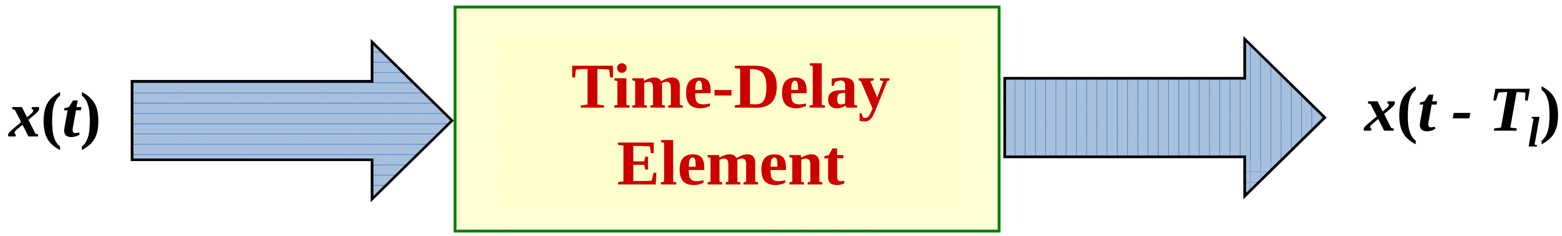
Control of Time-delay Systems

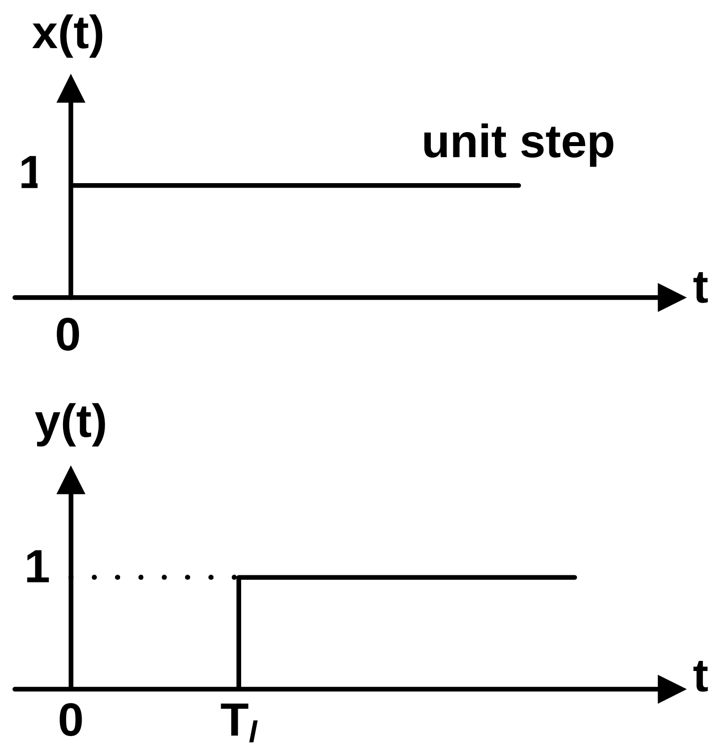
Prof. Anjan Rakshit and Prof. Amitava Chatterjee
Electrical Measurement and Instrumentation Laboratory,
Electrical Engineering Department,
Jadavpur University, Kolkata, India.

