or} \quad \frac{dC}{C} = -\frac{dx}{x}$$

Capacitor Microphones (contd…)

The Arrangement



$x_i \triangleq x - x_0$

- ✓ For stationary capacitor plates with separation x_0, $e_o = 0$.
- ✓ For small relative displacement x_i, $\frac{e_0(s)}{x_i(s)} = \frac{sK\tau}{1+s\tau}; K \triangleq \frac{E_b}{x_0}, \ \tau \triangleq \frac{\varepsilon AR}{x_0}$

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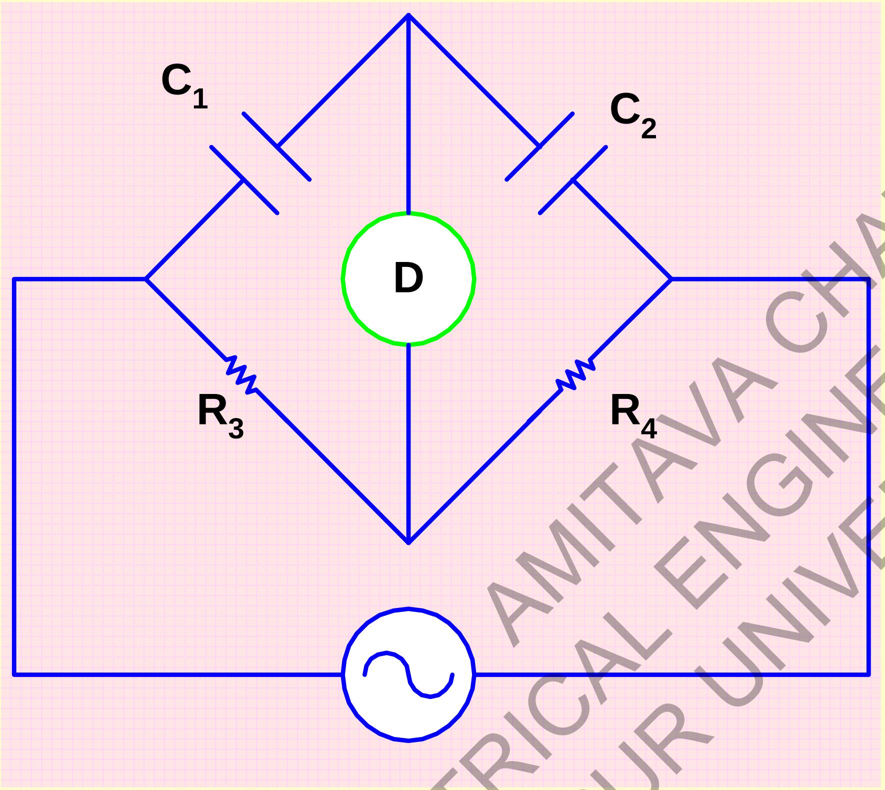
Bridge Circuits employed in connection with Capacitive Transducers

Null Method



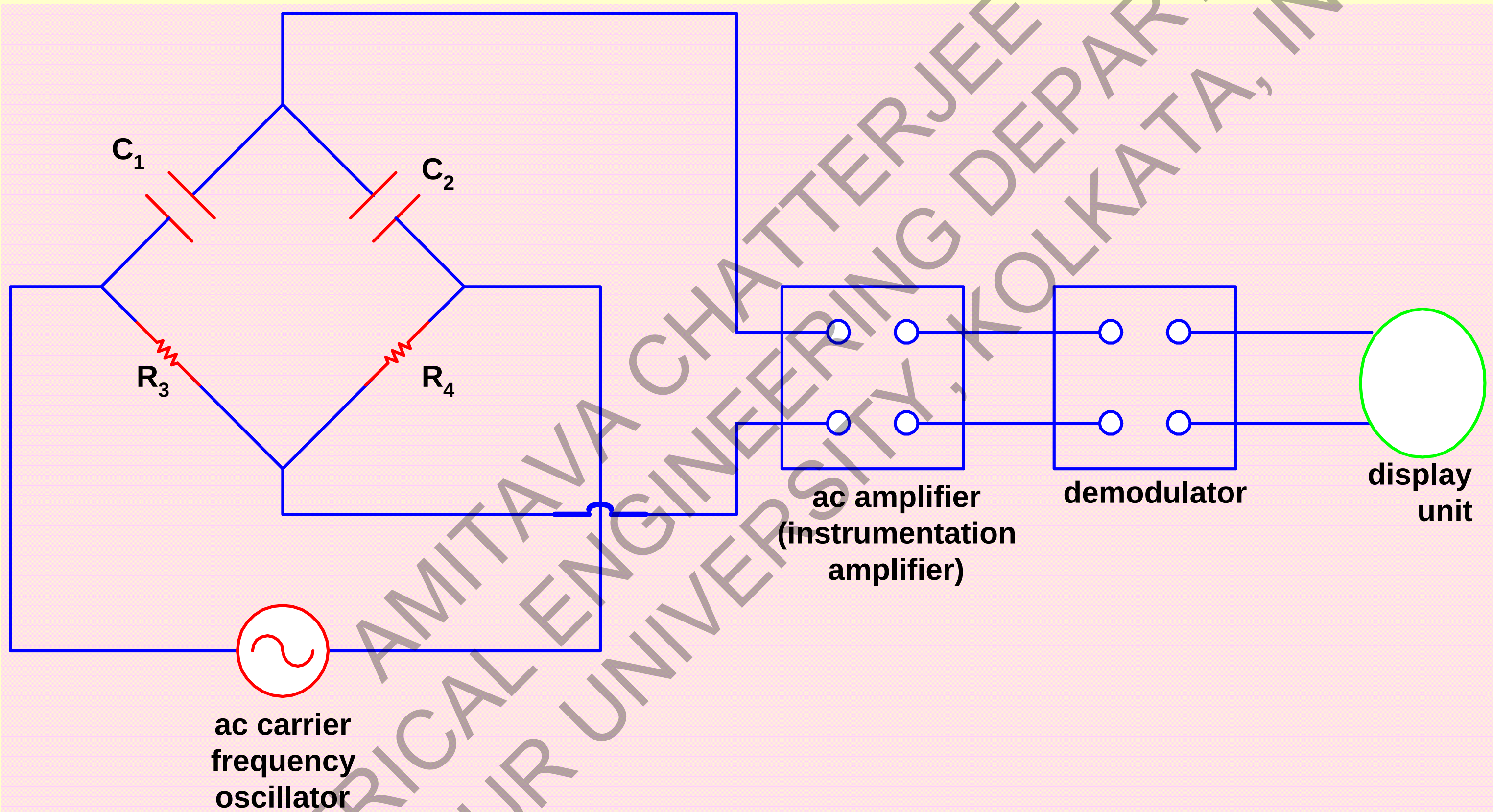
At balance,

$$C_1 = C_2 \frac{R_4}{R_3}$$

✓ **The balance can be achieved by varying either R_4 or R_3.**

Bridge Circuits employed in connection with Capacitive Transducers (contd…)