Time-Delay Element



T_l = Time-Delay

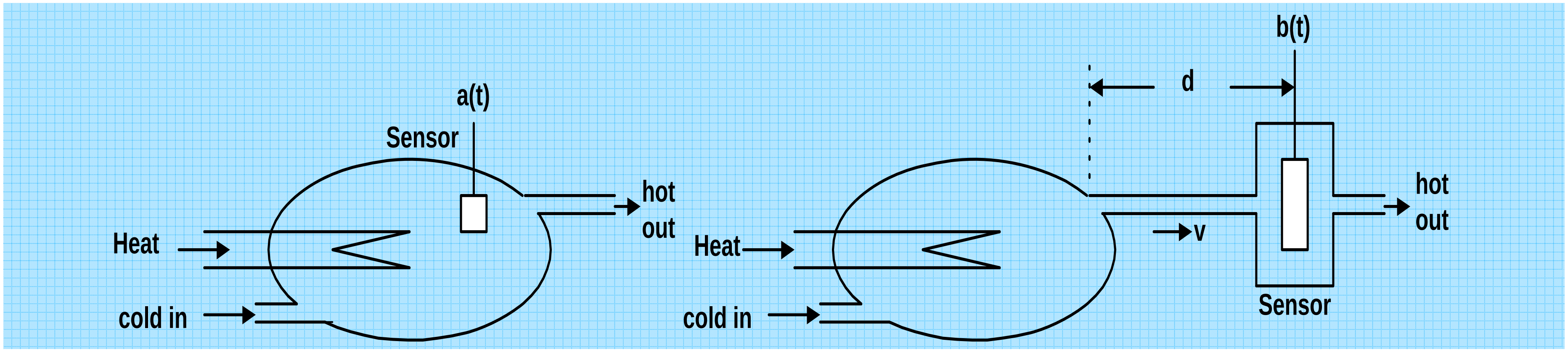
Time Response



Time-Delay Systems

Example 1

Temperature Measurement of Heat-Exchanger Output



v = velocity
d = distance

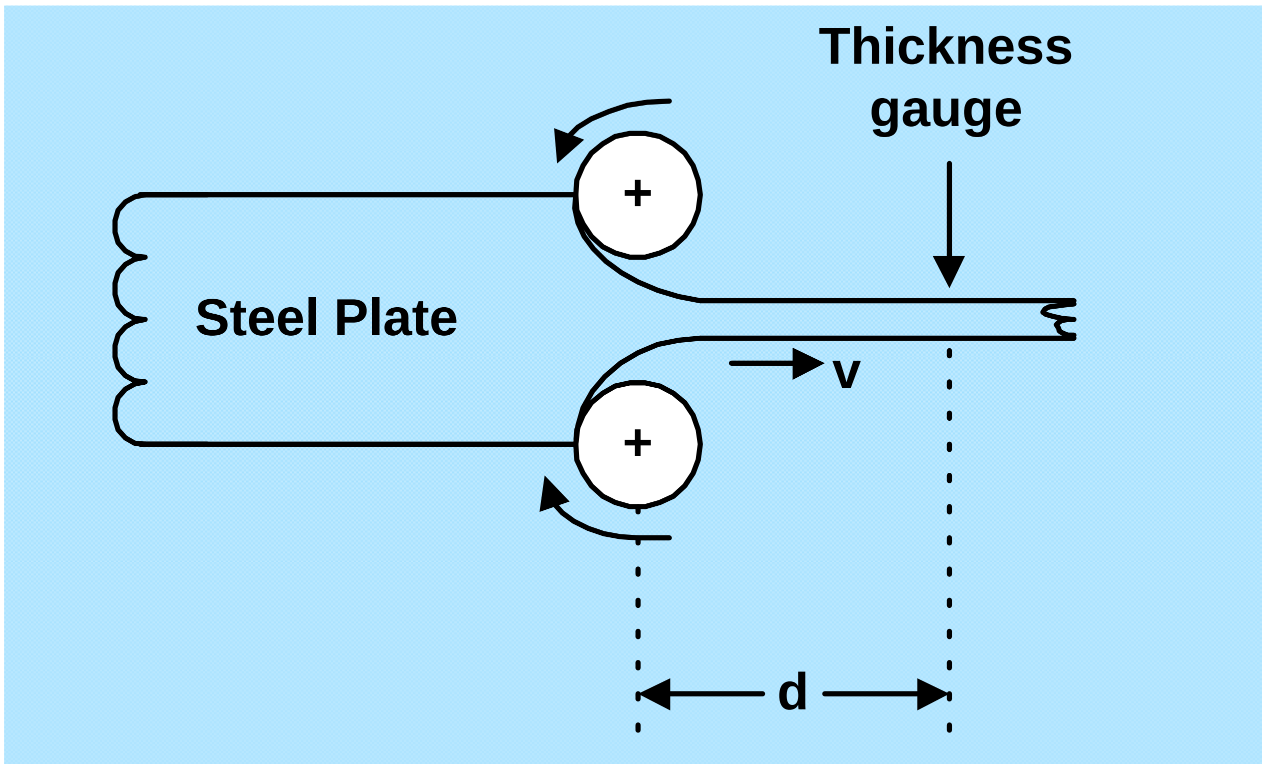
$b(t) = a(t - T_l)$

$$T_l = \frac{d}{v}$$

Time-Delay Systems

Example 2

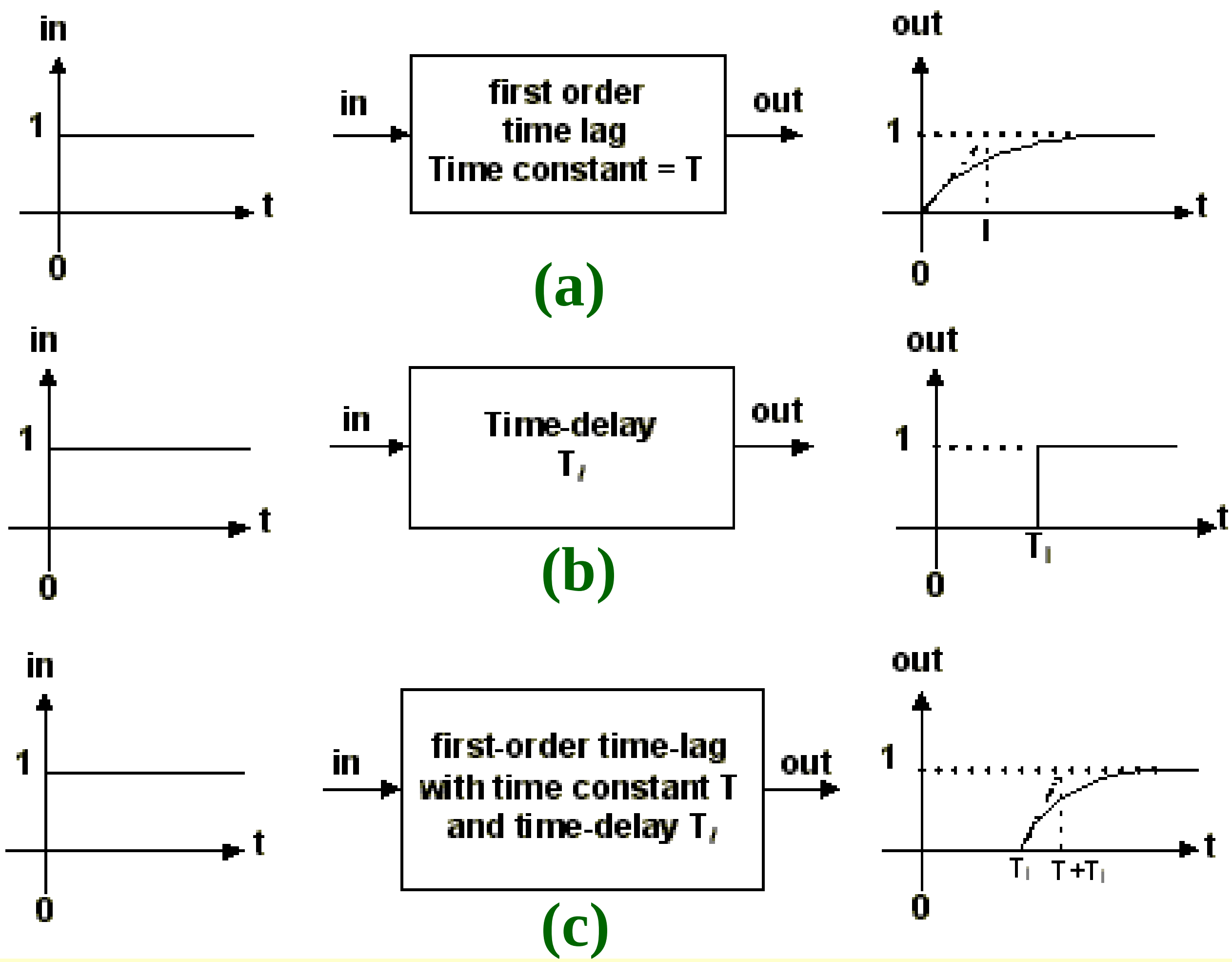
Thickness Measurement of Rolled Steel Plate



Time delay between change in thickness and measurement:

$$T_l = \frac{d}{v}$$

Difference Between Time-Lag and Time-Delay



✓In case (c), transfer function of the block = $G(s) = \dfrac{e^{-sT_l}}{1+sT}$,
T = time lag, T_l = delay

Problem due to Presence of Time-Delay in a Process Control Loop

- ✓ **Due to the presence of time-delay, any corrective action from the controller cannot be immediately applied to the process. Thus during that time, process may buildup a deviation, sometimes quite large, due to any load disturbance or change in set-point.**

Transfer Function of Time-Delay Element

Taking Laplace Transform:

$$\frac{Y(s)}{X(s)} = e^{-sT_l}$$

Transfer Function of a Time-Delay Element

Now, $$e^{-sT_l} = \frac{e^{-\frac{1}{2}sT_l}}{e^{\frac{1}{2}sT_l}} = \frac{1 - \frac{1}{2}sT_l + \frac{\left(\frac{1}{2}sT_l\right)^2}{2!} - - -}{1 + \frac{1}{2}sT_l + \frac{\left(\frac{1}{2}sT_l\right)^2}{2!} - - -}$$

Using First-Order Pade Approximation:

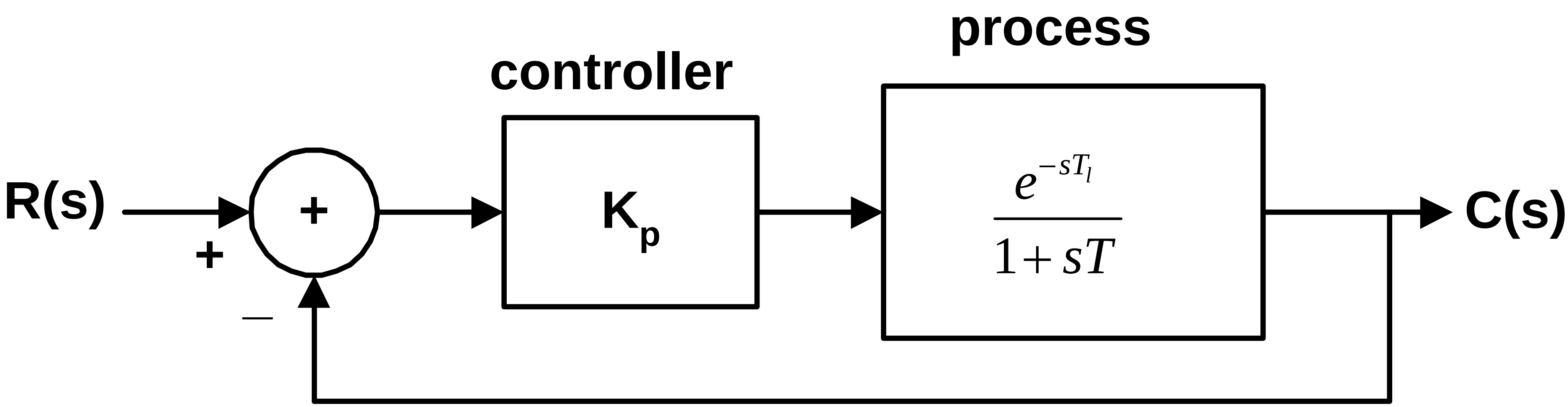
$$e^{-sT_l} \approx \frac{1 - \frac{1}{2}sT_l}{1 + \frac{1}{2}sT_l}$$

Effect of Time-Delay on Process Loop Response

T.F. of a Process with Time-Delay: ⇨ $G(s) = G'(s)e^{-sT_l}$

where $G'(s)$ is the transfer function of the delay free part of the process.

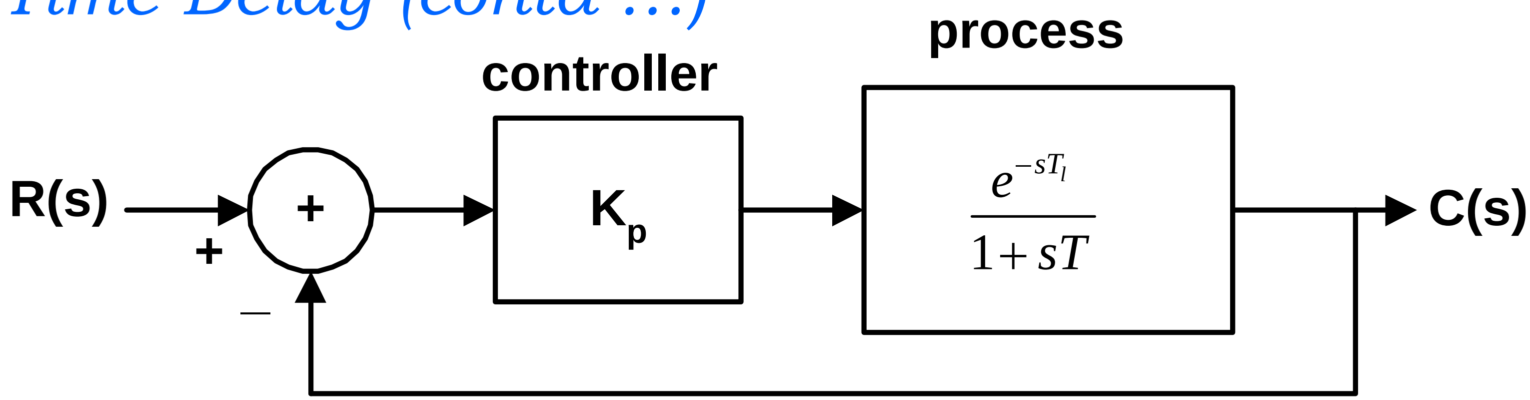
Case Study: Proportional Control of a First-Order Process with Time-Delay



K_p = Proportional Gain of the Controller

T = Process Time Constant

Case Study: Proportional Control of a First-Order Process with Time-Delay (contd …)



The Characteristic Eqn. of the C.L. System: $$1+\frac{K_p e^{-sT_l}}{1+sT}=0$$

Using First-Order Pade Approximation:

$$1+\frac{K_p\left(1-\frac{1}{2}sT_l\right)}{(1+sT)\left(1+\frac{1}{2}sT_l\right)}=0 \;\Rightarrow\; s^2+s\left(\frac{T+\frac{1}{2}T_l-\frac{1}{2}K_pT_l}{\frac{1}{2}TT_l}\right)+\left(\frac{K_p+1}{\frac{1}{2}TT_l}\right)=0$$

The standard form:

$$s^2+2\zeta\omega_n s+\omega_n^2=0,$$

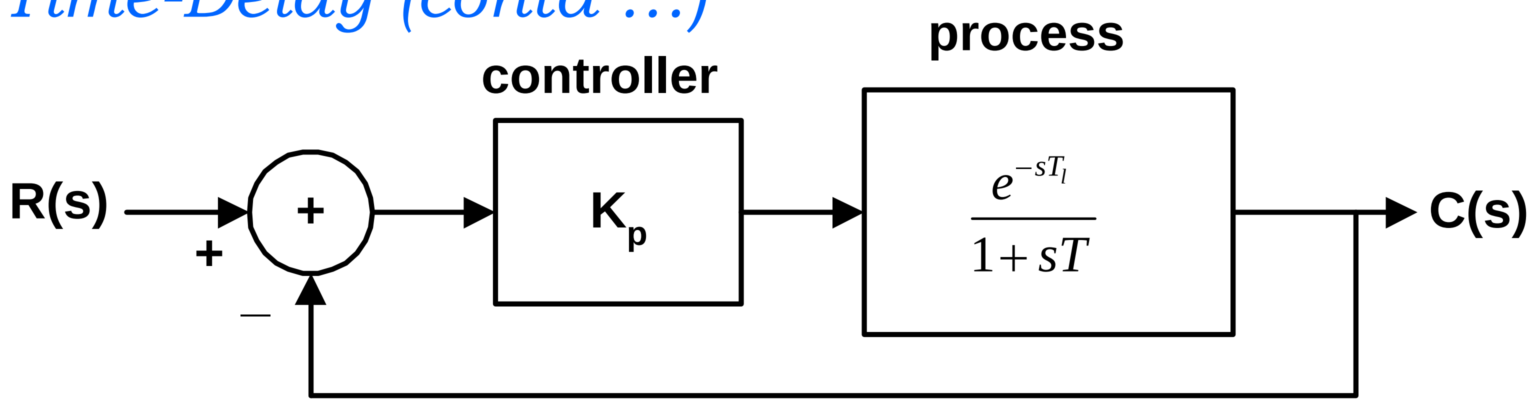
$$\omega_n=\sqrt{\frac{2(K_p+1)}{TT_l}},$$

The Natural Freq.

$$\zeta=\frac{T+\frac{1}{2}T_l-\frac{1}{2}K_pT_l}{\sqrt{2(K_p+1)TT_l}}$$

The Damping Ratio

Case Study: Proportional Control of a First-Order Process with Time-Delay (contd …)

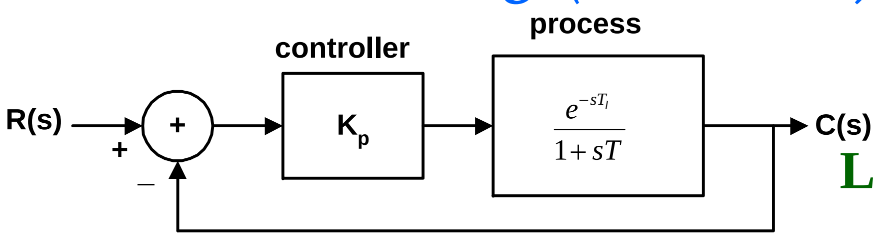


The damping ratio becomes zero when: $K_p = \left(\frac{2T + T_l}{T_l}\right)$

The system becomes unstable when: $K_p > \left(\frac{2T + T_l}{T_l}\right)$

✓ *Conclusion:* **Time-delay may cause instability even in first-order system for large values of K_p.**

Case Study: Proportional Control of a First-Order Process with Time-Delay (contd …)



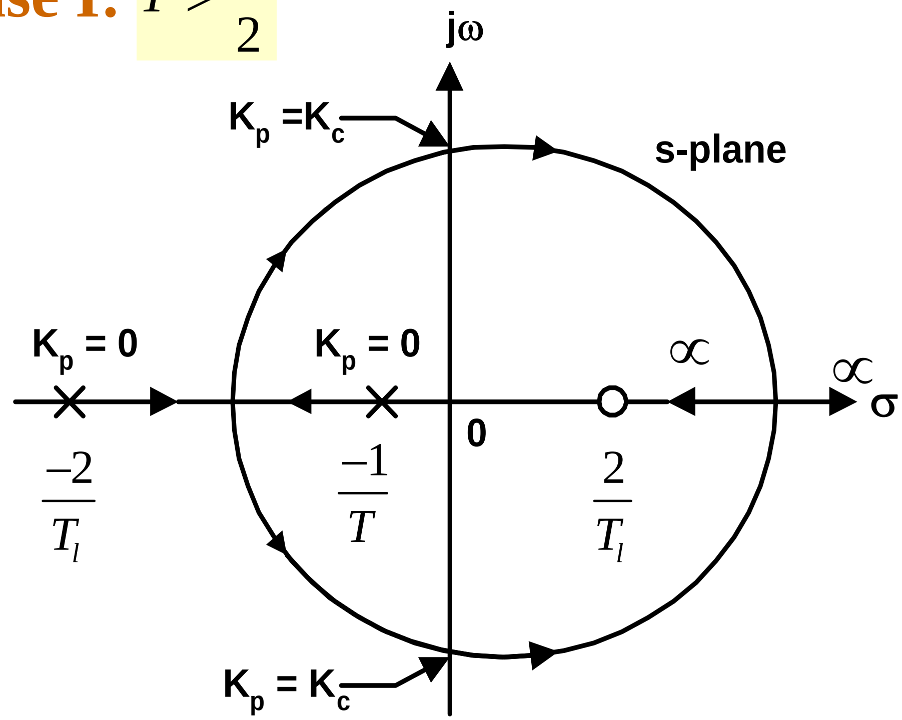
Root Locus Plot

Loop T.F. of the C. L. System: $\dfrac{K_p e^{-sT_l}}{1+sT}$

Using First-Order Pade Approximation:

$$\frac{K_p\left(1-\frac{sT_l}{2}\right)}{(1+sT)\left(1+\frac{sT_l}{2}\right)} = \frac{\frac{-K_p}{T}\left(s-\frac{2}{T_l}\right)}{\left(s+\frac{1}{T}\right)\left(s+\frac{2}{T_l}\right)}$$

Case I: $T > \dfrac{T_l}{2}$

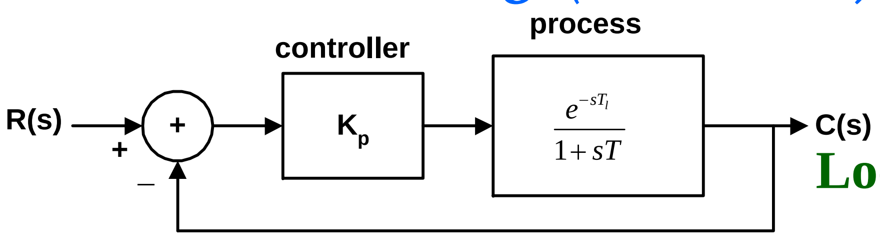


K_c: Critical Gain

$$K_c = \frac{2\left(T+\frac{T_l}{2}\right)}{T_l}$$

$$= \left(\frac{2T+T_l}{T_l}\right)$$

Case Study: Proportional Control of a First-Order Process with Time-Delay (contd …)