Direct Readout Method

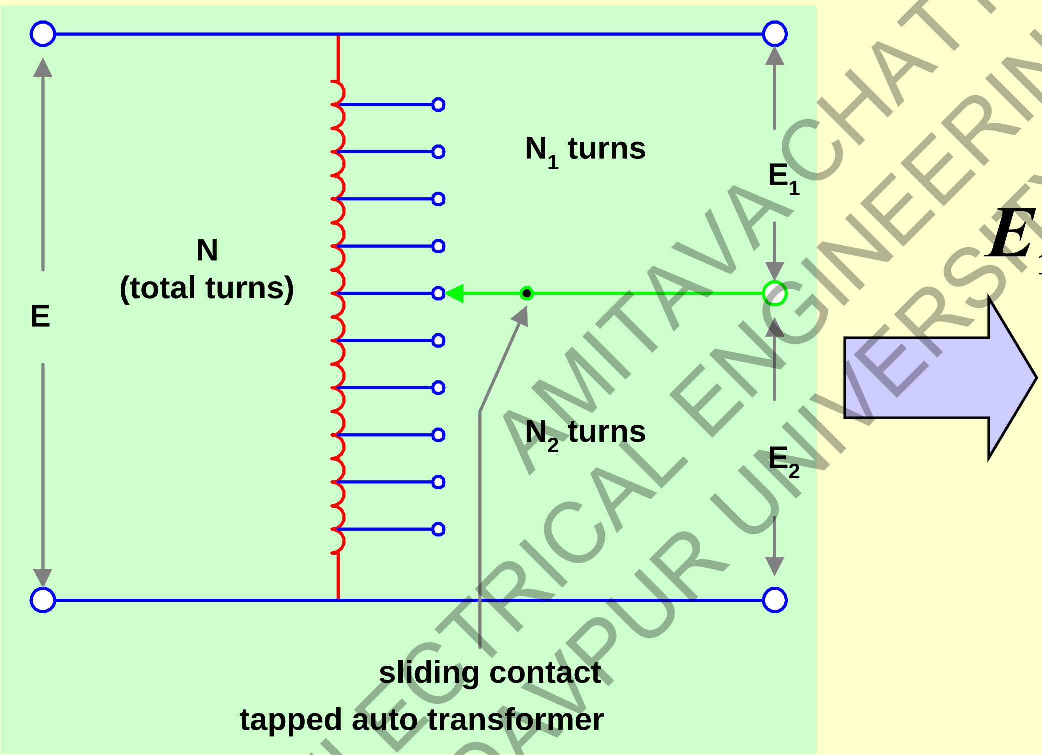


- ✓ **The unbalanced voltage measured can be used as a measure of the variable, unknown capacitance.**

Measurement of Capacitance by Transformer Ratio Bridges

Why Transformer Ratio Bridge?

✓ **It is a popular alternative to conventional a.c. bridges, because of the versatility and accuracy that the *ratio transformers* can offer.**



For an ideal auto transformer,

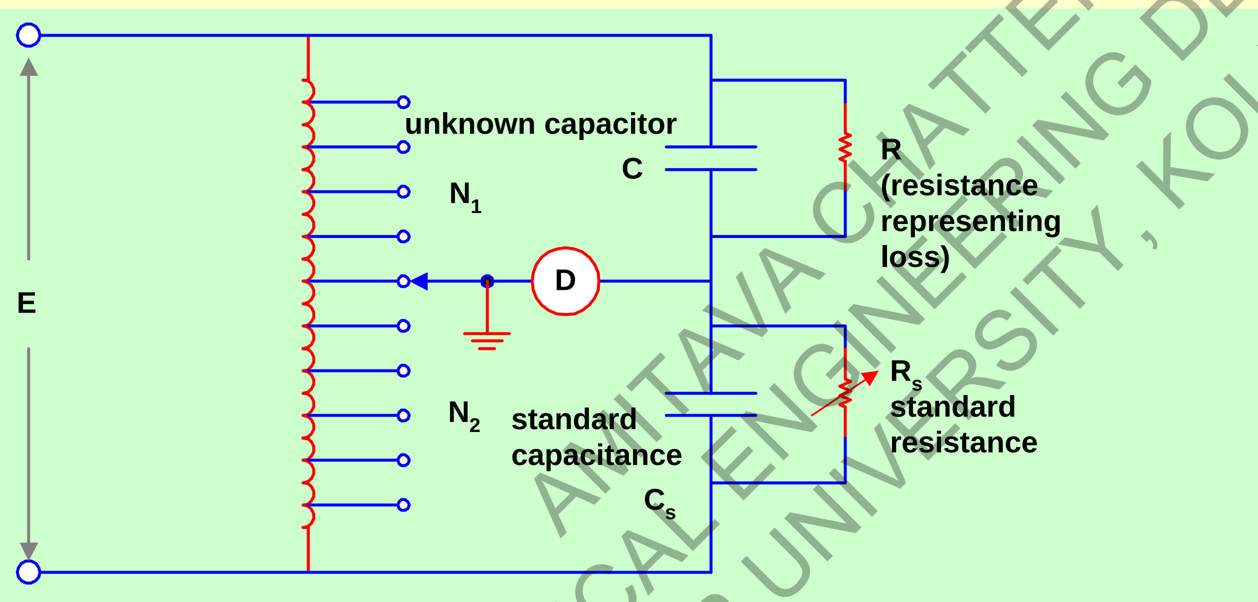
$$E_1 = E \cdot \frac{N_1}{N} \quad \text{and} \quad E_2 = E \cdot \frac{N_2}{N}$$

where

$$E = 4K_f \varphi_m f N$$

Measurement of Capacitance by Transformer Ratio Bridges (contd…)