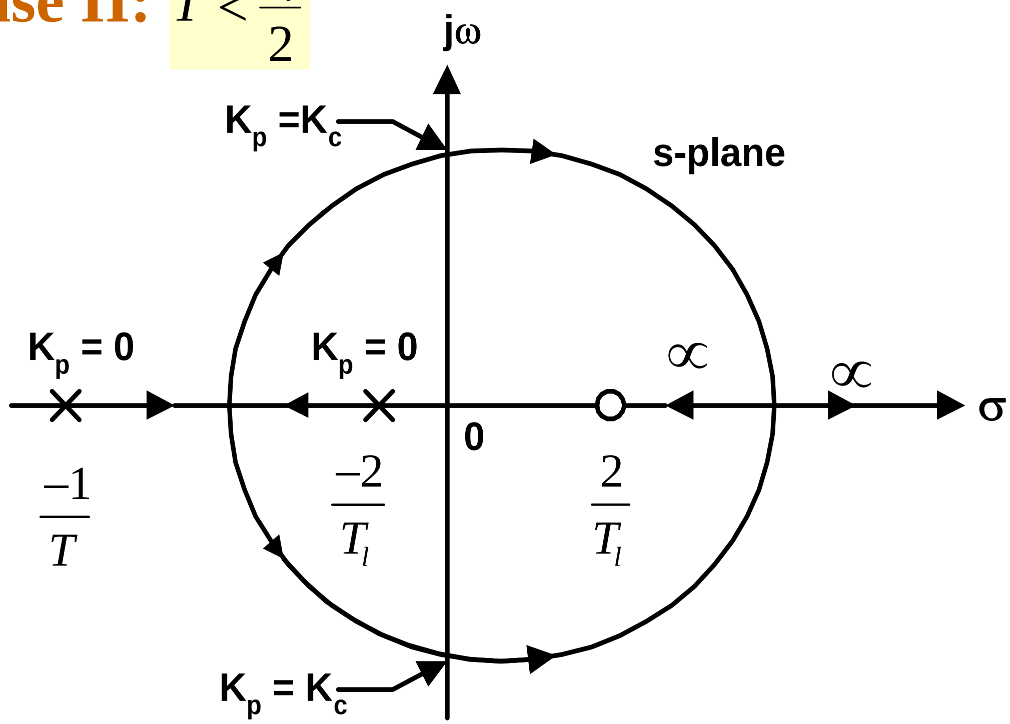
Root Locus Plot

Loop T.F. of the C. L. System: $\dfrac{K_p e^{-sT_l}}{1+sT}$

Using First-Order Pade Approximation:

$$\frac{K_p\left(1-\frac{sT_l}{2}\right)}{(1+sT)\left(1+\frac{sT_l}{2}\right)}=\frac{\frac{-K_p}{T}\left(s-\frac{2}{T_l}\right)}{\left(s+\frac{1}{T}\right)\left(s+\frac{2}{T_l}\right)}$$

Case II: $T<\dfrac{T_l}{2}$

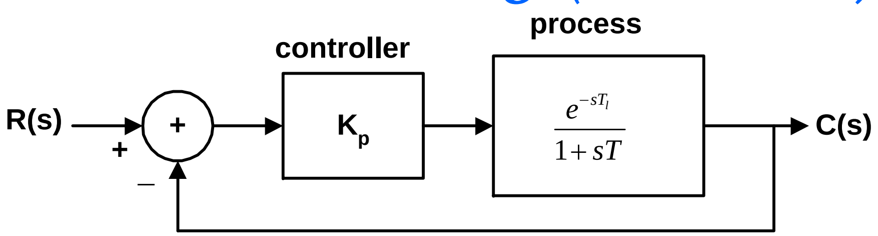


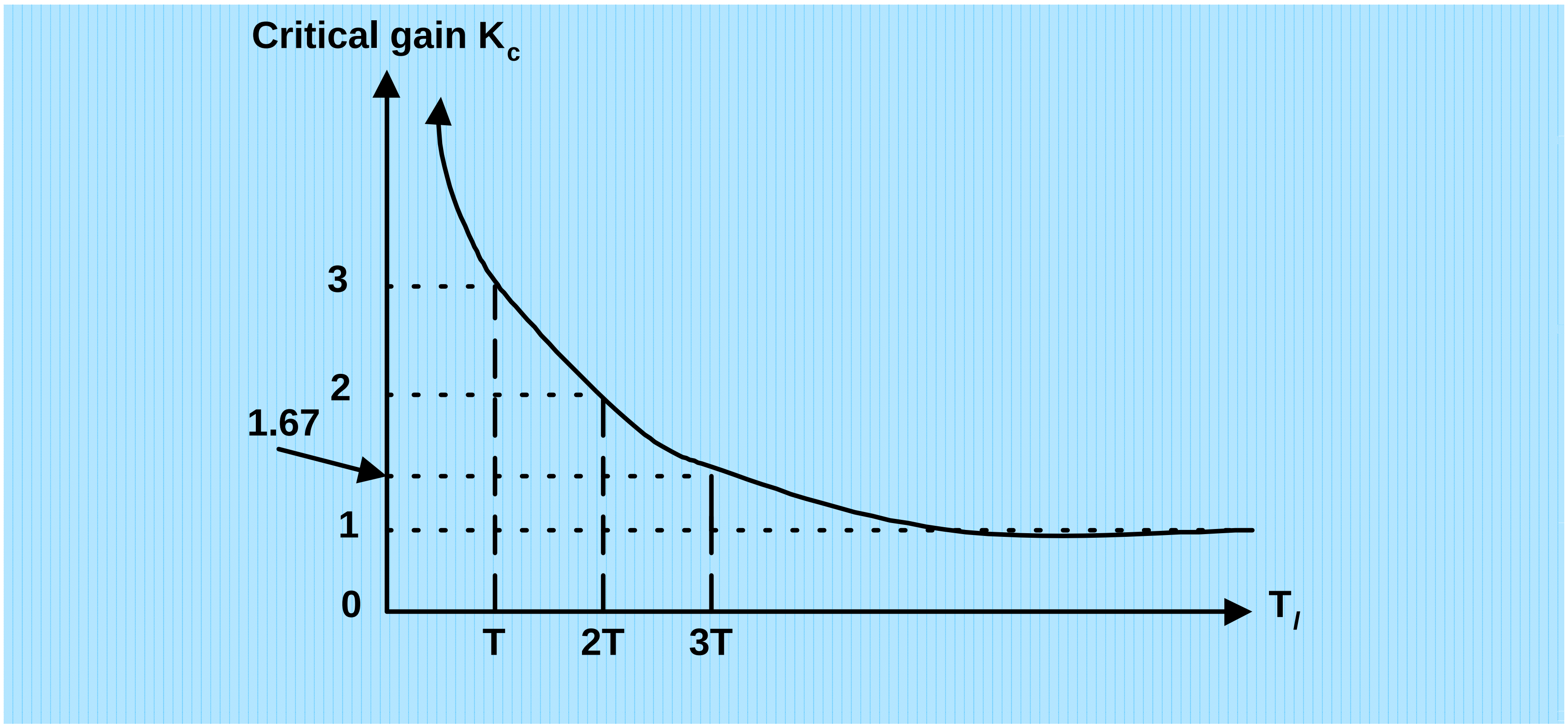
K_c: Critical Gain

$$K_c=\frac{2\left(T+\frac{T_l}{2}\right)}{T_l}$$

$$=\left(\frac{2T+T_l}{T_l}\right)$$

Case Study: Proportional Control of a First-Order Process with Time-Delay (contd ...)