✓ *Ratio transformers,* **when employed in ratio bridges, are very similar in operation to conventional a.c. bridges.**



measurement of capacitance
using ratio transformer

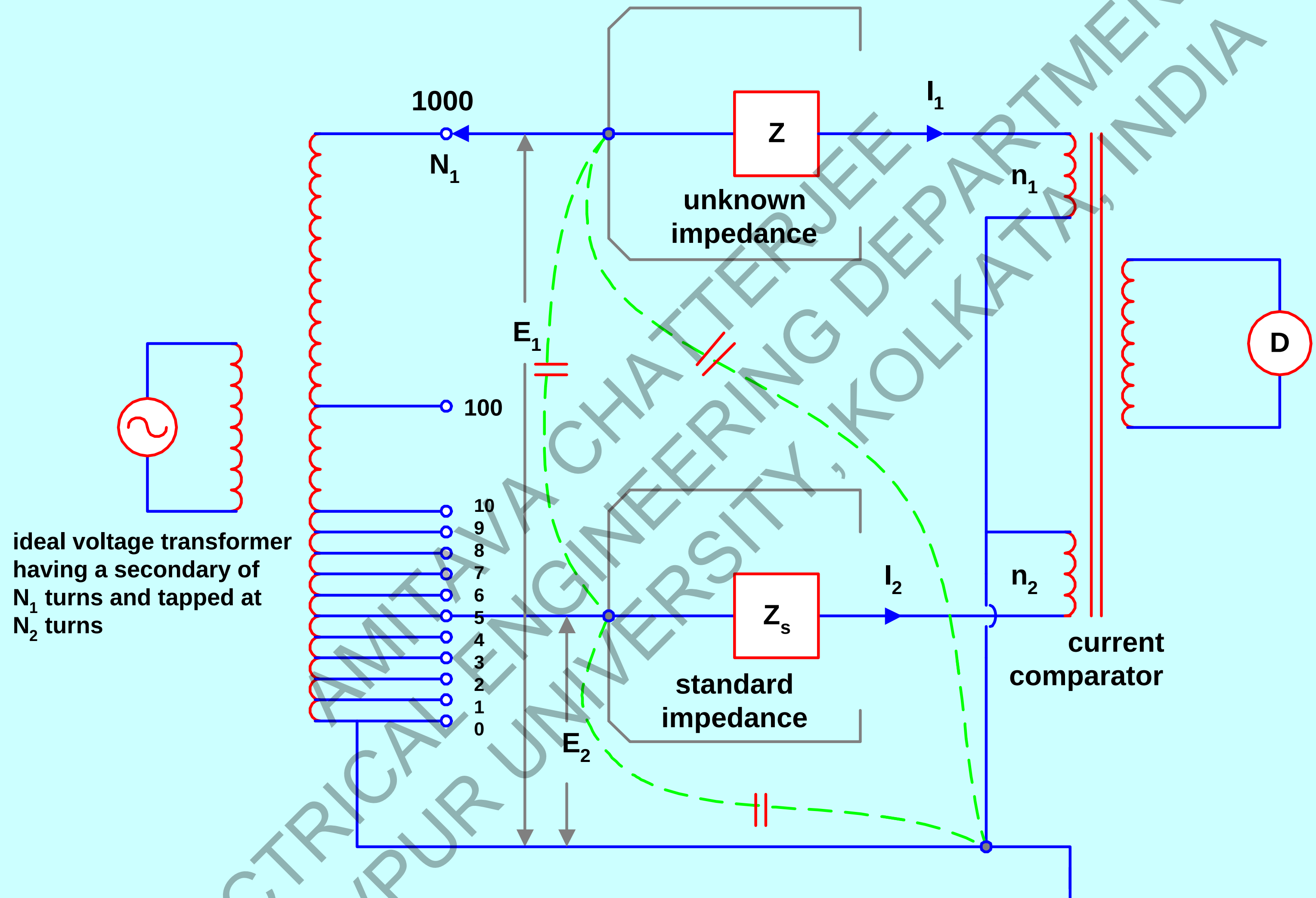
At balance:

$$C = \frac{N_2}{N_1} \cdot C_s$$

and

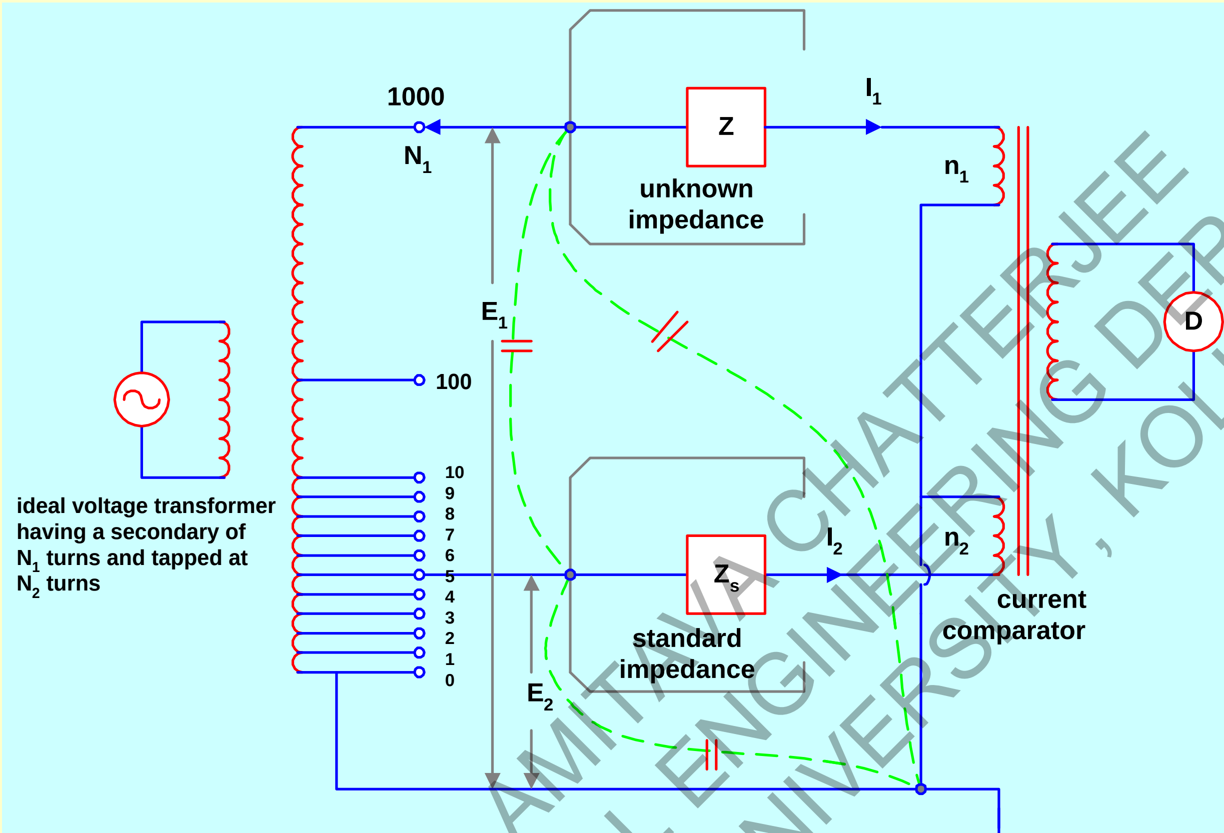
$$R = \frac{N_1}{N_2} \cdot R_s$$

Transformer Double Ratio Bridges



basic circuit for a double ratio transformer bridge

Transformer Double Ratio Bridges (contd…)



basic circuit for a double ratio transformer bridge

At balance:

$$I_1 n_1 = I_2 n_2$$

$$\therefore \frac{N_1 n_1}{Z} = \frac{N_2 n_2}{Z_s}$$

$$\text{or } \; Z = Z_s \frac{N_1 n_1}{N_2 n_2}$$

$$C = \frac{N_2 n_2}{N_1 n_1} \cdot C_s$$

Thank You