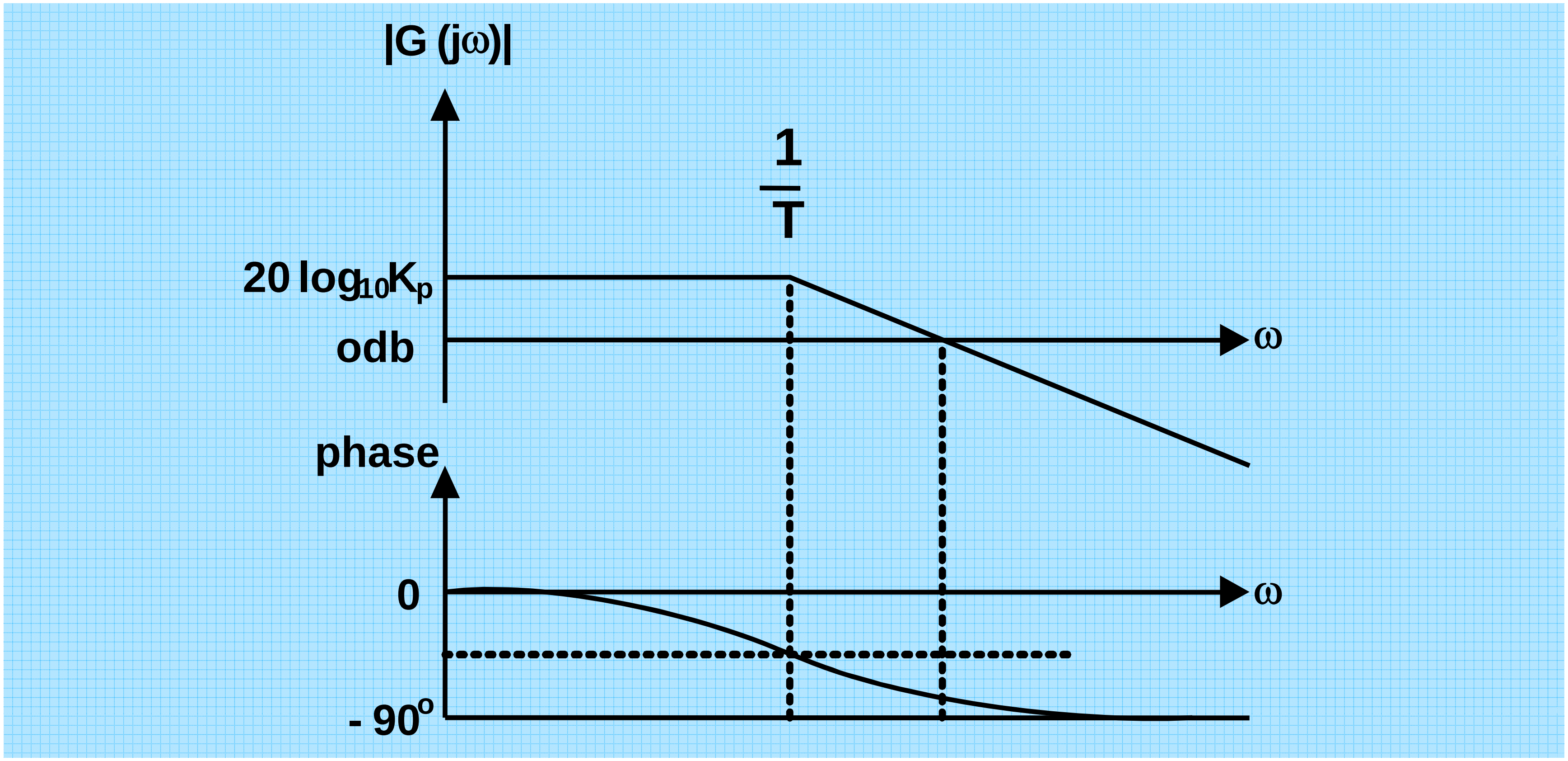


✓ **The Critical gain falls with increasing T_l.**

Bode Plot

Case I: First Order System without Time-Delay

Open Loop Transfer Function: $G(s) = \frac{K_p}{1 + sT} \Rightarrow G(j\omega) = \frac{K_p}{1 + j\omega T}$



✓ *Conclusion:* **The phase plot never crosses – 180° line, hence the system is always stable.**

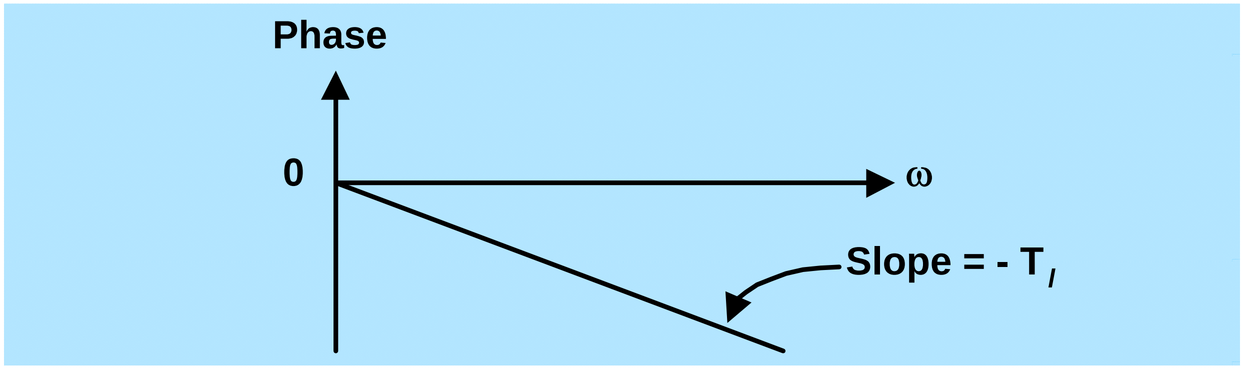
Bode Plot

Case II: First Order System with Time-Delay

Open Loop Transfer Function: $G(s) = \dfrac{K_p e^{-sT_l}}{1+sT} = \left(\dfrac{K_p}{1+sT}\right) e^{-sT_l} = G_1(s) G_2(s)$

Frequency Response of Time-Delay Unit:

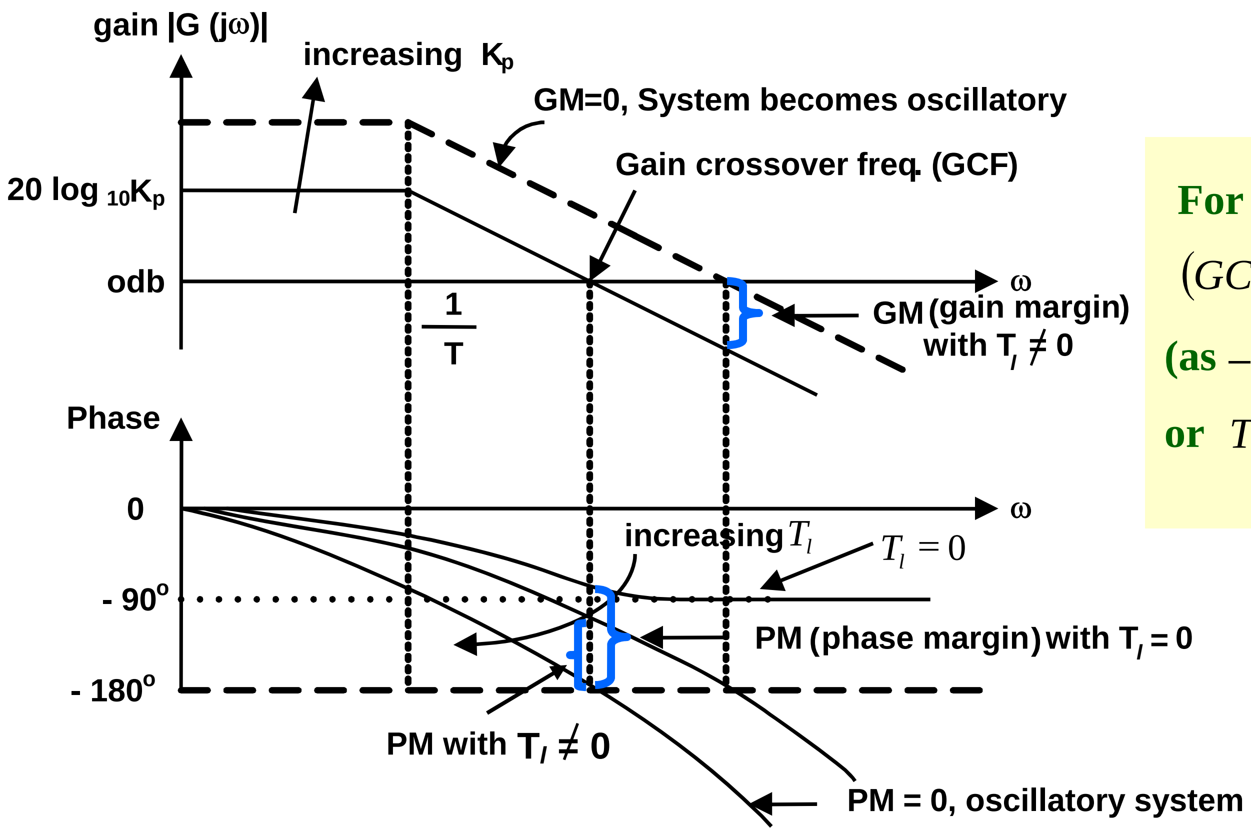
$$G_2(j\omega) = 1\angle -\omega T_l$$



Bode Plot

Case II: First Order System with Time-Delay

Open Loop Transfer Function: $G(s) = \dfrac{K_p e^{-sT_l}}{1+sT} = \left(\dfrac{K_p}{1+sT}\right)e^{-sT_l} = G_1(s)G_2(s)$



For a particular gain *Kp*, for stability,

$(GCF)\cdot T_l < PM$

(as $-\omega T_l$ is the phase of time-delay)

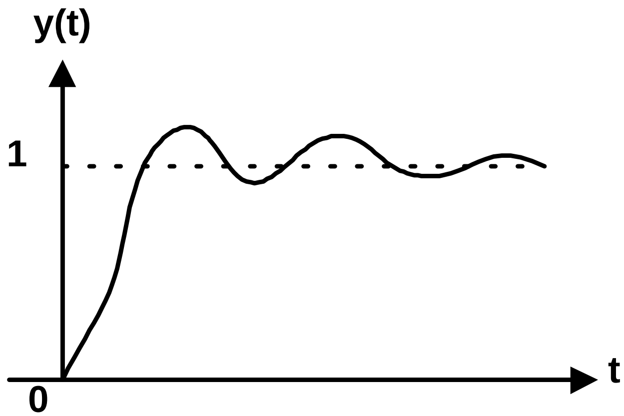
or $T_l < \dfrac{PM}{GCF}$

Smith's Principle for Control of Time-Delay Systems

Process with time-delay:

$$G(s) = G'(s)e^{-sT_l}$$

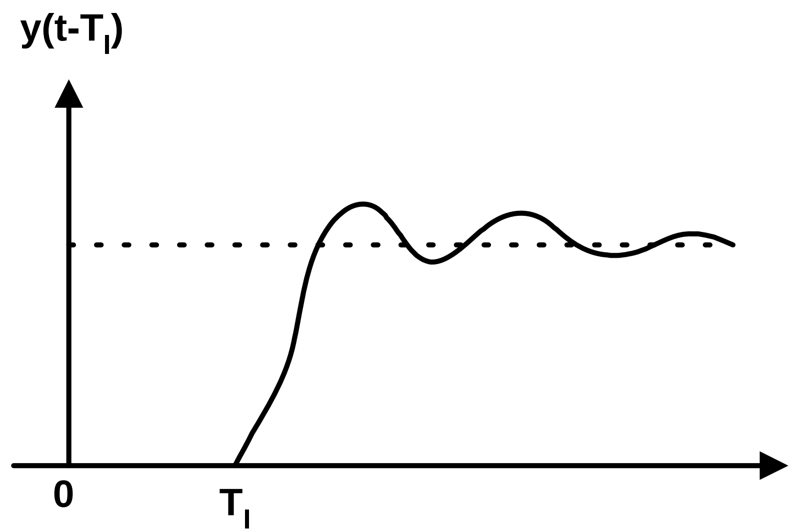
$G'(s)$ is the T.F. of the delay free part of the process and T_l is the time-delay.



$y(t)$ represents the unit step response of $G'(s)$ in a unity feedback closed loop system.

Smith's Principle:

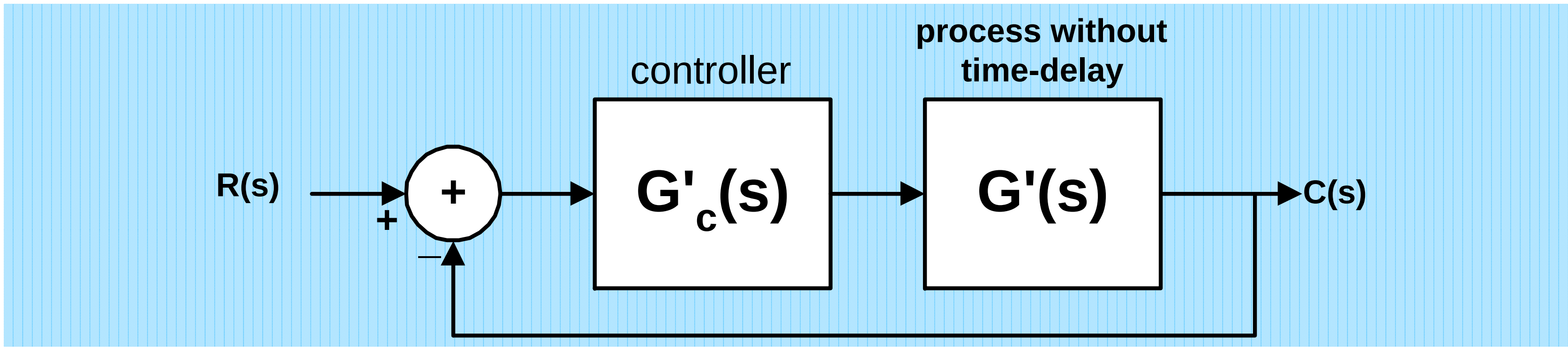
The unit step response desired from the closed loop system with the delay introduced in series with $G'(s)$ is $y(t - T_l)$.



If response $y(t)$ satisfies the design criteria for the delay-free case, then the response to be designed for the system with time-delay is the same response but delayed by T_l.

Smith's Method

✓ **Let the controller in the delay free case be $G_c'(s)$.**



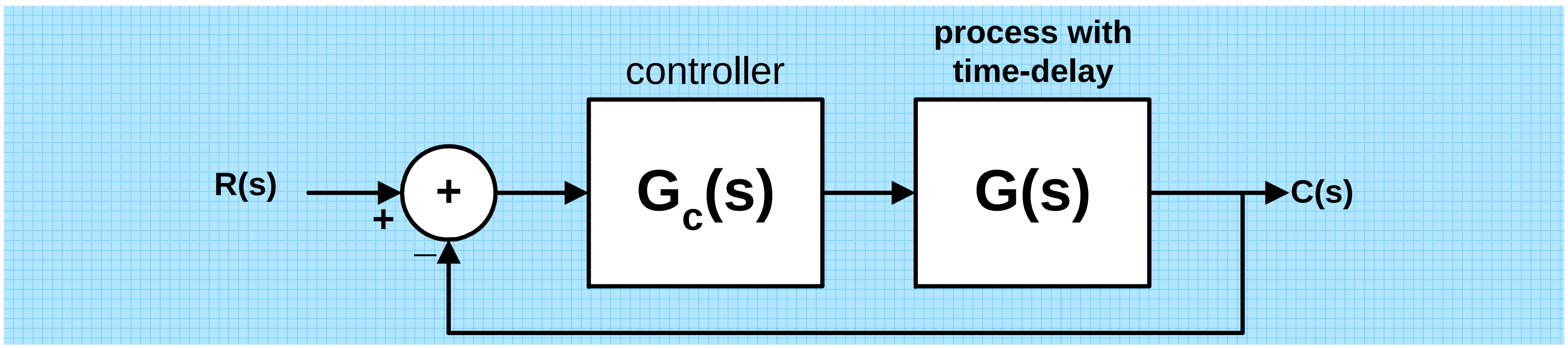
The closed loop transfer function: $\Rightarrow$ $$\frac{G_c'(s)G'(s)}{1+G_c'(s)G'(s)}$$ **with time response y(t).**

The time response from a system with closed loop transfer function :

$$\frac{G_c'(s)G'(s)}{1+G_c'(s)G'(s)}e^{-sT_l}$$ **will be y $(t - T_l)$.**

Smith's Method

✓ **Let the same response (i.e. y (t – T_l)) be available from a controller G_c(s) and the process with time-delay G(s) in a unity feedback closed loop system.**



The closed loop transfer function: $\frac{G_c(s)G(s)}{1+G_c(s)G(s)}$ **with time response y (t – T_l).**

According to Smith's Method

$$\frac{G_c(s)G(s)}{1+G_c(s)G(s)} = \frac{G_c^{/}(s)G^{/}(s)e^{-sT_l}}{1+G_c^{/}(s)G^{/}(s)}$$

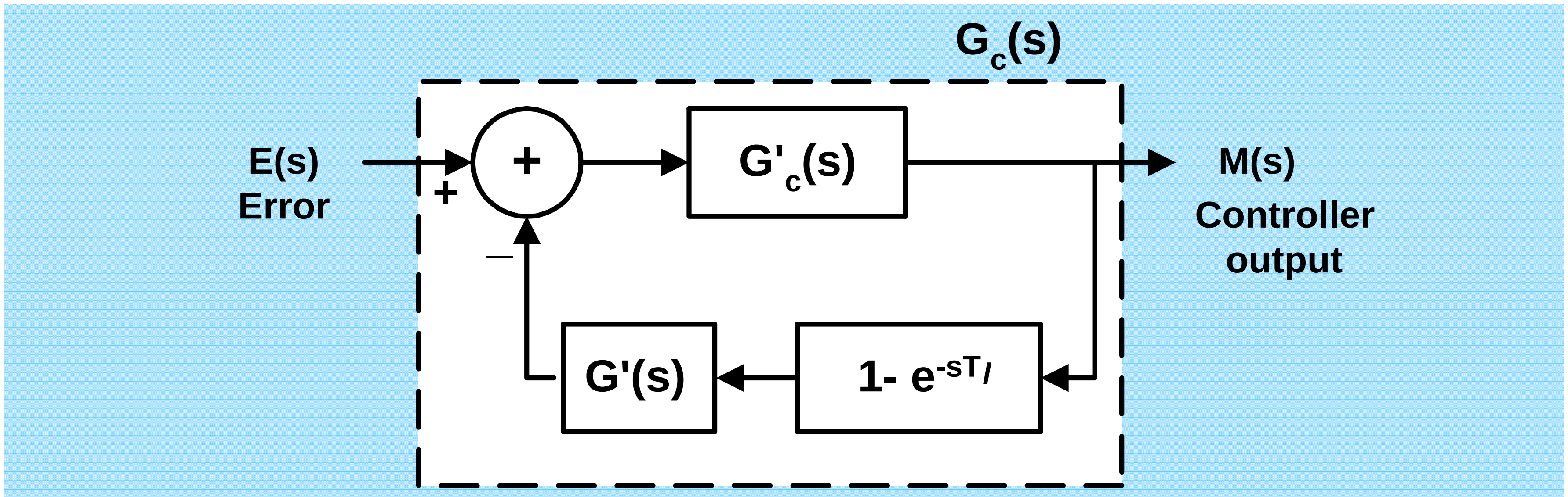
Substituting $G(s) = G'(s)e^{-sT_l}$

$$G_c(s) = \frac{G_c^{/}(s)}{1+G_c^{/}(s)G^{/}(s)\left(1-e^{-sT_l}\right)}$$

$G_c(s)$: **T.F. of the required controller for the process with time-delay**

Smith's Method

The Realization of the Controller $G_c(s)$

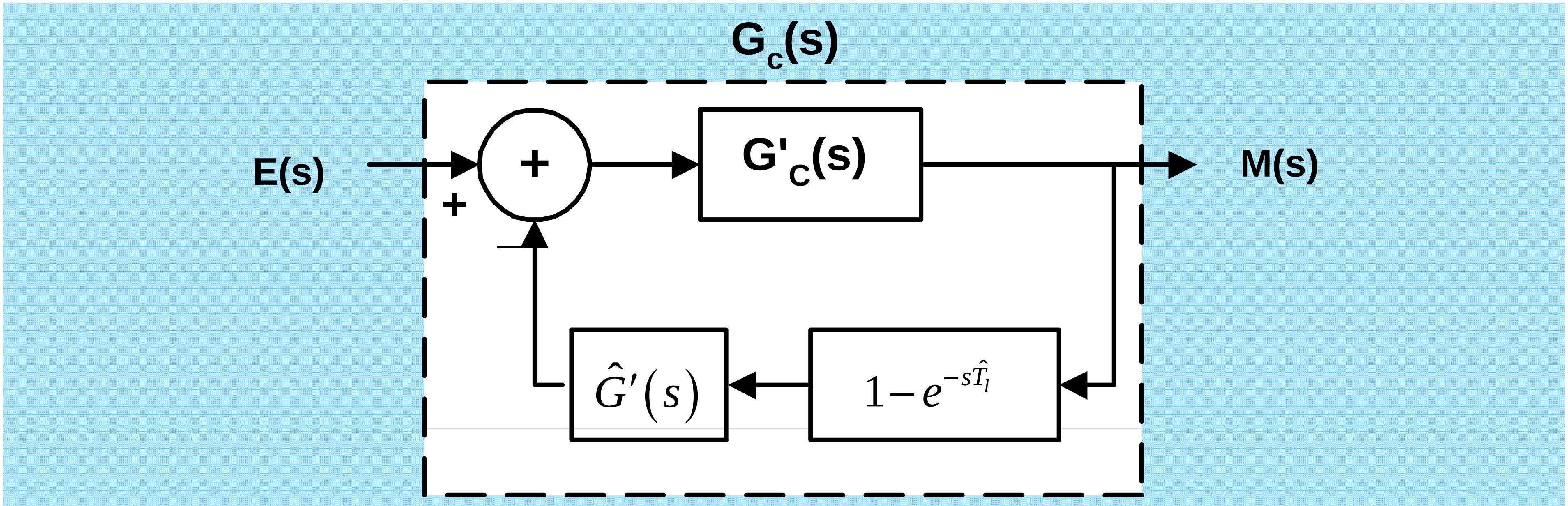


- ✓ **For the above realization of $G_c(s)$, knowledge of $G'(s)$ and T_l is required.**
- ✓ **In practice, model estimates of $G'(s)$ and T_l are used.**

Model Estimates:
- $\hat{G}'(s)$, **model of $G'(s)$**
- $\hat{T}_l$, **model of T_l**

Smith's Method

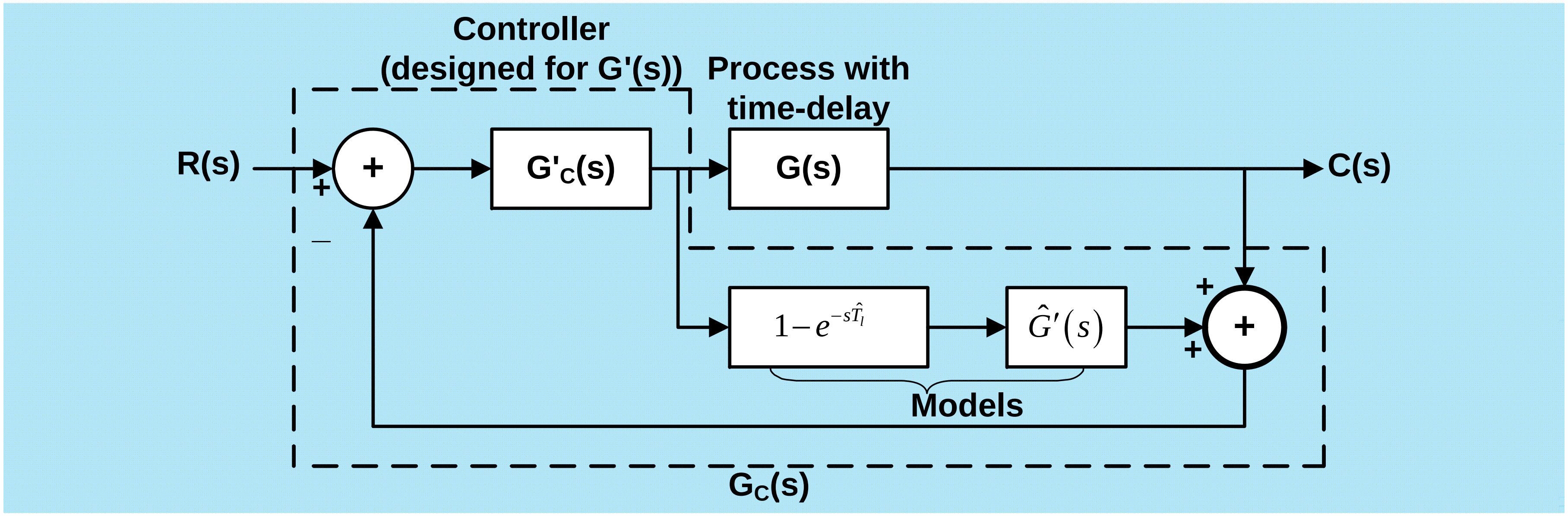
The Realization of the Controller $G_c(s)$



$G_c'(s)$ **: The Controller for Delay-Free Process $G'(s)$**

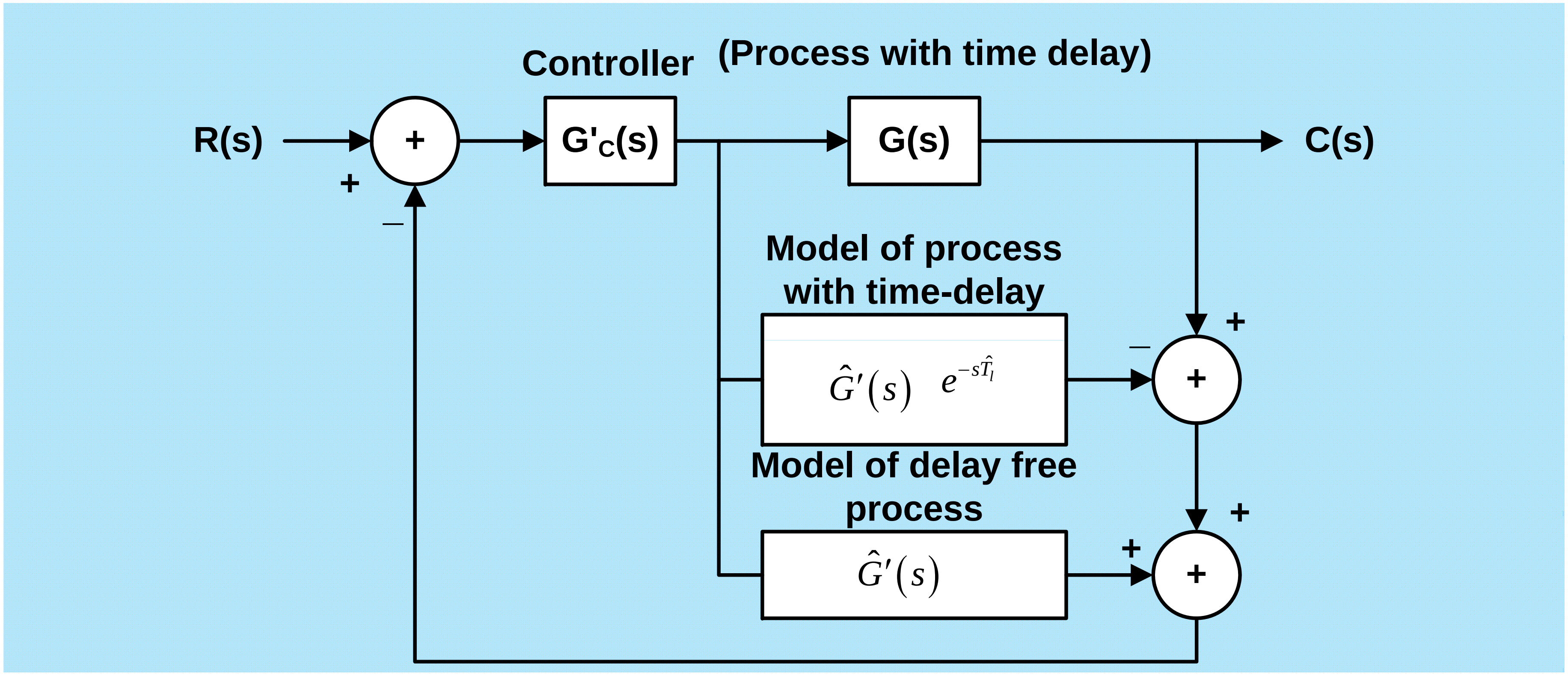
Realisation of the Closed Loop System with $G_c(s)$

Scheme I:



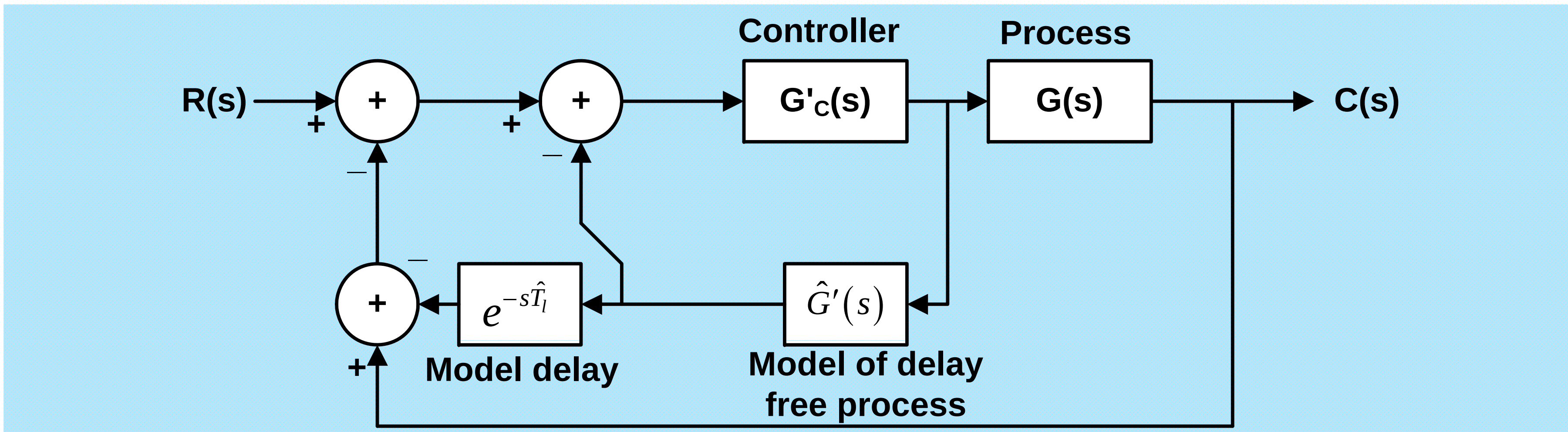
Realisation of the Closed Loop System with $G_c(s)$

Scheme II:



Realisation of the Closed Loop System with $G_c(s)$

Scheme III:

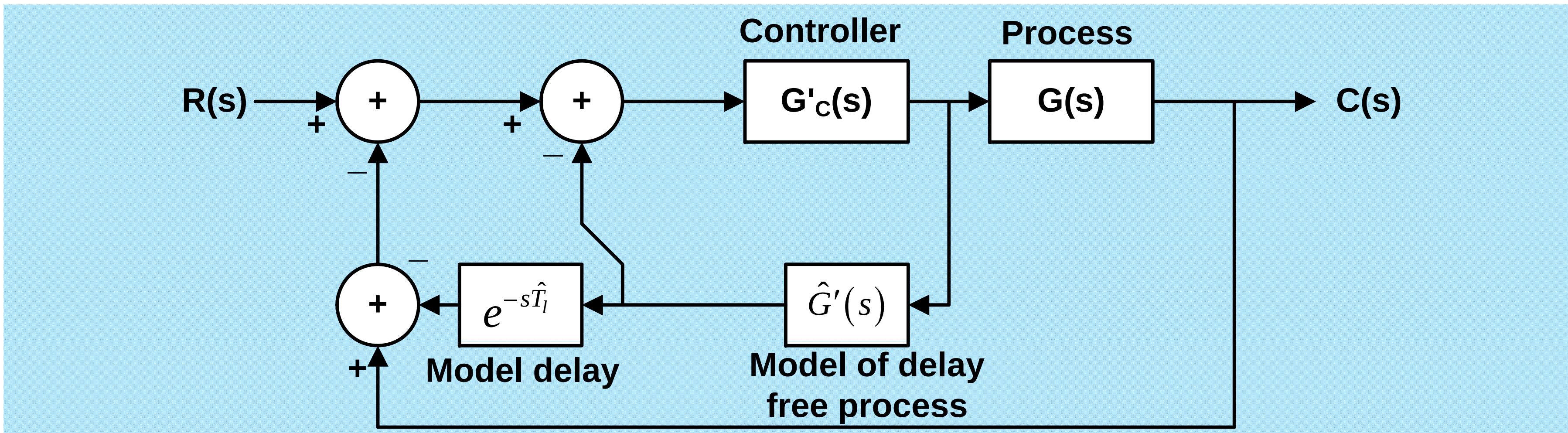


✓ **Smith's method assumes that models are exact, i.e.** $\hat{G}'(s) = G'(s)$ **and** $\hat{T}_l = T_l$**.**

✓ **In practice, there is always some mismatch between the model and the actual parameters, therefore, the closed-loop response may differ depending upon the degree of mismatch.**

Realisation of the Closed Loop System with $G_c(s)$

Scheme III:



✓ **The model of delay free process provides an estimate of output from delay free process for the controller. The estimate is, in fact, the prediction of the process output. Thus the term** *Smith's Predictor* **is used for such controllers.**

Thank You